Florentin Smarandache

(author and editor)

Collected Papers

Volume XVI

Neutrosophic Theories and Applications
Neutrosophic Statistics
Advances in Plithogenic Sets
Logic & Thought
Physics & Cosmology

Neutrosophic Science International Association (NSIA)
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Introductory Note

This *sixteenth volume* of **Collected Papers** is an extensive work, comprising 78 articles that span almost 1000 pages. The papers, originally published in various scientific journals, cover the broad and interdisciplinary fields of neutrosophics and other areas of study, including logic, philosophy, physics, mathematics, statistics, information fusion, and robotics. The volume features articles authored by Florentin Smarandache, either alone or in collaboration with 81 co-authors (from 21 different countries): Shumaila Abbas, Mohammed Abdel-Sattar, M. Modather M. Abdoug, Usama Afzal, Daud Ahmad, Abdulrahman AlAita, Shahbaz Ali, Fahad Alsharari, Suriana Alias, Saima Anis, Muhammad Aslam, Hamdi Ayed, M.B. Bera, Noel Batista-Hernández, Robert N. Boyd, Said Broumi, Claudia Camacho-Zuñiga, T. Daniel Chandra, Ebenezer Chifu, Victor Christianto, Mihaela Colhon, Shrouk El-Amir, Ibrahim El-Henawy, Takaaki Fujita, Nguyen Long Giang, Ana González-Marcos, Phung The Huan, Sadia Iqbal, Azhar Ali Janjua, Maissam Jdid, Madad Khan, Waris Khan, Muhammad Shahid Khan, Anamika Kumari, Luong Thi Hong Lan, M. Lathamaheswari, B.S. Mahapatra, G.S. Mahapatra, Muhammad Khalid Mahmood, Nivetha Martin, Pierre Millette, Mona Mohamed, M.K. Mondal, Bahnishikha Roy Muhuri, Norzieha Mustapha, Fermín Navaridas-Nalda, Tran Thi Ngan, Antonios Paraskevas, P. Prabakaran, Madiha Qayyum, Shazia Rana, Alina Reşceanu, Jesus Estupiñan Ricardo, Andreas Ries, Yaser Saber, Muhammad Saeed, Taoufik Saidani, P. Sathya, Noorazliyana Shafii, Muhammad Ahmed Shehzad, Rehan Ahmad Khan Sherwani, Rajesh Singh, Le Hoang Son, S. Sudha, Vu Duc Thai, Hugo Terashima-Marin, Nguyen Tho Thong, Pham Huy Thong, Monica Tilea, Tran Manh Tuan, Yunita Umniyati, Maikel Leyva Vázquez, Michael Gr. Voskoglou, Mohammad Kaif Wajid, Mohammad Saif Wajid, Mohd Anas Wajid, Roliza Md Yasin, Shouzhen Zeng, Muhammad Zeeshan, Alia Nur Izzah Zulkifli.

Keywords

Neutrosophic Set, Qualitative Comparative Analysis, Synergy of Giving, Tree of Life, Stewardship Ethics, Similarity measure, Entropy measure, Temporal Complex Neutrosophic Set, Neutrosophic representation, Decision-Making Model, Attribute Selection, Uncertainty Reasoning, Questionnaire Analysis, Youth Engagement, Correlation Measure, TOPSIS, Single-Valued Neutrosophic Set, Educational Innovation, Higher Education, Data Science, Critical Thinking, Distance Measure, Hybrid Distance Measure, Bipolar neutrosophic set, Frank Triangular Norms, Operational Laws, Aggregation Operators, Picture Fuzzy Set, Data Partition with Noises, Fuzzy Clustering, Health insurance, COVID-19, cosine similarity measure, Polynomial Neural Network, sustainability, Neutrosophic theory, Multi-Criteria Decision Making, Class Imbalance, Ranking Classifiers, Tree Soft Sets, Natural Resource Conservation, Non-Parametric Test, Mood's Median Test, Uncertain Data, Ranked Set Sampling, Population Mean, Demographic Data, Neutrosophic Median Ranked Set Sampling, Plithogenic Sociogram, Extended Plithogenic Sets, Food Packaging materials, Plithogenic Forest Hypersoft sets, Plithogenic Decision Making, Plithogenic Contradictions, Plithogenic Logic, Logical Operators, MultiPerspectivism Model, Pluralism, Multi-Neutrosophic Sets, Social Systems Theory, Philosophical Movements, Revisionism, Inspirationalism, Recurrentism, Sophisticalism, Rejectivism, Paradoxism, Philosophy of Contradiction, Affirmation, Negation, Neutrality, Philosophical Diversity, Existentialism, Dynamic Philosophy, Dialectics, Yin Yang.

FLORENTIN SMARANDACHE is an emeritus professor of Mathematics at the University of New Mexico, United States. He got his MSc in Mathematics and Computer Science from the University of Craiova, Romania, PhD in Mathematics from the State University of Kishinev, Moldova, and Postdoctoral in Applied Mathematics from Okayama University of Sciences, Japan, and The Guangdong University of Technology, Guangzhou, China.

He is the founder of neutrosophy (generalization of dialectics), neutrosophic set, logic, probability and statistics since 1995 and has published hundreds of papers and books on neutrosophic physics, superluminal and instantaneous physics, unmatter, quantum paradoxes, absolute theory of relativity, paradoxism, outerart, neutrosophy as a new branch of philosophy, Law of Included Multiple-Middle, multispace and multistructure, hypersoft set, IndetermSoft Set and IndetermHyperSoft Set, SuperHyperGraph, SuperHyperTopology, SuperHyperAlgebra, Neutrosophic SuperHyperAlgebra, degree of dependence and independence between neutrosophic components, refined neutrosophic set, neutrosophic over-under-off-set, plithogenic set / logic / probability / statistics, neutrosophic triplet and duplet structures, quadruple neutrosophic structures, extension of algebraic structures to NeutroAlgebra and AntiAlgebra, NeutroGeometry & AntiGeometry, Dezert-Smarandache Theory and so on.

In addition, he published many books of poetry, dramas, children' stories, translations, essays, a novel, folklore collections, traveling memories, and art albums.

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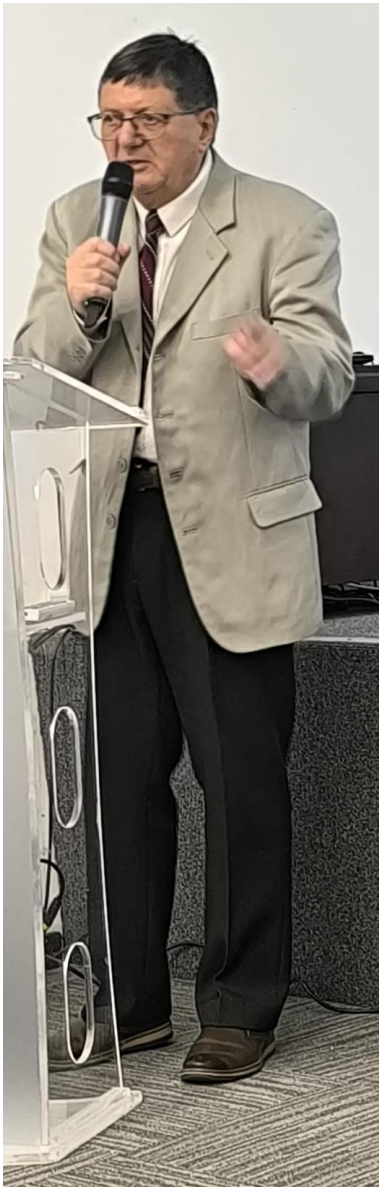

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Country	*Number of Authors*
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China	**2**
Ecuador	**2**
Egypt	**5**
Greece	**2**
India	**12**
Indonesia	**3**
Japan	**1**
Malaysia	**5**
Mexico	**4**
Morocco	**1**
Nigeria	**1**
Pakistan	**13**
Paraguay	**1**
Romania	**3**
Saudi Arabia	**5**
Spain	**2**
Syria	**2**
USA	**2**
Vietnam	**9**
Total	***82***

NEUTROSOPHIC THEORIES AND APPLICATIONS

Enhancing Set-Theoretic Research Methods with Neutrosophic Sets

Maikel Leyva Vázquez [1,*] , Jesus Estupiñan Ricardo [2] and Florentin Smarandache [3]

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Abstract

This study employed the integration of neutrosophic set theory with set-theoretic methods and Qualitative Comparative Analysis (QCA) to examine intricate social phenomena. Participants' opinions and attitudes were captured using neutrosophic Likert scales, which reflected elements of truth, indeterminacy, and falsity. This proposal is illustrated in a case study facilitated a more comprehensive and subtle examination, emphasizing the significance of variables such as motivation, instructional excellence, and educational resources in achieving academic accomplishment. The results of the necessary condition analysis and set coincidence analysis indicated that motivation and teaching quality have the highest individual impact. Nevertheless, the convergence of scholarly resources, drive, and instructional excellence markedly augmented the likelihood of achieving academic success, underscoring the significance of considering multiple factors collectively. The study proposes that future research should investigate additional sets associated with neutrosophic approaches, such as plithogenic sets, and employ more intricate neutrosophic scales to better capture the complexity of data. This approach provides a comprehensive and precise viewpoint for comprehending cause-and-effect relationships in social phenomena, offering a valuable instrument for research and policy development.

Keywords: Set-Theoretic Methods; Qualitative Comparative Analysis; fsQCA; Neutrosophic Sets.

1 | Introduction

Set-theoretic methods reflect a positivist approach to knowledge, as they are used to forecast and elucidate real-world occurrences by discerning patterns and causal connections among the elements [1]. Ragin initially introduced set-theoretic research methods in the late 1980s, which were primarily founded on crisp sets [2]. Set-theoretic methods, which focus on the antecedent conditions shared by instances of an outcome, are rooted in John Stuart Mill's method of agreement [3]. The set-theoretic perspective employs the concept of set membership to determine whether a case can be characterized by a particular concept or not. Within the context of set-theoretic methods, the process of concept formation carries a distinct meaning compared to traditional measurement theory. It primarily revolves around determining whether a particular case can be classified as belonging to a concept (or set) or not. The act of assigning set membership is also referred to as calibration [4].

Qualitative Comparative Analysis (QCA) [5] is a well-established method based on set theory. During the initial discussions of QCA in the 1980s and 1990s, it was only applicable to crisp sets, which means that a decision had to be made about whether a case belongs to a set or not. The necessity for "dichotomization" has prompted significant criticism of crisp-set QCA.

Fuzzy Set Qualitative Comparative Analysis (fsQCA) [6] is an advanced analytical technique used to explore the causal relationships between different conditions and outcomes within complex social phenomena. Unlike traditional quantitative methods that often rely on large datasets and assume linear relationships, fsQCA allows researchers to analyze data sets with a more nuanced and qualitative approach, capturing the complexity and richness of real-world scenarios.

There is a connection between fuzzy sets and neutrosophic Likert scales [7] in the way they handle imprecision and uncertainty. Neutrosophic sets and, thus, neutrosophic Likert scales enable an even broader representation that incorporates indeterminacy as a core component, whereas fuzzy sets enable the representation and manipulation of data that are not exactly specified. For survey responses in which participants' opinions not only vary across a spectrum (as accommodated by fuzzy sets) but also may include a degree of indecision or neutrality that is difficult to capture using traditional fuzzy logic or crisp Likert scales, neutrosophic Likert scales are, therefore, particularly well suited.

In this paper, we incorporate neutrosophic set theory into set-theoretic methods and Qualitative Comparative Analysis (QCA), integrating indeterminacy. We present a method for developing single-valued neutrosophic sets [8] from questionnaires applied to social groups to set-theoretic methods.

2 | Preliminaries

2.1 | Complexity Theory and Causality and Neutrosophic Sets

Complexity theory suggests that the relationships between variables are not always straightforward and can exhibit non-linear patterns [9]. This means that the same cause might lead to different effects depending on the context. Three key principles provide insight into this theory: conjunction, equifinality, and causal asymmetry [10]:

- Conjunction refers to the idea that antecedent conditions work together cooperatively to produce an outcome, rather than acting independently to explain variance.
- Equifinality implies that a system can achieve the same end state through multiple different initial conditions and pathways.
- Causal asymmetry indicates that the presence of an outcome might be driven by certain conditions, yet the absence of those conditions does not necessarily lead to the absence of the outcome. For instance, consider a restaurant with exceptional food quality. This high food quality might attract many diners. However, the same restaurant might also have low patronage due to its poor location or inadequate parking facilities. Conversely, a restaurant with mediocre food might still attract many customers if it has excellent service, a prime location, or unique entertainment options. This demonstrates that the relationship between conditions (food quality, location, service) and the outcome (number of diners) is not straightforward or fixed.

These principles highlight that the relationship between conditions and outcomes is complex and not fixed.

Neutrosophy can extend the notion of complex causality by incorporating the inherent indeterminacy and uncertainty in social phenomena [11]. Neutrosophic set theory, with its ability to handle indeterminacy, provides a more nuanced approach to understanding these complex relationships.

2.2 | Neutrosophic Liker Scales

Surveys utilizing neutrosophic Likert scales [12, 13] effectively measure the diversity of opinions and their influence on public policy and social discourse, capturing areas of consensus, disagreement, and ambivalence.

Next, we introduce the fundamental definitions and concepts related to neutrosophic sets and single-valued neutrosophic sets.

Definition 1. [14] Let U be a discourse universe. $N=\{(x, T(x), I(x), F(x)): x \in U\}$ is a neutrosophic set, denoted by a truth-membership function, $TN : U \rightarrow]0-, 1+[$; an indeterminacy-membership function, $IN : U \rightarrow]0-, 1+[$; and a falsity-membership function, $FN : U \rightarrow]0-, 1+[$.

Single-valued neutrosophic sets provide a way to represent and analyze possible elements in the discourse universe U

Definition 2. [15] Let U be a discourse universe. A single-valued neutrosophic set is defined as $N=\{(x, T(x), I(x), F(x)): x \in U\}$, which is identified by a truth-membership function, $TN : U \rightarrow [0, 1]$; indeterminacy-membership function, $IN : U \rightarrow [0, 1]$; and falsity-membership function, $FN : U \rightarrow [0, 1]$, with $0 \leq TN(x)+IN(x)+FN(x) \leq 3$

Using neutrosophic scales with single-valued neutrosophic sets, responses are categorized based on the total of the True, Indeterminate, and False components as follows:

- T+I+F<1: Incomplete
- T+I+F=1: Complete
- T+I+F>1: Contradictory

These values are obtained because, in many cases, opinions are incomplete or contradictory. This classification is one of the advantages of using neutrosophic methods, as it allows for a more nuanced understanding of varying degrees of truth, indeterminacy, and falsity in responses.

3 | Proposed Framework

To illustrate the detailed process to follow, Figure 1 is presented. This figure shows a flowchart that describes the key steps in the implementation model up to the analysis. Each step of the diagram is explained with a brief description, thus facilitating the understanding of the entire process.

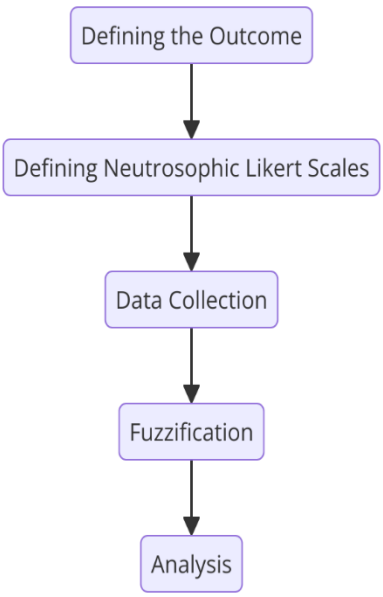

Figure 1. Proposed framework.

i). Defining the Outcome: Identify and clearly define the specific phenomenon, event, or condition that is the focus of the investigation.

ii). Defining Neutrosophic Likert Scales: Develop Neutrosophic Likert Scales to measure the outcome and variables. Unlike traditional Likert scales that measure responses on a fixed scale (e.g., 1 to 5), Neutrosophic Likert Scales incorporate elements of truth, indeterminacy, and falsity. Each response on the scale is represented by a triple (T, I, F), where T is the degree of truth, I is the degree of indeterminacy, and F is the degree of falsity. This allows for a more nuanced capture of respondent opinions and attitudes.

iii). Data Collection: Gather relevant data on the cases in terms of various indicators or measures related to the outcome. Ensure that the data collected are comprehensive and accurately reflect the variables being studied. Use the Neutrosophic Likert Scales in questionnaires and surveys to gather data on the outcome and relevant variables. This approach provides a richer dataset that captures the complexity of respondent opinions and attitudes.

iv). Fuzzification: Neutrosophic sets are converted to equivalent fuzzy membership sets used in this case in line with [16].

Let $AN = \{x, (TA(x), IA(x), FA(x)) : x \in X\}$ be an NS. Its equivalent fuzzy membership set is defined as $AF = \{(x, \mu A(x)) : x \in X\}$, where $\mu A(x) = s\left((TA(x), IA(x), FA(x)), (1,0,0)\right)$. So, using the equation of similarity proposed,

$$\mu A(x) = 1 - \frac{1}{2}[(1 - T_A(x)) + \max\{I_A(x), F_A(x)\}] \quad (1)$$

As the range of the similarity measure function is the unit interval [0,1], μA(x)∈[0,1] for all x∈X. Hence, the membership function of the derived fuzzy set belongs to [0,1] and thus it satisfies the property of a membership function of a fuzzy set (FS).

v). Analysis: Conduct fsQCA to identify which combinations of factors or conditions are associated with the presence or degree of the outcome. For fsQCA program for Windows is used for data processing [17, 18].

The validity of the configuration is assessed by measuring the consistency and coverage values. Consistency is the measure of how reliably the set of pathways produces the desired outcome. Coverage pertains to the degree to which the outcome is elucidated by this arrangement of pathways[19]:

$$\text{Consistency } (Y_i \leq X_i) = \frac{\sum \min(X_i, Y_i)}{\sum Y_i} \quad (2)$$

$$\text{Coverage } (Y_i \leq X_i) = \frac{\sum \min(X_i, Y_i)}{\sum X_i} \quad (3)$$

where:

X_i is the membership value of case i in the set of causal conditions.

Y_i is the membership value of case iii in the outcome set.

Both are utilized during the comparative analysis to measure established relationships between individual conditions, combinations of conditions, configurations of pathways, and the result. Typically, values that exceed 0.8 are regarded as strong [19].

4 | Case Study

The defined outcome is the perception of Academic Success (SUCCESS). A Likert scale is developed, represented as single-valued neutrosophic sets. The study also considers other variables: Academic Resources

(REC), Motivation (MOT), and Quality of Teaching (CAL). A survey was conducted with a group of 12 Software Engineering students at the University of Guayaquil (see Table 1).

Table 1. Survey data.

Case	REC	MOT	CAL	SUCCESS
1	(0.9,0.9, 0.2)	(0.6, 1, 0.5)	(0.3, 0.7, 0.3)	(0.8, 0.6, 0.7)
2	(0.5, 0.5,0.5)	(1,1,1)	(0.5,0.2,0.5)	(0.6, 0.6, 0.7)
3	(0.8, 0.7, 0.4)	(0.7, 0.9, 0.5)	(0.8, 0.5, 0.5)	(0.8, 0.5, 0.5)
4	(1,1,0)	(0.8,0.8,0)	(1,0.9,0.3)	(0.7, 1, 0.9)
5	(1,0.5,0)	(1,0.5,1)	(1,0.5,1)	(0.9, 0.6, 0.1)
6	(0.9, 0.9, 0.9)	(0.9, 0.9, 0.9)	(0.9, 0.9, 0.9)	(0.9, 0.9, 0.9)
7	(0.2, 0.5, 0.8)	(1,0,0)	(0.5, 0.5, 0.5)	(0.8, 0.5, 0.2)
8	(1, 0.9, 0.1)	(0.9, 0.9, 0.1)	(0.9, 0.9, 0.1)	(0.9, 0.9, 0.1)
9	(1,1,0)	(0.8, 0.8, 0)	(1,0,0)	(0.9, 0, 0)
10	(0.7, 1, 0.2)	(0.9, 0.4, 0)	(0.6, 0.9, 0.1)	(1,0,0)
11	(0.4, 0.7, 0.2)	(0.3, 0.9, 0.4)	(0.8, 0.4, 0.6)	(0.4, 0.8, 0.3)
12	(0.6, 1, 0.6)	(0.6, 0.5, 0.2)	(0.2, 0.5, 0.7)	(1,0,1)

The fuzzification process is developed using Eq. (1) (see Table 2).

Table 2. Fuzzified values.

Case	REC	MOT	CAL	SUCCESS
1	0.50	0.30	0.30	0.55
2	0.50	0.50	0.50	0.45
3	0.55	0.40	0.65	0.65
4	0.50	0.50	0.55	0.35
5	0.75	0.50	0.50	0.65
6	0.50	0.50	0.50	0.50
7	0.20	1.00	0.50	0.65
8	0.55	0.50	0.50	0.50
9	0.50	0.50	1.00	0.95
10	0.35	0.75	0.35	1.00
11	0.35	0.20	0.60	0.30
12	0.30	0.55	0.25	0.50

A necessary condition analysis is performed to test consistency and coverage (Table 3).

Table 3. Necessary condition analysis.

Conditions tested	Consistency	Coverage
REC	0.730496	0.927928
MOT	0.794326	0.903226
CAL	0.794326	0.903226
REC+MOT+CAL	0.957447	0.854430

According to the analysis of essential factors for success, the results show that the individual factors of REC, MOT, and CAL have a high occurrence rate, with each being present in around 90% of successful cases. This implies that these factors are frequently linked to favorable results. Nevertheless, their consistencies are marginally lower, with REC at 73%, and both MOT and CAL at around 79%, suggesting that although they are often present, they are not conclusive indicators of success on their own. The aggregate condition (REC + MOT + CAL) exhibits a significantly high level of consistency at 95%, highlighting its robust predictive capability when all three factors are simultaneously present. However, the coverage of this combined

condition decreases to 85%, indicating that while the combination of REC, MOT, and CAL is a strong indicator of success, it does not explain all instances of success. Hence, the combination of REC, MOT, and CAL not only holds individual importance but also greatly increases the probability of success, emphasizing the synergistic impact of these factors in attaining the intended result. A set coincidence analysis is performed (see Table 4).

Table 4. Set coincidence analysis.

Conditions	Coincidence
REC, MOT, CAL	0.594937
REC, MOT	0.678571
REC, CAL	0.740741
MOT, CAL	0.675676

The Set Coincidence procedure was utilized to assess the intersection of key variables: Academic Resources (REC), Motivation (MOT), and Quality of Teaching (CAL). The findings demonstrate that the level of overlap between REC and MOT is 0.678571, between REC and CAL is 0.740741, and between MOT and CAL is 0.675676. The combined overlap of REC, MOT, and CAL is precisely 0.594937. These findings indicate that although each variable is strongly associated with academic success, their combined impact exhibits a robust predictive ability, emphasizing the significance of considering multiple interacting factors in attaining academic achievement. The results of the set superset analysis are shown in Table 5.

Table 5. Subset/Superset analysis results.

Terms	Consistency	Coverage	Combined
REC**MOT****CAL**	0.957447	0.638298	0.794931
REC*MOT	0.957895	0.645390	0.799335
REC*CAL	0.950000	0.673759	0.812578
MOT*CAL	0.960000	0.680851	0.821001
REC	0.927928	0.730496	0.841773
MOT	0.903226	0.794326	0.873243
CAL	0.903226	0.794326	0.873243

The results of the subset/superset analysis indicate that motivation (MOT) and quality of teaching (CAL) are the strongest and most relevant conditions for predicting academic success individually. The combination of these two conditions is also very effective. Although the combination of the three conditions (REC, MOT, and CAL) has a high consistency, its coverage is smaller, suggesting that while it is a powerful combination, it is not the only way to achieve academic success. Academic resources (ACRs), both individually and in combination with other conditions, also play an important role, but not as dominant as MOT and CAL.

5 | Conclusions

This study demonstrates that the integration of set-theoretical methods with neutrosophic approaches enables a more sophisticated and comprehensive examination of intricate social phenomena. Neutrosophic Likert scales were employed to more accurately capture participants' opinions and attitudes, encompassing aspects of truth, indeterminacy, and falsehood. The findings from the necessary condition analysis, set matching, and subset/superset analysis indicate that motivation and quality of teaching are the most significant factors in predicting academic success, both independently and when considered together. Nevertheless, it was observed that the amalgamation of scholarly resources, drive, and excellence in instruction substantially enhances the likelihood of achievement, underscoring the synergistic significance of these factors. It is recommended that future research investigates sets associated with neutrosophic approaches, such as plithogenic sets, and employs more intricate neutrosophic scales to better capture the complexity of data.

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n-Norm and n-Conorm in Neutrosophic Logic and Set, and The Neutrosophic Topologies

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***Abstract**: In this paper we present the N-norms/N-conorms in neutrosophic logic and set as extensions of T-norms/T-conorms in fuzzy logic and set. Also, as an extension of the Intuitionistic Fuzzy Topology we present the Neutrosophic Topologies.*

***Keywords**: neutrosophic logic and set, N-norms, N-conorms, neutrosophic composition k-law, neutrosophic topologies.*

1. DEFINITION OF THE NEUTROSOPHIC LOGIC/SET

Let T, I, F be real standard or non-standard subsets of $]^{-}0,1^{+}[$, with $\sup T = t\sup$, $\inf T = t\inf$, $\sup I = i\sup$, $\inf I = i\inf$, $\sup F = f\sup$, $\inf F = f\inf$, and $n\sup = t\sup + i\sup + f\sup$, $n\inf = t\inf + i\inf + f\inf$.

Let U be a universe of discourse, and M a set included in U. An element x from U is noted with respect to the set M as x(T, I, F) and belongs to M in the following way: it is t% true in the set, i% indeterminate (unknown if it is or not) in the set, and f% false, where t varies in T, i varies in I, f varies in F.

Statically T, I, F are subsets, but dynamically T, I, F are functions/operators depending on many known or unknown parameters.

2. NEUTROSOPHIC LOGIC

In a similar way define the **Neutrosophic Logic**: A logic in which each proposition x is T% true, I% indeterminate, and F% false, and we write it x(T,I,F), where T, I, F are defined above.

3. N-NORMS AND N-CONORMS FOR THE NEUTROSOPHIC LOGIC AND SET

As a generalization of T-norm and T-conorm from the Fuzzy Logic and Set, we now introduce the **N-norms and N-conorms** for the Neutrosophic Logic and Set.

We define a *partial relation order* on the neutrosophic set/logic in the following way: $x(T_1, I_1, F_1) \le y(T_2, I_2, F_2)$ iff (if and only if) $T_1 \le T_2$, $I_1 \ge I_2$, $F_1 \ge F_2$ for crisp components.

And, in general, for subunitary set components: $x(T_1, I_1, F_1) \le y(T_2, I_2, F_2)$ iff

$\inf T_1 \le \inf T_2$, $\sup T_1 \le \sup T_2$,
$\inf I_1 \ge \inf I_2$, $\sup I_1 \ge \sup I_2$,
$\inf F_1 \ge \inf F_2$, $\sup F_1 \ge \sup F_2$.

If we have mixed - crisp and subunitary - components, or only crisp components, we can transform any crisp component, say "a" with a $\dot{I}$ [0,1] or a$\dot{I}$ $]^{-}0, 1^{+}[$, into a subunitary set [a, a]. So, the definitions for sub unitary set components should work in any case.

3.1. N-NORMS

$$Nn : (\,]\text{-}0,1+[\times]\text{-}0,1+[\times]\text{-}0,1+[\,)2\,?\,]\text{-}0,1+[\times]\text{-}0,1+[\times]\text{-}0,1+[\quad (1)$$

$$Nn\,(x(T1,I1,F1), y(T2,I2,F2)) = (NnT(x,y), NnI(x,y), NnF(x,y)) \quad (2)$$

where $N_nT(.,.)$, $N_nI(.,.)$, $N_nF(.,.)$ are the truth/membership, indeterminacy, and respectively falsehood/nonmembership components.

N_n have to satisfy, for any x, y, z in the neutrosophic logic/set M of the universe of discourse U, the following axioms:

a) Boundary Conditions:

$N_n(x, 0) = 0$, $N_n(x, 1) = x$.

b) Commutativity: $N_n(x, y) = N_n(y, x)$.

c) Monotonicity:

If $x \leq y$, then $N_n(x, z) \leq N_n(y, z)$.

d) Associativity:

$N_n(N_n(x, y), z) = N_n(x, N_n(y, z))$.

There are cases when not all these axioms are satisfied, for example the associativity when dealing with the neutrosophic normalization after each neutrosophic operation. But, since we work with approximations, we can call these **N-pseudo-norms**, which still give good results in practice.

N_n represent the *and* operator in neutrosophic logic, and respectively the *intersection* operator in neutrosophic set theory.

Let $J \in \{T, I, F\}$ be a component.

Most known N-norms, as in fuzzy logic and set the T-norms, are:

• The Algebraic Product N-norm:

$N_{n-algebraic}J(x, y) = x \cdot y$

• The Bounded N-Norm:

$N_{n-bounded}J(x, y) = \max\{0, x + y - 1\}$

• The Default (min) N-norm:

$N_{n-min}J(x, y) = \min\{x, y\}$.

A general example of N-norm would be this.

Let $x(T_1, I_1, F_1)$ and $y(T_2, I_2, F_2)$ be in the neutrosophic set/logic M. Then:

$$Nn(x, y) = (T1/\backslash T2, I1 \therefore /I2, F1 \therefore /F2) \quad (3)$$

where the "/\" operator, acting on two (standard or non-standard) subunitary sets, is a N-norm (verifying the above N-norms axioms); while the "\/" operator, also acting on two (standard or non-standard) subunitary sets, is a N-conorm (verifying the below N-conorms axioms).

For example, /\ can be the Algebraic Product T-norm/N-norm, so $T_1/\backslash T_2 = T_1 \cdot T_2$ (herein we have a product of two subunitary sets – using simplified notation); and \/ can be the Algebraic Product T-conorm/N-conorm, so $T_1\backslash/T_2 = T_1+T_2-T_1 \cdot T_2$ (herein we have a sum, then a product, and afterwards a subtraction of two subunitary sets).

Or /\ can be any T-norm/N-norm, and \/ any T-conorm/N-conorm from the above and below; for example the easiest way would be to consider the *min* for crisp components (or *inf* for subset components) and respectively *max* for crisp components (or *sup* for subset components).

If we have crisp numbers, we can at the end neutrosophically normalize.

3.2. N-CONORMS

$$Nc : (\,]-0,1+[\times]-0,1+[\times]-0,1+ \\ [\,)2\,?\,]-0,1+[\times]-0,1+[\times]-0,1+[\quad (4)$$

$$Nc\,(x(T1, I1, F1), y(T2, I2, F2)) = \\ (NcT(x, y), NcI(x, y), NcF(x, y)), \quad (5)$$

where $N_nT(.,.)$, $N_nI(.,.)$, $N_nF(.,.)$ are the truth/membership, indeterminacy, and respectively falsehood/nonmembership components.

N_c have to satisfy, for any x, y, z in the neutrosophic logic/set M of universe of discourse U, the following axioms:

a) Boundary Conditions:

$N_c(x, \mathbf{1}) = \mathbf{1}$, $N_c(x, \mathbf{0}) = x$.

b) Commutativity: $N_c(x, y) = N_c(y, x)$.

c) Monotonicity:

if $x \leq y$, then $N_c(x, z) \leq N_c(y, z)$.

d) Associativity:

$N_c(N_c(x, y), z) = N_c(x, N_c(y, z))$.

There are cases when not all these axioms are satisfied, for example the associativity when dealing with the neutrosophic normalization after each neutrosophic operation. But, since we work with approximations, we can call these **N-pseudo-conorms**, which still give good results in practice.

N_c represent the *or* operator in neutrosophic logic, and respectively the *union* operator in neutrosophic set theory.

Let $J \in \{T, I, F\}$ be a component.

Most known N-conorms, as in fuzzy logic and set the T-conorms, are:

• The Algebraic Product N-conorm:

$N_{c-algebraic}J(x, y) = x + y - x \cdot y$

• The Bounded N-conorm:

$N_{c-bounded}J(x, y) = \min\{1, x + y\}$

• The Default (max) N-conorm:

$N_{c-max}J(x, y) = \max\{x, y\}$.

A general example of N-conorm would be

this. Let $x(T_1, I_1, F_1)$ and $y(T_2, I_2, F_2)$ be in the neutrosophic set/logic M. Then:

$$Nn(x, y) = (T1 \therefore /T2, I1/\backslash I2, F1/\backslash F2) \quad (6)$$

Where – as above - the "/\" operator, acting on two (standard or non-standard) subunitary sets, is a N-norm (verifying the above N-norms axioms); while the "\/" operator, also acting on two (standard or non-standard) subunitary sets, is a N-conorm (verifying the above N-conorms axioms).

For example, /\ can be the Algebraic Product T-norm/N-norm, so $T_1/\backslash T_2 = T_1 \cdot T_2$ (herein we have a product of two subunitary sets); and \/ can be the Algebraic Product T-conorm/N-conorm, so $T_1 \backslash / T_2 = T_1 + T_2 - T_1 \cdot T_2$ (herein we have a sum, then a product, and afterwards a subtraction of two subunitary sets).

Or /\ can be any T-norm/N-norm, and \/ any T-conorm/N-conorm from the above; for example the easiest way would be to consider the *min* for crisp components (or *inf* for subset components) and respectively *max* for crisp components (or *sup* for subset components).

If we have crisp numbers, we can at the end neutrosophically normalize.

Since the min/max (or inf/sup) operators work the best for subunitary set components, let's present their definitions below. They are extensions from subunitary intervals {defined in [3]} to any subunitary sets. Analogously we can do for all neutrosophic operators defined in [3].

Let $x(T_1, I_1, F_1)$ and $y(T_2, I_2, F_2)$ be in the neutrosophic set/logic M.

Neutrosophic Conjunction/Intersection:

$$x/\backslash y = (T/\backslash, I/\backslash, F/\backslash) \quad (7)$$

where $\inf T_{\wedge} = \min\{\inf T_1, \inf T_2\}$
$\sup T_{\wedge} = \min\{\sup T_1, \sup T_2\}$
$\inf I_{\wedge} = \max\{\inf I_1, \inf I_2\}$
$\sup I_{\wedge} = \max\{\sup I_1, \sup I_2\}$
$\inf F_{\wedge} = \max\{\inf F_1, \inf F_2\}$
$\sup F_{\wedge} = \max\{\sup F_1, \sup F_2\}$

Neutrosophic Disjunction/Union:

$$x \backslash /y = (T \therefore /, I \therefore /, F \therefore /) \quad (8)$$

where $\inf T_{\vee} = \max\{\inf T_1, \inf T_2\}$
$\sup T_{\vee} = \max\{\sup T_1, \sup T_2\}$
$\inf I_{\vee} = \min\{\inf I_1, \inf I_2\}$
$\sup I_{\vee} = \min\{\sup I_1, \sup I_2\}$
$\inf F_{\vee} = \min\{\inf F_1, \inf F_2\}$
$\sup F_{\vee} = \min\{\sup F_1, \sup F_2\}$

Neutrosophic Negation/Complement:

$$C(x) = (TC, IC, FC) \quad (9)$$

where $T_C = F_1$
$\inf I_C = 1 - \sup I_1$
$\sup I_C = 1 - \inf I_1$
$F_C = T_1$

Upon the above Neutrosophic Conjunction /Intersection, we can define the Neutrosophic Containment.

Neutrosophic Containment:

We say that the neutrosophic set A is included in the neutrosophic set B of the universe of discourse U, iff for any $x(T_A, I_A, F_A) \in A$ with $x(T_B, I_B, F_B) \in B$ we have:

$\inf T_A \leq \inf T_B$; $\sup T_A \leq \sup T_B$;
$\inf I_A \geq \inf I_B$; $\sup I_A \geq \sup I_B$;
$\inf F_A \geq \inf F_B$; $\sup F_A \geq \sup F_B$.

3.3. REMARKS

a) The non-standard unit interval $]^-0, 1^+[$ is merely used for philosophical applications, especially when we want to make a distinction between relative truth (truth in at least one world) and absolute truth (truth in all possible worlds), and similarly for distinction between relative or absolute falsehood, and between relative or absolute indeterminacy.

But, for technical applications of neutrosophic logic and set, the domain of definition and range of the N-norm and N-conorm can be restrained to the normal standard real unit interval [0, 1], which is easier to use, therefore: N_n: $([0,1] \times [0,1] \times [0,1])^2 \rightarrow [0,1] \times [0,1] \times [0,1]$ and N_c: $([0,1] \times [0,1] \times [0,1])^2 \rightarrow [0,1] \times [0,1] \times [0,1]$.

b) Since in NL and NS the sum of the components (in the case when T, I, F are crisp numbers, not sets) is not necessary equal to 1 (so the normalization is not required), we can keep the final result un-normalized.

But, if the normalization is needed for special applications, we can normalize at the end by dividing each component by the sum all components. If we work with intuitionistic logic/set (when the information is incomplete, i.e. the sum of the crisp components is less than 1, i.e. *sub-normalized*), or with

paraconsistent logic/set (when the information overlaps and it is contradictory, i.e. the sum of crisp components is greater than 1, i.e. *over-normalized*), we need to define the neutrosophic measure of a proposition/set.

If x(T,I,F) is a NL/NS, and T,I,F are crisp numbers in [0,1], then the **neutrosophic vector norm** of variable/set x is the sum of its components:

$$\text{Nvector - norm}(x) = T + I + F \qquad (10)$$

Now, if we apply the N_n and N_c to two propositions/sets which maybe intuitionistic or paraconsistent or normalized (i.e. the sum of components less than 1, bigger than 1, or equal to 1), x and y, what should be the neutrosophic measure of the results $N_n(x,y)$ and $N_c(x,y)$?

Herein again we have more possibilities:

- either the product of neutrosophic measures of x and y: $N_{vector\text{-}norm}(N_n(x,y)) = N_{vector\text{-}norm}(x) \cdot N_{vector\text{-}norm}(y)$,
- or their average: $N_{vector\text{-}norm}(N_n(x,y)) = (N_{vector\text{-}norm}(x) + N_{vector\text{-}norm}(y))/2$,
- or other function of the initial neutrosophic measures: $N_{vector\text{-}norm}(N_n(x,y)) = f(N_{vector\text{-}norm}(x), N_{vector\text{-}norm}(y))$, where f(.,.) is a function to be determined according to each application. Similarly for $N_{vector\text{-}norm}(N_c(x,y))$.

Depending on the adopted neutrosophic vector norm, after applying each neutrosophic operator the result is neutrosophically normalized. We'd like to mention that "**neutrosophically normalizing**" doesn't mean that the sum of the resulting crisp components should be 1 as in fuzzy logic/set or intuitionistic fuzzy logic/set, but the sum of the components should be as above: either equal to the product of neutrosophic vector norms of the initial propositions/sets, or equal to the neutrosophic average of the initial propositions/sets vector norms, etc.

In conclusion, we neutrosophically normalize the resulting crisp components T`,I`,F` by multiplying each neutrosophic component T`, I` ,F` with S/(T`+I`+F`), where S = $N_{vector\text{-}norm}(N_n(x,y))$ for a N-norm or S = $N_{vector\text{-}norm}(N_c(x,y))$ for a N-conorm - as defined above.

c) If T, I, F are subsets of [0, 1] the problem of neutrosophic normalization is more difficult.

i) If sup(T)+sup(I)+sup(F) < 1, we have an *intuitionistic proposition/set*.

ii) If inf(T)+inf(I)+inf(F) > 1, we have a *paraconsistent proposition/set*.

iii) If there exist the crisp numbers $t \in T$, $i \in I$, and $f \in F$ such that t+i+f =1, then we can say that we have a *plausible normalized proposition/set*.

But in many such cases, besides the normalized particular case showed herein, we also have crisp numbers, say $t_1 \in T$, $i_1 \in I$, and $f_1 \in F$ such that $t_1+i_1+f_1 < 1$ (incomplete information) and $t_2 \in T$, $i_2 \in I$, and $f_2 \in F$ such that $t_2+i_2+f_2 > 1$ (paraconsistent information).

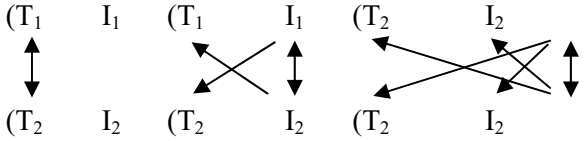
4. EXAMPLES OF NEUTROSOPHIC OPERATORS WHICH ARE N-NORMS OR N-PSEUDONORMS OR, RESPECTIVELY N-CONORMS OR N-PSEUDONORMS

We define a binary **neutrosophic conjunction (intersection)** operator, which is a particular case of a N-norm (neutrosophic norm, a generalization of the fuzzy T-norm):

$$c_N^{TIF} : ([0,1]\times[0,1]\times[0,1])^2 \to [0,1]\times[0,1]\times[0,1] \qquad (11)$$

$$c_N^{TIF}(x,y) = (T_1T_2, I_1I_2 + I_1T_2 + T_1I_2, F_1F_2 + F_1I_2 + F_1T_2 + F_2T_1 + F_2I_1) \qquad (12)$$

The neutrosophic conjunction (intersection) operator $x \wedge_N y$ component truth, indeterminacy, and falsehood values result from the multiplication $(T_1 + I_1 + F_1)\cdot(T_2 + I_2 + F_2)$ since we consider in a prudent way T p I p F, where "p " is a **neutrosophic relationship** and means "weaker", i.e. the products T_iI_j will go to I, T_iF_j will go to F, and I_iF_j will go to F for all i, j $\in \{1,2\}$, $i \neq j$, while of course the product T_1T_2 will go to T, I_1I_2 will go to I, and F_1F_2 will go to F (or reciprocally we can say that F prevails in front of I which prevails in front of T, and this neutrosophic relationship is transitive):



So, the truth value is T_1T_2, the indeterminacy value is $I_1I_2 + I_1T_2 + T_1I_2$ and the false value is $F_1F_2 + F_1I_2 + F_1T_2 + F_2T_1 + F_2I_1$. The norm of $x \breve{U}_N y$ is $(T_1 + I_1 + F_1) \times (T_2 + I_2 + F_2)$. Thus, if x and y are normalized, then $x \mathring{U}_N y$ is also normalized. Of course, the reader can redefine the neutrosophic conjunction operator, depending on application, in a different way, for example in a more optimistic way, i.e. I p T p F or $_T$ prevails with respect to I, then we get:

$$c_N^{TIF}(x,y) = (T_1T_2 + T_1I_2 + T_2I_1, I_1I_2, F_1F_2 + F_1I_2 + F_1T_2 + F_2T_1 + F_2I_1) \quad (13)$$

Or, the reader can consider the order T p F p I, etc.

Let's also define the unary **neutrosophic negation** operator:

$$n_N : [0,1]\times[0,1]\times[0,1] \to [0,1]\times[0,1]\times[0,1]$$
$$n_N(T,I,F) = (F,I,T) \quad (14)$$

by interchanging the truth T and falsehood F vector components.

Similarly, we now define a binary **neutrosophic disjunction (or union)** operator, where we consider the neutrosophic relationship F p I p T:

$$d_N^{FIT} : ([0,1]\times[0,1]\times[0,1])^2 \to [0,1]\times[0,1]\times[0,1] \quad (15)$$

$$d_N^{FIT}(x,y) = (T_1T_2 + T_1I_2 + T_2I_1 + T_1F_2 + T_2F_1, I_1F_2 + I_2F_1 + I_1I_2, F_1F_2) \quad (16)$$

We consider as neutrosophic norm of the neutrosophic variable x, where $NL(x) = T_1 + I_1 + F_1$, the sum of its components: $T_1 + I_1 + F_1$, which in many cases is 1, but can also be positive <1 or >1.

Or, the reader can consider the order F p T p I, in a pessimistic way, i.e. focusing on indeterminacy I which prevails in front of the truth T, or other **neutrosophic order** of the neutrosophic components T, I, F depending on the application.

Therefore,

$$d_N^{FIT}(x,y) = (T_1T_2 + T_1F_2 + T_2F_1, I_1F_2 + I_2F_1 + I_1I_2 + T_2I_1, F_1F_2) \quad (17)$$

4.1. NEUTROPHIC COMPOSITION k-LAW

Now, we define a more general neutrosophic composition law, named k-law, in order to be able to define neutrosophic k-conjunction/intersection and neutrosophic k-disjunction/union for k variables, where k ≥ 2 is an integer.

Let's consider k ≥ 2 neutrosophic variables, $x_i(T_i, I_i, F_i)$, for all $i \in \{1,2,\ldots k\}$. Let's denote

$$T = (T_1,...,T_k)$$
$$I = (I_1,...,I_k)$$
$$F = (F_1,...,F_k)$$

We now define a neutrosophic composition law o_N in the following way:

$$o_N : \{T,I,F\} \to [0,1] \quad (18)$$

If $z \in \{T,I,F\}$ then $z_{o_N} z = \prod_{i=1}^{k} z_i$

If z, $w \in \{T,I,F\}$ then

$$z_{o_N} w = \sum_{\substack{r=1 \\ \{i_1,...,i_r,j_{r+1},...,j_k\} \\ (i_1,...,i_r)\in C^r(1,2,...,k) \\ (j_{r+1},...,j_k)\in C^{k-r}(1,2,...,k)}}^{k-1} z_{i_1}\ldots z_{i_r} w_{j_r+1}\ldots w_{j_k} \quad (19)$$

where $C^r(1,2,...,k)$ means the set of combinations of the elements $\{1,2,...,k\}$ taken by r. [Similarly for $C^{k-r}(1,2,...,k)$.]

In other words, $z_{o_N} w$ is the sum of all possible products of the components of vectors z and w, such that each product has at least a z_i factor and at least a w_j factor, and each product has exactly k factors where each factor is a different vector component of z or of w. Similarly if we multiply three vectors:

$$T_{o_N} I_{o_N} F = \sum_{\substack{u,v,k-u-v=1 \\ \{i_1,...,i_u,j_{u+1},...,j_{u+v},l_{u+v+1},...,l_k\}\equiv\{1,2,...,k\} \\ (i_1,...,i_u)\in C^u(1,2,...,k),(j_{u+1},...,j_{u+v})\in \\ C^v(1,2,...,k),(l_{u+v+1},...,l_k)\in C^{k-u-v}(1,2,...,k)}}^{k-2} T_{i_1...i_u} I_{j_{u+1}...j_{u+v}} F_{l_{u+v+1}} \quad (20)$$

Let's see an example for $k = 3$.

x_1 (T_1, I_1, F_1)

x_2 (T_2, I_2, F_2)

$x_3\ (T_3, I_3, F_3)$

$T_{o_N} T = T_1T_2T_3, I_{o_N} I = I_1I_2I_3, F_{o_N} F = F_1F_2F_3$

$T_{o_N} I = T_1I_2I_3 + I_1T_2I_3 + I_1I_2T_3 + T_1T_2I_3 +$

$+ T_1I_2T_3 + I_1T_2T_3,$

$T_{o_N} F = T_1F_2F_3 + F_1T_2F_3 +$

$+ F_1F_2T_3 + T_1T_2F_3 + T_1F_2T_3 + F_1T_2T_3$

$I_{o_N} F = I_1F_2F_3 + F_1I_2F_3 + F_1F_2I_3 + I_1I_2F_3 +$

$+ I_1F_2I_3 + F_1I_2I_3,$

$T_{o_N} I_{o_N} F = T_1I_2F_3 +$

$+ T_1F_2I_3 + I_1T_2F_3 + I_1F_2T_3 + F_1I_2T_3 + F_1T_2I_3$

For the case when indeterminacy I is not decomposed in subcomponents {as for example $I = P \cup U$ where P = paradox (true and false simultaneously) and U = uncertainty (true or false, not sure which one)}, the previous formulas can be easily written using only three components as:

$$T_{o_N} I_{o_N} F = \sum_{i,j,r \in P(1,2,3)} T_i I_j F_r \qquad (21)$$

where $\mathcal{P}(1,2,3)$ means the set of permutations of $(1,2,3)$ i.e. {(1,2,3), (1,3,2), (2,1,3), (2,3,1), (3,1,2), (3,2,1)}.

$$z_{o_N} w = \sum_{\substack{i=1 \\ (i,j,r) \equiv (1,2,3) \\ (j,r) \in P^2(1,2,3)}}^{3} z_i w_j w_{j_r} + w_i z_j w_{j_r} \qquad (22)$$

This neurotrophic law is associative and commutative.

4.2. NEUTROPHIC LOGIC AND SET k-OPERATORS

Let's consider the neutrophic logic crispy values of variables x, y, z (so, for k = 3):

$NL(x) = (T_1, I_1, F_1)$ with $0 \le T_1, I_1, F_1 \le 1$

$NL(y) = (T_2, I_2, F_2)$ with $0 \le T_2, I_2, F_2 \le 1$

$NL(z) = (T_3, I_3, F_3)$ with $0 \le T_3, I_3, F_3 \le 1$

In neutrosophic logic it is not necessary to have the sum of components equals to 1, as in intuitionist fuzzy logic, i.e. $T_k + I_k + F_k$ is not necessary 1, for $1 \le k \le 3$.

As a particular case, we define the tri-nary conjunction neutrosophic operator:

$c_{3N}^{TIF} : ([0,1] \times [0,1] \times [0,1])^3 \to$

$[0,1] \times [0,1] \times [0,1]$

$c_{3N}^{TIF}(x, y, z) = (T_{o_N} T, I_{o_N} I + I_{o_N} T,$

$F_{o_N} F + F_{o_N} I + F_{o_N} T)$

If all x, y, z are normalized, then $c_{3N}^{TIF}(x, y, z)$ is also normalized.

If x, y, or y are non-normalized, then $\left| c_{3N}^{TIF}(x, y, z) \right| = |x| \times |y| \times |z|$, where |w| means norm of w.

c_{3N}^{TIF} is a 3-N-norm (neutrosophic norm, i.e. generalization of the fuzzy T-norm).

Again, as a particular case, we define the unary negation neutrosophic operator:

n_N:[0,1]x[0,1]x[0,1] → [0,1]x[0,1]x[0,1]

$n_N(x) = n_N(T_1, I_1, F_1) = (F_1, I_1, T_1)$

Let's consider the vectors:

$$T = \begin{pmatrix} T_1 \\ T_2 \\ T_3 \end{pmatrix}, I = \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} \text{ and } F = \begin{pmatrix} F_1 \\ F_2 \\ F_3 \end{pmatrix}$$

We note

$$T_{\overline{x}} = \begin{pmatrix} F_1 \\ T_2 \\ T_3 \end{pmatrix}, T_{\overline{y}} = \begin{pmatrix} T_1 \\ F_2 \\ T_3 \end{pmatrix}, T_{\overline{z}} = \begin{pmatrix} T_1 \\ T_2 \\ F_3 \end{pmatrix}, T_{\overline{xy}} = \begin{pmatrix} F_1 \\ F_2 \\ T_3 \end{pmatrix}$$

etc. and similarly

$$F_{\overline{x}} = \begin{pmatrix} T_1 \\ F_2 \\ F_3 \end{pmatrix}, F_{\overline{y}} = \begin{pmatrix} F_1 \\ T_2 \\ F_3 \end{pmatrix}, F_{\overline{xy}} = \begin{pmatrix} T_1 \\ F_2 \\ T_3 \end{pmatrix} \text{ etc.}$$

For shorter and easier notations let's denote $z_{o_N} w = zw$ and respectively $z_{o_N} w_{o_N} v = zwv$ for the vector neutrosophic law defined previously.

Then the neutrosophic trinary conjunction/ intersection of neutrosophic variables x, y, and z is:

$c_{3N}^{TIF}(x, y, z) = (TT, II + IT, FF + FI + FT + FIT) =$

$= (T_1T_2T_3, I_1I_2I_3 + I_1I_2T_3 + T_1I_2I_3 + I_1T_2T_3 + T_1I_2T_3 +$

$+ T_1T_2I_3, F_1F_2F_3 + F_1F_2I_3 + F_1I_2F_3 + I_1F_2F_3 + F_1I_2I_3 +$

$+ I_1F_2I_3 + I_1I_2F_3 + F_1F_2T_3 + F_1T_2F_3 + T_1F_2F_3 +$

$+ F_1T_2T_3 + T_1F_2T_3 + T_1T_2F_3 + T_1I_2F_3 + T_1F_2I_3 +$

$+ I_1F_2T_3 + I_1T_2F_3 + F_1I_2T_3 + F_1T_2F_3)$

Similarly, the neutrosophic tri-nary disjunction/union of neutrosophic variables x, y, and z is:

$d_{3N}^{TIF}(x,y,z) = (TT+TI+TF+TIF,\ II+IF,\ FF) =$
$= (T_1T_2T_3 + T_1T_2I_3 + T_1I_2T_3 + I_1T_2T_3 + T_1I_2I_3 +$
$+ I_1T_2I_3 + I_1I_2T_3 + T_1T_2F_3 + T_1F_2T_3 + F_1T_2T_3 +$
$+ T_1F_2F_3 + F_1T_2F_3 + F_1F_2T_3 + T_1I_2F_3 + T_1F_2I_3 +$
$+ I_1F_2T_3 + I_1T_2F_3 + F_1I_2T_3 + F_1T_2I_3, I_1I_2I_3 + I_1I_2F_3 +$
$+ I_1F_2I_3 + F_1I_2I_3 + I_1F_2F_3 + F_1I_2F_3 + F_1F_2I_3 +$
$+ F_1F_2F_3)$

Surely, other neutrosophic orders can be used for tri-nary conjunctions/intersections and respectively for tri-nary disjunctions/unions among the componenets T, I, F.

5. NEUTROSOPHIC TOPOLOGIES

A) General Definition of NT:

Let M be a non-empty set.

Let $x(T_A, I_A, F_A) \in A$ with $x(T_B, I_B, F_B) \in B$ be in the neutrosophic set/logic M, where A and B are subsets of M. Then (see Section 2.9.1 about N-norms / N-conorms and examples):

$A \cup B = \{x \in M, x(T_A \vee T_B, I_A \wedge I_B, F_A \wedge F_B)\}$,

$A \cap B = \{x \in M, x(T_A \wedge T_B, I_A \vee I_B, F_A \vee F_B)\}$,

$C(A) = \{x \in M, x(F_A, I_A, T_A)\}$.

A General Neutrosophic Topology on the non-empty set M is a family η of Neutrosophic Sets in M satisfying the following axioms:

- $\mathbf{0}(0,0,1)$ and $\mathbf{1}(1,0,0) \in \eta$;
- If $A, B \in \eta$, then $A \cap B \in \eta$;
- If the family $\{A_k, k \in K\} \subset \eta$, then $\bigcup_{k \in K} A_k \in \eta$.

B) An alternative version of NT

We cal also construct a Neutrosophic Topology on NT = $]^-0, 1^+[$, considering the associated family of standard or non-standard subsets included in NT, and the empty set ˜, called open sets, which is closed under set union and finite intersection.

Let A, B be two such subsets. The union is defined as:

$A \cup B = A+B-A \cdot B$, and the intersection as: $A \cap B = A \cdot B$. The complement of A, $C(A) = \{1^+\}-A$, which is a closed set. {When a non-standard number occurs at an extremity of an internal, one can write "]" instead of "(" and "[" instead of ")"}. The interval NT, endowed with this topology, forms a *neutrosophic topological space*.

In this example we have used the Algebraic Product N-norm/N-conorm. But other Neutrosophic Topologies can be defined by using various N-norm/N-conorm operators.

In the above defined topologies, if all x's are paraconsistent or respectively intuitionistic, then one has a Neutrosophic Paraconsistent Topology, respectively Neutrosophic Intuitionistic Topology.

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The Enduring Power of Giving and Caring from Cavemen to Kind Hearts: A Neutrosophic Multi-Criteria Decision Making Approach

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Abstract

Giving and caring are not just warm and fuzzy feelings; they are the bedrock of a healthy society. By understanding the neuroscience and sociology behind these behaviors, we can cultivate a world that prioritizes cooperation, compassion, and the well-being of all its members. We used the single valued neutrosophic sets (SVNSs) with the multi-criteria decision making (MCDM) method for evaluating the giving and caring impacts. We used TreeSoft to divide the problems as a tree. The SVNSs were used to overcome uncertain information in the evaluation process. The SVNSs were integrated with the DEMATEL method to compute the relationships between criteria. This study used nine criteria. The results show the criterion 5 has the highest impacts and criterion 1 has the lowest criterion.

Keywords: Generosity, Non-Diophantine, Synergy of Giving, Tree of life, Stewardship Ethics.

1 | Introduction

The importance of giving and caring in society might seem self-evident, but science sheds light on why these behaviors are not just niceties, but cornerstones of a thriving civilization. From the dawn of humanity, our brains and social structures have been wired for cooperation and empathy.

Neuroscience reveals the pleasure centers in our brains light up when we give to others. Studies have shown that acts of generosity trigger the release of dopamine, a neurotransmitter associated with reward and motivation. In essence, helping others makes us feel good! This positive reinforcement loop encourages prosocial behaviors, strengthening the bonds within a community.

Sociology adds another layer to the story. From the earliest hunter-gatherer bands, survival depended on collective action. Sharing food, caring for the sick and injured, and working together to secure resources were essential for group success. Societies that fostered cooperation and compassion thrived, while those prioritizing individual gain struggled.

Here's a breakdown of the enduring power of giving and caring:

- Evolutionary Advantage: Cooperation offered a clear survival benefit in our prehistoric past. Groups that looked after one another were more likely to raise healthy children, defend against threats, and innovate. This ingrained tendency to care for others continues to shape our social interactions.

- Stronger Communities: Giving and caring foster trust and reciprocity. When people feel supported and valued, they are more likely to cooperate, leading to a more stable and resilient society.
- Individual Well-being: Studies suggest that acts of generosity not only benefit the recipient but also the giver. Giving back can reduce stress, increase happiness, and even improve physical health.

The emphasis on giving and caring isn't limited to prehistoric times. Modern societies still rely on these fundamental human instincts. From charitable organizations to acts of everyday kindness, these behaviors create a ripple effect, strengthening the social fabric and fostering a sense of well-being for all.

1.1 | The Generosity Equation: Beyond Give and Take with Adam Grant

Adam Grant's seminal work, "Give and Take," sheds light on the power of giving in social dynamics. He classifies individuals into three categories: givers, takers, and matchers. Givers prioritize helping others, often at their own expense. Takers prioritize their own gain, sometimes exploiting others. Matchers seek a balance, reciprocating what they receive.

Grant's research demonstrates that givers, though sometimes taken advantage of, often rise to leadership positions due to the trust and loyalty they inspire. However, the concept of giving extends beyond the simple "give and take" model.

1.2 | The Non-Diophantine Arithmetic of Giving

Imagine giving not as a subtractive process, where you lose something by helping others, but as a process governed by non-Diophantine arithmetic [1, 2]. In traditional arithmetic, 1 - 1 = 0. In non-Diophantine arithmetic, however, the equation can yield a surprising outcome.

When we give, it creates a ripple effect. Our act of generosity inspires others to give back, creating a chain reaction of positive behavior. This synergy multiplies the impact of our initial gift, producing more than the sum of its parts.

1.3 | The Synergy of Giving

Think about open-source software projects. Developers contribute their time and expertise for free, creating a powerful resource that benefits everyone. This exemplifies the synergy of giving. The collective effort produces something far greater than what any individual could achieve alone.

Here's how the synergy of giving plays out:

- Inspiration: When we see others give, it motivates us to do the same. This domino effect creates a more generous society.
- Collaboration: Giving fosters a spirit of collaboration, where people share resources and ideas to achieve a common goal.
- Innovation: When people are free to give and share ideas, it sparks innovation and leads to new solutions and progress.

1.4 | Tree of life ethics since the era of Eden

According to the story of Eden in the Book of Genesis, stewardship ethics and tree of life ethics were both initiated by God himself, and they lay the foundation for a true City of God, which is a utopia envisioned by many religions.

- Stewardship Ethics: God placed humanity in the Garden of Eden as stewards, entrusting them with care for creation. This implies responsibility, not just dominion, requiring us to give back and nurture the environment.

- Tree of Life Ethics: The Tree of Life symbolizes flourishing life, attainable only through obedience to God's will. This will likely involve aspects of giving and caring, for true life thrives in a community that shares and supports its members.

These ethics, initiated by God, suggest that a City of God prioritizes the well-being of all its inhabitants. Giving and caring are not optional but essential for fulfilling God's purpose and creating a flourishing community.

2. | What is tree of life ethics and how does it function in today's society?

The concept of the Tree of Life, a source of immortality and flourishing, appears in various cultures and religions. But beyond its literal interpretation, the Tree of Life also represents a specific ethical framework with profound implications for building a better society. Let's explore what Tree of Life ethics are and how they function in the modern world.

2.1 | The Essence of Tree of Life Ethics

At its core, Tree of Life ethics emphasize creating conditions for a life that truly flourishes. This goes beyond mere survival; it encompasses well-being, fulfillment, and a sense of purpose for all members of a community.

Here are some key principles of this framework:

- Interdependence: Humans are interconnected, and the well-being of one is linked to the well-being of all.
- Compassion: Caring for others, especially those in need, is essential for a flourishing community.
- Sustainability: Living in harmony with nature and ensuring resources for future generations is crucial.
- Justice: Creating a fair and equitable society where everyone has the opportunity to thrive.

2.2 | Tree of Life Ethics in Action

While the story of Eden may be symbolic, the principles of Tree of Life ethics resonate with various social movements and initiatives today:

- Environmentalism: The growing focus on sustainability and environmental protection reflects the concern for preserving the foundation of all life.
- Social Justice: Movements advocating for equality and opportunity for all align with the concept of a just and flourishing society.
- Community Building: Initiatives fostering strong, supportive communities where individuals can contribute and reach their full potential embody the essence of interdependence.

2.3 | Challenges and Opportunities

Modern society faces numerous challenges: social inequality, environmental degradation, and political polarization. These issues highlight the need for a shift towards Tree of Life ethics.

3. | Evaluating the criteria of Giving and caring

This section presents the single valued neutrosophic sets with Tree soft [3, 4] to evaluate the criteria of giving and caring (GC). The single valued neutrosophic sets (SVNSs) are used to deal with uncertainty in the evaluation process.

Multi-attribute decision-making (MADM) situations are those that, in real life, typically include many schemes and numerous characteristics. The secret to solving this issue is figuring out how to rank and choose the options given the circumstances. Commonly, alternatives are assessed using precise values in conventional

MADM techniques. However, since people are vulnerable to subjectivity and because objective objects are complicated, MADM problems frequently involve uncertainty, which means that the decision information provided could be more transparent and precise.

Zadeh [5, 6] performed an in-depth study and developed the idea of the fuzzy set (FS) to define fuzzy notions. A tool for expressing fuzzy information is FS. Based on this, Atanassov added the non-membership degree as a parameter to specify the intuitionistic fuzzy set. Intuitionistic fuzzy sets consider both membership and non-membership degrees. Despite its widespread development and popularity, FS theory cannot address all forms of uncertainty that arise in everyday life, such as inconsistent and ambiguous information.

Smarandache [7–9] thus put out the idea of a neutrosophic set. The intuitionistic fuzzy set has three membership functions: truth, indeterminacy, and falsity. The neutrosophic set is an extension of this fuzzy set. Subsequently, several academics put out single-valued neutrosophic sets (SVNSs) [10, 11].

The SVNSs were integrated with the DEMATEL method to compute the relationships between criteria. The DEMATEL method is a MADM method used to analysis the criteria. Tree soft used to divide the problem into a tree to simply computation and calculation [12]. This study has two levels. In the first level the goal from this study. In the second level, the evaluation of the criteria by the SVN- DEMATEL method [13, 14]. The following are the steps of the SVN- DEMATEL method.

Step 1. Build the direct relation matrix between criteria. This study used three experts to evaluate the criteria. The three experts are used the linguistic terms of SVNSs to build the direct relation matrix.

Step 2. Convert the single valued neutrosophic numbers into a crisp value by using the score function.

Step 3. Combined the direct relation matrix into one matrix.

Step 4. Normalize the direct relation matrix.

$$N = k.x \tag{1}$$

$$k = \frac{1}{\max\limits_{1 \le i \le n} \sum_{j=1}^{n} x_{ij}} \quad ; i, j = 1,2,3, \dots, n \tag{2}$$

Step 5. Compute the total relation matrix

$$T = N(I - N)^{-1} \tag{3}$$

Step 6. Obtain the casual diagram between criteria.

3.1 | Results of Evaluation Giving and Caring

This section introduces the relationships between criteria. This study introduces 9 criteria to be evaluated as society, cooperation, empathy, positive reinforcement, prosocial behaviors, secure resources, Evolutionary, advantage, stronger communities, individual well-being.

Step 1. Three experts evaluated the criteria to build the direct relation matrix as shown in Table 1.

Step 2. Then single valued neutrosophic numbers are converted into a crisp value by using the score function.

Step 3. Then combined the direct relation matrix into one matrix.

Step 4. Then normalize the direct relation matrix as shown in Table 2 using Eqs. (1) and (2).

Step 5. Then compute the total relation matrix as shown in Table 3 using Eq. (3).

Step 6. Obtain the casual diagram between criteria as shown in Figure 1. Criterion 5 has the highest impact and criterion 1 has the lowest impact.

Table 1. Direct relation matrix.

	GC_1	GC_2	GC_3	GC_4	GC_5	GC_6	GC_7	GC_8	GC_9
GC_1	1	(0.9,0.1,0.2)	(0.8,0.2,0.3)	(0.6,0.4,0.5)	(0.5,0.5,0.5)	(0.4,0.6,0.7)	(0.2,0.7,0.8)	(0.1,0.8,0.9)	(0.9,0.1,0.2)
GC_2	1/(0.9,0.1,0.2)	1	(0.1,0.8,0.9)	(0.4,0.6,0.7)	(0.5,0.5,0.5)	(0.6,0.4,0.5)	(0.8,0.2,0.3)	(0.9,0.1,0.2)	(0.8,0.2,0.3)
GC_3	1/(0.8,0.2,0.3)	1/(0.1,0.8,0.9)	1	(0.9,0.1,0.2)	(0.8,0.2,0.3)	(0.6,0.4,0.5)	(0.5,0.5,0.5)	(0.1,0.8,0.9)	(0.6,0.4,0.5)
GC_4	1/(0.6,0.4,0.5)	1/(0.4,0.6,0.7)	1/(0.9,0.1,0.2)	1	(0.6,0.4,0.5)	(0.8,0.2,0.3)	(0.4,0.6,0.7)	(0.2,0.7,0.8)	(0.5,0.5,0.5)
GC_5	1/(0.5,0.5,0.5)	1/(0.5,0.5,0.5)	1/(0.8,0.2,0.3)	1/(0.6,0.4,0.5)	1	(0.9,0.1,0.2)	(0.2,0.7,0.8)	(0.4,0.6,0.7)	(0.4,0.6,0.7)
GC_6	1/(0.4,0.6,0.7)	1/(0.6,0.4,0.5)	1/(0.6,0.4,0.5)	1/(0.8,0.2,0.3)	1/(0.9,0.1,0.2)	1	(0.1,0.8,0.9)	(0.5,0.5,0.5)	(0.1,0.8,0.9)
GC_7	1/(0.2,0.7,0.8)	1/(0.8,0.2,0.3)	1/(0.5,0.5,0.5)	1/(0.4,0.6,0.7)	1/(0.2,0.7,0.8)	1/(0.1,0.8,0.9)	1	(0.6,0.4,0.5)	(0.9,0.1,0.2)
GC_8	1/(0.1,0.8,0.9)	1/(0.9,0.1,0.2)	1/(0.1,0.8,0.9)	1/(0.2,0.7,0.8)	1/(0.4,0.6,0.7)	1/(0.5,0.5,0.5)	1/(0.6,0.4,0.5)	1	(0.8,0.2,0.3)
GC_9	1/(0.9,0.1,0.2)	1/(0.8,0.2,0.3)	1/(0.6,0.4,0.5)	1/(0.5,0.5,0.5)	1/(0.4,0.6,0.7)	1/(0.1,0.8,0.9)	1/(0.9,0.1,0.2)	1/(0.8,0.2,0.3)	1
	GC_1	GC_2	GC_3	GC_4	GC_5	GC_6	GC_7	GC_8	GC_9
GC_1	1	(0.5,0.5,0.5)	(0.4,0.6,0.7)	(0.2,0.7,0.8)	(0.1,0.8,0.9)	(0.9,0.1,0.2)	(0.8,0.2,0.3)	(0.6,0.4,0.5)	(0.5,0.5,0.5)
GC_2	1/(0.5,0.5,0.5)	1	(0.4,0.6,0.7)	(0.5,0.5,0.5)	(0.6,0.4,0.5)	(0.8,0.2,0.3)	(0.9,0.1,0.2)	(0.8,0.2,0.3)	(0.4,0.6,0.7)
GC_3	1/(0.4,0.6,0.7)	1/(0.4,0.6,0.7)	1	(0.1,0.8,0.9)	(0.9,0.1,0.2)	(0.8,0.2,0.3)	(0.6,0.4,0.5)	(0.5,0.5,0.5)	(0.2,0.7,0.8)
GC_4	1/(0.2,0.7,0.8)	1/(0.5,0.5,0.5)	1/(0.1,0.8,0.9)	1	(0.6,0.4,0.5)	(0.8,0.2,0.3)	(0.4,0.6,0.7)	(0.4,0.6,0.7)	(0.9,0.1,0.2)
GC_5	1/(0.1,0.8,0.9)	1/(0.6,0.4,0.5)	1/(0.9,0.1,0.2)	1/(0.6,0.4,0.5)	1	(0.9,0.1,0.2)	(0.5,0.5,0.5)	(0.1,0.8,0.9)	(0.8,0.2,0.3)
GC_6	1/(0.9,0.1,0.2)	1/(0.8,0.2,0.3)	1/(0.8,0.2,0.3)	1/(0.8,0.2,0.3)	1/(0.9,0.1,0.2)	1	(0.6,0.4,0.5)	(0.4,0.6,0.7)	(0.6,0.4,0.5)
GC_7	1/(0.8,0.2,0.3)	1/(0.9,0.1,0.2)	1/(0.6,0.4,0.5)	1/(0.4,0.6,0.7)	1/(0.5,0.5,0.5)	1/(0.6,0.4,0.5)	1	(0.8,0.2,0.3)	(0.5,0.5,0.5)
GC_8	1/(0.6,0.4,0.5)	1/(0.8,0.2,0.3)	1/(0.5,0.5,0.5)	1/(0.4,0.6,0.7)	1/(0.1,0.8,0.9)	1/(0.4,0.6,0.7)	1/(0.8,0.2,0.3)	1	(0.4,0.6,0.7)
GC_9	1/(0.5,0.5,0.5)	1/(0.4,0.6,0.7)	1/(0.2,0.7,0.8)	1/(0.9,0.1,0.2)	1/(0.8,0.2,0.3)	1/(0.6,0.4,0.5)	1/(0.5,0.5,0.5)	1/(0.4,0.6,0.7)	1
	GC_1	GC_2	GC_3	GC_4	GC_5	GC_6	GC_7	GC_8	GC_9
GC_1	1	(0.4,0.6,0.7)	(0.2,0.7,0.8)	(0.1,0.8,0.9)	(0.5,0.5,0.5)	(0.2,0.7,0.8)	(0.9,0.1,0.2)	(0.8,0.2,0.3)	(0.5,0.5,0.5)
GC_2	1/(0.4,0.6,0.7)	1	(0.5,0.5,0.5)	(0.4,0.6,0.7)	(0.4,0.6,0.7)	(0.5,0.5,0.5)	(0.6,0.4,0.5)	(0.9,0.1,0.2)	(0.6,0.4,0.5)
GC_3	1/(0.2,0.7,0.8)	1/(0.5,0.5,0.5)	1	(0.2,0.7,0.8)	(0.4,0.6,0.7)	(0.5,0.5,0.5)	(0.1,0.8,0.9)	(0.2,0.7,0.8)	(0.4,0.6,0.7)
GC_4	1/(0.1,0.8,0.9)	1/(0.4,0.6,0.7)	1/(0.2,0.7,0.8)	1	(0.6,0.4,0.5)	(0.8,0.2,0.3)	(0.2,0.7,0.8)	(0.2,0.7,0.8)	(0.2,0.7,0.8)
GC_5	1/(0.5,0.5,0.5)	1/(0.4,0.6,0.7)	1/(0.4,0.6,0.7)	1/(0.6,0.4,0.5)	1	(0.9,0.1,0.2)	(0.4,0.6,0.7)	(0.2,0.7,0.8)	(0.1,0.8,0.9)
GC_6	1/(0.2,0.7,0.8)	1/(0.5,0.5,0.5)	1/(0.5,0.5,0.5)	1/(0.8,0.2,0.3)	1/(0.9,0.1,0.2)	1	(0.5,0.5,0.5)	(0.5,0.5,0.5)	(0.9,0.1,0.2)
GC_7	1/(0.9,0.1,0.2)	1/(0.6,0.4,0.5)	1/(0.1,0.8,0.9)	1/(0.2,0.7,0.8)	1/(0.4,0.6,0.7)	1/(0.5,0.5,0.5)	1	(0.4,0.6,0.7)	(0.8,0.2,0.3)
GC_8	1/(0.8,0.2,0.3)	1/(0.9,0.1,0.2)	1/(0.2,0.7,0.8)	1/(0.2,0.7,0.8)	1/(0.2,0.7,0.8)	1/(0.5,0.5,0.5)	1/(0.4,0.6,0.7)	1	(0.6,0.4,0.5)
GC_9	1/(0.5,0.5,0.5)	1/(0.6,0.4,0.5)	1/(0.4,0.6,0.7)	1/(0.2,0.7,0.8)	1/(0.1,0.8,0.9)	1/(0.9,0.1,0.2)	1/(0.8,0.2,0.3)	1/(0.6,0.4,0.5)	1

Table 2. Normalized direct relation matrix.

	GC_1	GC_2	GC_3	GC_4	GC_5	GC_6	GC_7	GC_8	GC_9
GC_1	0.206711	0.119433	0.094169	0.06431	0.078091	0.101059	0.12862	0.101059	0.12862
GC_2	0.405231	0.206711	0.068904	0.084981	0.098762	0.126324	0.151588	0.17226	0.117136
GC_3	0.573096	0.842506	0.206711	0.084981	0.137808	0.126324	0.082685	0.059717	0.080388
GC_4	0.933675	0.513647	0.891585	0.206711	0.117136	0.158479	0.066607	0.05742	0.110246
GC_5	0.792394	0.447322	0.357298	0.364785	0.206711	0.17915	0.075794	0.050529	0.087278
GC_6	0.562726	0.349277	0.349277	0.269624	0.238513	0.206711	0.082685	0.094169	0.107949
GC_7	0.464681	0.290974	0.776181	0.671141	0.621029	0.776181	0.206711	0.117136	0.146995
GC_8	0.728248	0.248883	0.949888	0.778523	1	0.463535	0.399389	0.206711	0.117136
GC_9	0.35512	0.399389	0.604816	0.512614	0.794573	0.717878	0.307187	0.399389	0.206711

Table 3. Total direct relation matrix.

	GC_1	GC_2	GC_3	GC_4	GC_5	GC_6	GC_7	GC_8	GC_9
GC_1	-0.35799	-0.29738	-0.20362	-0.11337	-0.09752	-0.07639	-0.01533	-0.02587	-0.00399
GC_2	-0.33583	-0.38281	-0.31848	-0.1453	-0.13246	-0.12059	-0.02417	0.005	-0.0505
GC_3	-0.26861	0.139515	-0.41522	-0.26066	-0.19965	-0.18273	-0.07195	-0.05839	-0.0687
GC_4	-0.16108	-0.10678	0.003397	-0.36278	-0.39548	-0.30584	-0.15893	-0.12565	-0.08611
GC_5	-0.09673	-0.19988	-0.2763	-0.10323	-0.26643	-0.2347	-0.14317	-0.12872	-0.08431
GC_6	-0.27861	-0.25557	-0.24439	-0.14184	-0.18126	-0.18449	-0.14023	-0.09343	-0.07614
GC_7	-0.87975	-0.59506	-0.25981	-0.12784	-0.26995	-0.0161	-0.29486	-0.29558	-0.21948
GC_8	-0.92186	-0.84694	-0.28579	-0.15358	-0.08971	-0.48051	-0.21266	-0.33974	-0.342
GC_9	-1.34435	-0.87332	-0.54584	-0.26945	-0.09709	-0.1771	-0.2809	-0.13029	-0.28518

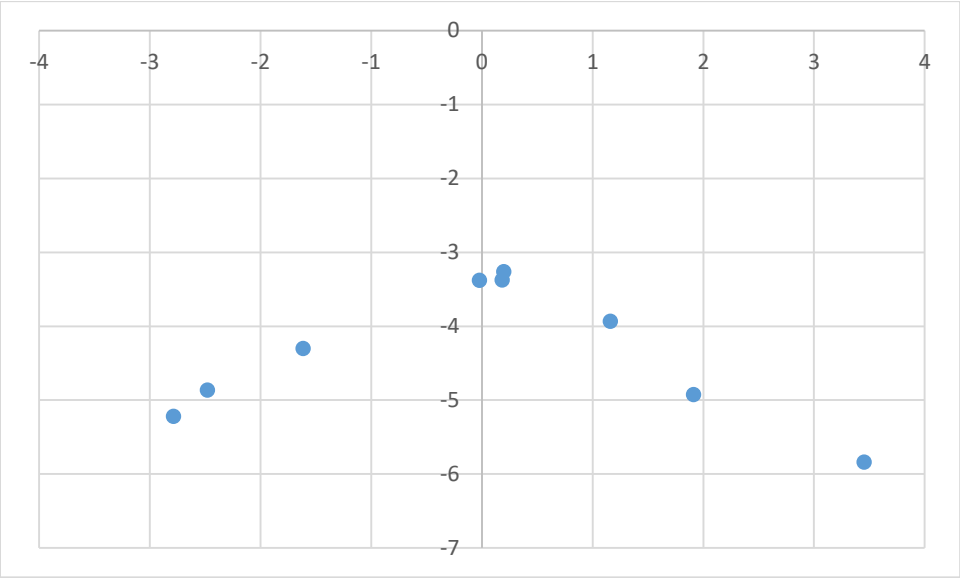

Figure 1. The relationships between criteria.

3.2 | Here's How We Can Embrace these Principles

- Individual Actions: Practicing compassion, making sustainable choices, and advocating for justice are all ways individuals can contribute.
- Community Initiatives: Supporting organizations that promote social good, environmental protection, and community development.
- Policy and Governance: Enacting policies that ensure equal opportunity, promote sustainability, and foster a sense of shared responsibility.

3.3 | Building a City of Life

Tree of Life ethics are not a utopian ideal, but a framework for building a better society, a "City of Life" where everyone has the opportunity to flourish. By practicing these principles on individual and collective levels, we can create a world that is more just, sustainable, and ultimately, a place where life truly thrives.

4 | Concluding Remark

In conclusion, giving and caring are not just warm and fuzzy feelings; they are the bedrock of a healthy society. By understanding the neuroscience and sociology behind these behaviors, we can cultivate a world that prioritizes cooperation, compassion, and the well-being of all its members.

Grant's "give and take" framework provides a valuable starting point, but the concept of giving goes beyond a tit-for-tat exchange. By embracing the non-Diophantine nature of giving and harnessing the synergy it creates, we can cultivate a world built on generosity, collaboration, and progress.

This study used the SVNSs with the MCDM method, and TreeSoft set. The SVNSs were used to overcome the uncertainty of the information. The SVNS has three memberships: tree, indeterminacy, and falsity. The SVNS was integrated with the DEMATEL method. The DEMATEL method is an MCDM method used to compute the relationships between criteria. The direct relation matrix was created by using the opinions of experts. Then, the single-valued neutrosophic numbers are used to build a direct relation matrix. Then, the score function is used to obtain the crisp values. Then, we combined these matrices into one matrix. Then, we applied the steps of the DEMATEL method. We used nine criteria. The results show that criterion 5 has the highest impact, and criterion 1 has the lowest impact.

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Author Contribution

All authors contributed equally to this work.

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Data Availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that there is no conflict of interest in the research.

Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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A New Development of Entropy and Similarity Measures in Temporal Complex Neutrosophic Environments for Tourist Destination Selection

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Abstract. In human existence, making decisions is a common event. Various techniques have been devised to tackle decision-making troubles in practical situations. Over the past ten years, a great deal of study has concentrated on quantifying the degree of ambiguity and unpredictability in knowledge using the concept of neutrosophic sets or extensions thereof. An efficient framework for handling information in decision-making problems involving uncertain, indeterminate, and time-related aspects is the Temporal Complex Neurosophic Set (TCNS). Measures of entropy and similarity can be helpful for evaluating data to solve multicriteria decision-making (MCDM) challenges in practical situations. However, TCNS information measurements were of no concern or relevance to the current technique. In the context of the TCNS, this work suggests multiple novel similarity and entropy measurements. The proposed metrics have been validated and shown to comply with the explicit definition of the entropy measure and similarity for the TCNS. The novel similarity and entropy measures on the TCNS environment are proposed in this research. The four similarity measures on the TCNS contain Dice, Jaccard, Cosine, and Cotangent. Also, a numerical example concerning selecting a Vietnam tourist destination is provided to validate the usefulness of the suggested measures. The practical application shows that proposed TCNS similarity and entropy metrics can produce accurate and significant outcomes for real-world decision-making problems.

Keywords: Similarity measure; Entropy measure; Neutrosophic set; Temporal Complex Neutrosophic Set; MCDM;

1. Introduction

In human activities, decision-making (DM) is a highly typical routine. A multi-attribute decision-making (MADM) problem is determining the most suitable option from a group of options that all need to be evaluated simultaneously. Numerous DM problems have been solved using MADM models, which have attracted the attention of multiple researchers in diverse domains [1, 2] However, real-world issues are excessively complex because they frequently need clarification, clarity, or insufficient data. As a result, decision-makers tackle those issues through the fuzzification process, which is a vital method to address humanistic systems that exist in real-world problems.

The concept of classical fuzzy sets (FS) was introduced by Zadeh [3] in 1965 as a way to handle uncertain data. By enabling elements to be a part of sets with varying levels of membership, FS successfully captures the fuzziness and ambiguity that are inherent in many common problems. After that, it was used by numerous scientists and scholars to look into various real-world events in different fields and recommend the best courses of action. However, the nature of covert human assessments of discontent was not explained by the FS theory, Atanassov [4] was prompted by this to suggest an intuitionistic fuzzy set (IFS), wherein the levels of each element's membership and non-membership are determined separately. Then, an extension of IFS, namely neutrosophic set (NS), is introduced by Smarandache [5] by introducing an uncertain membership function with degrees of falsity, indeterminacy, and truth into IFS. It's also an essential and effective technique for handling conflicting, ambiguous, and incomplete information in certain real-life situations [6, 7, 8].

Although FS or its extension versions have shown remarkable effectiveness in addressing issues emerging from vagueness and uncertainty in many real-world scenarios, they cannot solve the periodicity of some ambiguous information. It is challenging to express periodic forms using crisp sets, FS, IFS, or NS. In order to address this challenge, Ramot et al. [9] introduced the notion of the complex fuzzy set (CFS), which adds information to a phase component in order to provide details about a specific higher dimensional periodic problem. Since its introduction, many researchers have researched and developed CFS because it is considered a valuable tool for representing information with ambiguous, uncertain, and periodic components [10, 11].

Furthermore, the complex neutrosophic set (CNS) was described by Ali and Smarandache [12]. With the intention of integrating the "complex" component to make NS more flexible and sensitive to occasionally ambiguous information, CNS is an integrated version of the CFS and NS. Uncertainty, inconsistency, and indeterminacy are the three membership degree functions that define a CNS. These functions have a complex-valued range in the complex space unit circle. The CNS is limited if the total of the negative, abstinence, and positive grades is three

or less. Following its introduction, CNS demonstrated its value in characterizing data that included unpredictable, inconsistent, periodic, and indeterminate factors.

In real-world decision-making problems, we encounter complex natural phenomena that require adding a second dimension to represent the degree of membership. By establishing the second dimension, information is fully gathered, and the problem of information loss is minimized. Therefore, since the beginning of the CNS, many hybrid versions have been developed and used in a range of life domains, such as Interval CNS [13], Bipolar CNS [14], Complex neutrosophic soft set (CNSS) [15], Interval-valued complex neutrosophic soft set (IVCNSS) [16]... In light of the theories mentioned above, it is evident that many MCDM researchers have found the information's uncertainty and time-periodic element interesting. Many of the decision-making scenarios we encounter in real life involve periodic events, which call for the representation of various time intervals (e.g., seasonal variations in weather information). However, the present experiments have yet to take into account the effect of temporal factors by CNS in an MCDM model concurrently. Recently, Lan et al. [17] introduced an extension of CNS, namely Temporal Complex neutrosophic set (TCNS), to represent data with dynamic and temporal factors of several real-life decision-making contexts. It covers decision-making issues with uncertain, temporal, and periodic aspects. It is a novel opening with several practical applications that concern the time cycle and time element.

Making decisions involves systematically solving issues in the real world and selecting the best option after determining which feasible alternatives are available. All possible options are sorted, allowing users to choose the suitable solution. When faced with multi-criteria decision-making difficulties, decision-makers can select the best option primarily based on information measures, including cross-entropy, entropy, similarity, and distance measures. Because of this, entropy and similarity measurements are consistently two crucial subjects that have been investigated widely by many scholars from various backgrounds. Nevertheless, prior research has yet to focus on refining and strengthening the theory of the TCNS environments using similarity metrics, mathematical operations, and entropy measures. That is the primary motivation for carrying out this research.

(1) Define novel measures of entropy and similarity for the Temporal Complex Neutronosophic Set, which addresses the unpredictable, periodic, and temporal aspects of DM problems.

(2) Build the decision-making model using the proposed entropy and similarity measurements in the TCNS environment.

(3) Using a real-world case study of selecting a tourism location in Vietnam, illustrate the viability and logic of applying the suggested model's decision-making process.

(4) Prove our proposed model's effectiveness and potential for real-life case study through a comparative analysis with related MCDM techniques.

The remaining portions of the paper are organized in this manner. Section 2 outlines the research gap and a few connected works. Next, in Section 3, we go over a few of the significant concepts of the TCNS. Novel metrics of entropy and similarity depending on the TCNS framework are defined in Section 4. The multi-criteria technique for making decisions using proposed indicators is provided in Section 5. In Section 6, a comparative experiment is utilized to demonstrate how our theory might be used in practice in a particular tourist location. The conclusions are finally presented in Section 7.

2. **Literature review**

2.1. *Related works*

Similarity is essential to finding solutions in many intricate and unpredictable situations in human life. Both the entropy and similarity measurements were able to show how similar two objects were in an unexpected and uncertain environment. As a result, many scholars have concentrated on using NS or extensions to quantify fuzziness and ambiguity in data. Many different entropy and similarity metrics established and have effectively dealt with practical applications such as clustering analysis [18, 19], decision-making [20, 21], health diagnostics [22, 23] and pattern recognition [24, 25].

According to Hausdorff distance, Broumi and Smarandache [26] presented some similarities for NS in the neutrosophic environment. Several metrics of entropy and similarity of a single-valued NS were proposed by Majumdar et al. [27]. The approach based on score function and divergence measure is suggested to determine the weights of decision experts using a single-valued NS [28]. Bin Ji et al. [29] published the multi-parameter similarity measure for interval-valued neutrosophic sets based on the tangent function. The Q-Neutrosophic Soft set was enhanced with similarity and entropy tools by Abu Qamar and Hassan [30], who also evaluated how well they worked for issues involving medical diagnosis. Additionally, entropy and similarity are frequently employed as crucial metrics for resolving real-world multi-criteria decision-making issues [31, 32, 33]

Since entropy and similarity measurements are two significant topics in the CFS and CNS, they have been comprehensively investigated from a variety of perspectives in multiple studies. Utilizing mixed models to handle the ambiguities of periodical data—where time is a crucial component in its representation—researchers made essential contributions to the literature on similarity measures domains. Axiomatic formulations of the similarity measure and entropy for Complex Multi Fuzzy Soft Sets were presented by Al-Qudah and Hassan [34]. Using a complex fuzzy soft set, Ganeshsree Selvachandran et al. [35] introduced metrics of distance and similarity to address pattern recognition challenges involving digital photographs. The Cosine similarity measures were used by the authors in [36] to calculate the coefficients of

similarity and dissimilarity between two Complex linear Diophantine fuzzy sets. The Complex Multi-Fuzzy Hypersoft Set's Entropy and Similarity Measure concept was introduced in [37]. To demonstrate the validity and significance of the developed measures, some associated theorems and an application for a person's selection decision issue who intends to buy a car are established.

Keeping up with this development and improving the modeling of some real-world issues using our approach (CNS theory and its hybrid structures), novel dice similarity measurements utilizing CNS were proposed by Ali et al. [38]. The authors then used the pattern recognition model and the criteria they provided to test the validity and superiority of the pre-existing methods. Jaccard, Dice, and cosine are three examples of complex neutrosophic similarity metrics proposed by Mondal et al. [39]. A numerical case study demonstrates the suggested methods for handling multi-attribute decision-making scenarios in which students select an appropriate course for post-secondary education after passing a secondary exam. Xu et al. [40] introduced some distance measures based on Hamming, Hausdorff, and Euclidean metrics to deal with the interval CNS information. In order to help with decision-making in medical diagnosis, Faisal et al. [41] developed several similarity metrics of interval complex neutrosophic soft sets (ICNSS) based on the distance measurements: Euclidean and Hamming. An axiomatic definition of single-valued CNS entropy and normalized distance formulas are provided by [42]. Also, to demonstrate the usefulness and feasibility of our suggested entropy metric, we offer a real-world example that involves the choice of green suppliers.

Because physical problems are inherently complicated and unpredictable, the difficulty of researching MCDM has concentrated on the issues with criteria values in the form of intervals, drawing attention to this attractive study area. The research indicates they are effective at encapsulating and illustrating vague and ambiguous information, enabling decision-makers to make intelligent decisions.

2.2. *Research gap analysis*

The evaluations above suggest that these several measures of similarity and entropy and their applications play a significant role in efficiently managing uncertainty and inconsistency in decision-making contexts. However, there are several extant research gaps, as indicated below:

- All entropy and similarity measures in the literature are established based on the theory of CFS, CNS, and these hybrid structures.

- Entropy and similarity measures in the literature only deal with uncertain, imprecise, and periodic information. None of the above theories have the capability of managing information involving dynamic, temporal, and time-cycle factors in real-life problems.

- Previous theories focused on the time-varying component but neglected the temporal, time-cycle factor, and the impact of temporal factors.

- Many entropy measures exist for CFS, CNS, or extension of CNS in the literature that fail to deal with the problem of providing reasonable or consistent results to the decision-makers in the case of a TCNS environment.

- There hasn't been any study on entropy and similarity metrics especially created for TCNS information theory, even though they are essential to fuzzy set theory.

When taking into account the time cycle, periodic, and time components, TCNS is a novel opening with several practical applications. Unfortunately, previous studies have not concentrated on improving and solidifying the theory of the TCNS settings through entropy and similarity metrics. The above facts and the research gap in CNS neutrosophic information theory prompted us to pursue this work. Thus, the paper aims to develop novel entropy and similarity metrics for the TCNS.

3. **Preliminaries**

This section summarizes the core concepts of TCNS and the three operations (intersection, union, and complement) that will be applied in our study.

Definition 3.1 ([17])**.** Let U be a universal set, and let $\tilde{\tau} = \{\tau_1, \tau_2, ..., \tau_{n_\tau}\}$ be consecutive time periods. On U, a temporal complex neutrosophic set (TCNS) A is represented by form below:

$$\mathrm{A}(\theta, \tilde{\tau}) = \{\theta, \langle p(\theta, \tau_l).e^{j\mu(\theta,\tau_l)}, q(\theta, \tau_l).e^{j\nu(\theta,\tau_l)}, r(\theta, \tau_l).e^{j\eta(\theta,\tau_l)}\rangle \mid \theta \in U\} \tag{1}$$

Where $p(\theta, \tau_l)$, $q(\theta, \tau_l)$, $r(\theta, \tau_l) \in [0, 1]$ and $\mu(\theta, \tau_l)$, $\nu(\theta, \tau_l)$, $\eta(\theta, \tau_l) \in [0, 2\pi]$

Definition 3.2 ([17])**.** Suppose there are two TCNSs $\Omega_1(\theta, \tilde{\tau})$ and $\Omega_2(\theta, \tilde{\tau})$ respectively.

$$\Omega_1(\theta, \tilde{\tau}) = \left\{\theta, \left\langle p_{\Omega_1}(\theta, \tilde{\tau}) e^{j\mu_{\Omega_1}(\theta,\tilde{\tau})}, q_{\Omega_1}(\theta, \tilde{\tau}) e^{j\nu_{\Omega_1}(\theta,\tilde{\tau})}, r_{\Omega_1}(\theta, \tilde{\tau}) e^{j\eta_{\Omega_1}(\theta,\tilde{\tau})} \right\rangle\right\}$$
$$\Omega_2(\theta, \tilde{\tau}) = \left\{\theta, \left\langle p_{\Omega_2}(\theta, \tilde{\tau}) e^{j\mu_{\Omega_2}(\theta,\tilde{\tau})}, q_{\Omega_2}(\theta, \tilde{\tau}) e^{j\nu_{\Omega_2}(\theta,\tilde{\tau})}, r_{\Omega_2}(\theta, \tilde{\tau}) e^{j\eta_{\Omega_2}(\theta,\tilde{\tau})} \right\rangle\right\}$$

The following describes the fundamental functions of TCNS:

Complement:

$$C(\Omega_1(\theta, \tilde{\tau})) = \left\{\theta, \left\langle \begin{array}{l} r_{\Omega_1}(\theta, \tilde{\tau}) e^{j\left(2\pi - \eta_{\Omega_1}(\theta,\tilde{\tau})\right)}, \\ (1 - q_{\Omega_1}(\theta, \tilde{\tau})) e^{j\left(2\pi - v_{\Omega_1}(\theta,\tilde{\tau})\right)}, \\ p_{\Omega_1}(\theta, \tilde{\tau}) e^{j\left(2\pi - \mu_{\Omega_1}(\theta,\tilde{\tau})\right)} \end{array} \right\rangle \mid \theta \in U\right\} \tag{2}$$

Union:

$$\Omega_1(\theta, \tilde{\tau}) \cup \Omega_2(\theta, \tilde{\tau}) = \left\{\theta, \left\langle \begin{array}{l} (p_{\Omega_1}(\theta, \tilde{\tau}) \vee p_{\Omega_2}(\theta, \tilde{\tau})) e^{j\mu_{\Omega_1 \cup \Omega_2}(\theta,\tilde{\tau})}, \\ (q_{\Omega_1}(\theta, \tilde{\tau}) \wedge q_{\Omega_2}(\theta, \tilde{\tau})) e^{j\nu_{\Omega_1 \cup \Omega_2}(\theta,\tilde{\tau})}, \\ (r_{\Omega_1}(\theta, \tilde{\tau}) \wedge r_{\Omega_2}(\theta, \tilde{\tau})) e^{j\eta_{\Omega_1 \cup \Omega_2}(\theta,\tilde{\tau})} \end{array} \right\rangle \mid \theta \in U\right\} \tag{3}$$

Intersect:

$$\Omega_1(\theta,\tilde{\tau})\cap\Omega_2(\theta,\tilde{\tau})=\left\{\theta,\left\langle\begin{array}{l}(p_{\Omega_1}(\theta,\tilde{\tau})\wedge p_{\Omega_2}(\theta,\tilde{\tau}))\,e^{j\mu_{\Omega_1\cap\Omega_2}(\theta,\tilde{\tau})},\\(q_{\Omega_1}(\theta,\tilde{\tau})\vee q_{\Omega_2}(\theta,\tilde{\tau}))\,e^{j\nu_{\Omega_1\cap\Omega_2}(\theta,\tilde{\tau})},\\(r_{\Omega_1}(\theta,\tilde{\tau})\vee r_{\Omega_2}(\theta,\tilde{\tau}))\,e^{j\eta_{\Omega_1\cap\Omega_2}(\theta,\tilde{\tau})}\end{array}\right\rangle\mid\theta\in U\right\}\tag{4}$$

4. **Propose new measures of Temporal complex neutrosophic set**

The study presents a few TCNS similarity metrics in this part, including the Dice, Jaccard, Cosine, and Cotangent similarity measures. Additionally, it is suggested that decision-making models in temporally complex neutrosophic contexts start with an entropy assessment of TCNS.

4.1. *TCNS - Dice similarity measure*

Definition 4.1. Assume that $\Omega_1=\left\langle p_{\Omega_1}(\theta_i,\tau_l)\,e^{j\mu_{\Omega_1}(\theta_i,\tau_l)},q_{\Omega_1}(\theta_i,\tau_l)\,e^{j\nu_{\Omega_1}(\theta_i,\tau_l)},r_{\Omega_1}(\theta_i,\tau_l)\,e^{j\eta_{\Omega_1}(\theta_i,\tau_l)}\right\rangle$ and $\Omega_2=\left\langle p_{\Omega_2}(\theta_i,\tau_l)\,e^{j\mu_{\Omega_2}(\theta_i,\tau_l)},q_{\Omega_2}(\theta_i,\tau_l)\,e^{j\nu_{\Omega_2}(\theta_i,\tau_l)},r_{\Omega_2}(\theta_i,\tau_l)\,e^{j\eta_{\Omega_2}(\theta_i,\tau_l)}\right\rangle$ be are two TCNSs on universe of discourse U for $\theta_i\,(1,2,...,n_U)$ and $\tau_l\,(1,2,...,n_\tau)$.
Suppose $Sim_D(\Omega_1,\Omega_2)$ is a TCNS - Dice similarity measure between Ω_1 and Ω_2, $Sim_D(\Omega_1,\Omega_2)$ is defined as follows:

$$\begin{aligned}&Sim_D(\Omega_1,\Omega_2)\\&=\frac{1}{n_U*n_\tau}\sum_{i=1}^{n_U}\sum_{l=1}^{n_\tau}\frac{2*\left[\begin{array}{l}(p_{\Omega_1}(\theta_i,\tau_l)+\mu_{\Omega_1}(\theta_i,\tau_l))\,(p_{\Omega_2}(\theta_i,\tau_l)+\mu_{\Omega_2}(\theta_i,\tau_l))\\+(q_{\Omega_1}(\theta_i,\tau_l)+\nu_{\Omega_1}(\theta_i,\tau_l))\,(q_{\Omega_2}(\tau_l)+\nu_{\Omega_2}(\theta_i,\tau_l))\\+(r_{\Omega_1}(\theta_i,\tau_l)+\eta_{\Omega_1}(\theta_i,\tau_l))\,(r_{\Omega_2}(\theta_i,\tau_l)+\eta_{\Omega_2}(\theta_i,\tau_l))\end{array}\right]}{\left(\begin{array}{l}(p_{\Omega_1}(\theta_i,\tau_l)+\mu_{\Omega_1}(\theta_i,\tau_l))^2+(q_{\Omega_1}(\theta_i,\tau_l)+\nu_{\Omega_1}(\theta_i,\tau_l))^2\\+(r_{\Omega_1}(\theta_i,\tau_l)+\eta_{\Omega_1}(\theta_i,\tau_l))^2+(p_{\Omega_2}(\theta_i,\tau_l)+\mu_{\Omega_2}(\theta_i,\tau_l))^2\\+(q_{\Omega_2}(\theta_i,\tau_l)+\nu_{\Omega_2}(\theta_i,\tau_l))^2+(r_{\Omega_2}(\theta_i,\tau_l)+\eta_{\Omega_2}(\theta_i,\tau_l))^2\end{array}\right)}\end{aligned}\tag{5}$$

Example 4.2. Let $U=\{\theta_1,\theta_2\}$ be a universe of discourse; $\tilde{\tau}=\{\tau_1,\tau_2,\tau_3\}$ be time period; Ω_1 and Ω_2 be two TCNS in U

$$\Omega_1(\theta_1)=\left\{\begin{array}{l}\left\langle 0.1e^{j0.2},0.6e^{j0.45},0.3e^{j0.56}\right\rangle,\left\langle 0.8e^{j0.77},0.2e^{j0.86},0.4e^{j0.65}\right\rangle,\\\left\langle 0.6e^{j0.52},0.2e^{j0.2},0.36e^{j0.2}\right\rangle\end{array}\right\}$$

$$\Omega_1(\theta_2)=\left\{\begin{array}{l}\left\langle 0.5e^{j0.12},0.2e^{j0.65},0.5e^{j0.85}\right\rangle,\left\langle 0.3e^{j0.8},0.7e^{j0.23},0.1e^{j0.32}\right\rangle,\\\left\langle 0.74e^{j0.6},0.5e^{j0.15},0.66e^{j0.41}\right\rangle\end{array}\right\}$$

$$\Omega_2(\theta_1)=\left\{\begin{array}{l}\left\langle 0.6e^{j0.8},0.9e^{j0.25},0.4e^{j0.52}\right\rangle,\left\langle 0.6e^{j0.62},0.5e^{j0.24},0.3e^{j0.18}\right\rangle,\\\left\langle 0.3e^{j0.82},0.3e^{j0.6},0.4e^{j0.71}\right\rangle\end{array}\right\}$$

$$\Omega_2(\theta_2) = \left\{ \begin{array}{l} \left\langle 0.3e^{j0.22}, 0.4e^{j0.56}, 0.6e^{j0.26} \right\rangle, \left\langle 0.1e^{j0.38}, 0.5e^{j0.43}, 0.5e^{j0.24} \right\rangle, \\ \left\langle 0.2e^{j0.29}, 0.4e^{j0.72}, 0.5e^{j0.65} \right\rangle \end{array} \right\}$$

Then, the TCNS - Dice similarity measure between Ω_1 and Ω_2 can be: $Sim_D(\Omega_1, \Omega_2) = 0.87942$

Theorem 4.3. *Let Ω_1 and Ω_2 be two TCNSs then,*

(1) $0 \leq Sim_D(\Omega_1, \Omega_2) \leq 1$

(2) $Sim_D(\Omega_1, \Omega_2) = Sim_D(\Omega_2, \Omega_1)$

(3) $Sim_D(\Omega_1, \Omega_2) = 1$ *, if and only if* $\Omega_1 = \Omega_2$

(4) if Ω_3 is a TCNS in U and $\Omega_1 \subseteq \Omega_2 \subseteq \Omega_3$ then $Sim_D(\Omega_1, \Omega_3) \leq Sim_D(\Omega_1, \Omega_2)$ and $Sim_D(\Omega_1, \Omega_3) \leq Sim_D(\Omega_1, \Omega_2)$

Proof

(1) We can have,

$$2 * \left[\begin{array}{l} \left(p_{\Omega_1}(\theta_i, \tau_l) + \mu_{\Omega_1}(\theta_i, \tau_l)\right)\left(p_{\Omega_2}(\theta_i, \tau_l) + \mu_{\Omega_2}(\theta_i, \tau_l)\right) \\ + \left(q_{\Omega_1}(\theta_i, \tau_l) + \nu_{\Omega_1}(\theta_i, \tau_l)\right)\left(q_{\Omega_2}(\theta_i, \tau_l) + \nu_{\Omega_2}(\theta_i, \tau_l)\right) \\ + \left(r_{\Omega_1}(\theta_i, \tau_l) + \eta_{\Omega_1}(\theta_i, \tau_l)\right)\left(r_{\Omega_2}(\theta_i, \tau_l) + \eta_{\Omega_2}(\theta_i, \tau_l)\right) \end{array} \right]$$

$$\leq \left(\begin{array}{l} \left(p_{\Omega_1}(\theta_i, \tau_l) + \mu_{\Omega_1}(\theta_i, \tau_l)\right)^2 + \left(q_{\Omega_1}(\theta_i, \tau_l) + \nu_{\Omega_1}(\theta_i, \tau_l)\right)^2 \\ + \left(r_{\Omega_1}(\theta_i, \tau_l) + \eta_{\Omega_1}(\theta_i, \tau_l)\right)^2 + \left(p_{\Omega_2}(\theta_i, \tau_l) + \mu_{\Omega_2}(\theta_i, \tau_l)\right)^2 \\ + \left(q_{\Omega_2}(\theta_i, \tau_l) + \nu_{\Omega_2}(\theta_i, \tau_l)\right)^2 + \left(r_{\Omega_2}(\theta_i, \tau_l) + \eta_{\Omega_2}(\theta_i, \tau_l)\right)^2 \end{array} \right)$$

Hence, $0 \leq Sim_D(\Omega_1, \Omega_2) \leq 1$. Thus, the first inequality is shown to be true.

(2) It is easy to observe that this is true

(3) When $\Omega_1 = \Omega_2$, then it implies that $Sim_D(\Omega_1, \Omega_2) = 1$. On the other hand, if $Sim_D(\Omega_1, \Omega_2) = 1$ then,

$p_{\Omega_1}(\theta_i, \tau_l) = p_{\Omega_2}(\theta_i, \tau_l)$; $\mu_{\Omega_1}(\theta_i, \tau_l) = \mu_{\Omega_2}(\theta_i, \tau_l)$; $q_{\Omega_1}(\theta_i, \tau_l) = q_{\Omega_2}(\theta_i, \tau_l)$; $\nu_{\Omega_1}(\theta_i, \tau_l) = \nu_{\Omega_2}(\theta_i, \tau_l)$; $r_{\Omega_1}(\theta_i, \tau_l) = r_{\Omega_2}(\theta_i, \tau_l)$; $\eta_{\Omega_1}(\theta_i, \tau_l) = \eta_{\Omega_2}(\theta_i, \tau_l)$;

This implies that $\Omega_1 = \Omega_2$. Thus, the third inequality is proved.

(4) When $\Omega_1 \subseteq \Omega_2 \subseteq \Omega_3$, we can have

$p_{\Omega_1}(\theta_i, \tau_l) + \mu_{\Omega_1}(\theta_i, \tau_l) \leq p_{\Omega_2}(\theta_i, \tau_l) + \mu_{\Omega_2}(\theta_i, \tau_l) \leq p_{\Omega_3}(\theta_i, \tau_l) + \mu_{\Omega_3}(\theta_i, \tau_l)$;

$q_{\Omega_1}(\theta_i, \tau_l) + \nu_{\Omega_1}(\theta_i, \tau_l) \geq q_{\Omega_2}(\theta_i, \tau_l) + \nu_{\Omega_2}(\theta_i, \tau_l) \geq q_{\Omega_3}(\theta_i, \tau_l) + \nu_{\Omega_3}(\theta_i, \tau_l)$;

$r_{\Omega_1}(\theta_i, \tau_l) + \eta_{\Omega_1}(\theta_i, \tau_l) \geq r_{\Omega_2}(\theta_i, \tau_l) + \eta_{\Omega_2}(\theta_i, \tau_l) \geq r_{\Omega_3}(\theta_i, \tau_l) + \eta_{\Omega_3}(\theta_i, \tau_l)$;

So, $Sim_D(\Omega_1, \Omega_3) \leq Sim_D(\Omega_1, \Omega_2)$ and $Sim_D(\Omega_1, \Omega_3) \leq Sim_D(\Omega_2, \Omega_3)$. So, the fourth inequality is proved

4.2. *TCNS - Jaccard similarity measure*

Definition 4.4. Let two TCNSs

$\Omega_1 = \left\langle p_{\Omega_1}(\theta_i,\tau_l)\, e^{j\mu_{\Omega_1}(\theta_i,\tau_l)}, q_{\Omega_1}(\theta_i,\tau_l)\, e^{j\nu_{\Omega_1}(\theta_i,\tau_l)}, r_{\Omega_1}(\theta_i,\tau_l)\, e^{j\eta_{\Omega_1}(\theta_i,\tau_l)} \right\rangle$ and $\Omega_2 = \left\langle p_{\Omega_2}(\theta_i,\tau_l)\, e^{j\mu_{\Omega_2}(\theta_i,\tau_l)}, q_{\Omega_2}(\theta_i,\tau_l)\, e^{j\nu_{\Omega_2}(\theta_i,\tau_l)}, r_{\Omega_2}(\theta_i,\tau_l)\, e^{j\eta_{\Omega_2}(\theta_i,\tau_l)} \right\rangle$ and $\theta_i; i = 1,2,...,n_U$ and $\tau_l; l = 1,2,...,n_\tau$.

$Sim_J(\Omega_1,\Omega_2))$ is a TCNS - Jaccard similarity measure between Ω_1 and Ω_2 and $Sim_J(\Omega_1,\Omega_2))$ is defined as follows:

$$Sim_J(\Omega_1,\Omega_2) = \frac{1}{n_U * n_\tau}\sum_{i=1}^{n_U}\sum_{l=1}^{n_\tau}\frac{\begin{array}{l}(p_{\Omega_1}(\theta_i,\tau_l)+\mu_{\Omega_1}(\theta_i,\tau_l))(p_{\Omega_2}(\theta_i,\tau_l)+\mu_{\Omega_2}(\theta_i,\tau_l)) \\ +(q_{\Omega_1}(\theta_i,\tau_l)+\nu_{\Omega_1}(\theta_i,\tau_l))(q_{\Omega_2}(\theta_i,\tau_l)+q_{\Omega_2}(\theta_i,\tau_l)) \\ +(r_{\Omega_1}(\theta_i,\tau_l)+r_{\Omega_1}(\theta_i,\tau_l))(r_{\Omega_2}(\theta_i,\tau_l)+r_{\Omega_2}(\theta_i,\tau_l))\end{array}}{\begin{array}{l}(p_{\Omega_1}(\theta_i,\tau_l)+\mu_{\Omega_1}(\theta_i,\tau_l))^2+(q_{\Omega_1}(\theta_i,\tau_l)+\nu_{\Omega_1}(\theta_i,\tau_l))^2 \\ +(r_{\Omega_1}(\theta_i,\tau_l)+r_{\Omega_1}(\theta_i,\tau_l))^2+(p_{\Omega_2}(\theta_i,\tau_l)+\mu_{\Omega_2}(\theta_i,\tau_l))^2 \\ +(q_{\Omega_2}(\theta_i,\tau_l)+q_{\Omega_2}(\theta_i,\tau_l))^2+(r_{\Omega_2}(\theta_i,\tau_l)+r_{\Omega_2}(\theta_i,\tau_l))^2 \\ -\left(\begin{array}{l}(p_{\Omega_1}(\theta_i,\tau_l)+\mu_{\Omega_1}(\theta_i,\tau_l))(p_{\Omega_2}(\theta_i,\tau_l)+\mu_{\Omega_2}(\theta_i,\tau_l)) \\ +(q_{\Omega_1}(\theta_i,\tau_l)+\nu_{\Omega_1}(\theta_i,\tau_l))(q_{\Omega_2}(\theta_i,\tau_l)+q_{\Omega_2}(\theta_i,\tau_l)) \\ +(r_{\Omega_1}(\theta_i,\tau_l)+r_{\Omega_1}(\theta_i,\tau_l))(r_{\Omega_2}(\theta_i,\tau_l)+r_{\Omega_2}(\theta_i,\tau_l))\end{array}\right)\end{array}} \quad (6)$$

Example 4.5. Let $U = \{\theta_1,\theta_2\}$;$\tilde{\tau} = \{\tau_1,\tau_2,\tau_3\}$; and two TCNS Ω_1, Ω_2 in U.

$$\Omega_1(\theta_1) = \left\{\begin{array}{l}\left\langle 0.1e^{j0.2}, 0.6e^{j0.45}, 0.3e^{j0.56}\right\rangle, \left\langle 0.8e^{j0.77}, 0.2e^{j0.86}, 0.4e^{j0.65}\right\rangle, \\ \left\langle 0.6e^{j0.52}, 0.2e^{j0.2}, 0.36e^{j0.2}\right\rangle\end{array}\right\}$$

$$\Omega_1(\theta_2) = \left\{\begin{array}{l}\left\langle 0.5e^{j0.12}, 0.2e^{j0.65}, 0.5e^{j0.85}\right\rangle, \left\langle 0.3e^{j0.8}, 0.7e^{j0.23}, 0.1e^{j0.32}\right\rangle, \\ \left\langle 0.74e^{j0.6}, 0.5e^{j0.15}, 0.66e^{j0.41}\right\rangle\end{array}\right\}$$

$$\Omega_2(\theta_1) = \left\{\begin{array}{l}\left\langle 0.6e^{j0.8}, 0.9e^{j0.25}, 0.4e^{j0.52}\right\rangle, \left\langle 0.6e^{j0.62}, 0.5e^{j0.24}, 0.3e^{j0.18}\right\rangle, \\ \left\langle 0.3e^{j0.82}, 0.3e^{j0.6}, 0.4e^{j0.71}\right\rangle\end{array}\right\}$$

$$\Omega_2(\theta_2) = \left\{\begin{array}{l}\left\langle 0.3e^{j0.22}, 0.4e^{j0.56}, 0.6e^{j0.26}\right\rangle, \left\langle 0.1e^{j0.38}, 0.5e^{j0.43}, 0.5e^{j0.24}\right\rangle, \\ \left\langle 0.2e^{j0.29}, 0.4e^{j0.72}, 0.5e^{j0.65}\right\rangle\end{array}\right\}$$

$Sim_J(\Omega_1,\Omega_2)$ is The TTCNS - Jaccard similarity measure between Ω_1, Ω_2 can be: $Sim_J(\Omega_1,\Omega_2) = 0.78807$

Theorem 4.6. *Let Ω_1 and Ω_2 be two TCNSs then,*

(1) $0 \le Sim_J(\Omega_1,\Omega_2) \le 1$

(2) $Sim_J(\Omega_1,\Omega_2) = Sim_J(\Omega_2,\Omega_1)$

(3) $Sim_J(\Omega_1,\Omega_2) = 1$*, if and only if* $\Omega_1 = \Omega_2$

(4) if Ω_3 is a TCNS in U and $\Omega_1 \subseteq \Omega_2 \subseteq \Omega_3$ then $Sim_J(\Omega_1,\Omega_3) \le Sim_J(\Omega_1,\Omega_2)$ and $Sim_J(\Omega_1,\Omega_3) \le Sim_J(\Omega_2,\Omega_3)$

Proof

(1) Since:

$$\left(\begin{array}{l}\left(p_{\Omega_1}(\theta_i,\tau_l)+\mu_{\Omega_1}(\theta_i,\tau_l)\right)\left(p_{\Omega_2}(\theta_i,\tau_l)+\mu_{\Omega_2}(\theta_i,\tau_l)\right)\\+\left(q_{\Omega_1}(\theta_i,\tau_l)+\nu_{\Omega_1}(\theta_i,\tau_l)\right)\left(q_{\Omega_2}(\theta_i,\tau_l)+\nu_{\Omega_2}(\theta_i,\tau_l)\right)\\+\left(r_{\Omega_1}(\theta_i,\tau_l)+\eta_{\Omega_1}(\theta_i,\tau_l)\right)\left(r_{\Omega_2}(\theta_i,\tau_l)+\eta_{\Omega_2}(\theta_i,\tau_l)\right)\end{array}\right)$$

$$\leq\left(\begin{array}{l}\left(p_{\Omega_1}(\theta_i,\tau_l)+\mu_{\Omega_1}(\theta_i,\tau_l)\right)^2+\left(q_{\Omega_1}(\theta_i,\tau_l)+\nu_{\Omega_1}(\theta_i,\tau_l)\right)^2\\+\left(r_{\Omega_1}(\theta_i,\tau_l)+\eta_{\Omega_1}(\theta_i,\tau_l)\right)^2+\left(p_{\Omega_2}(\theta_i,\tau_l)+\mu_{\Omega_2}(\theta_i,\tau_l)\right)^2\\+\left(q_{\Omega_2}(\theta_i,\tau_l)+\nu_{\Omega_2}(\theta_i,\tau_l)\right)^2+\left(r_{\Omega_2}(\theta_i,\tau_l)+\eta_{\Omega_2}(\theta_i,\tau_l)\right)^2\\-\left[\begin{array}{l}\left(p_{\Omega_1}(\theta_i,\tau_l)+\mu_{\Omega_1}(\theta_i,\tau_l)\right)\left(p_{\Omega_2}(\theta_i,\tau_l)+\mu_{\Omega_2}(\theta_i,\tau_l)\right)\\+\left(q_{\Omega_1}(\theta_i,\tau_l)+\nu_{\Omega_1}(\theta_i,\tau_l)\right)\left(q_{\Omega_2}(\theta_i,\tau_l)+\nu_{\Omega_2}(\theta_i,\tau_l)\right)\\+\left(r_{\Omega_1}(\theta_i,\tau_l)+\eta_{\Omega_1}(\theta_i,\tau_l)\right)\left(r_{\Omega_2}(\theta_i,\tau_l)+\eta_{\Omega_2}(\theta_i,\tau_l)\right)\end{array}\right]\end{array}\right)$$

Hence, we have, $0\leq Sim_J(\Omega_1,\Omega_2)\leq 1$

(2) It is easily observed.

(3) When $\Omega_1=\Omega_2$, then, $Sim_J(\Omega_1,\Omega_2)=1$. On the other hand, if $Sim_J(\Omega_1,\Omega_2)=1$ then, $p_{\Omega_1}(\theta_i,\tau_l)=p_{\Omega_2}(\theta_i,\tau_l)$; $\mu_{\Omega_1}(\theta_i,\tau_l)=\mu_{\Omega_2}(\theta_i,\tau_l)$; $q_{\Omega_1}(\theta_i,\tau_l)=q_{\Omega_2}(\theta_i,\tau_l)$; $\nu_{\Omega_1}(\theta_i,\tau_l)=\nu_{\Omega_2}(\theta_i,\tau_l)$; $r_{\Omega_1}(\theta_i,\tau_l)=r_{\Omega_2}(\theta_i,\tau_l)$; $\eta_{\Omega_1}(\theta_i,\tau_l)=\eta_{\Omega_2}(\theta_i,\tau_l)$

This implies that $\Omega_1=\Omega_2$.

(4) When $\Omega_1\subseteq\Omega_2\subseteq\Omega_3$, we can write $p_{\Omega_1}(\theta_i,\tau_l)+\mu_{\Omega_1}(\theta_i,\tau_l)\leq p_{\Omega_2}(\theta_i,\tau_l)+\mu_{\Omega_2}(\theta_i,\tau_l)\leq p_{\Omega_3}(\theta_i,\tau_l)+\mu_{\Omega_3}(\theta_i,\tau_l)$;

$q_{\Omega_1}(\theta_i,\tau_l)+\nu_{\Omega_1}(\theta_i,\tau_l)\geq q_{\Omega_2}(\theta_i,\tau_l)+\nu_{\Omega_2}(\theta_i,\tau_l)\geq q_{\Omega_3}(\theta_i,\tau_l)+\nu_{\Omega_3}(\theta_i,\tau_l)$;

$r_{\Omega_1}(\theta_i,\tau_l)+\eta_{\Omega_1}(\theta_i,\tau_l)\geq r_{\Omega_2}(\theta_i,\tau_l)+\eta_{\Omega_2}(\theta_i,\tau_l)\geq r_{\Omega_3}(\theta_i,\tau_l)+\eta_{\Omega_3}(\theta_i,\tau_l)$;

So, $Sim_J(\Omega_1,\Omega_3)\leq S_J(\Omega_1,\Omega_2)$ and $Sim_J(\Omega_1,\Omega_3)\leq Sim_J(\Omega_2,\Omega_3)$.

4.3. *TCNS - Cosine Similarity*

The cosine similarity measure is one of the practical measures in the decision-making process. The similarity between two TCNS can be computed with the use of the temporal complex neutrosophic cosine measure. Time factors and time cycles are of interest to this metric, which employs complex neutrosophic numbers.

The cosine similarity measure for TCNSs is defined as follows:

Definition 4.7. Let $\Omega_1=\left\langle p_{\Omega_1}(\theta_i,\tau_l)\,e^{j\mu_{\Omega_1}(\theta_i,\tau_l)},q_{\Omega_1}(\theta_i,\tau_l)\,e^{j\nu_{\Omega_1}(\theta_i,\tau_l)},r_{\Omega_1}(\theta_i,\tau_l)\,e^{j\eta_{\Omega_1}(\theta_i,\tau_l)}\right\rangle$ and $\Omega_2=\left\langle p_{\Omega_2}(\theta_i,\tau_l)\,e^{j\mu_{\Omega_2}(\theta_i,\tau_l)},q_{\Omega_2}(\theta_i,\tau_l)\,e^{j\nu_{\Omega_2}(\theta_i,\tau_l)},r_{\Omega_2}(\theta_i,\tau_l)\,e^{j\eta_{\Omega_2}(\theta_i,\tau_l)}\right\rangle$ be two TCNSs and $\theta_i(1,2,3,...,n_U)$ belong to U and $\tau_l(1,2,3,...,n_\tau)$.

$Sim_{\cos}(\Omega_1,\Omega_2)$ is A TCNS - Cosine similarity measure between TCNSs Ω_1 and Ω_2 and $Sim_{\cos}(\Omega_1,\Omega_2)$ is proposed as follows:

$$
\begin{aligned}
&Sim_{\cos}(\Omega_1,\Omega_2)\\
&=\frac{1}{n_U * n_\tau}\sum_{i=1}^{n_U}\sum_{l=1}^{n_\tau}\frac{\begin{array}{l}(p_{\Omega_1}(\theta_i,\tau_l)+\mu_{\Omega_1}(\theta_i,\tau_l))(p_{\Omega_2}(\theta_i,\tau_l)+\mu_{\Omega_2}(\theta_i,\tau_l))\\ +(q_{\Omega_1}(\theta_i,\tau_l)+\nu_{\Omega_1}(\theta_i,\tau_l))(q_{\Omega_2}(\theta_i,\tau_l)+q_{\Omega_2}(\theta_i,\tau_l))\\ +(r_{\Omega_1}(\theta_i,\tau_l)+r_{\Omega_1}(\theta_i,\tau_l))(r_{\Omega_2}(\theta_i,\tau_l)+r_{\Omega_2}(\theta_i,\tau_l))\end{array}}{\begin{array}{l}\sqrt{\begin{array}{l}(p_{\Omega_1}(\theta_i,\tau_l)+\mu_{\Omega_1}(\theta_i,\tau_l))^2+(q_{\Omega_1}(\theta_i,\tau_l)+\nu_{\Omega_1}(\theta_i,\tau_l))^2\\ +(r_{\Omega_1}(\theta_i,\tau_l)+r_{\Omega_1}(\theta_i,\tau_l))^2\end{array}}\\ \times\sqrt{\begin{array}{l}(p_{\Omega_2}(\theta_i,\tau_l)+\mu_{\Omega_2}(\theta_i,\tau_l))^2+(q_{\Omega_2}(\theta_i,\tau_l)+q_{\Omega_2}(\theta_i,\tau_l))^2\\ +(r_{\Omega_2}(\theta_i,\tau_l)+r_{\Omega_2}(\theta_i,\tau_l))^2\end{array}}\end{array}}
\end{aligned}
\tag{7}
$$

Example 4.8. Let $U=\{\theta_1,\theta_2\};\tilde{\tau}=\{\tau_1,\tau_2,\tau_3\}$ and Ω_1 and Ω_2 are two TCNS in U

$$\Omega_1(\theta_1)=\left\{\begin{array}{l}\left\langle 0.1e^{j0.2},0.6e^{j0.45},0.3e^{j0.56}\right\rangle,\left\langle 0.8e^{j0.77},0.2e^{j0.86},0.4e^{j0.65}\right\rangle,\\ \left\langle 0.6e^{j0.52},0.2e^{j0.2},0.36e^{j0.2}\right\rangle\end{array}\right\}$$

$$\Omega_1(\theta_2)=\left\{\begin{array}{l}\left\langle 0.5e^{j0.12},0.2e^{j0.65},0.5e^{j0.85}\right\rangle,\left\langle 0.3e^{j0.8},0.7e^{j0.23},0.1e^{j0.32}\right\rangle,\\ \left\langle 0.74e^{j0.6},0.5e^{j0.15},0.66e^{j0.41}\right\rangle\end{array}\right\}$$

$$\Omega_2(\theta_1)=\left\{\begin{array}{l}\left\langle 0.6e^{j0.8},0.9e^{j0.25},0.4e^{j0.52}\right\rangle,\left\langle 0.6e^{j0.62},0.5e^{j0.24},0.3e^{j0.18}\right\rangle,\\ \left\langle 0.3e^{j0.82},0.3e^{j0.6},0.4e^{j0.71}\right\rangle\end{array}\right\}$$

$$\Omega_2(\theta_2)=\left\{\begin{array}{l}\left\langle 0.3e^{j0.22},0.4e^{j0.56},0.6e^{j0.26}\right\rangle,\left\langle 0.1e^{j0.38},0.5e^{j0.43},0.5e^{j0.24}\right\rangle,\\ \left\langle 0.2e^{j0.29},0.4e^{j0.72},0.5e^{j0.65}\right\rangle\end{array}\right\}$$

We have the following results:

$Sim_{\cos}(\Omega_1,\Omega_2)=0.73203$

Theorem 4.9. *Let Ω_1 and Ω_2 be two TCNSs then,*

(1) $0\le Sim_{\cos}(\Omega_1,\Omega_2)\le 1$

(2) $Sim_{\cos}(\Omega_1,\Omega_2)=Sim_{\cos}(\Omega_2,\Omega_1)$

(3) $Sim_{\cos}(\Omega_1,\Omega_2)=1$*, if and only if* $\Omega_1=\Omega_2$

(4) if Ω_3 is a TCNS in U and $\Omega_1\subseteq\Omega_2\subseteq\Omega_3$then $Sim_{\cos}(\Omega_1,\Omega_3)\le Sim_{\cos}(\Omega_1,\Omega_2)$ *and* $Sim_{\cos}(\Omega_1,\Omega_3)\le Sim_{\cos}(\Omega_2,\Omega_3)$

Proof

(1) We can have,

$$\left(\begin{array}{l} \left(p_{\Omega_1}(\theta_i,\tau_l)+\mu_{\Omega_1}(\theta_i,\tau_l)\right)\left(p_{\Omega_2}(\theta_i,\tau_l)+\mu_{\Omega_2}(\theta_i,\tau_l)\right) \\ +\left(q_{\Omega_1}(\theta_i,\tau_l)+\nu_{\Omega_1}(\theta_i,\tau_l)\right)\left(q_{\Omega_2}(\theta_i,\tau_l)+\nu_{\Omega_2}(\theta_i,\tau_l)\right) \\ +\left(r_{\Omega_1}(\theta_i,\tau_l)+\eta_{\Omega_1}(\theta_i,\tau_l)\right)\left(r_{\Omega_2}(\theta_i,\tau_l)+\eta_{\Omega_2}(\theta_i,\tau_l)\right) \end{array} \right)^2$$

$$\leq \left(\begin{array}{l} \left(p_{\Omega_1}(\theta_i,\tau_l)+\mu_{\Omega_1}(\theta_i,\tau_l)\right)^2+\left(q_{\Omega_1}(\theta_i,\tau_l)+\nu_{\Omega_1}(\theta_i,\tau_l)\right)^2 \\ +\left(r_{\Omega_1}(\theta_i,\tau_l)+\eta_{\Omega_1}(\theta_i,\tau_l)\right)^2+\left(p_{\Omega_2}(\theta_i,\tau_l)+\mu_{\Omega_2}(\theta_i,\tau_l)\right)^2 \\ +\left(q_{\Omega_2}(\theta_i,\tau_l)+\nu_{\Omega_2}(\theta_i,\tau_l)\right)^2+\left(r_{\Omega_2}(\theta_i,\tau_l)+\eta_{\Omega_2}(\theta_i,\tau_l)\right)^2 \end{array} \right)$$

Hence, $0 \leq S_{\text{Cos}}(\Omega_1,\Omega_2) \leq 1$

(2) It is easily observed.

(3) Since $\Omega_1=\Omega_2$, then $Sim_{\text{Cos}}(\Omega_1,\Omega_2)=1$. On the other hand, if $Sim_{\text{Cos}}(\Omega_1,\Omega_2)=1$, we have,

$p_{\Omega_1}(\theta_i,\tau_l)=p_{\Omega_2}(\theta_i,\tau_l)$;$\mu_{\Omega_1}(\theta_i,\tau_l)=\mu_{\Omega_2}(\theta_i,\tau_l)$; $q_{\Omega_1}(\theta_i,\tau_l)=q_{\Omega_2}(\theta_i,\tau_l)$.; $\nu_{\Omega_1}(\theta_i,\tau_l)=\nu_{\Omega_2}(\theta_i,\tau_l)$; $r_{\Omega_1}(\theta_i,\tau_l)=r_{\Omega_2}(\theta_i,\tau_l)$; $\eta_{\Omega_1}(\theta_i,\tau_l)=\eta_{\Omega_2}(\theta_i,\tau_l)$;

This implies that $\Omega_1=\Omega_2$

(4) When $\Omega_1\subseteq\Omega_2\subseteq\Omega_3$, we can have

$p_{\Omega_1}(\theta_i,\tau_l)+\mu_{\Omega_1}(\theta_i,\tau_l)\leq p_{\Omega_2}(\theta_i,\tau_l)+\mu_{\Omega_2}(\theta_i,\tau_l)\leq p_{\Omega_3}(\theta_i,\tau_l)+\mu_{\Omega_3}(\theta_i,\tau_l)$;

$q_{\Omega_1}(\theta_i,\tau_l)+\nu_{\Omega_1}(\theta_i,\tau_l)\geq q_{\Omega_2}(\theta_i,\tau_l)+\nu_{\Omega_2}(\theta_i,\tau_l)\geq q_{\Omega_3}(\theta_i,\tau_l)+\nu_{\Omega_3}(\theta_i,\tau_l)$;

$r_{\Omega_1}(\theta_i,\tau_l)+\eta_{\Omega_1}(\theta_i,\tau_l)\geq r_{\Omega_2}(\theta_i,\tau_l)+\eta_{\Omega_2}(\theta_i,\tau_l)\geq r_{\Omega_3}(\theta_i,\tau_l)+\eta_{\Omega_3}(\theta_i,\tau_l)$;

So, $Sim_J(\Omega_1,\Omega_3)\leq Sim_J(\Omega_1,\Omega_2)$ and $Sim_J(\Omega_1,\Omega_3)\leq Sim_J(\Omega_2,\Omega_3)$

Hence, Theorem 4.9 is proved.

4.4. *TCNS - Cotangent Similarity*

In this part, we utilize the special characteristics of the Cotangent function to present a novel method of similarity measurement in the context of TCNS.

Definition 4.10. Let two TCNSs $\Omega_1=\left\langle p_{\Omega_1}(\theta_i,\tau_l)\,e^{j\mu_{\Omega_1}(\theta_i,\tau_l)},q_{\Omega_1}(\theta_i,\tau_l)\,e^{j\nu_{\Omega_1}(\theta_i,\tau_l)},r_{\Omega_1}(\theta_i,\tau_l)\,e^{j\eta_{\Omega_1}(\theta_i,\tau_l)}\right\rangle$ and $\Omega_2=\left\langle p_{\Omega_2}(\theta_i,\tau_l)\,e^{j\mu_{\Omega_2}(\theta_i,\tau_l)},q_{\Omega_2}(\theta_i,\tau_l)\,e^{j\nu_{\Omega_2}(\theta_i,\tau_l)},r_{\Omega_2}(\theta_i,\tau_l)\,e^{j\eta_{\Omega_2}(\theta_i,\tau_l)}\right\rangle$ for all $\theta_i\,(1,2,3,...,n_U)$ belong to U and $\tau_l\,(1,2,3,...,n_\tau)$.

$Sim_{\text{Cot\_1}}(\Omega_1,\Omega_2)$ is a TCNS - Cotangent similarity measure between TCNSs Ω_1 and Ω_2 and $Sim_{\text{Cot\_1}}(\Omega_1,\Omega_2)$ is defined as follows:

$$Sim_{\text{Cot\_1}}(\Omega_1,\Omega_2) =$$

$$cot\left(\begin{array}{l}\dfrac{\pi}{4}+\dfrac{\pi}{8}\times\dfrac{1}{n_U*n_\tau}\displaystyle\sum_{i=1}^{n_U}\sum_{l=1}^{n_\tau}\left[\begin{array}{l}max\left(\begin{array}{l}\mid p_{\Omega_1}(\theta_i,\tau_l)-p_{\Omega_2}(\theta_i,\tau_l)\mid,\\ \mid q_{\Omega_1}(\theta_i,\tau_l)-q_{\Omega_2}(\theta_i,\tau_l)\mid,\\ \mid r_{\Omega_1}(\theta_i,\tau_l)-r_{\Omega_2}(\theta_i,\tau_l)\mid\end{array}\right)\\ +\dfrac{1}{2\pi}max\left(\begin{array}{l}\mid \mu_{\Omega_1}(\theta_i,\tau_l)-\mu_{\Omega_2}(\theta_i,\tau_l)\mid,\\ \mid \nu_{\Omega_1}(\theta_i,\tau_l)-\nu_{\Omega_2}(\theta_i,\tau_l)\mid,\\ \mid \eta_{\Omega_1}(\theta_i,\tau_l)-\eta_{\Omega_2}(\theta_i,\tau_l)\mid\end{array}\right)\end{array}\right]\end{array}\right) \tag{8}$$

$$Sim_{\text{Cot\_2}}(\Omega_1,\Omega_2) =$$

$$cot\left(\begin{array}{l}\dfrac{\pi}{4}+\dfrac{\pi}{24}\times\dfrac{1}{n_U*n_\tau}\displaystyle\sum_{i=1}^{n_U}\sum_{l=1}^{n_\tau}\left[\begin{array}{l}\left(\begin{array}{l}\mid p_{\Omega_1}(\theta_i,\tau_l)-p_{\Omega_2}(\theta_i,\tau_l)\mid\\ +\mid q_{\Omega_1}(\theta_i,\tau_l)-q_{\Omega_2}(\theta_i,\tau_l)\mid\\ +\mid r_{\Omega_1}(\theta_i,\tau_l)-r_{\Omega_2}(\theta_i,\tau_l)\mid\end{array}\right)\\ +\dfrac{1}{2\pi}max\left(\begin{array}{l}\mid \mu_{\Omega_1}(\theta_i,\tau_l)-\mu_{\Omega_2}(\theta_i,\tau_l)\mid\\ +\mid \nu_{\Omega_1}(\theta_i,\tau_l)-\nu_{\Omega_2}(\theta_i,\tau_l)\mid\\ +\mid \eta_{\Omega_1}(\theta_i,\tau_l)-\eta_{\Omega_2}(\theta_i,\tau_l)\mid\end{array}\right)\end{array}\right]\end{array}\right) \tag{9}$$

Example 4.11. Let $U=\{\theta_1,\theta_2\}$; $\tilde{\tau}=\{\tau_1,\tau_2,\tau_3\}$; Ω_1 and Ω_2 are two TCNS in U

$$\Omega_1(\theta_1)=\left\{\begin{array}{l}\left\langle 0.1e^{j0.2},0.6e^{j0.45},0.3e^{j0.56}\right\rangle,\left\langle 0.8e^{j0.77},0.2e^{j0.86},0.4e^{j0.65}\right\rangle,\\ \left\langle 0.6e^{j0.52},0.2e^{j0.2},0.36e^{j0.2}\right\rangle\end{array}\right\}$$

$$\Omega_1(\theta_2)=\left\{\begin{array}{l}\left\langle 0.5e^{j0.12},0.2e^{j0.65},0.5e^{j0.85}\right\rangle,\left\langle 0.3e^{j0.8},0.7e^{j0.23},0.1e^{j0.32}\right\rangle,\\ \left\langle 0.74e^{j0.6},0.5e^{j0.15},0.66e^{j0.41}\right\rangle\end{array}\right\}$$

$$\Omega_2(\theta_1)=\left\{\begin{array}{l}\left\langle 0.6e^{j0.8},0.9e^{j0.25},0.4e^{j0.52}\right\rangle,\left\langle 0.6e^{j0.62},0.5e^{j0.24},0.3e^{j0.18}\right\rangle,\\ \left\langle 0.3e^{j0.82},0.3e^{j0.6},0.4e^{j0.71}\right\rangle\end{array}\right\}$$

$$\Omega_2(\theta_2)=\left\{\begin{array}{l}\left\langle 0.3e^{j0.22},0.4e^{j0.56},0.6e^{j0.26}\right\rangle,\left\langle 0.1e^{j0.38},0.5e^{j0.43},0.5e^{j0.24}\right\rangle,\\ \left\langle 0.2e^{j0.29},0.4e^{j0.72},0.5e^{j0.65}\right\rangle\end{array}\right\}$$

Since Definitions 4.10, we have: $Sim_{Cot\_1}(\Omega_1,\Omega_2)=0.690485$; $Sim_{Cot\_2}(\Omega_1,\Omega_2)=0.82243$

Theorem 4.12. *Let Ω_1 and Ω_2 be two TCNSs then,*

(1) $0\le Sim_{Cot}(\Omega_1,\Omega_2)\le 1$

(2) $Sim_{Cot}(\Omega_1,\Omega_2)=Sim_{Cot}(\Omega_2,\Omega_1)$

(3) $Sim_{Cot}(\Omega_1,\Omega_2)=1$, *if and only if* $\Omega_1=\Omega_2$

(4) if Ω_3 *is a TCNS in* U *and* $\Omega_1 \subseteq \Omega_2 \subseteq \Omega_3$ *then* $Sim_{Cot}(\Omega_1, \Omega_3) \leq Sim_{Cot}(\Omega_1, \Omega_2)$ *and* $Sim_{Cot}(\Omega_1, \Omega_3) \leq Sim_{Cot}(\Omega_2, \Omega_3)$

Proof

The proof follows the same general approach as that used for Equations (5) and (6), with specific adaptations for this case.

(1) We have,

Hence, $0 \leq Sim_{Cot}(\Omega_1, \Omega_2) \leq 1$

(2) It is easily observed.

(3) When $\Omega_1 = \Omega_2$, then obviously$S_{Cot_2}(\Omega_1, \Omega_2) = 1$. On the other hand, if $S_{Cot\_2}(\Omega_1, \Omega_2) = 1$ then,

$p_{\Omega_1}(\theta_i, \tau_l) = p_{\Omega_2}(\theta_i, \tau_l)$;$\mu_{\Omega_1}(\theta_i, \tau_l) = \mu_{\Omega_2}(\theta_i, \tau_l)$;$q_{\Omega_1}(\theta_i, \tau_l) = q_{\Omega_2}(\theta_i, \tau_l)$; $\nu_{\Omega_1}(\theta_i, \tau_l) = \nu_{\Omega_2}(\theta_i, \tau_l)$; $r_{\Omega_1}(\theta_i, \tau_l) = r_{\Omega_2}(\theta_i, \tau_l)$; $\eta_{\Omega_1}(\theta_i, \tau_l) = \eta_{\Omega_2}(\theta_i, \tau_l)$;

This implies that $\Omega_1 = \Omega_2$

(4) When $\Omega_1 \subseteq \Omega_2 \subseteq \Omega_3$, wen can have

$p_{\Omega_1}(\theta_i, \tau_l) + \mu_{\Omega_1}(\theta_i, \tau_l) \leq p_{\Omega_2}(\theta_i, \tau_l) + \mu_{\Omega_2}(\theta_i, \tau_l) \leq p_{\Omega_3}(\theta_i, \tau_l) + \mu_{\Omega_3}(\theta_i, \tau_l)$;

$q_{\Omega_1}(\theta_i, \tau_l) + \nu_{\Omega_1}(\theta_i, \tau_l) \geq q_{\Omega_2}(\theta_i, \tau_l) + \nu_{\Omega_2}(\theta_i, \tau_l) \geq q_{\Omega_3}(\theta_i, \tau_l) + \nu_{\Omega_3}(\theta_i, \tau_l)$;

$r_{\Omega_1}(\theta_i, \tau_l) + \eta_{\Omega_1}(\theta_i, \tau_l) \geq r_{\Omega_2}(\theta_i, \tau_l) + \eta_{\Omega_2}(\theta_i, \tau_l) \geq r_{\Omega_3}(\theta_i, \tau_l) + \eta_{\Omega_3}(\theta_i, \tau_l)$;

So, $Sim_{Cot\_2}(\Omega_1, \Omega_3) \leq Sim_{Cot\_2}(\Omega_1, \Omega_2)$ and $Sim_{Cot\_2}(\Omega_1, \Omega_3) \leq Sim_{Cot\_2}(\Omega_2, \Omega_3)$. The Theorem 4.12 is proved.

4.5. *Entropy measures of Temporal complex neutrosophic set*

When Zadeh [3] first proposed the entropy metric, it was intended to quantify fuzziness in information. A measure of the uncertainty and fuzziness present in any set: fuzzy set, complex fuzzy set, neutrophilic, etc.—is the entropy measure. Since the TCNS can handle ambiguous and uncertain data in this work, finding a TCNS's entropy is particularly important.

Definition 4.13. A function E qualifies as a TCNS - Entropy measure on a finite set U if *E: TCNS(U)* $\rightarrow$ [0, 1] and it satisfies the following properties:

(a) $E(\Omega) = 0$ if Ω is a crisp set

(b) $E(\Omega) = 1$ if $p_\Omega(\theta, \tau) = 0.5$;$q_\Omega(\theta, \tau) = 0.5$;$r_\Omega(\theta, \tau) = 0.5$ and $\mu_\Omega(\theta, \tau) = \pi$;$\nu_\Omega(\theta, \tau) = \pi$;$\eta_\Omega(\theta, \tau) = \pi$ for $\forall \tau \in \tilde{\tau}; \forall \theta \in U$.

(c) $E(\Omega_1) \leq E(\Omega_2)$ if Ω_2 is more uncertain that Ω_1, that is $| p_{\Omega_1}(\theta, \tau) - 0.5 | \geq | p_{\Omega_2}(\theta, \tau) - 0.5 |$; $| q_{\Omega_1}(\theta, \tau) - 0.5 | \geq | q_{\Omega_2}(\theta, \tau) - 0.5 |$; $| r_{\Omega_1}(\theta, \tau) - 0.5 | \geq | r_{\Omega_2}(\theta, \tau) - 0.5 |$ and

$| \mu_{\Omega_1}(\theta, \tau) - \pi | \geq | \mu_{\Omega_2}(\theta, \tau) - \pi |$; $| \nu_{\Omega_1}(\theta, \tau) - \pi | \geq | \nu_{\Omega_2}(\theta, \tau) - \pi |$; $| \eta_{\Omega_1}(\theta, \tau) - \pi | \geq | \eta_{\Omega_2}(\theta, \tau) - \pi |$

(d) $E(\Omega) = E(\Omega^c)$, Ω^c is a TCNS complement of Ω

Theorem 4.14. *Let a Temporal Complex Neutrosophic Set Ω on U. A TCNS - The following equation suggests entropy metric of Ω:*

$$E(\Omega) = \tan\left(\frac{\pi}{6 * n_U * n_\tau} \sum_{i=1}^{n_U} \sum_{l=1}^{n_\tau} e_{il}(\Omega)\right) \tag{10}$$

Where

$$e_{il}(\Omega) = \begin{pmatrix} p_\Omega(\theta_i,\tau_l)(1 - p_\Omega(\theta_i,\tau_l)) + q_\Omega(\theta_i,\tau_l)(1 - q_\Omega(\theta_i,\tau_l)) \\ + r_\Omega(\theta_i,\tau_l)(1 - r_\Omega(\theta_i,\tau_l)) + \left(\frac{\mu_\Omega(\theta_i,\tau_l)}{2\pi}\right)\left(1 - \frac{\mu_\Omega(\theta_i,\tau_l)}{2\pi}\right) \\ + \left(\frac{\nu_\Omega(\theta_i,\tau_l)}{2\pi}\right)\left(1 - \frac{\nu_\Omega(\theta_i,\tau_l)}{2\pi}\right) + \left(\frac{\eta_\Omega(\theta_i,\tau_l)}{2\pi}\right)\left(1 - \frac{\eta_\Omega(\theta_i,\tau_l)}{2\pi}\right) \end{pmatrix}$$

Proof.

$E(\Omega)$ must strictly adhere to all conditions outlined in Definition 4.13.

(a) Assume that $E(\Omega) = 0$, it implies that $p(\theta_i,\tau_l) = 0$; $q(\theta_i,\tau_l) = 0$; $r(\theta_i,\tau_l) = 0$; $\mu(\theta_i,\tau_l) = 0$; $v(\theta_i,\tau_l) = 0$; $r(\theta_i,\tau_l) = 0$or $p(\theta_i,\tau_l) = 1$; $q(\theta_i,\tau_l) = 1$; $r(\theta_i,\tau_l) = 1$; $\mu(\theta_i,\tau_l) = 2\pi$; $v(\theta_i,\tau_l) = 2\pi$; $r(\theta_i,\tau_l) = 2\pi$;

(b) $E(\Omega) = 1$, if and only if $p(\theta_i,\tau_l) = 0.5$; $q(\theta_i,\tau_l) = 0.5$; $r(\theta_i,\tau_l) = 0.5$; $\mu(\theta_i,\tau_l) = \pi$; $v(\theta_i,\tau_l) = \pi$; $r(\theta_i,\tau_l) = \pi$

(c) Assum Ω_1, Ω_2 and Ω_2 more uncertain than that Ω_2. It implies that, if $p_{\Omega_1}(\theta,\tau) \leq p_{\Omega_2}(\theta,\tau) \leq 0.5$; $q_{\Omega_1}(\theta,\tau) \leq q_{\Omega_2}(\theta,\tau) \leq 0.5$; $r_{\Omega_1}(\theta,\tau) \leq r_{\Omega_2}(\theta,\tau) \leq 0.5$ or $p_{\Omega_1}(\theta,\tau) \geq p_{\Omega_2}(\theta,\tau) \geq 0.5$; $q_{\Omega_1}(\theta,\tau) \geq q_{\Omega_2}(\theta,\tau) \geq 0.5$; $r_{\Omega_1}(\theta,\tau) \geq r_{\Omega_2}(\theta,\tau) \geq 0.5$ for each $\theta_i \in X$.

Hence,

$p_{\Omega 1}(\theta_i,\tau_l)(1 - p_{\Omega_1}(\theta_i,\tau_l)) \leq p_{\Omega 2}(\theta_i,\tau_l)(1 - p_{\Omega_2}(\theta_i,\tau_l))$
$q_{\Omega 1}(\theta_i,\tau_l)(1 - q_{\Omega_1}(\theta_i,\tau_l)) \leq q_{\Omega 2}(\theta_i,\tau_l)(1 - q_{\Omega_2}(\theta_i,\tau_l))$;
$r_{\Omega 1}(\theta_i,\tau_l)(1 - r_{\Omega_1}(\theta_i,\tau_l)) \leq r_{\Omega 2}(\theta_i,\tau_l)(1 - r_{\Omega_2}(\theta_i,\tau_l))$;
and $\frac{\mu_{\Omega_1}(\theta_i,\tau_l)}{2\pi}\left(1 - \frac{\mu_{\Omega_1}(\theta_i,\tau_l)}{2\pi}\right) \leq \frac{\mu_{\Omega_2}(\theta_i,\tau_l)}{2\pi}\left(1 - \frac{\mu_{\Omega_2}(\theta_i,\tau_l)}{2\pi}\right)$;
$\frac{\nu_{\Omega 1}(\theta_i,\tau_l)}{2\pi}\left(1 - \frac{\nu_{\Omega_1}(\theta_i,\tau_l)}{2\pi}\right) \leq \frac{\nu_{\Omega 2}(\theta_i,\tau_l)}{2\pi}\left(1 - \frac{\nu_{\Omega_2}(\theta_i,\tau_l)}{2\pi}\right)$;
$\frac{\eta_{\Omega 1}(\theta_i,\tau_l)}{2\pi}\left(1 - \frac{\eta_{\Omega 1}(\theta_i,\tau_l)}{2\pi}\right) \leq \frac{\eta_{\Omega_2}(\theta_i,\tau_l)}{2\pi}\left(1 - \frac{\eta_{\Omega 2}(\theta_i,\tau_l)}{2\pi}\right)$;
which implies $E(\Omega_1) \leq E(\Omega_2)$

(d) $E(\Omega) = E(\Omega^c)$ is trivial

Example 4.15. Let $U = \{\theta_1, \theta_2\}$;$\tilde{\tau} = \{\tau_1, \tau_2, \tau_3\}$ and Ω_1 and Ω_2 are two TCNS in U

$$\Omega_1(\theta_1) = \left\{ \begin{array}{l} \left\langle 0.1e^{j0.2}, 0.6e^{j0.45}, 0.3e^{j0.56} \right\rangle, \left\langle 0.8e^{j0.77}, 0.2e^{j0.86}, 0.4e^{j0.65} \right\rangle, \\ \left\langle 0.6e^{j0.52}, 0.2e^{j0.2}, 0.36e^{j0.2} \right\rangle \end{array} \right\}$$

$$\Omega_1(\theta_2) = \left\{ \begin{array}{l} \left\langle 0.5e^{j0.12}, 0.2e^{j0.65}, 0.5e^{j0.85} \right\rangle, \left\langle 0.3e^{j0.8}, 0.7e^{j0.23}, 0.1e^{j0.32} \right\rangle, \\ \left\langle 0.74e^{j0.6}, 0.5e^{j0.15}, 0.66e^{j0.41} \right\rangle \end{array} \right\}$$

$$\Omega_2(\theta_1) = \left\{ \begin{array}{l} \left\langle 0.6e^{j0.8}, 0.9e^{j0.25}, 0.4e^{j0.52} \right\rangle, \left\langle 0.6e^{j0.62}, 0.5e^{j0.24}, 0.3e^{j0.18} \right\rangle, \\ \left\langle 0.3e^{j0.82}, 0.3e^{j0.6}, 0.4e^{j0.71} \right\rangle \end{array} \right\}$$

$$\Omega_2(\theta_2) = \left\{ \begin{array}{l} \left\langle 0.3e^{j0.22}, 0.4e^{j0.56}, 0.6e^{j0.26} \right\rangle, \left\langle 0.1e^{j0.38}, 0.5e^{j0.43}, 0.5e^{j0.24} \right\rangle, \\ \left\langle 0.2e^{j0.29}, 0.4e^{j0.72}, 0.5e^{j0.65} \right\rangle \end{array} \right\}$$

Entropy values can be estimated by applying Definition 7 as follows: $E_{\Omega_1} = 0.44476$; $E_{\Omega_2} = 0.47580$;

5. **Multi-Criteria Decision Making**

Consider that $\tilde{A}$, $\tilde{C}$, $\tilde{D}$ are sets of alternatives, requirements and decision makers; where $\tilde{A} = \{A_1, A_2, ..., A_{n_A}\}$ and $\tilde{C} = \{C_1, C_2, ..., C_{n_C}\}$ and $\tilde{D} = \{D_1, D_2, ..., D_{n_D}\}$. With regard to the decision maker D_{i_d}; $i_d = 1, 2, ..., n_D$, the evaluation description of an alternatives A_{i_a}; $i_a = 1, 2, ..., n_A$ on an attribute C_{i_c}; $i_c = 1, 2, ..., n_C$ in given time period τ_l; $l = 1, 2, ..., n_\tau$ is represented by matrix $U^{i_d}(\tau_l) = \left(\theta^{i_d}_{i_a i_c}(\tau_l)\right)_{m \times n}$. where $\theta^{i_d}_{i_a i_c}(\tau_l)$ chosen as the language identifier of CNS by given period τ_l.

Let
$\theta_{i_a i_c i_d}(\tau_l) = \left\langle p_{i_a i_c i_d}(\tau_l) e^{j\mu_{i_a i_c i_d}(\tau_l)}, q_{i_a i_c i_d}(\tau_l) e^{j\nu_{i_a i_c i_d}(\tau_l)}, r_{i_a i_c i_d}(\tau_l) e^{j\eta_{i_a i_c i_d}(\tau_l)} \right\rangle$

Step 1. In the absence of any pre-existing information regarding criterion weights, we proceed to calculate the weight w_{i_c} of criterion C_{i_c} as follows:

Equation (11) can be used to evaluate the decision makers' averaged rating.

$$\theta_{i_a i_c} = \frac{1}{n_D * n_\tau} \overset{n_\tau}{\underset{l=1}{\oplus}} \overset{n_D}{\underset{i_a=1}{\oplus}} x_{i_a i_c i_d}(\tau_l) = \left\langle \tilde{T}_{i_a i_c}, \tilde{I}_{i_a i_c}, \tilde{F}_{i_a i_c} \right\rangle \tag{11}$$

The weight w_{i_c} of criterion C_{i_c} is calculated by Equation (12)

$$w_{i_c} = \frac{\left(1 - E_{C_{i_c}}\right)}{n_c - \sum_{i=1}^{n_c} E_{C_{i_c}}} \tag{12}$$

Where $E_{C_{i_c}} = \frac{1}{n_A} \sum_{i_a=1}^{n_A} E(\theta_{i_a i_c})$; each $E(\theta_{i_a i_c})$ is calculated using Equation (10)

Step 2. The temporal complex neutrosophic positive ideal solution and temporal complex neutrosophic negative ideal solution are represented by two values, $TCN-PIS$ and $TCN-NIS$, respectively, and are determined in the following way:

$$TCN-PIS=\left\{\theta,\left\langle \max\left(p_{i_ai_c}\left(\theta,\tilde{\tau}\right)\right)e^{j\max(\mu_{i_ai_c}(\theta,\tilde{\tau}))},0,0\right\rangle\right\} \tag{13}$$

$$TCN-NIS= \left\{\theta,\left\langle 0,\min\left(q_{i_ai_c}\left(\theta,\tilde{\tau}\right)\right)e^{j\min(\nu_{i_ai_c}(\theta,\tilde{\tau}))},\min\left(r_{i_ai_c}\left(\theta,\tilde{\tau}\right)\right)e^{j\min(\eta_{i_ai_c}(\theta,\tilde{\tau}))}\right\rangle\right\} \tag{14}$$

Step 3 For alternatives to $TCN-PIS$ and $TCN-NIS$, we can analyze the weighted similarity index using Theorems 4.3-4.12. And equation (15) is used to calculate the weighted similarity measure $S_{i_a}^{+}$ and $S_{i_a}^{-}$ of A_{i_a} $(i_a=1,2,...,n_A)$-(16)

$$S_{i_a}^{+}=\overset{n_A}{\underset{i_a=1}{\oplus}} w_{i_c}Sim\left(\theta_{i_ai_c},TCN-PIS\right) \tag{15}$$

$$S_{i_a}^{-}=\overset{n_A}{\underset{i_a=1}{\oplus}} w_{i_c}S\left(\theta_{i_ai_c},TCN-NIS\right) \tag{16}$$

Step 4. We determine the relative closeness of alternative A_{i_a} to the ideal solution using the following calculation

$$RR_{i_a}=\frac{S_{i_a}^{-}}{S_{i_a}^{+}+S_{i_a}^{-}} \tag{17}$$

Step 5. Through a comparison based on relative closeness values, we have identified the most promising alternatives, listed in descending order of their suitability.

6. **A case study for choosing a tourist destination**

A practical use of recommended metrics to a decision problem analyzing a real-world scenario is shown in this section. The case study is a particular illustration of the challenge of selecting a travel destination in Vietnam. This model can be applied to travel companies supporting and advising tourists.

6.1. *Problem description*

The viability and efficacy of the suggested decision technique are illustrated by an example provided by Lan et al. [17] regarding the selection of a tourism location in Vietnam. Figure 1 presents an MCDM model for choosing a tourism destination in Vietnam. Assume for the moment that a business is looking for a tourism attraction in Vietnam. For example, L_i; $i=1,2,3,4,5$ is one of the five tourist locations that the company recommended. There are twenty sub-criteria that are specific to each place and three group criterion that determine which site is the best tourist destination. This enables a more precise evaluation of the different locations.

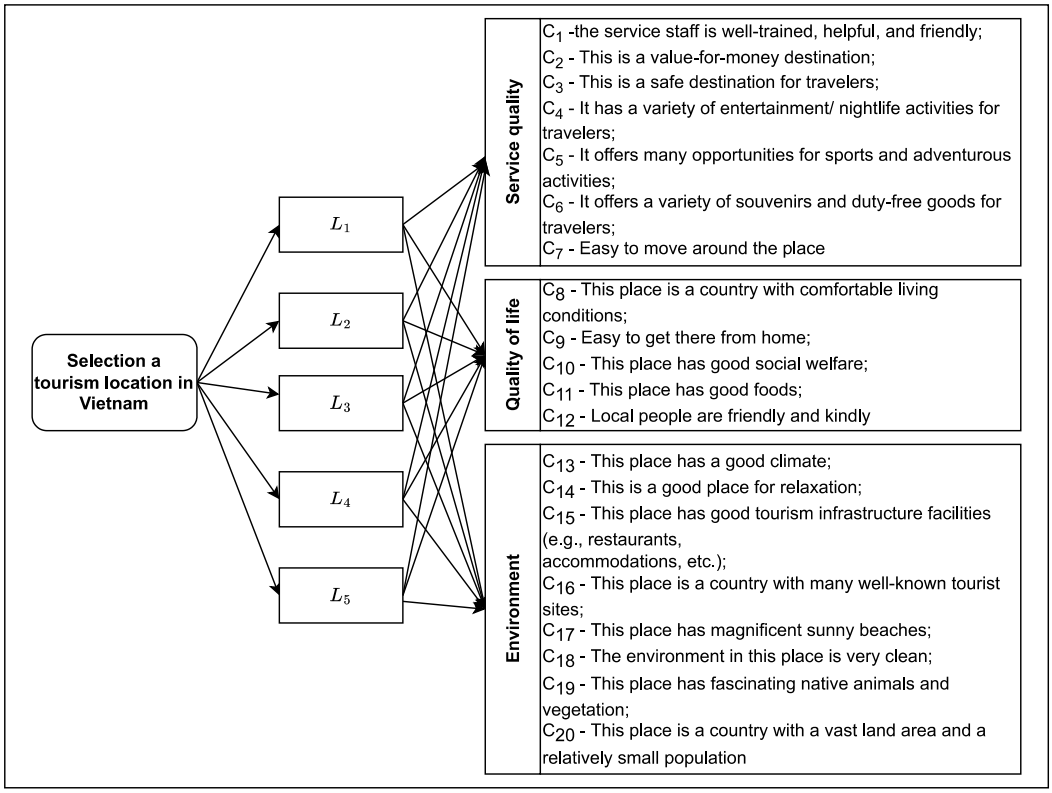
Selection a tourism location in Vietnam
L_1
L_2
L_3
L_4
L_5
Service quality
C_1 -the service staff is well-trained, helpful, and friendly;
C_2 - This is a value-for-money destination;
C_3 - This is a safe destination for travelers;
C_4 - It has a variety of entertainment/ nightlife activities for travelers;
C_5 - It offers many opportunities for sports and adventurous activities;
C_6 - It offers a variety of souvenirs and duty-free goods for travelers;
C_7 - Easy to move around the place
Quality of life
C_8 - This place is a country with comfortable living conditions;
C_9 - Easy to get there from home;
C_{10} - This place has good social welfare;
C_{11} - This place has good foods;
C_{12} - Local people are friendly and kindly
Environment
C_{13} - This place has a good climate;
C_{14} - This is a good place for relaxation;
C_{15} - This place has good tourism infrastructure facilities (e.g., restaurants, accommodations, etc.);
C_{16} - This place is a country with many well-known tourist sites;
C_{17} - This place has magnificent sunny beaches;
C_{18} - The environment in this place is very clean;
C_{19} - This place has fascinating native animals and vegetation;
C_{20} - This place is a country with a vast land area and a relatively small population

FIGURE 1. Selection a tourism location in Vietnam

TABLE 1. Language variables

Label	Number
Very low (VL)	$\langle 0.15e^{j0.55}, 0.65e^{j0.45}, 0.65e^{j0.35}\rangle$
Low (L)	$\langle 0.25e^{j0.65}, 0.55e^{j0.55}, 0.65e^{j0.45}\rangle$
Medium (M)	$\langle 0.40e^{j0.75}, 0.50e^{j0.65}, 0.45e^{j0.55}\rangle$
High (H)	$\langle 0.55e^{j0.85}, 0.45e^{j0.75}, 0.35e^{j0.65}\rangle$
Very high (VH)	$\langle 0.65e^{j0.95}, 0.25e^{j0.85}, 0.25e^{j0.75}\rangle$

Three experts with varying backgrounds and specialties have been invited to conduct a thorough study of the four locations. The estimates made by the expert are represented as TCN numbers.

6.2. *Solution approach using the proposed measures*

TCNS is a technology that helps characterize periodic, ambiguous information that we encounter in our daily lives. In order to show that the suggested similarity measures are useful in resolving real-world issues, such as selecting tourism sites in multi-criteria decision-making problems, we provide a real-world instance in the TCNS environment in this section.

TABLE 2. Similarity measure Values at step 3

Measures	Ideal solution	Alternatives				
		L_1	L_2	L_3	L_4	L_5
Sim_D	S^+	0.63893	0.63931	0.63727	0.63702	0.63459
	S^-	0.65671	0.65388	0.65677	0.66025	0.66090
Sim_j	S^+	0.46947	0.46989	0.46768	0.46740	0.46479
	S^-	0.48892	0.48580	0.48899	0.49283	0.49354
Sim_{Cos}	S^+	0.58630	0.58890	0.58585	0.58270	0.58158
	S^-	0.55933	0.55967	0.56067	0.56004	0.56200
Sim_{Cot_1}	S^+	0.64133	0.62623	0.63643	0.66239	0.65205
	S^-	0.59517	0.57743	0.59477	0.62114	0.61690
Sim_{Cot_2}	S^+	0.718926	0.71437	0.71740	0.72466	0.72094
	S^-	0.78670	0.76771	0.78476	0.81370	0.80613

Step 1. Equations 11 and 12 can be used for calculating the weight of each of the twenty criteria.

$w_1 = 0.04920; w_2 = 0.05787; w_3 = 0.05274; w_4 = 0.04981; w_5 = 0.03842;$

$w_6 = 0.04216; w_7 = 0.04805; w_8 = 0.05547; w_9 = 0.05323; w_{10} = 0.04190;$

$w_{11} = 0.04967; w_{12} = 0.0439; w_{13} = 0.04418; w_{14} = 0.04968; w_{15} = 0.05861;$

$w_{16} = 0.05000; w_{17} = 0.053227; w_{18} = 0.05968; w_{19} = 0.05397; w_{20} = 0.05745;$

Step 2. The positive ideal solution (TCN-PIS) and negative ideal solution (TCN-NIS) of the temporal complex neutrosophic should be estimated. Here's how the values are derived:

$$TCN-PIS = \left\{ \begin{array}{l} \left\langle 0.696238.\ e^{j0.916667}, 0, 0 \right\rangle, \left\langle 0.696238.e^{j0.916667}, 0, 0 \right\rangle, \\ \left\langle 0.696238.e^{j0.916667}, 0, 0 \right\rangle, \left\langle 0.696238.e^{j0.916667}, 0, 0 \right\rangle \end{array} \right\}$$

$$TCN-NIS = \left\{ \begin{array}{l} \left\langle 0, 0.30411.e^{j0.583333}, 0.279672.e^{0.483333} \right\rangle, \\ \left\langle 0, 0.30411.e^{j0.583333}, 0.279672.e^{0.483333} \right\rangle, \\ \left\langle 0, 0.30411.e^{j0.583333}, 0.279672.e^{0.483333} \right\rangle, \\ \left\langle 0, 0.30411.e^{j0.583333}, 0.279672.e^{0.483333} \right\rangle \end{array} \right\}$$

Step 3. Determine TCN-PIS and TCN-NIS's similarity metrics for five different places as shown in Table 2.

Step 4. Calculate the proximity of the optimal solution to five different places. They are described in Table 3.

Step 5. Table 3 displays the five places that are ordered according to their relative proximity.

TABLE 3. The relative proximity of places determined via similarity metrics

Mesuares		Alternatives				
		L_1	L_2	L_3	L_4	L_5
Sim_D	RR	0.50686	0.50563	0.50753	0.50895	**0.51015**
	In Order	4	5	3	2	1
Sim_J	RR	0.51015	0.50832	0.51114	0.51324	**0.51500**
	In Order	4	5	3	2	1
Sim_{Cos}	RR	0.48823	0.48728	0.48902	0.49009	**0.49144**
	In Order	4	5	3	2	1
Sim_{Cot_1}	RR	0.48134	0.47973	0.48308	0.48393	**0.48615**
	In Order	4	5	3	2	1
Sim_{Cot_2}	RR	0.52251	0.51800	0.52242	**0.52894**	0.52789
	In Order	3	5	4	1	2

6.3. *Result and Discussion*

In this section, We validate the proposed model's efficacy through comparison with Ali et al.'s models (2020) [38] in temporal complex neutrosophic environments. In Table 3, the ranking results of 5 locations at four different periods, $\tau_1, \tau_4, \tau_3, \tau_4$, are suitably explained for the problem of selecting a tourist destination. According to Table 4, the rankings of five locations at four times each of τ_1, τ_4, τ_3, and τ_4 are $L_5 \succ L_4 \succ L_1 \succ L_3 \succ L_2$; $L_5 \succ L_4 \succ L_3 \succ L_1 \succ L_2$; $L_5 \succ L_4 \succ L_3 \succ L_1 \succ L_2$ and $L_5 \succ L_4 \succ L_1 \succ L_3 \succ L_2$ and in all four time periods $L_5$is assigned as the optimal location. In the meantime, the suggested method's overall outcome at four different periods based on the metrics: Dice, Jaccard, Cosine, and Cotangent similarity, respectively, are $L_5 \succ L_4 \succ L_3 \succ L_1 \succ L_2$; $L_5 \succ L_4 \succ L_3 \succ L_1 \succ L_2$; $L_5 \succ L_4 \succ L_3 \succ L_1 \succ L_2$; $L_4 \succ L_5 \succ L_1 \succ L_3 \succ L_2$. The best location according to the combined results of the Cosine, Jaccard, and Dice similarity measures is L_5; the best location according to the Cotangent similarity measure is L_4.

This finding demonstrates how the benefits and practical application of the suggested measures handle time-related decision-making challenges in a TCNS. Furthermore, in TCN contexts, it is more flexible and generalized than the technique of Ali et al. [38]. The ability to handle periodic and temporal factors in the data effectively is one of TCNS's advantages over standard CNS. The proposed approach has the considerable advantage of taking into account the impact of temporal aspects in CNS disorders while also reducing information loss. In other words, we argue that new similarity and distance measures will provide decision-makers with various options based on their optimistic and pessimistic behavior during the decision-making process.

Table 4. Results of Ali et al. method for locations at four periods

Measures		Alternitives				
		L_1	L_2	L_3	L_4	L_5
τ_1	RR	0.81427	0.80866	0.81220	0.81627	**0.81948**
	In Order	3	5	4	2	1
τ_2	RR	0.81330	0.81059	0.81335	0.81762	**0.82221**
	In Order	4	5	3	2	1
τ_3	RR	0.81413	0.80887	0.81348	0.81707	**0.82197**
	In Order	4	5	3	2	1
τ_4	RR	0.81316	0.80860	0.81309	0.81665	**0.81934**
	In Order	3	5	4	2	1

7. Conclusion, Limitations, and Future Works

TCNS theory is a valuable tool for solving problems concerning the uncertain, temporal, and periodical factors of decision-making. In this paper, we have introduced four similarity measures: Dice, Jaccard, Cosine, and cotangent in the context of the TCNS. Furthermore, we have established an entropy measure of TCNS to ascertain the weights of unknown attributes in MCDM. Next, based on suggested entropy and similarity metrics in the TCNS environment, a new MCDM technique has also been devised. Lastly, the TCN environment provides a numerical example of decision-making issues when selecting a tourist location in Vietnam. It is provided to highlight the benefits and real-world suitability of the suggested actions. It is demonstrated through the case study that the suggested TCNS entropy and similarity measurements can yield credible outcomes for decision-making issues. However, the proposed measures only use the discrete temporal variables and apply to only a practical problem of tourist destinations chosen in Vietnam. Hence, in future follow-up research, we can use TCNS to account for continuous time variables and apply our model to other real-life problems, including selecting workers, medical treatment, logistics center choosing, etc.

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A Neutrosophic Decision-Making Model for Determining Young People's Active Engagement

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Young people's skills and attitudes must be observed, studied and evaluated in order to create appropriate models that would serve an educational purpose. In this paper, we propose a decision-making model with the aim to detect certain attitudinal and behavioral patterns of actively engaged young people. The data used in this research resulted from a questionnaire drawn up by a group of researchers from six European countries with the aim to investigate the youth's awareness about the *Sustainable Devel-opment Goals* and their engagement as active agents of development and change at regional level. For the purpose of this study, we selected the regional results obtained from administering this questionnaire in Dolj County (Romania) and La Rioja (Spain). We developed a neutrosophic model that determines the *Mindchanger profile* of the respondents based on a minimal set of questions, which is dependent on the inclusion or exclusion of the nationality-specific traits. The resulting decisions were then compared with the respondents' self-evaluations, yielding high precisions (more than 0.83) for all the investigated evaluation scenarios. Our results were significantly better than the ones provided by several machine learning models applied on the same set of data. A direct impact of our model is that it can be applied to questionnaires which include linguistic responses that express, among others, unclear or vague thoughts. Additionally, it offers the possibility to identify the minimal set of questions that impact the respondent's answer choice to a target question.

1. Introduction

Current realities of the globalized communities have pointed to the stringent need for a world of greater justice, equality and human rights for all its citizens, a world in which all actors of society are responsible for reaching the *Sustainable Development Goals* (SDGs) included in the 2030 Agenda for Sustainable Development.

Young citizens play a key role in addressing issues of global concern at local and regional levels and in improving the critical understanding of the communities and the wider public opinion about these issues. They are crucial in promoting awareness messages, changes of behavior and active engagement due to their ability to innovate, communicate and influence their peers by using specific tools that overcome geographical distances. Moreover, the EU consensus recognized youths as agents of development and change and, as such, as essential contributors to the 2030 Agenda. In this context, youths can be considered as actual "Mindchangers" that can actively support the Decade of Action announced in 2019 by the United Nations.

In order to identify the elements that foster the active engagement of young people in six European regions, the questionnaire "Youth engagement in climate change and migration issues" (henceforth YECCM) was drawn up within the DEAR-funded project "Mindchangers: Regions and Youth for Planet and People" and was administered to young people aged 15–35 from Wallonie–Bruxelles (Belgium), Auvergne–Rhône–Alpes (France), Baden–Wurttemberg (Germany), Piemonte (Italy), Dolj County (Romania) and La Rioja (Spain).

According to recent Eurobarometers (*Special Eurobarometer 441*[1] and *Special Eurobarometer 476*[2]), EU citizens are hardly aware about the exact dimensions of poverty and global environmental emergencies, and even less willing to engage

themselves and change their behavior and attitudes to contribute to the reaching of SDGs. 63% of EU citizens have never heard about the SDGs (*Special Eurobarometer 441*) and just over half of all respondents agree that, as individuals, they can play a role in tackling poverty in developing countries (53%), whereas four in ten disagree (43%) and only 44% claim to be personally involved in helping developing countries (*Special Eurobarometer 476*). The YECCM questionnaire complements these data, collected at national level, with references about the local and regional context. Moreover, it complements the investigation of the engagement and commitment of local and regional governments to SDGs, carried out in 2021 by the European Committee of the Regions (CoR) and the Organisation for Economic Co-operation and Development (OECD), with an analysis of young people's engagement and commitment in the above-mentioned six European regions.

In order to obtain an overview of the current situation at the regional level, the questionnaire was drawn up with the objective to examine the youths' awareness about SDGs and their engagement as active agents of development and change, in the field of global challenges, specifically on two topics: *climate change* and *migration*. Similarly to the Eurobarometers, about which it was emphasized that "*it is not a tool to collect statistics, it rather provides a snapshot of public opinion perceptions at a given time*", this questionnaire offers a snapshot of young people's attitudes, perspectives, motivations, preferences and reflections on active engagement in the six European regions, partners in the Mindchangers project. It additionally provides a general profile of a Mindchanger, i.e., of a young person willing to get actively engaged in civic activities. However, if a local authority or a civil society organization needs to analyze the likelihood to become a Mindchanger, the YECCM presents the disadvantage of being time-consuming because of the large number of questions, resulting from its wide range of objectives. Therefore, despite its complexity, when taken in its entirety, the YECCM may not be considered an effective tool for local authorities and civil society organizations interested in working with young people who already have or are willing to reach a high level of engagement.

Considering all the above, our study addresses the following research questions:

(1) *Is there a minimal set of questions in the YECCM questionnaire that allow us to determine, irrespective of the national context, the young people's self-evaluation of the likelihood to become Mindchangers?*
(2) *Are there any nationality-based characteristics that can influence this self-evaluation?*

To answer these questions, we created a decision-making model that can accurately process the respondents' answers to a set of questions selected from the YECCM questionnaire and, based on these answers, to determine for each respondent his/her self-evaluation of the likelihood to become a Mindchanger or shortly, his/her *Mindchanger profile*.

There are decision situations in which the information cannot be assessed precisely in a quantitative form, but it may be qualitatively evaluated using labels such

as "*rather agree*", "*rather disagree*", etc. As a consequence, Fuzzy Sets (FS)[3] are widely used in decision-making problems due to their ability to represent uncertainty or vagueness. FS were extended to Intuitionistic Fuzzy Sets (IFS)[4,5] which provide a better way to manage the inaccuracy, dubiousness, and vulnerabilities that are usually found in human information. Designed especially to deal with imprecision and uncertainty in decision-making, Neutrosophic Sets (NS)[6] generalize the concepts of classical sets, FS and IFS by adding three grades of truth, falsehood and indeterminacy, being thus more indeterminacy tolerant as compared to the fuzzy counterparts. The significant level of uncertainty corresponding to the answers in the questionnaire led us to design and implement a neutrosophic model for the questionnaire data representation and processing. Our neutrosophic model responds to the need to investigate the impact of the uncertainty degrees of the respondents' answer choices to a target question.

The model we propose selects a minimal set of questions from the YECCM questionnaire, based on which the young people's *Mindchanger profile* can be determined with maximum precision. Feature selection is a dimension-reduction method that diminishes dataset size while maintaining model performance.[7] The obtained precision is quite significant (0.83) and stable in the sense that it is not altered by different training sets with the same distribution,[8] in our case by national or regional characteristics. Still, in order to select only those nationality-based characteristics that could result in a higher precision, we explored, for each of the two countries' datasets, additional questions from the questionnaire. Our efforts paid off as we increased the model precision for both datasets considered separately. The whole study has been implemented and run with the Python environment.

Subsequently, the model presented in this paper can be applied to questionnaires which include linguistic responses that express, among others, unclear or vague thoughts, with the aim to predict the respondents' answers to a specific question, named in our study the target question.

The paper is organized as follows. Section 2 provides a short overview of similar studies existing in the literature, Sec. 3 presents the structure and the administration of the YECCM questionnaire, Sec. 4 is dedicated to the proposed decision-making model, Sec. 5 presents the conducted experiments for our model on different scenarios and the obtained results, while Sec. 6 summarizes the main conclusions of the study and opens future research directions.

2. Related Works

The use of questionnaires to collect data about people's opinions, behaviors or preferences is a common practice in behavioral and social science research. Even if there are some disadvantages, such as potential social desirability biases and excessively rationalized responses,[9] there are also advantages, such as the easiness to administer them, the ability to obtain data from different places with distinct characteristics (wide coverage) and the low cost, among others.[10] The questions included in a questionnaire can be: dichotomous, nominal, or ordinal, the well-known Likert

scales. Many studies were carried out on the number of predefined answers on Likert scale in order to discuss its influence on their reliability and validity.[11,12] To facilitate the analysis of the collected responses, questions are usually designed in closed-response format involving linguistic variable representations.[11]

For the data analysis stage, statistical methods, such as descriptive statistics, correlations, multivariate analysis of variance,[13] linear and logistic regression[14,15] and structural equation modeling,[16] are the most prominent approaches to draw inferences and reach conclusions from the datasets. Recently, machine learning techniques, such as random forests,[17,18] auto regressive integrated moving average (ARIMA),[19] decision trees and support vector machines (SVM),[20] have also been applied to survey analysis in social sciences.

In contrast to the statistical techniques, these data-driven approaches do not require prior knowledge about the relationship under study. Furthermore, they are able to identify complex relationships in data. Thus, the application of machine learning in survey research can help to tackle research problems that usually involve specifying parametric regression models.[21]

Despite the advantages that machine learning techniques can bring to the analysis of survey data, we must keep in mind that the rating scales typically used in survey questions are linguistic variables,[22] which are naturally vague.[23] As a direct consequence, the techniques to analyze Likert-type data for statistical purposes are quite limited.[10,24] Indeed, representing these responses as crisp variables and calculating statistics or training models with them may result in unreasonable bias due to the ambiguity and uncertainty in the respondent's mind.[25]

Regarding the decision processes, it is obvious that questionnaire data could not represent the unique source for the decision-making, as these data are usually full of noise, incompleteness and inconsistency.[26] These problems can be overcome by the use of FS,[27–29] as they provide ways to deal with uncertainty in information and to minimize the uncertainty in the decision-making problems.[30] In a fuzzy-based approach, responses are encoded by means of appropriate linguistic descriptors which are assumed to be identified with fuzzy numbers/intervals[31]: T represents the truth, while $F = 1 - T$ is the nonmembership (or the falsehood). IFS[4] extend FS concept by modeling the vagueness or incomplete data and, thus, are more effective in representing the essence of responses in this type of questionnaires.[32,33] In IFS, besides the membership (T) and nonmembership (F), the indeterminacy called hesitancy is also represented: $H = 1 - T - F$.[34]

NS are even more suitable than IFS to represent the responses to questionnaires, allowing for more accurate representations of the respondents' true feelings or thoughts. In neutrosophy, we also consider Indeterminacy membership function (I)[35–37] along with the membership and nonmembership, all three components being totally independent from one another, as their sum is $0 \leq T + I + F \leq 3$. If compared with IFS, NS allow the uncertainty measurement, apart from the truth and falsity degrees.[38] NS can be restrained to IFS, but the independence of components is lost and the results of the aggregation operators are totally different in

the neutrosophic environment as compared to the intuitionistic fuzzy environment, since Indeterminacy is ignored by IFS operators (indeterminacy is ignored by the intuitionistic fuzzy aggregation operators). Even if $T + I + F = 1$, there is still a distinction between IFS and NS: in IFS the operators (union, intersection, complement/negation, difference, etc.) are defined for T and F only, not for I, while in NS the operators (union, intersection, complement/negation, difference, etc.) are defined with respect to all components T, I, F.

In 2019, a new research direction was proposed in the literature under the name of *Neutrosophic sociology* or *Neutrosociology*,[39] addressing the studies of sociology by means of neutrosophic methods. In this case, due to the membership function of indeterminacy,[35] the use of NS is suitable for modeling the lack of knowledge, doubts or contradictions that can be found in the answers of a survey.[40]

3. Youth Engagement in Climate Change and Migration Issues Questionnaire

The design of the YECCM questionnaire was made according to the stages of survey design defined by Lavrakas.[41] The formulation of the objectives and of the research questions was followed by the definition of a set of key concepts related to engagement and the development of a conceptual framework, including the issues to be addressed in the questionnaire.[42,43] Three main organizing concepts were identified for youth engagement: *affective*, *cognitive* and *behavioral*.[44,45] The concepts included in the conceptual framework were operationalized into questionnaire items. The conceptual framework was refined leading to the creation of 64 items that cover the most important issues regarding youth engagement. Two types of variables were used: nominal and ordinal, in the form of Likert-type questions with four and five-point rating scales.

3.1. *Structure of the questionnaire*

The structure of the questionnaire consists of a short introduction to the project and the research objectives, and the five sections detailed below:

- Section 1. Socio-demographic data (10 questions)
- Section 2. SDGs (6 questions)
- Section 3. Climate change (23 questions)
- Section 4. Migration (23 questions)
- Section 5. Mindchangers (2 questions)

Each question has a unique ID of the form:

$$Q.\langle \#\ of\ section\rangle.\langle \#\ of\ question\rangle.\langle \#\ of\ option\rangle^*$$

where $\langle \#\ of\ option\rangle*$, when applicable, corresponds to the structure of the question and represents the number of the items targeted by the question.

The questions included in Sec. 1 collected demographic data about the surveyed group: age, gender, nationality, residence, education, employment a.o. Moreover, in question $Q.1.9$, young people were asked to describe themselves by self-evaluating a list of personal characteristics based on a 1–5 rating scale (1 being the lowest and 5, the highest). The 24 characteristics corresponding to questions $Q.1.9.1$–$Q.1.9.24$ were established by the researchers and considered key characteristics for an actively engaged young person. As detailed in Sec. 5, the self-assessment of these characteristics will contribute to the improvement of the precision rate for determining the *Mindchanger profile*.

Section 2 included a question evaluating the general awareness regarding the SDGs (question $Q.2.1$), alongside questions aimed at offering a perspective on regional circumstances, capacities and levels of responsibility with regard to the implementation of the SGDs.

Sections 3 and 4 were divided in three scenarios, A, B and C. In Scenario A entered young people engaged in actions on several occasions (more than twice), whereas Scenario B included young people engaged in actions at least once or twice. In Scenario C entered only the young people who have never been engaged in actions addressing climate change/migration. Sections 3 and 4 were randomly sequenced, in the sense that some respondents got Sec. 3 first, others got Sec. 4, and, at the end of each section, they were asked if they wanted to continue with the other section.

Section 5 referred to the characteristics of a Mindchanger and to the participants' self-evaluation of the likelihood to achieve a high level of engagement in actions addressing climate change and migration.

3.2. *Questionnaire administration*

To facilitate its administration at the regional level, the YECCM questionnaire was translated into five languages and made available online using the LimeSurvey tool.[46] Data cleaning was done in accordance with the decision reached by the regional researchers that only the answers that exceeded 20 min were taken into consideration (at least 10 min per section).

The recommended number of respondents at regional level was approximately 100 young people. The research was organized in compliance with the EU's 2016 General Data Protection Regulation (EU Reg. 2016/679). Prior to their participation, the respondents were informed that the data collected would be processed and disclosed exclusively in aggregated form, that is without examining or describing in any way characteristics and opinions individually; the results obtained would be used only for research purposes; the data collected would be kept in a protected place, and would not be disseminated in any way outside the research group. Moreover, this research got the ethical approval from the Bioethical Committee of the University of Torino (Decision no. 0438557/13.07.2021).

In this study, we refer to the answers provided by the respondents from Dolj County (Romania) and La Rioja (Spain). Table 1 shows demographic information

Table 1. Demographics of Romanian and Spanish respondents participating in the survey.

Country	N	Gender			Age			
		Female	Male	Other	15–20	21–25	26–30	31–35
Romania	100	71	29	0	33	44	11	12
Spain	102	74	27	1	5	61	18	18

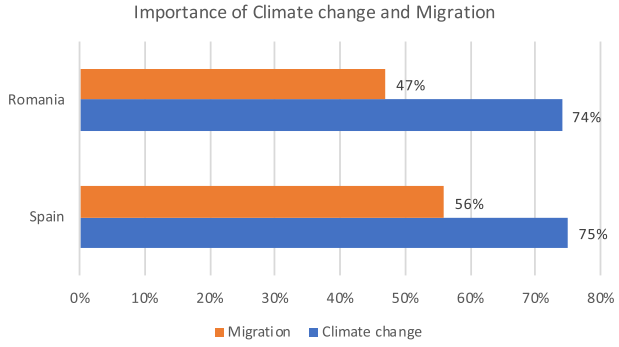


Fig. 1. Importance of climate change and migration, based on YECCM data.

about the respondents. The average age of the Romanian respondents was 24 years and of the Spanish ones 25–26 years.

The questionnaire data showed that young people paid a greater importance to the issues of climate change as opposed to migration and even the level of their personal concern about climate action is higher (see Fig. 1).

Moreover, when asked to rate the importance of young people's engagement in addressing climate change ($Q.3.14$) or migration ($Q.4.14$) issues by using a four-level Likert scale (1: *not at all*, 2: *not important*, 3: *important*, 4: *very important*), a great majority of the respondents admitted that their engagement is important,

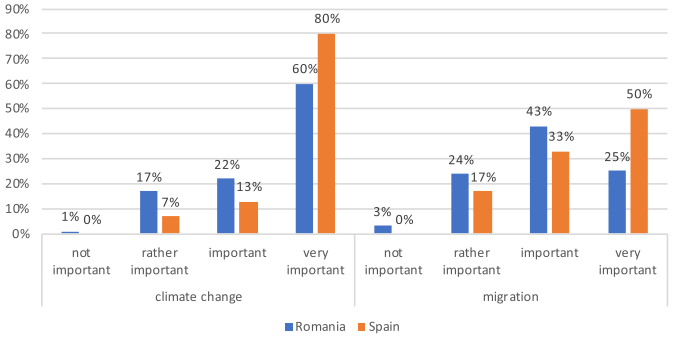
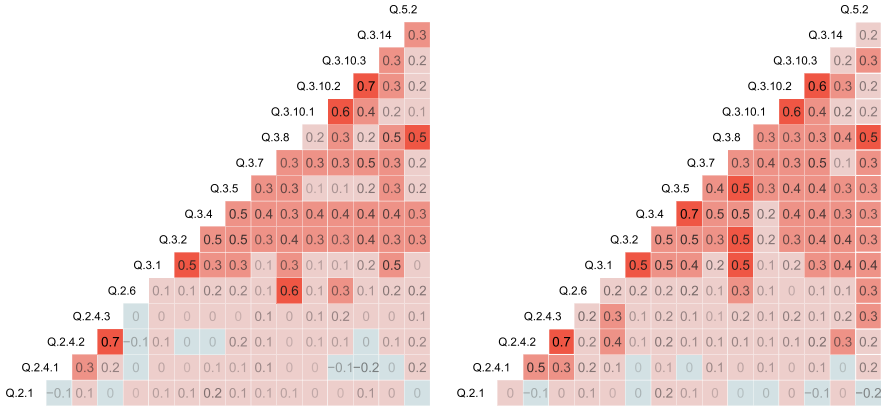


Fig. 2. Importance of young people's engagement on climate change and migration, based on YECCM data.



(a) Romanian data (b) Spanish data

Fig. 3. Correlations between questions from Secs. 2 and 3 and Mindchanger options for each dataset.

with more importance attributed to climate change issues than to migration. Both Romanian and Spanish respondents consider that young people's actions have a higher impact on climate change issues than on migration (Fig. 2).

Considering the above, the study presented in this paper and detailed in the following sections refers to young people's engagement in climate change actions. We create our model taking into consideration the questions related to this topic which are included in Sec. 3 of the questionnaire, as well as those included in Sec. 2, referring to the SDGs.

We built the correlation matrix (see Fig. 3) for the questions of Secs. 2 and 3 of the questionnaire in terms of responses' correlation with the target question $Q.5.2$, which asks the respondents to evaluate their *Mindchanger profile*. In this case, Spearman's correlation (r) was used because we dealt with Likert-scale items. Overall, as illustrated in Fig. 3, the relationship between questions on the SDGs ($Q.2.x$) and on climate change ($Q.3.x$) with $Q.5.2$ is weak. There is only a moderate positive relationship between the attractiveness to participate in climate change-related actions in the future ($Q.3.8$) and the *Mindchanger profile* of the respondents ($Q.5.2$) ($r = 0.51, p < 0.001$ for both datasets).

Regarding the questions in Sec. 2, respondents from both countries agree on the role that both national and regional authorities should play in the fight against global inequalities and poverty ($Q.2.4.2$ and $Q.2.4.3$): $r = 0.72, p < 0.001$ for Romanian respondents and $r = 0.68, p < 0.001$ for Spanish respondents. On the other hand, there is a moderate positive relationship between the importance of climate change for the respondents ($Q.3.1$) and their concern about this issue ($Q.3.2$) ($r = 0.51, p < 0.001$ and $r = 0.54, p < 0.001$ for Romanian and Spanish data, respectively). Finally, for both Romanian and Spanish respondents, there is a moderate positive relationship between their awareness of measures/projects addressing

climate change at the three levels considered: local (Q.3.10.1), national (Q.3.10.2) and international (Q.3.10.3).

To sum up, the correlation values are poor for both Romanian and Spanish data. Nevertheless, with the neutrosophic approach detailed in the following section we achieve more than 0.8 precision in determining the respondents' *Mindchanger profile* based on their responses to a set of selected questions from Secs. 2 and 3.

4. A Neutrosophic Decision-Making Model to Determine the *Mindchanger Profiles*

NS theory focuses on the importance of indeterminacy, while in other set theories this is completely or partially ignored. This theory is considered the right tool to handle and represent data with a high degree of indeterminacy, such as the answers to Likert scale questions[40] because we can define not only membership functions of truthfulness and falseness, but also indeterminacy, independently of the membership function of falseness.[37,40]

We construct a neutrosophic decision-making model by means of the following two algorithms:

- **Algorithm 1.** To determine the *Mindchager profile* of a specific respondent by computing an overall score based on his/her answers to a set of selected questions. For each respondent, this overall score will be used to decide one of the four possible values (alternatives) of the *Mindchager profiles* using a Ranking algorithm for single-valued neutrosophic triplets. The alternatives of the decision are as follows: *unlikely/rather likely/likely/most likely to become a Mindchanger*.
- **Algorithm 2.** To identify the minimal set of questions from Secs. 2 and 3 of the YECCM questionnaire which lead to the best precision rate for determining the *Mindchanger profile*. The answers to these questions will represent the attributes under consideration in the proposed decision-making model.

We test the proposed model on several datasets: two datasets specific to a certain country (in our case Spain and Romania), but also one combined dataset, which includes the previous two. The results show that the proposed model achieves its highest precision (more than 80%) on the combined dataset but also maintains the precision on the Romanian and Spanish datasets taken separately, which means that the model's precision is stable regardless of the nationality of the respondents.

In what follows, we describe the proposed model in detail.

4.1. *Questionnaire answers as single-valued neutrosophic triplets*

For a specific question $Q.x$ of the YECCM questionnaire, we can have multiple answer options, written as: $Ans^i_{Q.x}$, $i \geq 2$.

In the YECCM questionnaire, the user's choices when answering the questions are expressed in natural language using linguistic terms. The linguistic approach is

the solution to qualify the user's perception, which, in natural language, is often expressed with words instead of numerical values.

For example, let us consider a question $Q.x$ from the questionnaire with the following set of possible answers: *Not at all*, *Not much*, *Much*, *Very much*. As shown below, this combination of answers is frequently encountered in the questionnaire. In our approach, each answer is represented as a *neutrosophic fuzzy set* commonly named *single-valued neutrosophic triplet number* because the components are single-valued numbers. Thus, each alternative for the $Q.x$ answers is represented as $Ans^i_{Q.x} = (T^i_{Q.x}, I^i_{Q.x}, F^i_{Q.x})$, $i = \overline{1,4}$, such that $T^i_{Q.x}, I^i_{Q.x}, F^i_{Q.x} \in [0,1]$, where

- $T^i_{Q.x}$ is the truth (or membership) component, quantifying the positive quality of the answer;
- $I^i_{Q.x}$ is the degree of indeterminacy (neutrality), more precisely the unclear or confused aspect of the answer, which is neither true nor false;
- $F^i_{Q.x}$ is the degree of falsehood (or nonmembership) of the ith answer of question $Q.x$, which is opposed to the truth degree $T^i_{Q.x}$.

This representation follows the neutrosophy's triad ($\langle A\rangle$, $\langle neutA\rangle$, $\langle antiA\rangle$), where $\langle A\rangle$ is an item (concept, idea, proposition, etc.), $\langle antiA\rangle$ is its opposite, while $\langle neutA\rangle$ is the neutrality (indeterminacy) between these opposites. There are many such neutrosophic triads (triplets) in our everyday life, such as: (positive, neutral, negative), (win, draw, lose), (accept, undecided, reject), (membership, indeterminacy, nonmembership), (truth, indeterminacy, falsehood), etc.[47]

We must point out that if a question $Q.x$ has only two dual opposite answers, such as *Yes* and *No*, the model can also be applied on this crisp data, but no indeterminacy will be represented, because the answer *Yes* determines the $(1,0,0)$ neutrosophic number representation, where $T^1_{Q.x} = 1$ and $I^1_{Q.x} = F^1_{Q.x} = 0$, while *No* determines the $(0,0,1)$ representation, $F^2_{Q.x} = 1$ and $I^2_{Q.x} = T^2_{Q.x} = 0$, so in both cases the indeterminacy is 0.

In the NS literature, there are several cases that can be encountered for the sum of the components T, I and F as NS is an extension of IFS, even if the sum of single-valued neutrosophic components is < 1, or > 1, or $= 1$[48]:

- when T, I, F are independent from one another, their sum is $0 \leq T + I + F \leq 3$,
- when T, I, F are totally-dependent, as in IFS, the sum is $0 \leq T + I + F \leq 1$,
- when T, I, F are partially independent and partially-dependent, then $1 \leq T + I + F \leq 3$.

As presented in Tables 2 and 3 below, the neutrosophic representations we propose consider that all the questionnaire answers are in the totally-dependent case in which $T^i_{Q.x} + I^i_{Q.x} + F^i_{Q.x} = 1$ where $Ans^i_{Q.x} = (T^i_{Q.x}, I^i_{Q.x}, F^i_{Q.x})$, $i \geq 2$, is the representation for the ith answer to the question $Q.x$. In this way, the answers of all considered questions have equal importance in the study, even the crisp ones.

But even when the components are totally-dependent and their sum is 1, as in our case when $T^i_{Q.x} + I^i_{Q.x} + F^i_{Q.x} = 1$ for any ith answer to any question $Q.x$ from

Table 2. The four *Mindchanger profiles* and their representations.

Profile	Neutrosophic representation	Linguistic value
$Profile_1$	$(0.35, 0.3, 0.35)$	*unlikely to become a Mindchanger*
$Profile_2$	$(0.7, 0.2, 0.1)$	*rather likely to become a Mindchanger*
$Profile_3$	$(0.85, 0.1, 0.05)$	*likely to become a Mindchanger*
$Profile_4$	$(1, 0, 0)$	*most likely to become a Mindchanger*

Table 3. Neutrosophic representations for the answer options of the $Core\_Set$ questions.

Question	Options	Neutrosophic triplet number		
		Truth	Indeterminacy	Falsehood
$Q.2.6$	Not at all	0	0	1
	Not much	0.25	0.5	0.25
	Much	0.75	0.25	0
	Very much	1	0	0
$Q.3.2$	Not at all	0	0	1
	Not much	0.25	0.5	0.25
	Much	0.75	0.25	0
	Very much	1	0	0
$Q.3.5$	Never	0	0	1
	Rarely	0.25	0.5	0.25
	Often	0.75	0.25	0
	Always	1	0	0
$Q.3.8$	Not at all	0	0	1
	Not enough	0.25	0.5	0.25
	Enough	0.75	0.25	0
	A lot	1	0	0
$Q.3.14$	Not important	0	0	1
	Rather important	0.25	0.5	0.25
	Important	0.75	0.25	0
	Very important	1	0	0

the questionnaire, we can apply the neutrosophic operators and get different results than in the case of applying the intuitionistic fuzzy operators.[49,50] This is due to the fact that in NS the indeterminacy is included in the operators' formulas, but in IFS the indeterminacy (hesitancy) is not present in the IF operators (hesitancy is completely ignored).

All neutrosophic-triplet numbers that represent the questionnaire answers take into account the level of truthfulness, indeterminacy and falsity corresponding to the semantics of the answers' linguistic descriptors. The existing literature suggests taking the elements of a linguistic term set as primary ones and distributing them on a scale on which a total order is defined.[51] For example, a set of seven terms S:

$$S = \{s_0 = \text{none}; s_1 = \text{very low}; s_2 = \text{low}; s_3 = \text{medium}; s_4 = \text{high};$$
$$s_5 = \text{very high}; s_6 = \text{perfect}\}, \quad s_a < s_b \text{ if and only if } a < b$$

is often required to satisfy the following[51]:

(1) There is a negation operator, e.g., $Neg(s_i) = s_j$; $j = T - i$ ($T + 1$ is the cardinality);
(2) There is a maximization operator: $Max(s_i; s_j) = s_i$ if $s_i \geq s_j$;
(3) There is a minimization operator: $Min(s_i; s_j) = s_i$ if $s_i \leq s_j$.

For each question $Q.x$, the set of the linguistic terms of $Q.x$ answers form an n-terms set noted as $S_{Q.x}$,$n \geq 2$. In line with the existing literature, we consider the neutrosophic representation of the $Q.x$ answers based on the cardinality of the term set, that is $\mid S_{Q.x} \mid$, and according to the semantics of its elements as shown below.

The set of single-valued neutrosophic triplet numbers corresponding to a respondent's answers to a selected set of questions $Q_1, \ldots, Q_n$, $n \geq 1$ are aggregated into a single-valued neutrosophic triplet. When applying a neutrosophic aggregation operator we can obtain a different result from that obtained when applying intuitionistic fuzzy operators, since the intuitionistic fuzzy operators ignore the indeterminacy, while the neutrosophic aggregation operators take into consideration the indeterminacy at the same level as truth-membership and falsehood-nonmembership are taken.[36]

Most of the aggregation operators are linear combinations of the fuzzy t-norm (denoted by $\wedge_F$) and fuzzy t-conorm (denoted by $\vee_F$), but nonlinear combinations may as well be constructed. In this study, we use the arithmetic average aggregation operator.

Let us consider a specific respondent $\mathcal{U}$ and its answers to a set of selected questions extracted from the questionnaire: $Q_1, \ldots, Q_n$ represented as

$$Ans^{\mathcal{U}}_{Q_1}, \ldots, Ans^{\mathcal{U}}_{Q_n},$$

where the superscript denotes the answer option selected by the user $\mathcal{U}$ to each of the questions Q_i.

We consider the answers of the user $\mathcal{U}$ represented as a set of n single-valued neutrosophic triplet numbers. We obtain

$$Answers_{\mathcal{U}} = \bigcup_{i=1}^{n} \{Ans^{\mathcal{U}}_{Q_i} | Ans^{\mathcal{U}}_{Q_i} = (T^{\mathcal{U}}_{Q_i}, I^{\mathcal{U}}_{Q_i}, F^{\mathcal{U}}_{Q_i})\}.$$

We aggregate these answers representations into a single-valued neutrosophic-triplet number and we note this operation by means of Agg function defined on the set of n tuples of $Answers_{\mathcal{U}}$:

$$Agg(Answers_{\mathcal{U}}) = \left(\frac{\sum_{i=1}^{n} T^{\mathcal{U}}_{Q_i}}{n}, \frac{\sum_{i=1}^{n} I^{\mathcal{U}}_{Q_i}}{n}, \frac{\sum_{i=1}^{n} F^{\mathcal{U}}_{Q_i}}{n} \right).$$

Based on the resulted single-valued neutrosophic-triplet number $Agg(Answers_{\mathcal{U}})$ the model decides the best choice of the *Mindchanger profile* value for the user $\mathcal{U}$.

Let us use the following notations for the possible answer options of the target question QT: *How likely are you to become a Mindchanger?*:

- Ans^1_{QT} corresponding to the answer "*unlikely to become a Mindchanger*";
- Ans^2_{QT} corresponding to the answer "*rather likely to become a Mindchanger*";
- Ans^3_{QT} corresponding to the answer "*likely to become a Mindchanger*";
- Ans^4_{QT} corresponding to the answer "*most likely to become a Mindchanger*".

As previously pointed out, these answers represent the four possible values of the *Mindchager profile* and also the four alternatives of the proposed decision-making model.

The neutrosophic representation corresponding to the questionnaire answers is defined based on their level of truthfulness, indeterminacy and falsity. In order to identify a proper representation for the QT answers, we mapped the linguistic terms of $S_{QT} = unlikely, rather\ likely, likely, most\ likely$ on an ordered structure of 7 linguistic terms: $S_{7-\text{terms}} = s_0 = not\ at\ all, s_1 = very\ unlikely, s_2 = unlikely, s_3 = rather\ likely, s_4 = likely, s_5 = most\ likely, s_6 = definite$[51] in which $s_i < s_j$ if $i < j$. We obtain

$$s_0 < s_1 < s_2 = \text{unlikely} < s_3 = \text{rather likely} < s_4 = \text{likely} < s_5 = \text{most likely} < s_6$$

in which the terms of S_{QT} are put mostly in the positive half-part of $S_{7-\text{terms}}$ structure.

To ensure a proper granularity among the neutrosophic representations of the QT answers we considered the following triplet numbers: $Ans^1_{QT} = (0.35, 0.3, 0.35)$, $Ans^2_{QT} = (0.7, 0.2, 0.1)$, $Ans^3_{QT} = (0.85, 0.1, 0.05)$, $Ans^4_{QT} = (1, 0, 0)$ (see Fig. 4). Because of the high level of positiveness for the QT answers, we cannot define the

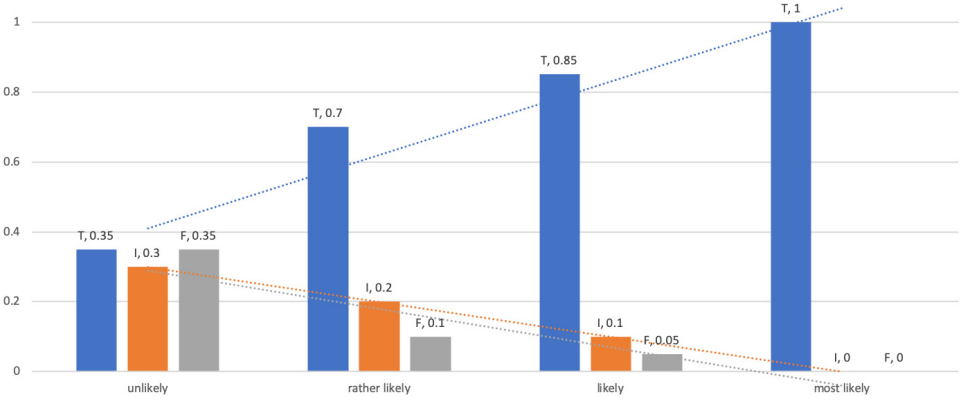


Fig. 4. The single-valued neutrosophic-triplet numbers corresponding to the linguistic variables *unlikely*, *rather likely*, *likely*, *most likely*.

negation operator on these neutrosophic representations as suggested in Ref. 51 but we can define the Max and Min operators using the neutrosophic intersection and union:

(1) $\text{Min}(Ans^i_{QT} = (T^i_{QT}, I^i_{QT}, F^i_{QT}),\ Ans^j_{QT} = (T^j_{QT}, I^j_{QT}, F^j_{QT})) = \wedge_N (Ans^i_{QT}, Ans^j_{QT}) = (\min\{T^i_{QT}, T^j_{QT}\},\ \max\{I^i_{QT}, I^j_{QT}\},\ \max\{F^i_{QT}, F^j_{QT}\}) = Ans^i_{QT}$ for $i < j$, $i, j \in \{1, \ldots, 4\}$ where $\wedge_N$ denotes the neutrosophic intersection.

(2) $\text{Max}(Ans^i_{QT} = (T^i_{QT}, I^i_{QT}, F^i_{QT}),\ Ans^j_{QT} = (T^j_{QT}, I^j_{QT}, F^j_{QT})) = \vee_N (Ans^i_{QT}, Ans^j_{QT}) = (\max\{T^i_{QT}, T^j_{QT}\},\ \min\{I^i_{QT}, I^j_{QT}\},\ \min\{F^i_{QT}, F^j_{QT}\}) = Ans^j_{QT}$ for $i < j$, $i, j \in \{1, \ldots, 4\}$ where $\vee_N$ denotes the neutrosophic union.

By making use of the single-valued score, accuracy and certainty functions introduced by Smarandache,[36] we can determine a total order on the set of neutrosophic triplets (T, I, F). This total order is used in our study to determine, for a specific user, the *Mindchanger profile* value corresponding to his/her answers to a set of selected questions from the YECCM questionnaire.

4.2. *Decision-making model using ranking algorithm for single-valued neutrosophic triplets*

In our model, the possible answers to the target question QT represent the four *Mindchanger profile* values (linguistic descriptions) that the proposed model must determine, denoted by $Profile_i$, for $i = \overline{1,4}$. Table 2 summarizes these profiles together with their neutrosophic representations in our study.

In order to determine the *Mindchanger profile* for a specific user $\mathcal{U}$ we apply a total order relation for the value of $Agg(Answers_{\mathcal{U}})$ and for the values corresponding to the neutrosophic representations of the four profiles: $Profile_1, \ldots, Profile_4$.

To this aim, we make use of single-valued score, accuracy and certainty functions defined for single-valued neutrosophic triplet numbers that provide a total order on the set of these numbers.[36] All these functions are used by the Ranking algorithm for single-valued neutrosophic triplets introduced by Smarandache.[36] We apply this algorithm to determine the respondents' Mindchanger profiles.

If M is a set of single-valued neutrosophic triplet numbers, $M = \{(T, I, F) \mid T, I, F \in [0, 1], 0 \leq T+I+F \leq 3\}$, the *Single-Valued Neutrosophic Score Function*[36] was defined as $s : M \to [0, 1]$ such that

$$s(T, I, F) = \frac{T + (1 - I) + (1 - F)}{3} = \frac{2 + T - I - F}{3}.$$

The minimum possible value of the score function is 0 (zero), which occurs when $(T, I, F) = (0, 1, 1)$ — the maximum neutrosophic falsehood value, which is opposite of $(1, 0, 0)$ — the maximum neutrosophic truth value.

If M is a set of single-valued neutrosophic triplet numbers, $M = \{(T, I, F) \mid T, I, F \in [0,1], 0 \leq T + I + F \leq 3\}$, the *Single-Valued Neutrosophic Accuracy Function*[36] was defined as $a : M \to [-1, 1]$, such that

$$a(T, I, F) = T - F.$$

If M is a set of single-valued neutrosophic triplet numbers, $M = \{(T, I, F) \mid T, I, F \in [0,1], 0 \leq T + I + F \leq 3\}$, the *Single-Valued Neutrosophic Certainty Function*[36] $c : M \to [0, 1]$ was defined such that

$$c(T, I, F) = T.$$

If we consider (T_1, I_1, F_1) and (T_2, I_2, F_2) two single-valued neutrosophic triplets from M, i.e., $T_1, I_1, F_1, T_2, I_2, F_2 \in [0, 1]$, the *Ranking algorithm for Single-Valued Neutrosophic Triplets*[36] was defined as follows:

Apply the Neutrosophic Score Function.

(1) If $s(T_1, I_1, F_1) > s(T_2, I_2, F_2)$, then $(T_1, I_1, F_1) > (T_2, I_2, F_2)$.
(2) If $s(T_1, I_1, F_1) < s(T_2, I_2, F_2)$, then $(T_1, I_1, F_1) < (T_2, I_2, F_2)$.
(3) If $s(T_1, I_1, F_1) = s(T_2, I_2, F_2)$ apply the Neutrosophic Accuracy Function:
  (3.1) If $a(T_1, I_1, F_1) > a(T_2, I_2, F_2)$, then $(T_1, I_1, F_1) > (T_2, I_2, F_2)$.
  (3.2) If $a(T_1, I_1, F_1) < a(T_2, I_2, F_2)$, then $(T_1, I_1, F_1) < (T_2, I_2, F_2)$.
  (3.3) If $a(T_1, I_1, F_1) = a(T_2, I_2, F_2)$ apply the Neutrosophic Certainty Function.
    (3.3.1) If $c(T_1, I_1, F_1) > c(T_2, I_2, F_2)$, then $(T_1, I_1, F_1) > (T_2, I_2, F_2)$.
    (3.3.2) If $c(T_1, I_1, F_1) < c(T_2, I_2, F_2)$, then $(T_1, I_1, F_1) < (T_2, I_2, F_2)$.
    (3.3.1) If $c(T_1, I_1, F_1) = c(T_2, I_2, F_2)$, then $(T_1, I_1, F_1) = (T_2, I_2, F_2)$, i.e., $T_1 = T_2$, $I_1 = I_2$, $F_1 = F_2$.

By using the Ranking algorithm defined by means of the single-valued neutrosophic score, accuracy and certainty functions, we can locate the value corresponding to the scores of a respondent's answers on the *Mindchanger profiles* scale (see Fig. 5). We note that the YECCM questionnaire does not provide a possible profile for the maximum neutrosophic falsehood value. Nevertheless, we consider the threshold τ_0 in order to obtain a complete scale for all the possible score function values.

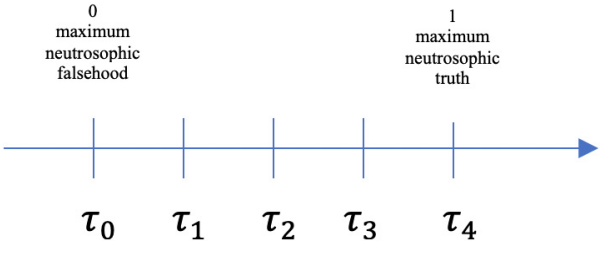


Fig. 5. The five threshold values used for the *Mindchanger profiles* scale.

Algorithm 1. Algorithm for determining the *Mindchanger profile* value of a user $\mathcal{U}$ using his/her answers to $Q_1, \ldots, Q_n$ questions:

function getMindchangerProfile($\mathcal{U}$, $\{Q_1, \ldots, Q_n\}, QT$)

1. Take $Answers_{\mathcal{U}}$: $\{(T^{\mathcal{U}}_{Q_i}, I^{\mathcal{U}}_{Q_i}, F^{\mathcal{U}}_{Q_i}) \mid Ans^{\mathcal{U}}_{Q_i} = (T^{\mathcal{U}}_{Q_i}, I^{\mathcal{U}}_{Q_i}, F^{\mathcal{U}}_{Q_i}), i = \overline{1,n}\}$
2. Take $Profile_1, \ldots, Profile_4$: $Profile_i = (T^i_{QT}, I^i_{QT}, F^i_{QT}), i = \overline{1,4}$
3. Aggregate the answers of user $\mathcal{U}$: $Agg(Answers_{\mathcal{U}}) = (T^{\mathcal{U}}, I^{\mathcal{U}}, F^{\mathcal{U}})$ where $T^{\mathcal{U}} = \frac{\sum_{i=1}^{n} T^{\mathcal{U}}_{Q_i}}{n}$, $I^{\mathcal{U}} = \frac{\sum_{i=1}^{n} I^{\mathcal{U}}_{Q_i}}{n}$, $F^{\mathcal{U}} = \frac{\sum_{i=1}^{n} F^{\mathcal{U}}_{Q_i}}{n}$
4. Compute the score function of user $\mathcal{U}$:

$$S_{\mathcal{U}} = S(T^{\mathcal{U}}, I^{\mathcal{U}}, F^{\mathcal{U}}) = \frac{T^{\mathcal{U}} + (1 - I^{\mathcal{U}}) + (1 - F^{\mathcal{U}})}{3}$$

5. Locate the value of $S_{\mathcal{U}}$ among the five thresholds values defined as follows: $\tau_0 = S(0,1,1)$, $\tau_1 = S(Profile_1)$, $\tau_2 = S(Profile_2)$, $\tau_3 = S(Profile_3)$, $\tau_4 = S(Profile_4)$:
6. Identify the pair of consecutive thresholds $(\tau_i, \tau_{i+1}), i = \overline{0,3}$ such that $\tau_i \leq S(T^{\mathcal{U}}, I^{\mathcal{U}}, F^{\mathcal{U}}) \leq \tau_{i+1}$ and then compute the percentage distances between $S(T^{\mathcal{U}}, I^{\mathcal{U}}, F^{\mathcal{U}})$ and τ_i and τ_{i+1}, respectively:

   $\overline{S_{\mathcal{U}} Profile_i} = \frac{S(T^{\mathcal{U}}, I^{\mathcal{U}}, F^{\mathcal{U}}) - \tau_i}{\tau_{i+1} - \tau_i}$ represents the percentage distance of $S(T^{\mathcal{U}}, I^{\mathcal{U}}, F^{\mathcal{U}})$ from τ_i, $i > 0$ and
   $\overline{S_{\mathcal{U}} Profile_{i+1}} = \frac{\tau_{i+1} - S(T^{\mathcal{U}}, I^{\mathcal{U}}, F^{\mathcal{U}})}{\tau_{i+1} - \tau_i}$ represents the percentage distance of $S(T^{\mathcal{U}}, I^{\mathcal{U}}, F^{\mathcal{U}})$ from τ_{i+1}
7. **return** $Profile_i$, if and only if $\overline{S_{\mathcal{U}} Profile_i}$ is minimal, with $i > 0$.

endfunction

Therefore, at Step 5 of Algorithm 1 we consider as thresholds the values of the score functions corresponding to the neutrosophic representations of *Mindchanger profiles*. For a specific user $\mathcal{U}$, the values $T^{\mathcal{U}}$, $I^{\mathcal{U}}$ and $F^{\mathcal{U}}$ corresponding to the aggregated triplet $Agg(Answers_{\mathcal{U}})$ (Step 3) can be different from the resulted threshold values, but we got four intervals: $[\tau_0, \tau_1]$, $[\tau_1, \tau_2]$, $[\tau_2, \tau_3]$, $[\tau_3, \tau_4]$ and we compute the score function of the triplet $S_{\mathcal{U}} = S(T^{\mathcal{U}}, I^{\mathcal{U}}, F^{\mathcal{U}})$ we get from the user' questionnaire answers (Step 4) and place it into the interval in which it fits and then we test if the resulted score function $S_{\mathcal{U}}$ is closer to the left or right side of the interval it belongs to (Step 6). As a direct consequence, we approximate the result corresponding to a user $\mathcal{U}$ with its closer threshold, and thus we determine our prediction for the user's *Mindchanger profile*.

For example, if we get the following aggregated values for a user $\mathcal{U}$: $Agg(Answers_{\mathcal{U}}) = (0.6, 0.3, 0.1)$, its score function is $S_{\mathcal{U}} = S(T^{\mathcal{U}}, I^{\mathcal{U}}, F^{\mathcal{U}}) = (0.6+0.7+0.9)/3 = 0.733$. But 0.733 belongs to the interval $[\tau_1, \tau_2] = [0.57, 0.8]$ and it is closer to 0.8, since 0.733 is above the average of the interval. So, we approximate the result to $0.8 = \tau_2 = S(Profile_2)$ and the algorithm returns $Profile_2 =$ *rather likely to become a Mindchanger* for the user $\mathcal{U}$.

4.3. *Selection of the relevant questions from the YECCM questionnaire*

To select the minimal set of attributes that maximizes the decision rate of the *Mindchanger profile* value, we run Algorithm 1 in a question selection mechanism described in Algorithm 2. The scope of the subsequent steps of Algorithm 2 is to identify the minimal set of relevant questions from the YECCM questionnaire that influence the *Mindchanger profile* decision of the respondents. The resulted set of such questions is represented as $Core\_Set$.

Algorithm 2. Algorithm for attribute selection by considering a set of questions from the YECCM questionnaire and a target question QT.

function CoreSetSelection(*Set of YECCM questions*, *Dataset*, QT)

1. Generate all subsets of the *Set of YECCM questions*.
1.1. For each subset S:
1.1.1. For each respondent $\mathcal{U}$ from Dataset:
1.1.2. Take $Profile^{\mathcal{U}}$ = **getMindchangerProfile(**$\mathcal{U}$**,** S**,** QT**)**
1.1.3. If $Profile^{\mathcal{U}}$ equates to $Ans^{\mathcal{U}}_{QT}$ then increase the number of correct decisions
1.2. Take $Precision_S = \frac{\#\ of\ correct\ decisions}{\#\ of\ respondents}$
2. Take $Core\_Set$ = the minimal set S with a maximal $Precision_S$ value
3. **return** $Core\_Set$, $Precision_{Core\_Set}$

endfunction

As already specified in the previous section, in order to obtain a good decision rate, the questions upon which our model is built are selected according to a specific topic and according to their correlation values with the target question QT. The statistics performed on the overall data of the questionnaire guided us through the questions included in Secs. 2 and 3 (see Fig. 3).

By applying Algorithm 2 on this data, we obtained a set of five questions that, taken together, determine the maximum decision score, more precisely:

$$Core\_Set, Precision = \textbf{CoreSetSelection}(Questions\ of\ Sect.\ 2\ and\ 3, RO\ and\ SP\ Datasets, Q.5.2)$$

where $Core\_Set = \{Q.2.6, Q.3.2, Q.3.5, Q.3.8, Q.3.14\}$ and $Precision = 0.8366$.

The questions of the $Core\_Set$ are as follows:

- **Q.2.6** *How much are you willing to change your personal behavior to reduce the impact of global issues?* (with possible answers *Not at all/Not much/Much/Very much*)
- **Q.3.2** *How much do you feel concerned about climate change?* (with possible answers *Not at all/Not much/Much/Very much*)
- **Q.3.5** *Do you keep up with the news about climate change?* (with possible answers *Never/Rarely/Often/Always*)

- **Q.3.8** *How attractive is the idea of participating in actions addressing climate change in the future?* (with possible answers *Not at all/Not enough/Enough/A lot*)
- **Q.3.14** *How much do you think young people's engagement on climate change is important to address the issue?* (with possible answers *Not important/Rather important/Important/Very important*)

In Table 3, we present the questions of the *Core_Set* together with their possible answers and the corresponding neutrosophic representations. Because all the questions of the *Core_Set* have dual answers, $Ans^1_{Q.x}$ is the negation of $Ans^4_{Q.x}$ and $Ans^2_{Q.x}$ is the negation of $Ans^3_{Q.x}$. The neutrosophic-triplet numbers are defined using the scale [0, 0.25, 0.5, 0.75, 1] and cover all the operators suggested for linguistic terms set[51]:

(1) $Neg(Ans^i_{QT}) = Ans^j_{QT}$ with $j = 5 - i$,
(2) $\text{Min}(Ans^i_{QT}, Ans^j_{QT}) = Ans^i_{QT}$ for $i < j$, $i, j \in \{1, \ldots, 4\}$,
(3) $\text{Max}(Ans^i_{QT}, Ans^j_{QT}) = Ans^j_{QT}$ for $i < j$, $i, j \in \{1, \ldots, 4\}$.

As a direct consequence, for a specific respondent, the entire set of selected questions from the YECCM questionnaire generates, in our internal representation, a set of single-valued neutrosophic triplet numbers (Step 1 of Algorithm 1) which characterizes the respondent's knowledge about the investigated questions targeting global problems (hunger, poverty, etc.) or climate change.

5. Experiments and Results

The evaluation process of the proposed decision-making model was designed in two phases. The main purpose of *Phase 1* was to find the minimal set of questions based on which the model decision rate for the whole dataset (including both Romanian and Spanish datasets) is maximum. As already specified, the best precision was obtained for the questions of the *Core_Set*:

$$Core\_Set = \{Q.2.6, Q.3.2, Q.3.5, Q.3.8, Q.3.14\}.$$

As can be seen in Table 4, for the whole dataset (in short ALL Dataset), the obtained precision is 0.8366. We have also evaluated the precision obtained with the

Table 4. Neutrosophic model evaluation results.

Evaluation phase	Set of questions	RO Dataset	SP Dataset	ALL Dataset
Phase 1	*Core_Set*	0.84	0.8333	0.8366
Phase 2	*Core_Set* ∪ {*Q*.1.9.07, *Q*.1.9.10, *Q*.1.9.11, *Q*.1.9.19, *Q*.2.1}	0.87	0.67	0.73
	Core_Set ∪ {*Q*.1.9.07, *Q*.1.9.14, *Q*.1.9.19, *Q*.1.9.20, *Q*.3.1}	0.78	0.8725	0.8267

Core_Set questions by considering each dataset separately (Romanian and Spanish datasets). We found out that the precision score is stable as it is preserved when the Romanian and Spanish datasets are evaluated separately.

Phase 1 led to a successful result since it provided the minimal set of questions based on which the precision of the model reached its maximum value, without being altered by the nationality-specifics traits of the respondents.

The *Core_Set* questions are consistent with the three main components that are the base of youth engagement, which "*consists of a behavioral component* (*such as spending time doing the activity*), *an affective component* (*for example*, *deriving pleasure from participating*) *and a cognitive component* (*for example*, *knowing about the activity*)".[44] It includes the minimal set of questions that need to be addressed in order to determine the *Mindchanger profile* of the respondents. In other words, this selection emphasizes that it is necessary to interrogate young people about all three components of engagement in order to determine their *Mindchanger profiles* with maximum accuracy:

— the affective component ($Q.2.6$ and $Q.3.2$),
— the cognitive component ($Q.3.5$) and
— the behavioral component ($Q.3.8$ and $Q.3.14$).

This interplay between the three components also confirms that engagement is a multidimensional construct[52] and an ongoing process.

In *Phase 2* of the evaluation, we considered each dataset separately. We aimed at identifying other questions that, added to the *Core_Set* questions, increase the precision of the model by considering the particularities of each national group.

We ran Algorithm 2 for the Romanian dataset (in short RO Dataset) and we obtained the following results:

$$Core\_Set_{RO}, Precision_{RO} = \textbf{CoreSetSelection}(Q.1.9.x \cup \textit{Questions of Sect. 2 and 3}, \textit{RO Dataset}, Q.5.2)$$

where $Core\_Set_{RO} = Core\_Set \cup \{Q.1.9.07, Q.1.9.19,\ Q.1.9.10, Q.1.9.11, Q.2.1\}$ and $Precision_{RO} = 0.87$.

$Q.1.9.x$ denotes the set of questions $Q.1.9.1$, ..., $Q.1.9.24$ targeting the 24 characteristics that the respondents were asked to self-evaluate. As can be seen, we obtained 0.87 precision for the Romanian Dataset adding supplementary questions to the *Core_Set*.

For the Spanish dataset (in short SP Dataset) the maximum precision of 0.8725 was obtained for the following set of questions:

$$Core\_Set_{SP}, Precision_{SP} = \textbf{CoreSetSelection}(Q.1.9.x \cup \textit{Questions of Sect. 2 and 3}, \textit{SP Dataset}, Q.5.2)$$

where $Core\_Set_{SP} = Core\_Set \cup \{Q.1.9.07, Q.1.9.19, Q.1.9.14, Q.1.9.20, Q.3.1\}$ and $Precision_{SP} = 0.8725$.

The maximum precision scores are represented in Table 4 in green rectangles, while minimal precision scores are represented in red rectangles.

It is worth highlighting the two important variables that may largely explain these results, as well as some of the differences found between the two analyzed datasets.

The model determines with a precision score of 0.87 the *Mindchanger profile* for a young person about whom we know, besides the answers to the $Core\_Set$, the level of awareness about the SDGSs (RO Dataset) or the importance of climate change (SP Dataset), and the self-attributed rate for the characteristics considered in *Phase 2* of our evaluation (Table 5). The characteristics resulting from *Phase 2* are targeted by the following questions:

— for Romania: Questions $Q.1.9.x \cap Core\_Set_{RO}$,
— for Spain: questions $Q.1.9.x \cap Core\_Set_{SP}$.

Table 5 shows the self-attributed characteristics of the young people from Dolj County (RO Dataset) and La Rioja (SP Dataset) considered in *Phase 2* of the evaluation. In this analysis, there are two common personality traits that influence the decision rate (i.e., responsible and open to international solidarity), and also two different ones, which can be explained by looking into the regional/cultural context.

5.1. *A comparison between the neutrosophic model and different machine learning models*

In this section, we compare the results of the neutrosophic model with several machine learning (ML) models, such as decision trees, random forests and SVM. The performance of the ML models was evaluated using five-fold cross-validation. In addition, to avoid overfitting, we performed hyperparameter tuning for each model via grid search using five-fold cross-validation with five repetitions. The procedure

Table 5. The additional questions resulted from Phase 2 of the model evaluation.

	Romania (Dolj county)	Spain (La Rioja)
Personal characteristics	Responsible (Q.1.9.07)	
	Open to international solidarity (Q.1.9.19)	
	Good communicator with young people from my own culture (Q.1.9.10)	Extroverted (Q.1.9.14)
	Good communicator with young people from other cultures (Q.1.9.11)	Interested in global issues (Q.1.9.20)
Awareness level	Have you ever heard or read about the SDGs agreed by the international community? (Q.2.1)	How important do you consider the issue of climate change? (Q.3.1)

Table 6. Machine learning models evaluation results.

Set of questions	Trained model	RO Dataset	SP Dataset	ALL Dataset
$Core\_Set$	Decision Tree	0.6784	0.7553	0.773
	Random Forest	0.6669	0.7858	0.7472
	SVM	0.6659	0.8358	0.7679
$Core\_Set \cup \{Q.1.9.07, Q.1.9.10,$ $Q.1.9.11,\ Q.1.9.19, Q.2.1\}$	Decision Tree	0.6157	0.7337	0.7267
	Random Forest	0.6268	0.7232	0.701
	SVM	0.6449	0.7126	0.6806
$Core\_Set \cup \{Q.1.9.07, Q.1.9.14,$ $Q.1.9.19,\ Q.1.9.20, Q.3.1\}$	Decision Tree	0.6479	0.7958	0.7004
	Random Forest	0.6359	0.8047	0.7370
	SVM	0.6750	0.7942	0.7112

for each ML model and dataset is as follows:

(1) The grid search is created.
(2) The dataset is split into five subsets or folds.
(3) The previously created grid and four-folds from Step 2 (training data) are used to find the optimal hyperparameters:
   (a) The training data are again split five-fold.
   (b) For each possible combination of parameters, a model is trained with four-folds from Step 3.a.
   (c) Each model is evaluated on the remaining part of the data, i.e., the fold that was not used in Step 3.b.
   (d) Steps 3.b and 3.c are repeated five times (each time using a different fold for evaluation). This process is repeated five times. Thus, in total, 25 models are trained and evaluated for each combination of parameters.
(4) The model trained with the optimal hyperparameters is used to predict the target question QT for the fifth fold (test data), which was not used in Step 3.
(5) Steps 3 and 4 are repeated five times (each time using a different fold for evaluation) and the performance measure is calculated by averaging the values obtained in the loop.

The results presented in Table 6 show that the ML models are not able to reach the precision scores obtained with the neutrosophic model. For the whole dataset, the best precision is 0.773, while for the Romanian dataset, the precision is even lower, with a value equal to 0.6784. Finally, for the Spanish dataset, the best precision is 0.8358, which is not bad, but also lower than the best score obtained with the neutrosophic approach. It should be noted that the best results were obtained with the $Core\_Set$, i.e., with the fewest number of questions.

6. Conclusions and Future Work

Communication, a transversal skill of sustainable education,[53] is fundamental for any kind of human interaction or interpersonal exchange of information, ideas and

emotions. Being a key competence included in the Romanian core curriculum (as part of the Language and communication area of learning), it lays at the foundation of the entire educational system. Thus, the Romanian young people are encouraged to develop communication-associated skills from a very early age. Moreover, further emphasis is placed on intercultural communication, which has become a priority for the Romanian educational system and is aimed at helping to integrate young Romanians in the present world.[54,55] Recent migration phenomena have brought along the need to interact and adapt, especially since half of the young Romanian respondents reported to have experienced migration stories in their families. Additionally, the Romanian society is characterized by cultural diversity, so intercultural communication has always been a prerequisite in mediating relationships in diverse social and cultural environments. On the other hand, the fact that more than half of the sample are graduates of higher education and one in three is employed may impact their concern for their future and their conscientiousness about the world they live in, especially with regard to sustainable solidarity goals in general, and climate change in particular. Moreover, recent international events confirmed the level of responsibility and solidarity of young people from Dolj, since in the particular case of the unfortunate events in Ukraine, which brought massive waves of refugees to Romania, numerous young volunteers and young representatives of the civil society organizations initiated local actions and offered their support. All these characteristics may lead to a good precision rate for the Romanian group if they are investigated in correlation not only with the $Core\_set$, but also with the level of awareness about the SDGs.

In the specific case of La Rioja (Spain), where almost nine out of ten respondents are employed and about 73% of the participants have a Bachelor's or Master's degree, we refer to the possible influences of the socio-economic context and the personal characteristics associated with the young respondents. In terms of context, the social and economic development of the region is oriented towards agriculture, the food industry and the service sector. In this sense, the perceived negative effects for this socio-economic development as a consequence of global phenomena, such as climate change, may determine the high impact of some questions on climate change in predicting the young people's Mindchanger profile. On the other hand, some personal characteristics related to the age, employment status and university education of most of the young people surveyed (e.g., responsibility), seem to be determinant in the transition from awareness to active participation that will enable their personal and professional development, as well as the common welfare within a framework of sustainable development. Finally, and closely related to the idiosyncrasies of La Rioja, some distinctive personality traits, such as extroversion, seem to have an impact on the self-estimated likelihood to become a Mindchanger.

The statistical analysis of the 24 characteristics from questions $Q.1.9.x$ depicted in Fig. 6 shows the mean score and standard deviation of all the characteristics analyzed, ordered from the highest to the lowest overall score (represented by the

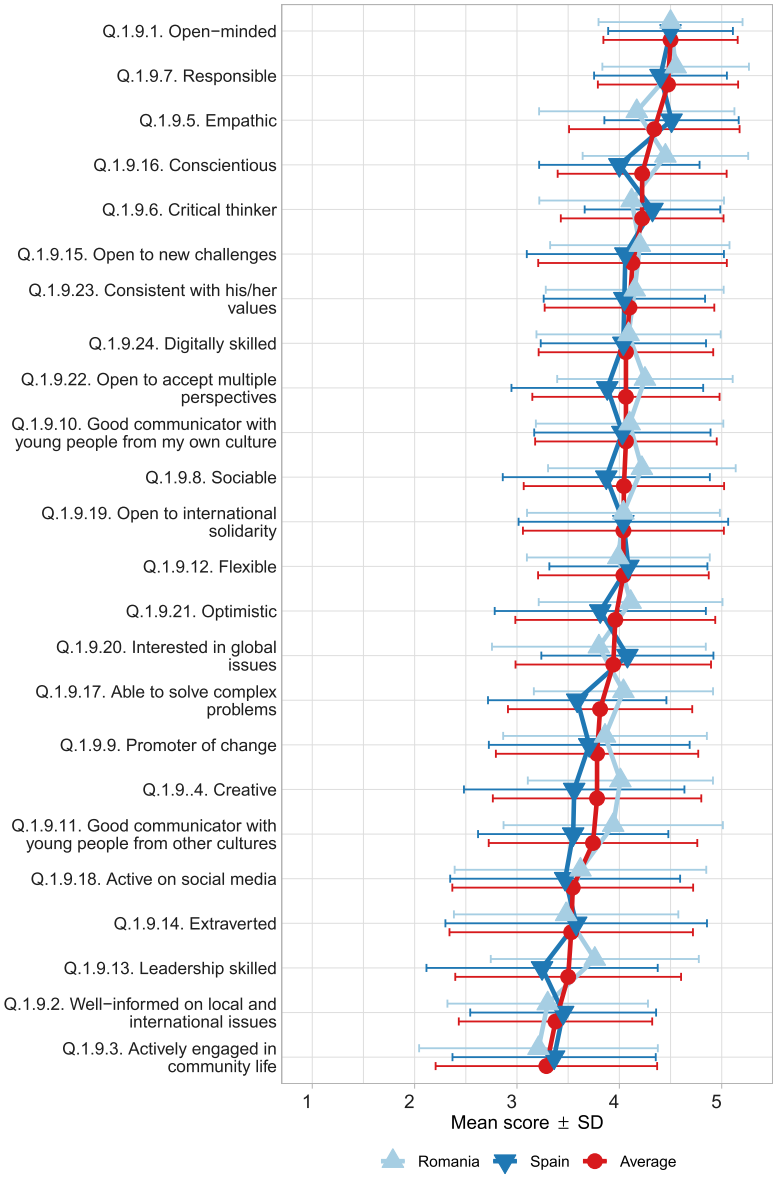


Fig. 6. Mean values of the respondents' self-evaluated characteristics (questions $Q.1.9.x$).

red line). In addition, the mean scores per country are represented. Although there are no significant differences between the responses of the two countries, Romanian participants scored higher than Spanish participants in almost 63% of the characteristics analyzed. The highest consensus (lower standard deviation) and the highest scores (around 4.5 points out of 5) in the self-assessments made by all participants correspond to the personal traits open-minded and responsible. In

contrast, the lowest overall scores correspond to actively engaged in community life and well-informed on local and international issues. Moreover, in the Spanish data, the lowest mean score is given to leadership skilled.

The $Q.1.9.x$ characteristics selected by Algorithm 2 in Phase 2 of the evaluation, namely, $Q.1.9.x \cap Core\_Set_{RO}$ and $Q.1.9.x \cap Core\_Set_{SP}$, only partially coincide with the ones statistically-determined to have the highest consensus and the highest scores. This means that the highest scored characteristics are not the most relevant for determining the *Mindchanger profile* and confirm the fact that the proposed decision-making model complements the statistical analysis, providing a specific tool for obtaining a high precision rate.

Therefore, this study was aimed at increasing the understanding of what makes young people actively engaged with SDGs and of what determines them to develop a pro-environmental behavior.

Based on the data collected through the YECCM questionnaire, we explored the likelihood of young people to act as Mindchangers, that is to reach a high level of civic engagement. By means of a neutrosophic approach we determined the minimal set of questions that allows for a high decision rate of the respondents' *Mindchanger profile*. We demonstrated that the neutrosophic model offers a quite significant and stable precision rate (0.83), in the sense that it is not altered by national or regional characteristics. In order to improve the decision rate, we further explored, for each of the two countries' datasets, additional questions targeting specific national features. This decision-making model proved to be successful for both datasets, considered separately.

In conclusion, we highlight several advantages of using the proposed model on questionnaire data. First, this model can be applied to questionnaires which involve linguistic responses that express, among others, unclear or vague thoughts. The model can be configured to work on any combinations of questions, for each combination the precision being calculated by considering the sum of correct decisions versus the number of respondents. In this manner, one could easily identify the minimum set of characteristics based on which the decision of the target choice (in this study denoted as *Mindchanger profile*) can be done with a maximum precision. Also, the model can be configured to work on different datasets in order to identity the particular combination of questions which maintains its precision on the considered data.

A direct impact of our approach is the possibility to identify the minimal set of questions that influence the respondent's answer choice to a target question. More precisely, the proposed decision-making model allows the identification of a minimal set of attributes (e.g., attitudes, beliefs, behavior) that can generate a high rate of precision. In the context of youth engagement, the obtained $Core_Set$s correspond to the conceptual framework of engagement (affective, cognitive, behavioral) and can be used by a trainer or a teacher to enrich young people's attitudes and beliefs that impact their choices to become Mindchangers.

Our study was performed on a set of specific topic questions which included only four-point Likert scale. Further investigation can be made to generalize the decision-making model and consequently develop it in order to process other types of questions, such as open-ended questions, whose answers are natural language texts, or five and seven-point Likert scale questions, which give a better reflection of the respondents' opinions.

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Health Insurance Provider Selection Through Novel Correlation Measure of Neutrosophic Sets Using TOPSIS

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Abstract: Risk monitoring aims to recognize and control potential threats to our assets or health activities. Insurance is vital, as it compensates for any unexpected loss of property or life. Clients selecting the best health insurance provider must consider various factors and their relative significance to their circumstances. Multi-criteria decision-making (MCDM) helps people analyze and compare policy alternatives to find the best option. This research aims to assist potential health policy purchasers by addressing the selection of health insurance providers as an MCDM problem. The correlation coefficient is useful for identifying the importance of several conflicting criteria. The idea of correlation coefficients is extended in a neutrosophic context to capture the indeterminacy and incomplete information in the relationship among the criteria. The technique for order preference by similarity to an ideal solution (TOPSIS) approach is a useful and straightforward approach to solving MCDM problems. However, it often became ambiguous to researchers due to its involvement in the distance measure technique. The proposed neutrosophic correlation measure may also replace the ambiguity of using a suitable distance measure in the TOPSIS approach. This study extends the TOPSIS method by using the proposed neutrosophic correlation coefficient on single-valued neutrosophic sets (SVNSs). The criteria preferences are computed using a method based on the removal effects of the criteria (MEREC) approach. Some valuable concepts, like the weighted closeness measure of type I and type II and the weighted index parameter, are introduced with their properties to establish the proposed neutrosophic TOPSIS approach. An MCDM approach for health insurance providers has been constructed to illustrate the proposed approach numerically. The proposed method suggest that the health insurance provider Υ_2 is the most beneficial alternative, whereas Υ_1 is the least suitable. The client considers the terms and conditions for non-coverage and the facilities provided for pediatric and maternity care while buying health insurance. The comparative analysis of the suggested technique demonstrates the merit of the research in terms of consistency. The sensitivity analysis demonstrates the flexibility and robustness of the obtained results.

***Keywords*:** correlation measure, TOPSIS, single-valued neutrosophic set, MCDM, health insurance, MEREC

MSC: 90B50, 90C29, 91B06

1. Introduction

Risk management is the rational creation and implementation of a loss mitigation strategy [1]. Risk monitoring entails detecting possible risks in advance, analyzing them, and taking preventative measures to mitigate them [2]. Insurance is an essential component of risk management in modern society, and its primary objective is to protect human life. An individual can take life insurance in two ways: health insurance for medical care and endowment insurance with a built-in savings component. Health insurance covers the costs associated with medical treatment for policyholders who sustain injuries or illnesses due to a catastrophe within the policy period [3]. Health insurance is popular because it offers financial protection, access to quality healthcare, preventive care services, peace of mind, and tax benefits.

After the globalization of India in the early 1990s, private players became interested in investing in the health sector. As a result, the rapid growth of private health insurance providers (HIPs) is entering the market, offering various health insurance products [4]. The Indian government launched Ayushman Bharat in 2018 to provide health coverage to over 100 million vulnerable families [5]. People who do not belong to the vulnerable class have discovered that they need to insure themselves to obtain better medical treatment than government hospitals. Many people need help choosing HIPs due to the lack of information regarding the plans and the fact that, even if such information were available, it would only describe the HIPs' features using linguistic terms that they struggle to categorize. In this process, ambiguity and incompleteness are introduced into the decision-making procedure. Therefore, it is highly challenging for a decision-maker (DM) to evaluate the characteristics of the HIPs and rank them in order of importance. This article formulates the issue of picking the best HIP as a multi-criteria decision-making (MCDM) problem, aiming to assist the health insurance buyer.

The MCDM offers a systematic technique for decision-making involving multiple conflicting criteria, which assists DMs by considering a broader overview of factors and their interrelationships [6]. The TOPSIS approach, introduced by Hwang and Yoon [7] and improved by Hwang et al. [8], is a popular MCDM technique for sound logic, greater flexibility, a more straightforward calculation procedure, and a distance measure simultaneously for the most preferable and least desirable outcomes [9]. The description of the criteria often needs to be completed or is indeterminate in the MCDM process, and the quantitative assessment of these criteria often depends on the subjective judgments of the DMs. Hence, it is only possible to accurately assess this complete information through the subjective assessment of the DMs. TOPSIS offers a methodical and adaptable MCDM framework, effectively managing uncertainty by incorporating subjective assessments and facilitating a transparent decision-making procedure. TOPSIS ranks the alternatives by identifying the ideal solutions through distance measurement. In an uncertain context, there needs to be a definitive guideline regarding the optimal distance measurement among various alternatives (e.g., Euclidean, Hamming, and Taxicab), which brings ambiguity into the decision-making procedure.

Data and information on the criteria of an option are frequently combined with hesitancy, indeterminacy, and uncertainty in the MCDM problem. In particular, MCDM techniques are vulnerable to the subjectivity of the experts when they use linguistic ideas for assessment. The idea of fuzzy sets (FSs) [10] with a membership degree offers a better option to handle these uncertain MCDM problems. The intuitionistic fuzzy set (IFS) [11], Pythagorean fuzzy set [12], and fermatean fuzzy set [13] are extensions of FS with two membership functions describing the acceptance and rejection degree with certain conditions. These FSs can handle ambiguous and partial data, not indeterminate or inconsistent information [14]. Smarandache [15] introduced the neutrosophic set (NS) with the neutrosophy ideology [16] that every idea has some degree of falsehood and indeterminacy in addition to truth. The single-valued neutrosophic sets (SVNSs) [17] are generalizations of FSs, IFSs, PFSs, and FFSs. It has three independent membership functions, all of which lie in $[0, 1]$ and reflect truth, indeterminacy, and falsity. As a result, SVNS is a better choice for representing the linguistic evaluation of the criteria in MCDM.

The purpose and aim of this article are as follows:

• The HIP selection problem can be formulated as a decision-making problem analyzing the key features of the different HIPs.

• The TOPSIS method can incorporate a neutrosophic correlation to eliminate the uncertainty of selecting distance measures in an ambiguous context.

• A criteria weight determination procedure may be used to evaluate the objective relevance of a criterion, eliminating the expert's subjective judgment of the criteria.

• An integrated TOPSIS approach can be introduced to deal with uncertain MCDM issues for a more accurate and reliable evaluation of decision-making.

To track the remaining article, Section 2 writes up the related literature that highlights the study's reason and research gap. In Section 3, the fundamental idea of the neutrosophic set is discussed. The concept of the neutrosophic correlation coefficient is proposed in Section 4. An integrated neutrosophic correlation-based TOPSIS approach is suggested in Section 5. The numerical illustration of HIP selection, comparison, and sensitivity analysis is presented in Section 6. The findings of this study are discussed in Section 7 along with a few potential suggestions for future work.

2. Literature review

This part provides a related study on MCDM application in the healthcare industry and the development of neutrosophic MCDM approaches.

2.1 *Selection of health insurance using MCDM*

The MCDM model can be a valuable process for selecting any service provider for performance analysis. Some researchers tried to apply crisp and fuzzy decision-making approaches to assess the performance of insurance companies. Gharizadeh et al. [18] introduced an integrated approach for performance analysis of insurance companies through AHP and principal component analysis. Susanto and Utama [19] considered fuzzy and non-fuzzy parameters in the claim settlement of the insurance companies, where the AHP approach determines the criteria preferences. Zulkifly et al. [20] proposed a fuzzy TOPSIS approach to find the rank of HIPs. A combined fuzzy AHP and fuzzy TOPSIS approach was proposed by Kahraman et al. [21] to assist potential buyers of health insurance. Erdebilli et al. [22] applied q-rof fuzzy set-based TOPSIS and VIKOR approaches to assess private health insurance plans. Yang et al. [23] introduced an algorithm that depends on q-rof sets, interactive, and Maclaurin symmetric mean operators for the financial performance of insurance companies. Mishra et al. [24] proposed a divergence measure and extended the TODIM [Portuguese acronym for Interactive Multi-Criteria Decision Making] approach using the proposed measure for vehicle insurance quality assessment. A group decision-making (GDM) framework was formulated by Yucenur and Demirel [25] using the extended VIKOR approach to determine the suitable HIPs for forest investment. An intuitionistic fuzzy method based on the removal effects of criteria (MEREC)-MARCOS approach was developed for GDM to assess the operational effectiveness of the insurance companies [26].

2.2 *Neutrosophic TOPSIS approach*

The neutrosophic fuzzy set [15], which Smarandache proposed as a generalization of the IFS, is capable of handling inaccurate information (uncertain, inaccurate, incomplete, and vague information) and is based on truth, indeterminacy, and falsity. Several distance measures, operators, and correlation measures were implemented on SVNS to solve the MCDM problem. Among them, Luo et al. [27] established a distance measure on SVNS for pattern recognition. Zhang et al. [28] proposed a neutrosophic TODIM-BSC method to assess the efficiency of insurance firms. Singh et al. [29] suggested an inter-valued neutrosophic fuzzy approach through principal component analysis.

There are several articles [30] that include in-depth simulation-based comparisons and mathematical analysis of TOPSIS to clear the ambiguity over which one should be used to solve MCDM problems. TOPSIS is a popular and effective way to handle MCDM issues in an uncertain environment. The conventional TOPSIS method considers only distance measures, not similarity or probability. Biswas et al. [31] solved TOPSIS by SVN Euclidean distance measure. Karasan and Kaya [32] proposed a neutrosophic TOPSIS approach for evaluating different network controllers and relays for aerial vehicles. Hezam et al. [33] applied neutrosophic TOPSIS to rank COVID-19 vaccine alternatives. Pouresmaeil et al. [34] constructed a score function to solve interval neutrosophic MCDM problems using the TOPSIS approach.

Mollaoglu et al. [35] identified alternate fuel sources for ship investment choices using the neutrosophic TOPSIS approach. Some recent applications of the neutrosophic TOPSIS approach are summarised in Table 1.

Table 1. Recent applications of neutrosophic TOPSIS approach

Approach	Weight determination	Applications	Reference
Neutrosophic TOPSIS	ANP	Tourist destination selection	[36]
Neutrosophic TOPSIS and COPRAS	AHP	Women university selection	[37]
Neutrosophic TOPSIS	Average method	Surfactant-free microemulsion fuel selection	[38]
Single-valued neutrosophic TOPSIS	Term presence and term frequency	Doctor selection	[39]
Neutrosophic TOPSIS	CRITIC	Aircraft selection	[40]
Neutrosophic TOPSIS	CRITIC and entropy	Best player selection	[41]
Triangular neutrosophic TOPSIS	Average method	Renewable energy source selection	[42]
Single-valued neutrosophic TOPSIS	Neutrosophic rating	Cloud service selection	[43]
Neutrosophic TOPSIS	Neutrosophic rating	Nutritional education strategies selection	[44]
Neutrosophic TOPSIS	AHP	Ideal manufacture selection	[45]
Neutrosophic TOPSIS		Smartphone selection	[46]
Single-valued neutrosophic TOPSIS		Software development company selection	[47]
Neutrosophic TOPSIS	Single-valued neutrosophic weighted averaging operator	Logistic selection	[48]
Neutrosophic AHP and TOPSIS	AHP	Smart village selection	[49]
Single-valued neutrosophic TOPSIS	Single-valued neutrosophic weighted averaging operator	Tablet selection	[50]
Single-valued neutrosophic TOPSIS	Subject	e-commerce development strategies	[51]
Single-valued neutrosophic TOPSIS	Single-valued neutrosophic weighted averaging operator	Software project selection	[52]
		Logistic centre location selection	[53]

Implementing various neutrosophic TOPSIS approaches in the MCDM and multi-criteria GDM problems is convenient, as seen in Table 1. However, there are few papers on insurance policy selection treated as an MCDM problem and solved by fuzzy TOPSIS: Sehhat et al. [54] used a crisp TOPSIS approach; Mimovic et al. [55] used interval fuzzy rough sets-based TOPSIS; Sekar et al. [56] and Chu and Le [57] used fuzzy TOPSIS. Neutrosophic TOPSIS has a wide range of applications, although HIP selection is not mentioned as an MCDM problem in any of the current literature, according to Table 1.

The evaluation of the significance of the criterion is of utmost relevance in MCDM procedures. The criteria weights may be decided by subjective evaluation by an expert or objective assessment using the decision matrix. The subjective evaluation often entails individual preferences and the expert's inability to judge accurately. To eliminate this issue, using an objective evaluation method using statistical or mathematical techniques is preferable. The MEREC technique has several benefits, such as identifying duplicate criteria, simplicity and intuitiveness, flexibility in adjusting decisions, resilience to uncertainty, transparency and involvement of stakeholders, and compatibility with other methods. The collective advantages of these aspects make the MEREC technique more effective and appropriate for determining criterion preference.

2.3 *Fuzzy correlation-based TOPSIS approach*

In practice, correlation coefficients are often employed to assess relationships between variables and are utilized in various disciplines. Karl Pearson first introduced the correlation coefficient to deal with crisp numbers. Chiang and Lin [58] introduced the fuzzy correlation coefficient to show whether the fuzzy sets are positively or negatively correlated. Ye [59] proposed intuitionistic decision-making based on weighted correlation coefficients. Lin et al. [60] developed a Pythagorean correlation coefficient-based TOPSIS approach for inpatient stroke rehabilitation analysis. Golui et al. [61] suggested a correlation for the fermatean fuzzy-based TOPSIS approach considering the hesitant components and applied it for ranking electric vehicles. Zhang et al. [62] introduced a hesitant fuzzy-based correlation for supplier selection. Liu et al. [63] proposed a hesitant fuzzy correlation coefficient for medical diagnosis challenges. Ye [64, 65] proposed the SVNS correlation coefficient and extended this idea to cosine similarity measures to solve MCDM problems. Zeng et al. [66] proposed a correlation between SVNSs and applied TOPSIS for software selection.

Except for Zeng et al. [66], all correlation coefficients reported in previous studies fall within the $[0, 1]$ range. Consequently, we need to represent the negative correlation between SVNSs accurately. This work suggests a novel idea for the SVNS correlation coefficient to address the drawbacks of the current correlation coefficients. The main advantage of this approach is that the proposed SVNS correlation coefficient lies in $[-1, 1]$, which is identical to the traditional statistical measure.

According to the research gap, the following are the article's contributions:

• A more robust correlation coefficient was devised to quantify the strength of the associations between SVNSs, encompassing both positive and negative relationships.

• A correlation-based integrated TOPSIS method is proposed for addressing uncertain MCDM problems.

• The MEREC approach's weight determination replaces the TOPSIS approach's subjective weight assessment of the criteria.

• A healthcare provider analysis is developed as an MCDM problem by identifying viable alternatives and their associated criteria.

• The proposed approach is compared with distance-based TOPSIS approaches to check its consistency.

• Sensitivity analysis generates a collection of uncertain decision-making by varying the parameters.

3. Preliminaries

This section provides the basic arithmetic operations on SVNSs.

Definition 1 Single-valued neutrosophic set (SVNS): [17] Suppose $\xi_{\tilde{P}}, \zeta_{\tilde{P}}, \kappa_{\tilde{P}}$: $\mathbf{R} \to [0, 1]$ denote the truth, hesitation, and falsity membership functions, respectively, then an SVNS $\tilde{P}$ of a single-valued independent variable t is defined by $\tilde{P} = \{\langle t; [\, \xi_{\tilde{P}}(t), \zeta_{\tilde{P}}(t), \kappa_{\tilde{P}}(t)] \,\rangle : t \in T\}$, where T is the universal set.

A diagram of the SVNS is given in Figure 1.

For simplicity, we use $\tilde{p} = (\xi_p, \zeta_p, \kappa_p)$ as single-valued neutrosophic element (SVNE) instead of SVNS.

Definition 2 Arithmetic operations of SVNEs: [67] Suppose $\tilde{p} = (\xi_p, \zeta_p, \kappa_p)$ and $\tilde{q} = (\xi_q, \zeta_q, \kappa_q)$ be any two SVNEs and $\lambda \neq 0$ be real constant. The algebraic operations are described as follows:

$$\tilde{p} \bigoplus \tilde{q} = (\xi_p + \xi_q - \xi_p \xi_q, \zeta_p \zeta_q, \kappa_p \kappa_q)$$

$$\tilde{p} \bigotimes \tilde{q} = (\xi_p \xi_q, \zeta_p + \zeta_q - \zeta_p \zeta_q, \kappa_p + \kappa_q - \kappa_p \kappa_q)$$

$$\lambda \tilde{p} = (1 - (1 - \xi_p)^{\lambda}, \zeta_p^{\lambda}, \kappa_p^{\lambda})$$

$$\tilde{p}^{\lambda} = (\xi_p^{\lambda},\, 1-(1-\zeta_p)^{\lambda},\, 1-(1-\kappa_p)^{\lambda})$$

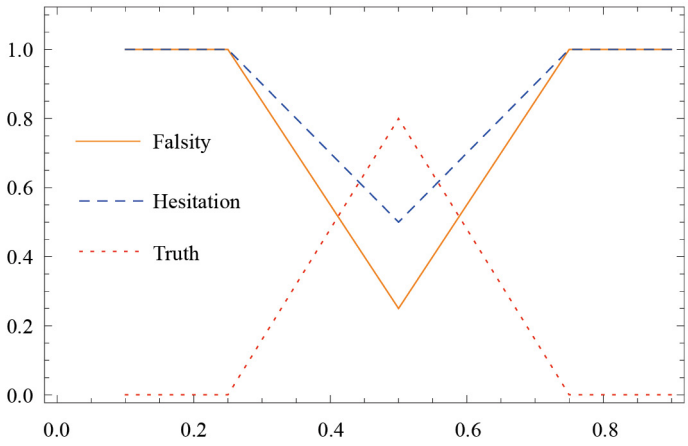

Figure 1. Diagram of a single-valued neutrosophic sets

Definition 3 Score function: [68] The score function of a SVNE $\tilde{p} = (\xi_p,\, \zeta_p,\, \kappa_p)$ is described by:

$$\mathfrak{S}_p = \frac{3+\xi_p-2\zeta_p-\kappa_p}{4};\ \mathfrak{S} \in [0,\, 1] \tag{1}$$

4. Proposed single-valued neutrosophic correlation coefficient

This part proposes a neutrosophic correlation coefficient and its distinctive characteristics. The weighted neutrosophic correlation coefficient is also introduced and established by describing its properties.

Let us consider m distinct alternatives defined as $\Upsilon = \{\Upsilon_1, \Upsilon_2, ..., \Upsilon_m\}$ and n criteria $\Psi = \{\Psi_1, \Psi_2, ..., \Psi_n\}$, where Ψ_B and Ψ_C respectively, be the collection of benefit-based and cost-based criteria, such that $\Psi = \Psi_B \cup \Psi_C$ and $\Psi_B \cap \Psi_C = \emptyset$. Let $\varsigma = (\varsigma_1, \varsigma_2, ..., \varsigma_n)$ be the criteria weights such that $\sum_{i=1}^{n} \varsigma_j = 1$. Each entry in the decision matrix $\mathbb{M} = (m_{ij})_{m\times n}$ is SVNS, as shown below:

$$\mathbb{M} = \begin{array}{c} \\ \Upsilon_1 \\ \Upsilon_2 \\ \vdots \\ \Upsilon_m \end{array} \begin{array}{c} \begin{array}{ccccc} \Psi_1 & \Psi_2 & \Psi_3 & \dots & \Psi_n \end{array} \\ \begin{pmatrix} (\xi_{11},\, \zeta_{11},\, \kappa_{11}) & (\xi_{12},\, \zeta_{12},\, \kappa_{12}) & (\xi_{13},\, \zeta_{13},\, \kappa_{13}) & \dots & (\xi_{1n},\, \zeta_{1n},\, \kappa_{1n}) \\ (\xi_{21},\, \zeta_{21},\, \kappa_{21}) & (\xi_{22},\, \zeta_{22},\, \kappa_{22}) & (\xi_{23},\, \zeta_{23},\, \kappa_{23}) & \dots & (\xi_{2n},\, \zeta_{2n},\, \kappa_{2n}) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ (\xi_{m1},\, \zeta_{m1},\, \kappa_{m1}) & (\xi_{m2},\, \zeta_{m2},\, \kappa_{m2}) & (\xi_{m3},\, \zeta_{m3},\, \kappa_{m3}) & \dots & (\xi_{mn},\, \zeta_{mn},\, \kappa_{mn}) \end{pmatrix} \end{array} \tag{2}$$

The element $m_{ij} = (\xi_{ij},\ \zeta_{ij},\ \kappa_{ij})$ denotes the ij^{th} rating of SVNS in the decision matrix (2) by the expert. The characteristic m_i corresponding to the alternative Υ_i in the i^{th} row is represented as

$$m_i = \{(c_1,\ \xi_{i1},\ \zeta_{i1},\ \kappa_{i1}),\ (c_2,\ \xi_{i2},\ \zeta_{i2},\ \kappa_{i2}),\ ...,\ (c_n,\ \xi_{in},\ \zeta_{in},\ \kappa_{in})\}.$$

Definition 4 Neutrosophic correlation coefficient: Let Υ_s and Υ_t represent two neutrosophic characteristics of an alternative in the neutrosophic decision matrix $\mathbb{M}$ respectively, where

$$\Upsilon_s = \{(c_1,\ \xi_{s1},\ \zeta_{s1},\ \kappa_{s1}),\ (c_2,\ \xi_{s2},\ \zeta_{s2},\ \kappa_{s2}),\ ...,\ (c_n,\ \xi_{sn},\ \zeta_{sn},\ \kappa_{sn})\}$$

$$\Upsilon_t = \{(c_1,\ \xi_{t1},\ \zeta_{t1},\ \kappa_{e1}),\ (c_2,\ \xi_{t2},\ \zeta_{t2},\ \kappa_{t2}),\ ...,\ (c_n,\ \xi_{tn},\ \zeta_{tn},\ \kappa_{tn})\}$$

Suppose $\bar{\xi}_j = \sum_{i=1}^{m} \xi_{ij}/m$, $\bar{\zeta}_j = \sum_{i=1}^{m} \zeta_{ij}/m$, and $\bar{\kappa}_j = \sum_{i=1}^{m} \kappa_{ij}/m$, then the neutrosophic correlation coefficient between Υ_s and Υ_t is defined as follows:

$$\vartheta(\Upsilon_s, \Upsilon_t) = \frac{1}{3}\left[\rho_\xi(\Upsilon_s, \Upsilon_t) + \rho_\zeta(\Upsilon_s, \Upsilon_t) + \rho_\kappa(\Upsilon_s, \Upsilon_t)\right] \tag{3}$$

where

$$\rho_\xi(\Upsilon_s, \Upsilon_t) = \frac{\sum_{j=1}^{n}\left[\xi_{sj}^2 - \bar{\xi}_j^2\right]\left[\xi_{tj}^2 - \bar{\xi}_j^2\right]}{\sqrt{\sum_{j=1}^{n}\left[\xi_{sj}^2 - \bar{\xi}_j^2\right]^2}\sqrt{\sum_{j=1}^{n}\left[\xi_{tj}^2 - \bar{\xi}_j^2\right]^2}},$$

$$\rho_\zeta(\Upsilon_s, \Upsilon_t) = \frac{\sum_{j=1}^{n}\left[\zeta_{sj}^2 - \bar{\zeta}_j^2\right]\left[\zeta_{tj}^2 - \bar{\zeta}_j^2\right]}{\sqrt{\sum_{j=1}^{n}\left[\zeta_{sj}^2 - \bar{\zeta}_j^2\right]^2}\sqrt{\sum_{j=1}^{n}\left[\zeta_{tj}^2 - \bar{\zeta}_j^2\right]^2}},$$

and

$$\rho_\kappa(\Upsilon_s, \Upsilon_t) = \frac{\sum_{j=1}^{n}\left[\kappa_{sj}^2 - \bar{\kappa}_j^2\right]\left[\kappa_{tj}^2 - \bar{\kappa}_j^2\right]}{\sqrt{\sum_{j=1}^{n}\left[\kappa_{sj}^2 - \bar{\kappa}_j^2\right]^2}\sqrt{\sum_{j=1}^{n}\left[\kappa_{tj}^2 - \bar{\kappa}_j^2\right]^2}}.$$

We assume that the denominators of $\rho_\xi(\Upsilon_s, \Upsilon_t)$, $\rho_\zeta(\Upsilon_s, \Upsilon_t)$, $\rho_\kappa(\Upsilon_s, \Upsilon_t)$ are not equal to zero.

Definition 5 Weighted neutrosophic correlation coefficient: Let Υ_s, Υ_t be the two neutrosophic characteristics in the neutrosophic decision matrix $\mathbb{M}$ and $\varsigma = (\varsigma_1, \varsigma_2, ..., \varsigma_n)$, $\sum_{j=1}^{n} \varsigma_j = 1$ be the weight vector related to the criteria. Then, the weighted neutrosophic correlation coefficient between Υ_s, Υ_t is defined as follows:

$$\vartheta^{\varsigma}(\Upsilon_s, \Upsilon_t) = \frac{1}{3}\left[\rho_{\xi}^{\varsigma}(\Upsilon_s, \Upsilon_t) + \rho_{\zeta}^{\varsigma}(\Upsilon_s, \Upsilon_t) + \rho_{\kappa}^{\varsigma}(\Upsilon_s, \Upsilon_t)\right] \tag{4}$$

where,

$$\rho_{\xi}^{\varsigma}(\Upsilon_s, \Upsilon_t) = \frac{\sum_{j=1}^{n} \varsigma_j \left[\xi_{sj}^2 - \bar{\xi_j^2}\right] \cdot \left[\xi_{tj}^2 - \bar{\xi_j^2}\right]}{\sqrt{\sum_{j=1}^{n} \varsigma_j \left[\xi_{sj}^2 - \bar{\xi_j^2}\right]^2} \cdot \sqrt{\sum_{j=1}^{n} \varsigma_j \left[\xi_{tj}^2 - \bar{\xi_j^2}\right]^2}} \tag{5}$$

$$\rho_{\zeta}^{\varsigma}(\Upsilon_s, \Upsilon_t) = \frac{\sum_{j=1}^{n} \varsigma_j \left[\zeta_{sj}^2 - \bar{\zeta_j^2}\right] \cdot \left[\zeta_{tj}^2 - \bar{\zeta_j^2}\right]}{\sqrt{\sum_{j=1}^{n} \varsigma_j \left[\zeta_{sj}^2 - \bar{\zeta_j^2}\right]^2} \cdot \sqrt{\sum_{j=1}^{n} \varsigma_j \left[\zeta_{tj}^2 - \bar{\zeta_j^2}\right]^2}} \tag{6}$$

$$\rho_{\kappa}^{\varsigma}(\Upsilon_s, \Upsilon_t) = \frac{\sum_{j=1}^{n} \varsigma_j \left[\kappa_{sj}^2 - \bar{\kappa_j^2}\right] \cdot \left[\kappa_{tj}^2 - \bar{\kappa_j^2}\right]}{\sqrt{\sum_{j=1}^{n} \varsigma_j \left[\kappa_{sj}^2 - \bar{\kappa_j^2}\right]^2} \cdot \sqrt{\sum_{j=1}^{n} \varsigma_j \left[\kappa_{tj}^2 - \bar{\kappa_j^2}\right]^2}}. \tag{7}$$

The denominators of the equations (5), (6), and (7) are assumed to be non-zero.

Theorem 1 The membership component $\rho_{\xi}^{\varsigma}(\Upsilon_s, \Upsilon_t)$ of $\vartheta^{\varsigma}(\Upsilon_s, \Upsilon_t)$ satisfy the following characteristics:

(i) $\rho_{\xi}^{\varsigma}(\Upsilon_s, \Upsilon_t) = \rho_{\xi}^{\varsigma}(\Upsilon_t, \Upsilon_s)$

(ii) $\rho_{\xi}^{\varsigma}(\Upsilon_s, \Upsilon_t) = 1$ if $\xi_{sj} = \xi_{tj}\ \forall\ c_j \in \Psi$

(iii) $|\rho_{\xi}^{\varsigma}(\Upsilon_s, \Upsilon_t)| \leq 1$

Proof. The properties (i) and (ii) are trivial as $\rho_{\xi}^{\varsigma}(\Upsilon_s, \Upsilon_t) = \frac{\sum_{j=1}^{n} \varsigma_j \left[\xi_{sj}^2 - \bar{\xi_j^2}\right]^2}{\left[\sqrt{\sum_{j=1}^{n} \varsigma_j \left[\xi_{sj}^2 - \bar{\xi_j^2}\right]^2}\right]^2} = 1$. To prove (iii), it is known that $-1 \leq \left((\xi_{sj})^2 - \left(\bar{\xi_j}\right)^2\right) \cdot \left((\xi_{tj})^2 - \left(\bar{\xi_j}\right)^2\right) \leq 1$. Thus, $-1 \leq \sum_{j=1}^{n} \varsigma_j \left((\xi_{sj})^2 - \left(\bar{\xi_j}\right)^2\right) \cdot \left((\xi_{tj})^2 - \left(\bar{\xi_j}\right)^2\right) \leq 1$, since $\sum_{j=1}^{n} \varsigma_j = 1$.

The denominator of (5) is $0 \leq \sum_{j=1}^{n} \left((\xi_{sj})^2 - \left(\bar{\xi_j}\right)^2\right)^2 \leq n$ and $0 \leq \sum_{j=1}^{n} \left((\xi_{tj})^2 - \left(\bar{\xi_j}\right)^2\right)^2 \leq n$.

So it is obvious that $0 \leq \sum_{j=1}^{n} \varsigma_j \left((\xi_{sj})^2 - \left(\bar{\xi_j}\right)^2\right)^2 \leq 1$ and $0 \leq \sum_{j=1}^{n} \varsigma_j \left((\xi_{tj})^2 - \left(\bar{\xi_j}\right)^2\right)^2 \leq 1$ since $\sum_{j=1}^{n} \varsigma_j = 1$.

Hence, $\sqrt{\sum_{j=1}^{n} \varsigma_j \left((\xi_{sj})^2 - \left(\bar{\xi_j}\right)^2\right)^2} \cdot \sqrt{\sum_{j=1}^{n} \varsigma_j \left((\xi_{tj})^2 - \left(\bar{\xi_j}\right)^2\right)^2} \leq \sqrt{1}.\sqrt{1} = 1$.

Similarly, it can be shown $-1 \leq \rho_{\xi}(\Upsilon_s, \Upsilon_t) \leq 1$, this establishes the theorem.

5. Proposed integrated neutrosophic TOPSIS approach

The current section illustrates the suggested neutrosophic TOPSIS methodology to solve uncertain MCDM issues.

Definition 6 Neutrosophic positive ideal solution (PIS) and negative ideal solution (NIS): Let Υ_+ and Υ_- denote the PIS and NIS represented as $\Upsilon_+ = \{(c_1, \Upsilon_{+1}), (c_2, \Upsilon_{+2}), ..., (c_n, \Upsilon_{+n})\}$ and $\Upsilon_- = \{(c_1, \Upsilon_{-1}), (c_2, \Upsilon_{-2}), ..., (c_n, \Upsilon_{-n})\}$, where Υ_{+j}, and Υ_{-j} are defined as $\Upsilon_{+j} = (\xi_{+j}, \zeta_{+j}, \kappa_{+j})$ where

$$(\xi_{+j},\, \zeta_{+j},\, \kappa_{+j}) = \begin{cases} \left(\max\limits_{i=1}^{m} \xi_{ij},\, \min\limits_{i=1}^{m} \zeta_{ij},\, \min\limits_{i=1}^{m} \kappa_{ij} \right) & if\ c_j \in \Psi_B \\ \left(\min\limits_{i=1}^{m} \xi_{ij},\, \max\limits_{i=1}^{m} \zeta_{ij},\, \max\limits_{i=1}^{m} \kappa_{ij} \right) & if\ c_j \in \Psi_C \end{cases} \tag{8}$$

And $\Upsilon_{-j} = (\xi_{-j},\, \zeta_{-j},\, \kappa_{-j})$ where

$$(\xi_{-j},\, \zeta_{-j}\ \kappa_{-j}) = \begin{cases} \left(\min\limits_{i=1}^{m} \xi_{ij},\, \max\limits_{i=1}^{m} \zeta_{ij},\, \max\limits_{i=1}^{m} \kappa_{ij} \right) & if\ c_j \in \Psi_B \\ \left(\max\limits_{i=1}^{m} \xi_{ij},\, \min\limits_{i=1}^{m} \zeta_{ij},\, \min\limits_{i=1}^{m} \kappa_{ij} \right) & if\ c_j \in \Psi_C \end{cases} \tag{9}$$

Definition 7 Type I and type II closeness measures: Let $\mathfrak{C}(\Upsilon_i,\, \Upsilon_+)$ and $\mathfrak{C}(\Upsilon_i,\, \Upsilon_-)$ represent the neutrosophic correlation coefficients for PIS and NIS respectively, for Υ_i. Also, let $\mathfrak{M}_I(\Upsilon_i)$, $\mathfrak{M}_{II}(\Upsilon_i)$ represent the type I and type II closeness measures, respectively, where

$$\mathfrak{M}_I(\Upsilon_i) = \frac{1 - \mathfrak{C}(\Upsilon_i,\, \Upsilon_-)}{2 - \mathfrak{C}(\Upsilon_i,\, \Upsilon_+) - \mathfrak{C}(\Upsilon_i,\, \Upsilon_-)} \tag{10}$$

$$\mathfrak{M}_{II}(\Upsilon_i) = \frac{1 + \mathfrak{C}(\Upsilon_i,\, \Upsilon_+)}{2 + \mathfrak{C}(\Upsilon_i,\, \Upsilon_+) + \mathfrak{C}(\Upsilon_i,\, \Upsilon_-)}. \tag{11}$$

It is assumed that the denominator of equations (10) and (11) is not zero.

Theorem 2 The type I closeness measure $\mathfrak{M}_I(\Upsilon_i)$ for every neutrosophic characteristic Υ_i meet the characteristics listed below:

(i) $0 \le \mathfrak{M}_I(\Upsilon_i) \le 1$

(ii) $\mathfrak{M}_I(\Upsilon_i) = 1$ if $\mathfrak{C}(\Upsilon_i,\, \Upsilon_+) = 1$

(iii) $\mathfrak{M}_I(\Upsilon_i) = 0$ if $\mathfrak{C}(\Upsilon_i,\, \Upsilon_-) = 1$

Proof. For (i), we know that $-1 \le \mathfrak{C}(\Upsilon_i,\, \Upsilon_+) \le 1$ and $-1 \le \mathfrak{C}(\Upsilon_i,\, \Upsilon_-) \le 1$.

So, $0 \le 1 - \mathfrak{C}(\Upsilon_i,\, \Upsilon_+) \le 2$ and $0 \le 1 - \mathfrak{C}(\Upsilon_i,\, \Upsilon_-) \le 2$ and $0 \le 2 - \mathfrak{C}(\Upsilon_i,\, \Upsilon_+) - \mathfrak{C}(\Upsilon_i,\, \Upsilon_-) \le 4$. This proves (i).

The proof of (ii) can be verified using $\mathfrak{C}(\Upsilon_i,\, \Upsilon_+)$ since we have $\mathfrak{C}(\Upsilon_i,\, \Upsilon_+) = 1$. Similarly, (iii) can be proved.

If $\mathfrak{C}(\Upsilon_i,\, \Upsilon_+) = 1$ gives a strong positive relationship, as consequently, type I closeness measure $\mathfrak{M}_I = 1$ gives that the corresponding alternative Υ_i is better than the rest of the alternative. $\mathfrak{C}(\Upsilon_i,\, \Upsilon_-) = 1$ provides a stronger positive relation between Υ_i and Υ_- implies type I closeness measure $\mathfrak{M}_I = 0$, signifies that the alternative Υ_i is inferior.

Theorem 3 The type II closeness measure $\mathfrak{M}_{II}(\Upsilon_i)$ for every neutrosophic characteristic Υ_i possesses the following characteristics:

(i) $0 \le \mathfrak{M}_{II}(\Upsilon_i) \le 1$

(ii) $\mathfrak{M}_{II}(\Upsilon_i) = 0$ if $\mathfrak{C}(\Upsilon_i,\, \Upsilon_+) = -1$

(iii) $\mathfrak{M}_{II}(\Upsilon_i) = 1$ if $\mathfrak{C}(\Upsilon_i,\, \Upsilon_-) = -1$

(iv) $\mathfrak{M}_{II}(\Upsilon_-) \le \mathfrak{M}_{II}(\Upsilon_+)$

Proof. For (i), we know that $-1 \leq \mathfrak{C}(\Upsilon_i, \Upsilon_+) \leq 1$ and $-1 \leq \mathfrak{C}(\Upsilon_i, \Upsilon_-) \leq 1$. So, $0 \leq 1+\mathfrak{C}(\Upsilon_i, \Upsilon_+) \leq 2$ and $0 \leq 1+\mathfrak{C}(\Upsilon_i, \Upsilon_-) \leq 2$ and $0 \leq 2+\mathfrak{C}(\Upsilon_i, \Upsilon_+)+\mathfrak{C}(\Upsilon_i, \Upsilon_-) \leq 4$. This proves (i).

The proof of (ii) can be verified using $\mathfrak{C}(\Upsilon_i, \Upsilon_+)$ since we have $\mathfrak{C}(\Upsilon_i, \Upsilon_+) = -1$, then $\mathfrak{M}_{II} = 0$.

The proof of (iii) can be verified using $\mathfrak{C}(\Upsilon_i, \Upsilon_-)$ since we have $\mathfrak{C}(\Upsilon_i, \Upsilon_-) = -1$, then $\mathfrak{M}_{II} = 1$.

For (iv), it is known from the previous results that $\mathfrak{C}(\Upsilon_-, \Upsilon_+) = \mathfrak{C}(\Upsilon_+, \Upsilon_-)$ and $\mathfrak{C}(\Upsilon_i, \Upsilon_i) = 1$. Using previous definitions, we have

$$\mathfrak{M}_{II}(\Upsilon_-) = \frac{1+\mathfrak{C}(\Upsilon_-, \Upsilon_+)}{2+\mathfrak{C}(\Upsilon_-, \Upsilon_+)+\mathfrak{C}(\Upsilon_-, \Upsilon_-)} = \frac{1+\mathfrak{C}(\Upsilon_-, \Upsilon_+)}{3+\mathfrak{C}(\Upsilon_-, \Upsilon_+)}$$

$$\mathfrak{M}_{II}(\Upsilon_+) = \frac{1+\mathfrak{C}(\Upsilon_-, \Upsilon_+)}{2+\mathfrak{C}(\Upsilon_+, \Upsilon_+)+\mathfrak{C}(\Upsilon_+, \Upsilon_-)}$$

$$\leq \frac{1+1}{3+\mathfrak{C}(\Upsilon_+, \Upsilon_-)} = \frac{2}{3+\mathfrak{C}(\Upsilon_-, \Upsilon_+)}.$$

As $\mathfrak{C}(\Upsilon_-, \Upsilon_+) \leq 1$, we can conclude that $\mathfrak{M}_{II}(\Upsilon_-) \leq \mathfrak{M}_{II}(\Upsilon_+)$.

Definition 8 Index value: Let ε denote a closeness parameter, where $0 \leq \varepsilon \leq 1$. The neutrosophic correlation-based index value is defined by $\mathscr{I}(\Upsilon_i) = \varepsilon\mathfrak{M}_I(\Upsilon_i) + (1-\varepsilon)\mathfrak{M}_{II}(\Upsilon_i)$. The type I and type II closeness measurements impact the parameter ε. The larger value of ε indicates that neutrosophic correlation closeness $\mathscr{I}(\Upsilon_i)$ would focus on $\mathfrak{M}_I(\Upsilon_i)$ and the smaller value of ε indicates type II focus on $\mathfrak{M}_{II}(\Upsilon_i)$.

Theorem 4 The index value $\mathscr{I}(\Upsilon_i)$ fulfils the following conditions for every neutrosophic characteristic Υ_i in the neutrosophic decision matrix $\mathbb{M}$:

(i) $0 \leq \mathscr{I}(\Upsilon_i) \leq 1$

(ii) $\mathscr{I}(\Upsilon_i) = \mathfrak{M}_I(\Upsilon_i)$ if $\varepsilon = 1$

(iii) $\mathscr{I}(\Upsilon_i) = \mathfrak{M}_{II}(\Upsilon_i)$ if $\varepsilon = 0$

(iv) $\mathscr{I}(\Upsilon_-) \leq \mathscr{I}(\Upsilon_+)$

Definition 9 Weighted type I and type II closeness measures: Suppose the neutrosophic PIS and NIS of the neutrosophic characteristic of an alternative Υ_i be Υ_+, Υ_-. Also, let Υ_i be any neutrosophic characteristic. Let ς_j be the weight of the criteria Ψ_i where $0 \leq \varsigma_j \leq 1$ and $\sum_{j=1}^{n} \varsigma_j = 1$. Let $\mathfrak{M}_I^{\varsigma}(\Upsilon_i)$ and $\mathfrak{M}_{II}^{\varsigma}(\Upsilon_i)$ denotes the weighted closeness measure of type I and type II, then

$$\mathfrak{M}_I^{\varsigma}(\Upsilon_i) = \frac{1-\mathfrak{C}^{\varsigma}(\Upsilon_i, \Upsilon_-)}{2-\mathfrak{C}^{\varsigma}(\Upsilon_i, \Upsilon_+)-\mathfrak{C}^{\varsigma}(\Upsilon_i, \Upsilon_-)} \tag{12}$$

$$\mathfrak{M}_{II}^{\varsigma}(\Upsilon_i) = \frac{1+\mathfrak{C}^{\varsigma}(\Upsilon_i, \Upsilon_+)}{2+\mathfrak{C}^{\varsigma}(\Upsilon_i, \Upsilon_+)+\mathfrak{C}^{\varsigma}(\Upsilon_i, \Upsilon_-)}. \tag{13}$$

It is assumed that the denominators of equations (12) and (13) are non-zero.

Theorem 5 The weighted type I closeness measure $\mathfrak{M}_I^{\varsigma}(\Upsilon_i)$ satisfies following properties for each neutrosophic characteristic Υ_i in the neutrosophic decision matrix $\mathbb{M}$:

(i) $0 \leq \mathfrak{M}_1^{\varsigma}(\Upsilon_i) \leq 1$

(ii) $\mathfrak{M}_I^{\varsigma}(\Upsilon_i) = 1$ if $\mathfrak{C}^{\varsigma}(\Upsilon_i, \Upsilon_+) = 1$

(iii) $\mathfrak{M}_I^{\varsigma}(\Upsilon_i) = 0$ if $\mathfrak{C}^{\varsigma}(\Upsilon_i, \Upsilon_-) = 1$

(iv) $\mathfrak{M}_I^{\varsigma}(\Upsilon_-) = 0$ if $\mathfrak{M}_1^{\varsigma}(\Upsilon_+) = 1$

(v) $\mathfrak{M}_I^{\varsigma}(\Upsilon_i) = \mathfrak{M}_I(\Upsilon_i)$ if $\varsigma = (1/n, 1/n, ..., 1/n)$

Proof. From previous theorems, we know that $-1 \leq \mathfrak{C}(\Upsilon_i, \Upsilon_+) \leq 1$ and $-1 \leq \mathfrak{C}(\Upsilon_i, \Upsilon_-) \leq 1$. So, $0 \leq 1 - \mathfrak{C}(\Upsilon_i, \Upsilon_+) \leq 2$, $0 \leq 1 - \mathfrak{C}(\Upsilon_i, \Upsilon_-) \leq 2$ and $0 \leq 2 - \mathfrak{C}(\Upsilon_i, \Upsilon_+) - \mathfrak{C}(\Upsilon_i, \Upsilon_-) \leq 4$. Therefore, $0 \leq \mathfrak{M}_I^{\varsigma}(\Upsilon_i) \leq 1$. Hence (i) is true.
The proofs of (ii), (iii), (iv), and (v) are obvious.

Theorem 6 The weighted type II closeness measure $\mathfrak{M}_{II}^{\varsigma}(\Upsilon_i)$ satisfies following properties for each neutrosophic characteristic Υ_i in the neutrosophic decision matrix $\mathbb{M}$:

(i) $0 \leq \mathfrak{M}_{II}^{\varsigma}(\Upsilon_i) \leq 1$

(ii) $\mathfrak{M}_{II}^{\varsigma}(\Upsilon_i) = 0$ if $\mathfrak{C}^{\varsigma}(\Upsilon_i, \Upsilon_+) = -1$

(iii) $\mathfrak{M}_{II}^{\varsigma}(\Upsilon_i) = 1$ if $\mathfrak{C}^{\varsigma}(\Upsilon_i, \Upsilon_-) = -1$

(iv) $\mathfrak{M}_{II}^{\varsigma}(\Upsilon_-) \leq \mathfrak{M}_{II}^{\varsigma}(\Upsilon_+)$

(v) $\mathfrak{M}_{II}^{\varsigma}(\Upsilon_i) = \mathfrak{M}_{II}(\Upsilon_i)$ if $\varsigma = (1/n, 1/n, ..., 1/n)$

Proof. From previous theorems, we know that $-1 \leq \mathfrak{C}(\Upsilon_i, \Upsilon_+) \leq 1$ and $-1 \leq \mathfrak{C}(\Upsilon_i, \Upsilon_-) \leq 1$. So, $-2 \leq \mathfrak{C}(\Upsilon_i, \Upsilon_+) + \mathfrak{C}(\Upsilon_i, \Upsilon_-) \leq 2$ and $0 \leq 2 + \mathfrak{C}(\Upsilon_i, \Upsilon_+) + \mathfrak{C}(\Upsilon_i, \Upsilon_-) \leq 4$. Therefore, $0 \leq \mathfrak{M}_{II}^{\varsigma}(\Upsilon_i) \leq 1$. Hence (i) is true.
The proofs of (ii), (iii), (iv), and (v) are obvious.

Definition 10 Weighted index value: Let ε, $0 \leq \varepsilon \leq 1$ be a closeness parameter, then the weighted index value $\mathscr{I}^{\varsigma}(\Upsilon_i)$ of Υ_i is given by:

$$\mathscr{I}^{\varsigma}(\Upsilon_i) = \varepsilon \mathfrak{M}_I^{\varsigma}(\Upsilon_i) + (1 - \varepsilon)\mathfrak{M}_{II}^{\varsigma}(\Upsilon_i) \tag{14}$$

Theorem 7 The weighted index value $I^{\varsigma}(\Upsilon_i)$ fulfills the following properties for each neutrosophic characteristic Υ_i in the neutrosophic decision matrix $\mathbb{M}$.

(i) $0 \leq \mathscr{I}^{\varsigma}(\Upsilon_i) \leq 1$

(ii) $\mathscr{I}^{\varsigma}(\Upsilon_i) = \mathfrak{M}_I^{\varsigma}(\Upsilon_i)$ if $\varepsilon = 1$

(iii) $\mathscr{I}^{\varsigma}(\Upsilon_i) = \mathfrak{M}_{II}^{\varsigma}(\Upsilon_i)$ if $\varepsilon = 0$

(iv) $\mathscr{I}^{\varsigma}(\Upsilon_-) \leq \mathscr{I}^{\varsigma}(\Upsilon_+)$

(v) $\mathscr{I}^{\varsigma}(\Upsilon_i) = \mathscr{I}(\Upsilon_i)$ if $\varsigma = (1/n, 1/n, ..., 1/n)$

Proof. According to equation (13), since both $0 \leq \mathfrak{M}_I \leq 1$ and $0 \leq \mathfrak{M}_{II} \leq 1$ it is evident that $0 \leq \mathscr{I}^{\varsigma}(\Upsilon_i) \leq 1$. This proves (i). It is obvious to prove (ii) and (iii).

For (iv), it is already proven that $\mathfrak{M}_{II}(\Upsilon_-) \leq \mathfrak{M}_{II}(\Upsilon_+)$. Hence, $\mathscr{I}^{\varsigma}(\Upsilon_-) \leq \mathscr{I}^{\varsigma}(\Upsilon_+)$. Similarly, the proof of (v) is valid.

5.1 *Algorithm of the proposed neutrosophic TOPSIS*

The steps of the proposed neutrosophic TOPSIS are as follows:

Step I: Identify m alternatives $\Upsilon = \{\Upsilon_1, \Upsilon_2, ..., \Upsilon_m\}$ and n criteria $\Psi = \{\Psi_1, \Psi_2, ..., \Psi_n\}$, partitioned into Ψ_B and Ψ_C to contract a MCDM problem.

Step II: Formulate a neutrosophic decision matrix $\mathbb{M} = (m_{ij})_{m \times n}$ (According to equation (2)) using the neutrosophic rating m_{ij} from Table 2.

Step III: Convert the neutrosophic decision matrix $\mathbb{M}$ to a crisp decision matrix using equation (1). Apply the MEREC approach to compute the set of criteria weights $\varsigma = \{\varsigma_1, \varsigma_2, ..., \varsigma_n\}$.

Step IV: Normalize the neutrosophic decision matrix $\mathbb{M}$ according to vector normalization and identify the neutrosophic PIS Υ_+ and neutrosophic NIS Υ_- using the equations (8) and (9).

Step V: Compute the membership component $\rho_{\xi}^{\varsigma}(\Upsilon_i, \Upsilon_+)$, $\rho_{\xi}^{\varsigma}(\Upsilon_i, \Upsilon_-)$ and the hesitancy component $\rho_{\zeta}^{\varsigma}(\Upsilon_i, \Upsilon_+)$, $\rho_{\zeta}^{\varsigma}(\Upsilon_i, \Upsilon_-)$ and the non-membership component $\rho_{\kappa}^{\varsigma}(\Upsilon_i, \Upsilon_+)$, $\rho_{\kappa}^{\varsigma}(\Upsilon_i, \Upsilon_-)$ of the weighted correlation coefficient using the equations (5), (6) and (7) for every $\Upsilon_i \in \Upsilon$.

Step VI: Apply equation (4) to calculate the weighted neutrosophic correlation coefficient $\vartheta^{\varsigma}(\Upsilon_i, \Upsilon_+)$ and $\vartheta^{\varsigma}(\Upsilon_i, \Upsilon_-)$ between Υ_i, Υ_+, and Υ_i, Υ_- respectively for every $\Upsilon_i \in \Upsilon$.

Step VII: Use equations (12), (13) to calculate the weighted Type I and Type II closeness measure $\mathfrak{M}_I^{\varsigma}(\Upsilon_i)$ and $\mathfrak{M}_{II}^{\varsigma}(\Upsilon_i)$ for every $\Upsilon_i \in \Upsilon$.

Step VIII: Assign closeness parameter ε, $0 \leq \varepsilon \leq 1$ and compute the weighted index value $\mathscr{I}^{\varsigma}(\Upsilon_i)$ using equation (14) for each $\Upsilon_i \in \Upsilon$.

Step IX: Rank the m options in decreasing order of $\mathscr{I}^{\varsigma}(\Upsilon_i)$ values.

Table 2. Linguistic scale and their SVNS rating

Linguistic Variable	SVNS
Extremely significant (ES)	(0.92, 0.11, 0.12)
Very Significant (VS)	(0.78, 0.21, 0.23)
Significant (S)	(0.64, 0.32, 0.35)
Reasonable (R)	(0.50, 0.50, 0.50)
Insignificant (I)	(0.36, 0.38, 0.68)
Very Insignificant (VI)	(0.22, 0.27, 0.82)
Extremely Insignificant (EI)	(0.11, 0.21, 0.93)

6. Numerical example: Comparison of health coverage company

An MCDM problem is developed so that health insurance providers can examine their performance while considering numerous significant parameters. The proposed neutrosophic correlation-based TOPSIS approach is utilized to solve the MCDM problem.

6.1 *Problem description*

This part addresses a decision-making problem in selecting healthcare assurance providers. An insurance policy is connected to several characteristics, including flexible premium payment choices, a wide range of coverage options, a reputable insurance firm with a high claim settlement ratio, a reasonable premium, and outstanding client feedback. Several researchers work with a variety of these types of criteria. Here are a few criteria often examined in the literature on health insurance. The impact of the criterion premium cost and the number of network hospitals were examined by Ecer and Pamucar [26]. The claim settlement ratio [19] serves as an indicator of the brand strength of any company. Erdebilli et al. [22] examine the criterion brand-strength and analyze its influence on purchasers. Yang et al. [23] work with the criterion of the company's local reputation. The company has many branches that facilitate its operations. The whole amount of assets that the firm has is used to determine its solvency ratio. Gharizadeh et al. [18] work with total assets and several sites. Twelve criteria, including some new ones, have been carefully considered and analyzed for this problem.

Claim settlement ratio (Ψ_1): An insurance provider's efficacy can be assessed by examining its claim settlement ratio, which represents the proportion of settled claims among the total cases filed.

Solvency ratio (Ψ_2): This ratio may be used to assess an organization's capacity to satisfy its future responsibilities and financial liabilities.

Number of network hospitals (Ψ_3): Every insurance provider works in conjunction with several hospitals. Customers will receive the service at the designated number of hospitals at the time of their claim.

Number of branches (Ψ_4): The organization offers its services through branches spread around the world. The number of branches is an essential criterion.

Coverage area (Ψ_5): When travelling or residing in many locations, it is crucial to determine if the insurance company offers nationwide or international coverage.

Renewal process (Ψ_6): The company's renewal procedure is a component that allows them to offer their services. To make it easy for customers, the process should be flexible.

Maternity and pediatric care (Ψ_7): For individuals desiring to have children, the policy provides coverage for maternity care and pediatric services.

limitations (Ψ_8): The supplier's policy is linked to the many limitations and obstacles imposed by the provider.

Premium (Ψ_9): The insurance company must pay the sum to remain eligible for health coverage.

Deductible (Ψ_{10}): A person's out-of-pocket spending for covered medical bills is the deductible. This amount must be paid before insurance begins paying for those expenses.

Claim settlement process (Ψ_{11}): There are various steps in the health insurance claim settlement procedure to get payment for covered medical expenditures. A greater number of steps in the process causes trouble for the clients.

Customer grievance (Ψ_{12}): A grievance is a way for clients to convey their displeasure or complaint with a company's background, service, or goods.

The first seven of the twelve criteria are categorized as beneficial, highlighting their favorable nature or good influence out of the total twelve. On the other hand, the remaining five criteria are classified as non-beneficial, indicating factors that would not favorably impact the evaluation or objective as a whole.

At this point, we are considering twelve HIPs: $\Upsilon = \{\Upsilon_1, \Upsilon_2, \Upsilon_3, \Upsilon_4, \Upsilon_5, \Upsilon_6, \Upsilon_7, \Upsilon_8, \Upsilon_9, \Upsilon_{10}, \Upsilon_{11}, \Upsilon_{12}\}$ as alternatives for the implementation of our suggested neutrosophic TOPSIS framework in a fuzzy environment.
The description of the providers is as follows: Υ_1: This supplier has a good claim settlement and solvency ratio. However, network hospitals and branches are limited. Υ_2: This supplier has an excellent solvency ratio and handles maternity circumstances. The price of the premium is fair. Υ_3: The claim settlement procedure is lengthy, and the organization has positive relationships with many hospitals. Υ_4: This provider has insufficient numbers of connected hospitals and branches. Care for clients is acceptable. Υ_5: It is good for this provider to have a high solvency ratio and an easy renewal process. Easy settlement and a modest premium attract clients. υ_6: This provider has a high claim settlement ratio and provides excellent pediatric care. This company has a straightforward claim payment method and low pocket spending. Υ_7: This company has a good name but only has a few offices and different places where claims are settled. Υ_8: This provider with many branches has plenty of settlement and solvency ratios. However, there is no flexibility in the renewal procedure. Υ_9: Almost all factors are statistically significant on average. Υ_{10}: Settlement and solvency ratios are acceptable for this provider, whereas the number of branches is unsatisfactory. Υ_{11}: This service has a great name, but the stability ratio is insufficient. Υ_{12}: The corporation has a big branch network. The settlement procedure consists of many phases to accomplish a claim.

6.2 *Solution procedure by the proposed method*

Step I: The twelve HIPs $\Upsilon = \{\Upsilon_1, \Upsilon_2, \Upsilon_3, \Upsilon_4, \Upsilon_5, \Upsilon_6, \Upsilon_7, \Upsilon_8, \Upsilon_9, \Upsilon_{10}, \Upsilon_{11}, \Upsilon_{12}\}$ are identified and their twelve criteria are $\Psi = \{\Psi_1, \Psi_2, \Psi_3, \Psi_4, \Psi_5, \Psi_6, \Psi_7, \Psi_8, \Psi_9, \Psi_{10}, \Psi_{11}, \Psi_{12}\}$, where $\Psi_B = \{\Psi_1, \Psi_2, \Psi_3, \Psi_4, \Psi_5, \Psi_6, \Psi_7\}$ and $\Psi_C = \{\Psi_8\ \Psi_9, \Psi_{10}, \Psi_{11}, \Psi_{12}\}$.

Step II: The neutrosophic decision matrix $\mathbb{M} = (m_{ij})_{12\times 12}$ is constructed according to equation (2), considering the neutrosophic rating from Table 2.

Step III: With the help of equation (1), the crisp decision matrix be represent as

	Ψ_1	Ψ_2	Ψ_3	Ψ_4	Ψ_5	Ψ_6	Ψ_7	Ψ_8	Ψ_9	Ψ_{10}	Ψ_{11}	Ψ_{12}
Υ_1	0.783	0.895	0.465	0.480	0.663	0.783	0.480	0.440	0.465	0.480	0.663	0.500
Υ_2	0.663	0.783	0.465	0.465	0.500	0.663	0.895	0.663	0.500	0.500	0.895	0.663
Υ_3	0.895	0.663	0.895	0.465	0.663	0.500	0.480	0.663	0.480	0.663	0.895	0.500
Υ_4	0.663	0.480	0.465	0.465	0.480	0.480	0.783	0.783	0.500	0.480	0.663	0.663
Υ_5	0.480	0.783	0.480	0.480	0.500	0.783	0.480	0.663	0.480	0.663	0.480	0.500
Υ_6	0.783	0.663	0.500	0.465	0.500	0.500	0.895	0.500	0.895	0.480	0.480	0.663
Υ_7	0.895	0.663	0.480	0.440	0.663	0.663	0.480	0.500	0.500	0.500	0.895	0.663
Υ_8	0.895	0.783	0.663	0.895	0.500	0.480	0.480	0.500	0.500	0.663	0.663	0.783
Υ_9	0.500	0.663	0.465	0.663	0.663	0.663	0.480	0.500	0.500	0.500	0.895	0.663
Υ_{10}	0.783	0.895	0.465	0.480	0.500	0.500	0.480	0.500	0.500	0.480	0.663	0.500
Υ_{11}	0.783	0.440	0.465	0.663	0.663	0.500	0.480	0.500	0.500	0.500	0.895	0.500
Υ_{12}	0.440	0.500	0.440	0.895	0.500	0.663	0.480	0.500	0.440	0.480	0.895	0.500

Applying MEREC approach, the corresponding criteria weights are as follows: $\varsigma_1 = 0.135$, $\varsigma_2 = 0.123$, $\varsigma_3 = 0.043$, $\varsigma_4 = 0.067$, $\varsigma_5 = 0.045$, $\varsigma_6 = 0.059$, $\varsigma_7 = 0.044$, $\varsigma_8 = 0.103$, $\varsigma_9 = 0.166$, $\varsigma_{10} = 0.068$, $\varsigma_{11} = 0.060$, $\varsigma_{12} = 0.087$.

Step IV: The neutrosophic PIS Υ_+ and the neutrosophic NIS Υ_- are provided in the Table 4.

Step V: The weighted correlation components for membership, hesitant, and non-membership functions of neutrosophic PIN and NIS are shown in the Table 5.

Step VI: Weighted neutrosophic correlation coefficients are computed using equation 4 and provided in Table 6.

Step VII: Weighted type I and type II closeness measures are calculated with the help of equations (12) and (13) and provided in Table 6.

Step VIII: To get the weighted index value, we use $\varepsilon = 0.5$ to lend equal emphasis to type I and type II closeness measures.

Step IX: The ranking order of the twelve alternatives is shown in Table 7, with the index values arranged in decreasing order.

Table 3. Linguistic decision matrix based on neutrosophic rating

Criteria $\Longrightarrow$ / Alternatives $\downarrow$	Ψ_1	Ψ_2	Ψ_3	Ψ_4	Ψ_5	Ψ_6	Ψ_7	Ψ_8	Ψ_9	Ψ_{10}	Ψ_{11}	Ψ_{12}
Υ_1	VS	ES	VI	I	S	VS	I	EI	VI	I	S	R
Υ_2	S	VS	VI	VI	R	S	ES	S	R	R	ES	S
Υ_3	ES	S	ES	VI	S	R	I	S	I	S	ES	R
Υ_4	S	I	VI	VI	I	I	VS	VS	R	I	S	S
Υ_5	I	VS	I	I	R	VS	I	S	I	S	I	R
Υ_6	VS	S	R	VI	R	R	ES	R	ES	I	I	S
Υ_7	ES	S	I	EI	S	S	I	R	R	R	ES	S
Υ_8	ES	VS	S	ES	R	I	I	R	R	S	S	VS
Υ_9	R	S	VI	S	S	S	I	R	R	R	ES	S
Υ_{10}	VS	ES	VI	I	R	R	I	R	R	I	S	R
Υ_{11}	VS	EI	VI	S	S	R	I	R	R	R	ES	R
Υ_{12}	EI	R	EI	ES	R	S	I	R	EI	I	ES	R

Table 4. Neutrosophic PIS Υ_+ and Neutrosophic NIS Υ_-

Solutions	Components	Ψ_1	Ψ_2	Ψ_3	Ψ_4	Ψ_5	Ψ_6	Ψ_7	Ψ_8	Ψ_9	Ψ_{10}	Ψ_{11}	Ψ_{12}
Υ_+	ξ_{+j}	0.370	0.391	0.640	0.522	0.334	0.385	0.494	0.589	0.461	0.623	0.652	0.617
	ζ_{+j}	0.119	0.110	0.103	0.110	0.218	0.157	0.094	0.857	0.925	0.775	0.875	0.854
	κ_{+j}	0.083	0.080	0.048	0.053	0.218	0.145	0.058	0.868	0.942	0.818	0.902	0.844
Υ_-	ζ_{-j}	0.044	0.047	0.076	0.062	0.188	0.177	0.193	0.942	0.936	0.788	0.864	0.755
	ζ_{-j}	0.540	0.499	0.469	0.380	0.340	0.374	0.325	0.659	0.660	0.649	0.567	0.651
	κ_{-j}	0.645	0.619	0.373	0.407	0.424	0.428	0.330	0.466	0.548	0.646	0.445	0.660

The weighted closeness index ε is taken at 0.5 to find the optimal preference order of the alternatives. Since the weighted closeness index $\varepsilon = 0.5$, i.e. the DM gives equal importance to the weighted closeness measure of type I and Type II. The best option is Υ_2, and the worst is Υ_1 according to the proposed neutrosophic TOPSIS approach.

Table 5. Correlation coefficient components for neutrosophic PIS and NIS

Solution	Components	Υ_1	Υ_2	Υ_3	Υ_4	Υ_5	Υ_6	Υ_7	Υ_8	Υ_9	Υ_{10}	Υ_{11}	Υ_{12}
PIS	$\rho_{\zeta}^{\xi}(\Upsilon_i, \Upsilon_+)$	-0.691	0.476	0.150	0.231	-0.266	0.600	0.308	0.574	0.381	-0.044	0.158	-0.626
	$\rho_{\xi}^{\xi}(\Upsilon_i, \Upsilon_+)$	0.496	0.052	0.360	-0.075	-0.118	0.519	-0.204	-0.228	-0.251	-0.793	-0.442	0.437
	$\rho_{\kappa}^{\xi}(\Upsilon_i, \Upsilon_+)$	-0.809	0.676	0.075	0.261	-0.331	0.462	0.494	0.405	0.488	-0.112	0.092	-0.470
NIS	$\rho_{\zeta}^{\xi}(\Upsilon_i, \Upsilon_-)$	0.877	-0.621	-0.075	-0.445	0.252	-0.477	-0.410	-0.234	-0.364	-0.027	-0.234	0.639
	$\rho_{\xi}^{\xi}(\Upsilon_i, \Upsilon_-)$	-0.212	-0.158	-0.478	0.277	0.440	-0.025	-0.161	-0.028	0.410	0.359	-0.027	-0.105
	$\rho_{\kappa}^{\xi}(\Upsilon_i, \Upsilon_-)$	0.478	-0.643	-0.378	0.017	0.444	-0.116	-0.525	-0.254	-0.266	-0.097	0.157	0.556

Table 6. Correlation coefficients and closeness measures

Alternatives	$\vartheta^{\varsigma}(\Upsilon_i, \Upsilon_+)$	$\vartheta^{\varsigma}(\Upsilon_i, \Upsilon_-)$	$\mathfrak{M}_I(\Upsilon_i)$	$\mathfrak{M}_{II}(\Upsilon_i)$
Υ_1	-0.335	0.381	0.317	0.325
Υ_2	0.401	-0.474	0.711	0.727
Υ_3	0.195	-0.310	0.619	0.634
Υ_4	0.139	-0.050	0.550	0.545
Υ_5	-0.238	0.379	0.334	0.356
Υ_6	0.527	-0.206	0.718	0.658
Υ_7	0.199	-0.365	0.630	0.654
Υ_8	0.250	-0.172	0.610	0.602
Υ_9	0.206	-0.074	0.575	0.565
Υ_{10}	-0.317	0.078	0.412	0.388
Υ_{11}	-0.064	-0.034	0.493	0.492
Υ_{12}	-0.220	0.363	0.343	0.364

Table 7. HIP alternative index values and ranking

Alternatives	Υ_1	Υ_2	Υ_3	Υ_4	Υ_5	Υ_6	Υ_7	Υ_8	Υ_9	Υ_{10}	Υ_{11}	Υ_{12}
$\mathscr{I}^{\varsigma}$	0.321	0.719	0.627	0.547	0.345	0.688	0.642	0.606	0.570	0.400	0.493	0.354
Ranking	12	1	4	7	11	2	3	5	6	9	8	10

6.3 *Comparison study*

To test the merit and consistency of the suggested technique, the acquired result is compared to similar methods such as TOPSIS, VIKOR, WASPAS, and COPRAS. We convert the neutrosophic decision matrix to a crisp decision matrix using equation (1) to apply these approaches. The exact weight set of the criteria has been used as the MEREC approach in Step III of Section 6.2 does. For all approaches, we used vector normalization for the similarity of the comparison. Table 8 displays the results of different approaches.

Table 8. Index values of HIP alternatives

Alternatives	Υ_1	Υ_2	Υ_3	Υ_4	Υ_5	Υ_6	Υ_7	Υ_8	Υ_9	Υ_{10}	Υ_{11}	Υ_{12}
TOPSIS	0.38841	0.40229	0.42577	0.33610	0.32161	0.61210	0.40120	0.47126	0.29365	0.39486	0.31522	0.24841
VIKOR	0.92943	0.49704	0.60326	0.33062	0.53614	0.09799	0.59968	0.57092	0.48946	0.66734	0.54316	0.67091
WASPAS	0.39253	0.26313	0.28243	0.19857	0.25639	0.23658	0.30294	0.29179	0.26018	0.32919	0.28101	0.26487
COPRAS	0.48899	0.50490	0.50870	0.48189	0.47831	0.52004	0.49695	0.51899	0.48285	0.48305	0.47542	0.46234
Proposed method	0.32104	0.71908	0.62665	0.54746	0.34495	0.68806	0.64212	0.60566	0.57013	0.39986	0.49262	0.35354

The outcomes displayed in Table 8 indicate that the TOPSIS approach assigned the highest rank to HIP Υ_6 and the lowest rank to HIP Υ_{12}. In both VIKOR and WASPAS, the HIP Υ_1 is deemed the most favourable option; however, variations in the least favourable HIPs exist. HIP Υ_8 is assigned the highest ranking, whereas HIP Υ_{12} is positioned last, aligning with the outcome of the TOPSIS methodology. The proposed neutrosophic TOPSIS method assigns the highest priority to the HIP Υ_2 and the lowest priority to the HIP Υ_1. The TOPSIS method determines the ranking order primarily by the claim settlement ratio for HIP features, maternity and pediatric care, and the claim settlement process. The criteria for ranking orders in the WASPAS and VIKOR approaches are the renewal process and solvency ratio. When ranking HIPs, the COPRAS method considers the following factors: claim settlement ratio, solvency ratio, premium amount, and customer complaints. The suggested method ranks the HIPs while considering the quality of maternity and pediatric treatment, service limits, and indifference to the claim settlement procedure. The index value comparison of the HIPs in these approaches is depicted in Figure 2.

It is visible in Figure 2 that the VIKOR approach exhibits the most significant variation in index values, whereas the COPRAS approach demonstrates the slightest variation. The moderate variation in index values for HIPs between the two TOPSIS approaches demonstrates that the ranking variation of TOPSIS and the proposed neutrosophic TOPSIS are comparable. Considerable variability is evident in the index values of HIP Υ_1 and Υ_6. The HIP with the slightest variation in index values is Υ_{11}, whereas the index values of the remaining HIPs vary to a moderate degree. Despite the similarity in outcomes between the crisp techniques, discernible discrepancies persist in their index values. The potential reason for the discrepancy between the proposed and crisp approaches is that the more comprehensive evaluation criteria merit. Table 9 presents the ranking of the HIP alternatives.

The ranking order of the alternative in the proposed neutrosophic TOPSIS shows that the second alternative Υ_2 ranked top and the first alternative Υ_1 ranked last. The reason is the comparatively better rating of the Υ_2 alternatives in the cost-based criteria than the benefit-based. This fact reveals that customers give more importance to premiums, claim settlement processes, and customer grievances than to claim settlement ratios, company networks, and the number of branches.

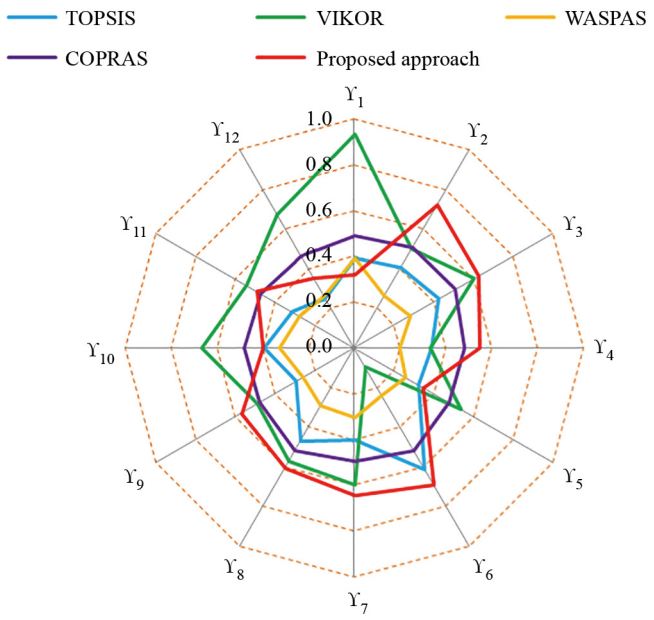

Figure 2. A comparison of index values of the HIPs alternatives

Table 9. Ranking comparison of the suggested and existing approaches

Approach	Results
TOPSIS	$\Upsilon_6 \succ \Upsilon_8 \succ \Upsilon_3 \succ \Upsilon_2 \succ \Upsilon_7 \succ \Upsilon_{10} \succ \Upsilon_1 \succ \Upsilon_4 \succ \Upsilon_5 \succ \Upsilon_{11} \succ \Upsilon_9 \succ \Upsilon_{12}$
VIKOR	$\Upsilon_1 \succ \Upsilon_{12} \succ \Upsilon_{10} \succ \Upsilon_3 \succ \Upsilon_7 \succ \Upsilon_8 \succ \Upsilon_{11} \succ \Upsilon_5 \succ \Upsilon_2 \succ \Upsilon_9 \succ \Upsilon_4 \succ \Upsilon_6$
WASPAS	$\Upsilon_1 \succ \Upsilon_{10} \succ \Upsilon_7 \succ \Upsilon_8 \succ \Upsilon_3 \succ \Upsilon_{11} \succ \Upsilon_{12} \succ \Upsilon_2 \succ \Upsilon_9 \succ \Upsilon_5 \succ \Upsilon_6 \succ \Upsilon_4$
COPRAS	$\Upsilon_6 \succ \Upsilon_8 \succ \Upsilon_3 \succ \Upsilon_2 \succ \Upsilon_7 \succ \Upsilon_1 \succ \Upsilon_{10} \succ \Upsilon_9 \succ \Upsilon_4 \succ \Upsilon_5 \succ \Upsilon_{11} \succ \Upsilon_{12}$
Proposed method	$\Upsilon_2 \succ \Upsilon_6 \succ \Upsilon_7 \succ \Upsilon_3 \succ \Upsilon_8 \succ \Upsilon_9 \succ \Upsilon_4 \succ \Upsilon_{11} \succ \Upsilon_{10} \succ \Upsilon_{12} \succ \Upsilon_5 \succ \Upsilon_1$

6.4 *Sensitivity analysis*

This section assesses the stability of the proposed neutrosophic TOPSIS technique by analyzing the sensitivity of criterion weight determination and closeness parameter variation.

6.4.1 *Criteria weights variation*

The weights derived using CRiteria Importance Through Intercriteria Correlation (CRITIC) [69] and ranking exponent techniques [70, 71] are compared with the weights by the MEREC method. When applying the ranking exponent technique, the decision expert rates each criterion according to the relevance of the criteria as a whole. According to the significance of the criteria, the decision expert must rank each criterion to apply the ranking method. Using Equation (1), the criteria are ranked based on the average SVNS score across every alternative.

Let r_j and ς_j be the rank and weight of the j^{th} criterion, respectively, where $j = 1, 2, 3, \ldots, n$, then rank exponent weight

$$\varsigma_j = \frac{(n - r_j + 1)^p}{\sum_{j=1}^{n}(n - r_j + 1)^p},\ p \in \Re$$

Table 10 displays the criteria weights determined by MEREC, CRITIC, and rank exponent weight ($p = 0, 0.5, 1, 2$).

Table 10. Criteria weights by several weight determination methods

Criteria	Ψ_1	Ψ_2	Ψ_3	Ψ_4	Ψ_5	Ψ_6	Ψ_7	Ψ_8	Ψ_9	Ψ_{10}	Ψ_{11}	Ψ_{12}
MEREC	0.135	0.123	0.043	0.067	0.045	0.059	0.044	0.103	0.166	0.068	0.060	0.087
CRITIC	0.077	0.066	0.065	0.085	0.095	0.125	0.106	0.059	0.057	0.098	0.092	0.074
$p = 0$	0.083	0.083	0.083	0.083	0.083	0.083	0.083	0.083	0.083	0.083	0.083	0.083
$p = 0.5$	0.113	0.108	0.034	0.084	0.076	0.103	0.090	0.068	0.048	0.059	0.118	0.097
$p = 1$	0.141	0.128	0.013	0.077	0.064	0.115	0.090	0.051	0.026	0.038	0.154	0.103
$p = 2$	0.186	0.154	0.002	0.055	0.038	0.125	0.075	0.025	0.006	0.014	0.222	0.098

There is no similarity in the criteria weights between the MEREC and the CRITIC approaches as seen from Table 10. There are three scenarios identified where the criteria weights for p vary depending on the rank-exponent approach. If p increases, the weights of the criteria Ψ_1, Ψ_2, Ψ_6, Ψ_{11}, and Ψ_{12} also increase. The weight of the criteria Ψ_3, Ψ_5, Ψ_8, Ψ_9, and Ψ_{10} decreases when p increases. But the criteria weights of Ψ_4 and Ψ_7 have mixed variation when p increases. Figure 3 shows these variations in weight determination.

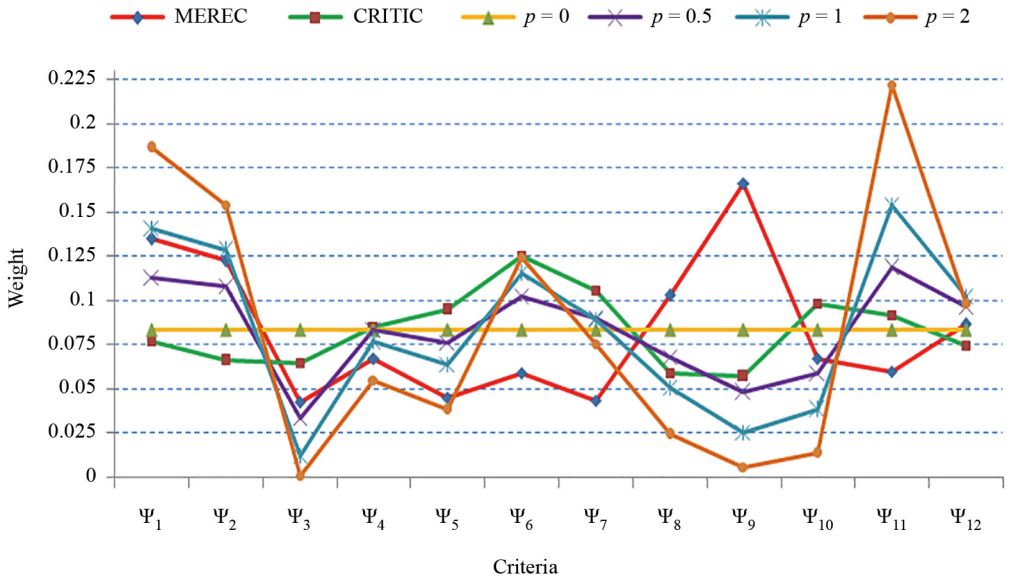

Figure 3. A comparison of the weights of the criterion

The maximum variation in criteria weights is visible from Figure 3 in the rank-exponent approach for $p = 2$. The criteria weights converge to equal weight when p decreases. The criteria weights fluctuate more in the MEREC approach than in the CRITIC approach. The CRITIC method allocates moderate weight to the criteria, whereas the MEREC approach has several ups and downs in the criteria weights. The ranking comparison of the proposed neutrosophic TOPSIS approach for criteria weight variation is shown in Figure 4.

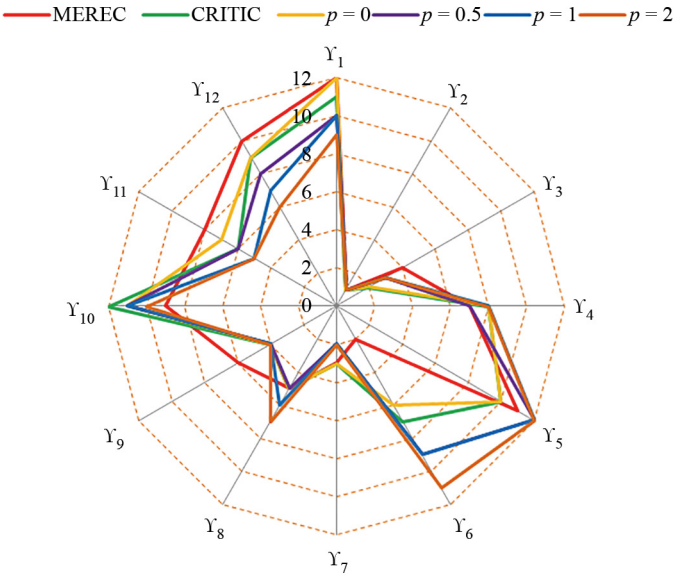

Figure 4. Ranking comparison of the suggested method with varying weight distributions

From Figure 4, it is evident that the ranking of the HIPs is always dominated by the alternative Υ_2 since this HIP consistently ranks first among all the approaches. The second position is obtained by either the seventh or third HIP. The fourth position is almost consistently obtained by Υ_9 HIP. The last phase of the ranking orders is obtained by the HIP alternatives Υ_1, Υ_5, or Υ_{10}. The HIP Υ_5 has maximum fluctuation in ranking from 11th to second. Among these HIPs, the alternatives Υ_2, Υ_7, Υ_4, and Υ_9 have consistency in ranking position among these weight determination processes.

6.4.2 *Closeness parameter variation*

We choose the variation in the closeness parameter ε to check the decision-making in several uncertain situations. The weights of the criteria remain unchanged in this process. The suggested neutrosophic technique is used to establish the HIPs rank sequence, and we observe that the ranking order of the HIPs remains unchanged for ε variation. Table 11 contains an overview of the acquired results.

It is seen from Table 11 that there is a slight variation of the index value $\mathscr{I}^{\varsigma}$ for the variation of ε. Consequently, the ranking sequence of the HIPs is unaffected by ε variation. This scenario happens due to the almost identical values of type I and type II closeness measures. This means the proposed approach slightly depends on DM choices when ranking the alternatives. Figure 5 demonstrates the ranking classification for ε variation.

Table 11. Index values of HIP alternatives for various ε

HIPs	$\varepsilon = 0.1$	$\varepsilon = 0.2$	$\varepsilon = 0.3$	$\varepsilon = 0.4$	$\varepsilon = 0.5$	$\varepsilon = 0.6$	$\varepsilon = 0.7$	$\varepsilon = 0.8$	$\varepsilon = 0.9$
Υ_1	0.324334	0.323512	0.322689	0.321867	0.321044	0.320222	0.319399	0.318577	0.317754
Υ_2	0.725423	0.723839	0.722254	0.720669	0.719085	0.717500	0.715916	0.714331	0.712746
Υ_3	0.632505	0.631041	0.629576	0.628112	0.626647	0.625183	0.623718	0.622253	0.620789
Υ_4	0.545772	0.546194	0.546615	0.547036	0.547457	0.547878	0.548299	0.548720	0.549141
Υ_5	0.353643	0.351470	0.349297	0.347123	0.344950	0.342777	0.340603	0.338430	0.336257
Υ_6	0.663900	0.669939	0.675978	0.682017	0.688056	0.694094	0.700133	0.706172	0.712211
Υ_7	0.651556	0.649196	0.646837	0.644477	0.642118	0.639758	0.637398	0.635039	0.632679
Υ_8	0.602349	0.603177	0.604004	0.604831	0.605659	0.606486	0.607314	0.608141	0.608968
Υ_9	0.566420	0.567348	0.568275	0.569203	0.570131	0.571058	0.571986	0.572913	0.573841
Υ_{10}	0.390324	0.392709	0.395093	0.397478	0.399863	0.402248	0.404633	0.407018	0.409403
Υ_{11}	0.492331	0.492403	0.492476	0.492549	0.492621	0.492694	0.492766	0.492839	0.492912
Υ_{12}	0.361945	0.359845	0.357744	0.355643	0.353543	0.351442	0.349341	0.347240	0.345140

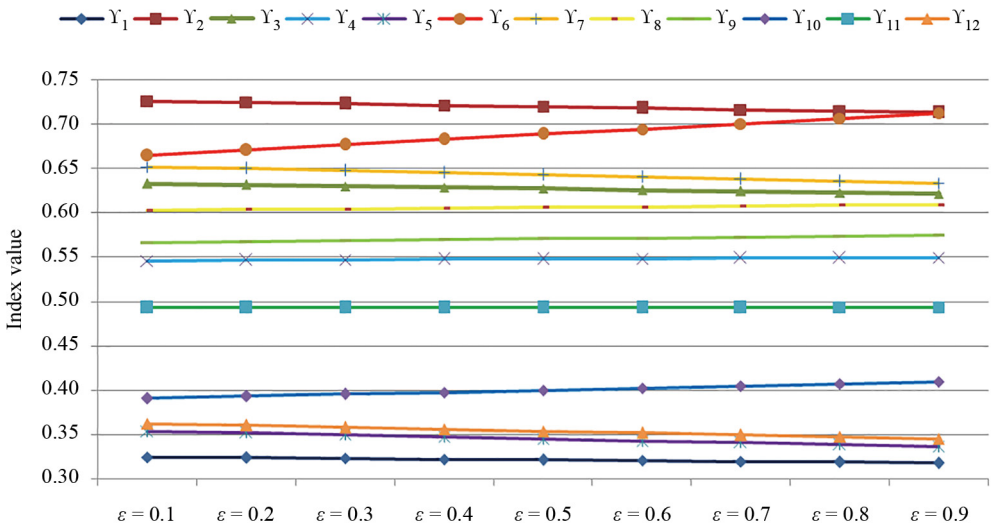

Figure 5. HIP alternatives' index values for different ε

As shown in Figure 5, the index values for most HIPs undergoing ε variation remain unchanged. The index value for HIP Υ_6 exhibits the greatest positive variation; however, this variation is not substantial enough to alter its current ranking order. As ε increases, the index value of HIP Υ_{10} marginally increases, whereas the index values of Υ_5, Υ_7 and Υ_{12} decrease slightly. The ranking positions of the HIP alternatives remain unaffected by this variation.

7. Theoretical and practical contribution

Theoretical contributions in this article include (i) novel neutrosophic correlation coefficients with similar characteristics to classical correlation coefficients. This neutrosophic correlation may be utilized to substitute distance measurements for several MCDM techniques. (ii) An integrated TOPSIS method that incorporates indeterminate information regarding the description of criteria, determination of objective weights, and ranking of alternatives.

The practical implications of this article include (i) health insurance customers' understanding of the characteristics to consider when purchasing health insurance. (ii) The customer is aware of the specific importance of the characteristics of health insurance companies. (iii) The suggested technique assists prospective health insurance customers by giving a ranking of HIPs. (iv) When modifying a plan, the HIP companies may consider client preferences.

8. Conclusion

In this study, we develop a correlation measure of SVNS by considering equal preferences for the components of SVNS. The correlation measure is $[-1, 1]$, corresponding to the traditional correlation coefficient. The weighted closeness measures of type I and type II are proposed based on the weighted correlation measure of the components of SVNS. The weighted closeness index is constructed to extend the TOPSIS technique using weighted closeness measures. The numerical results demonstrate that the proposed approach can solve uncertain MCDM issues. The HIP Υ_2 consistently receives the first position, and Υ_1 acquires the last option. An analysis compared to established methodologies is provided to demonstrate the usefulness and effectiveness of the proposed approach. The sensitivity analysis shows alternative decision-making in uncertain situations.

The suggested methodology offers numerous advantages, including the convergence of the proposed correlation coefficient of SVNS to the classical coefficient, the effective substitution of the distance measure in the TOPSIS approach with the suggested correlation measure, the integration of the weight determination process into the proposed TOPSIS methodology, and the capability of the proposed approach to handle any form of uncertain decision-making problem.

The suggested methodologies have the following limitations: single DM frequently deals with subjective and biased evaluations of the criteria; score function transformation to a crisp decision matrix. Diverse score functions for SVNS can be found in the literature, causing confusion among researchers. Additionally, the closeness parameter is entirely determined by the DM, whose subjectivity may influence the decision-making process.

Based on these limitations, the future research prospects are as follows: The idea of introducing correlation measures may be extended to more powerful fuzzy sets, such as Type II fuzzy sets, hesitant bi-fuzzy sets, complex spherical fuzzy sets, and Diophantine fuzzy sets. In the proposed approach, we determine the criteria of the weights using the MEREC approach, which can be extended by considering either a subjective assessment of the criteria weights or a fuzzy weight determination process. The number of DMs can be increased to make the proposed approach more convenient. A GDM framework can be formulated to solve such decision-making problems by incorporating a panel of DMs. Also, the proposed method can be implemented to solve any real-life MCDM problems, including risk evaluation, green energy selection, supplier selection, the Internet of Things, and renewable energy.

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TEC 21 Model and Critical Thinking: An NCM-based Neutrosophic Analysis in Higher Education

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Abstract: With a growing emphasis on developing transverse competencies in students, many educational institutions are trying to revolutionize the traditional system of education. One among others is TEC de Monterrey in Mexico which refines its educational models so that the students can be nourished with the skills that prepare them not only for the present but for years to come. Among skill development transverse competencies are considered a key to the successful career development of a student that emphasizes on critical thinking aspect. So mainly the the educational models try to inculcate or enhance the critical thinking of students. Towards this goal, TEC introduced the TEC21 model that is oriented towards critical thinking skill development of the students. The current work in this regard utilizes real-time student data collected using the eOpen instrument and takes into consideration experts' opinions to analyze the TEC21 model. The determinate factors are taken from the TEC21 model and indeterminate factors are identified from the literature. As real-time data especially educational data contain much uncertain, indeterminate, and unknown information, therefore this research introduces the use of Neutrosophic Logic to address the uncertainty of the data. Through the use of neutrosophy, we have shown how indeterminate factors when considered for analysis give a better representation of information. We have also compared neutrosophic cognitive maps with used earlier fuzzy cognitive maps to show their effectiveness in this regard. Overall system developed using NCM gives a better analysis of educational models like TEC21. This is the only work now that has utilized neutrosophy to understand and analyze the TEC21 model.

Keywords: Educational Innovation, Higher Education, TEC21 Model, Neutrosophy, Data Science, Critical Thinking

1. Introduction

The need of society pushes towards the development of not only physical infrastructure but also a lot of development is needed towards the development of the human mind for efficient and effective utilization of resources. Keeping this in mind prominent educational institution TEC de Monterrey keeps on enhancing its educational models to better prepare its students for a better

tomorrow. In light of this TEC has proposed an educational model TEC21 which is mainly focused on developing transverse competencies among students [8]. Though this model is in its development phase, here in the present work we try to analyze the model through a very popular approach of neutrosophic cognitive maps (NCMs) [9] [17]. NCMs are the extension of Fuzzy Cognitive Maps with an addition that allows us to quantify the uncertainty that holds a very important position when it comes to modeling uncertain, unknown, and indeterminate factors. Neutrosophy is a branch of philosophy that deals with indeterminacy, contradiction, and incomplete information. The neutrosophic set theory allows for the representation of indeterminate, imprecise, and inconsistent information, which can be useful in situations where traditional binary logic and crisp sets may not fully capture the complexities of the analysis. In a nutshell, we can say that NCM analysis explicitly considers the degree of indeterminacy or ambiguity associated with each factor whereas traditional FCM analysis typically treats factors as either present or absent, without explicitly accounting for degrees of uncertainty. Overall, NCM analysis provides a more flexible and nuanced approach to analyzing the external environment, particularly in situations where information is incomplete or uncertain.

It can help decision-makers better understand the complexities of the external environment and make more informed decisions in response to changes in these factors. Therefore in the present work we have used NCMs to analyze the effect of uncertain and indeterminate factors on the currently running TEC21 model in Tecnologico de Monterrey, Monterrey, Mexico to show how we can take into consideration unknown factors in analyzing any situation. We'll use neutrosophic numbers to represent the degrees of truth, indeterminacy, and falsehood for each factor.

The motivation for employing NCMs for analyzing the TEC 21 model lies in its ability to handle the uncertainties and complexities of the model's external environment. The TEC 21 model operates within a dynamic educational landscape where factors such as technology, policies, and societal trends are constantly evolving. NCM offers a more comprehensive approach by not only considering the presence or absence of these factors but also their degrees of truth, indeterminacy, and falsehood. This nuanced analysis can help in identifying potential risks and opportunities related to the TEC 21 model, enabling organizations to make more informed decisions and develop adaptive strategies. Furthermore, employing a novel approach like NCM can contribute to research and innovation in the field of educational technology, leading to new insights and methodologies for analyzing and improving educational models.

1.1. TEC 21 Model

TEC de Monterrey has always focussed on nourishing young minds to make them aware of what the current world requires. It has always focussed on training our young people and future professionals to have a strong emphasis on addressing the current problems that society demands to address. Tecnologico de Monterrey is known for leading in educational models and innovations. Their new model focuses on practical education, blending theory with skills. Starting in 2019, this model is being gradually implemented in all courses, aligning with Education 4.0. It offers students engaging projects that develop new skills, competencies, and knowledge. The TEC21 Educational Model, implemented on 26 campuses, aims to enhance student´s competitiveness through comprehensive training and active learning activities. It fosters entrepreneurship, leadership, and innovation competencies, with dedicated spaces like libraries, learning commons, and InnovAction Gym for collaborative work. Challenge-based learning is central to the TEC21 model, addressing industry needs and developing digital-based solutions. This model emphasizes the importance of incentives and proper distribution channels for industry collaboration. Academic institutions need to immerse students in real challenges to achieve better results. The TEC21 model focuses on developing competencies for work and lifelong learning, including disciplinary and transversal

competencies necessary for professional practice in various sectors. This goal of TEC has enabled it from time to time to come up with different strategies to refine its education models. In light of this TEC has generated a differentiating strategy in its new TEC21 Educational Model. The new model has its focus on building transverse competencies that future professionals must develop in both the disciplinary and personal aspects. The model's main aim is towards the development of these competencies through four major pillars that promote student transformation. These major pillars are: A) Challenging and interactive learning experiences, B) Flexibility in the teaching-learning process, C) The building of a memorable university experience, and, D) Inspiring innovative teachers [8]. The TEC21 Educational Model integrates the purposes of the vision, defines and links the actors and components that participate in the teaching-learning process and takes advantage of opportunities to offer students a comprehensive education of international quality. Its objective is to provide comprehensive training and improve the competitiveness of students in their professional field by enhancing the skills of future generations to develop the competencies required to enable them to become leaders who will face the challenges and opportunities of the 21st century. In August 2019, the Tecnologico de Monterrey (Tec Mty) began to implement the Tec21 Educational Model [6] [7]. Since its inception, many studies have been conducted to ascertain the objectives of this model. The present work in this regard has focused mainly on the influence of uncertainties and indeterminacy on the successful implementation of this model in TEC de Monterrey. The TEC21 model at Tecnológico de Monterrey is an innovative educational model focused on developing competencies through challenge-based learning. The model emphasizes the following key factors and these are the well-defined and measurable aspects of the TEC21 model. For the current study, they are considered as determinate factors throughout this paper.

These are as follows [8].

- Critical Thinking Skills (C1)
- Problem-Solving Ability (C2)
- Creativity (C3)
- Collaboration (C4)
- Communication Skills (C5)
- Self-Regulation (C6)
- Reflective Thinking (C7)

The rest of the paper is divided into 5 sections. Section 2 and Section 3 present extensive literature surveys and methodology respectively. Section 4 is focused on results and discussion whereas Section 5 presents the conclusion of the present work.

2. Literature Survey

With the growing need to bring evolution to education, there have always been efforts to propose new methodologies for refining the education systems across the globe. These efforts are put not only by the academicians but also a lot more is done by the researchers who continuously study the scenarios and come up with different strategies to propose new methods to refine the education system by taking insights from the real-time data. In light of this many researchers have studied the newly introduced TEC21 model in TEC de Monterrey, Monterrey, Mexico. Hugo A López et al. [10] studied the model and provided a way to integrate Education 4.0 with the present model to prepare students for the needs of Industry 4.0. Their research presented a case study of a Capstone project developed with undergraduate engineering students. Through this, they contributed towards showing that a suitable educational framework is needed for making students capable of addressing the demands of Industry 4.0. Guillermo Gándara Fierro et al. [11] have analyzed this model by taking a case study based on the first semester results. Through this work, they proposed the possibility of

helping the community's transformations. Under the new Tec21 Educational Model, Cintia Smith et al. [12] designed a strategy for the "Citizenship and Technology" course for the students. The design was proposed in a mixed-model scheme through which they detected areas of opportunity that signaled the need to make important adjustments in the activities design. They claimed that these changes in the strategy are needed to fulfill the following four major goals. These goals are to generate evidence of competence development; make adjustments to class dynamics for the correct teaching process, simplification of the evaluation mechanisms, and reconceptualization of special guests profile. Their study forms the basis for showing that the TEC21 model is not static but is open to changes. As the focus of the model is on the development of transverse competencies among students, there is a clear need for the methods for its development. Helena Belchior-Rocha et al. [13] have studied the importance of transverse competencies among the students enrolled in higher education courses. The authors in their latest work have stated that the acquisition of transversal competencies is affected by social, economic, technological, and political changes in the surrounding environment. Jean Cushen [14] evaluated the significance of the development of transversal competencies among the students of higher education. Through a comprehensive evaluation of its importance in the industry, the authors concluded that transversal competencies are expected to play a definitive role in future work scenarios. They analyzed the decisions and impacts surrounding the integration of transversal competencies into higher education assessments. They also focussed on the changes that higher education leaders must make. Jesús García-Álvarez et al. [15] provided a systematic review of the transversal competencies for employability in university graduates from an employer's perspective, with consideration of the importance of the topic in the cross-national context. The authors claimed the importance of these competencies through data collected from published articles from Scopus and Web of Science journals. For this purpose, the authors classified 41 transversal competencies into five dimensions. Hanesová et al. [16] focussed their research mainly on the development of transverse competencies together with taking into consideration the changing the forms of education so that they lead to the development of these competencies. The author's main objective was to design a new framework for mastering transversal competencies in a higher education environment.

Later they proposed that their framework could be updated based on processes of critical thinking and reflection. The extensive literature survey has given us more insights and data that influence the determinate factors in a significant manner. Like authors [1] Deci and Ryan discussed how intrinsic and extrinsic motivations drive student´s behaviors and learning outcomes. They emphasize that motivation is influenced by various internal and external factors, making it a complex and variable aspect of education. This is considered indeterminate as motivation fluctuates based on personal interests, external rewards, and situational factors, introducing uncertainty into its measurement and influence on educational outcomes. On the other hand, Hattie highlights the significant impact of different teaching methods on student achievement [2]. The effectiveness of these methods varies depending on context, student needs, and implementation strategies. This is considered an indeterminate factor in current work as the variability in the teaching method´s effectiveness, dependent on numerous contextual factors, introduces uncertainty in their impact on student learning. Garrison and Kanuka explored the transformative potential of blended learning, which combines online and face-to-face instruction [3]. The impact of technological tools varies based on their integration, student engagement, and technological proficiency. This factor is indeterminate as the effectiveness of technological tools is uncertain due to differences in implementation quality, technological accessibility, and student´s adaptability to technology. The renowned researcher Fraser discusses the development and importance of classroom environment instruments, emphasizing how physical and psychological classroom conditions affect student learning and behavior [4]. They emphasized that the classroom environment is influenced by dynamic interactions between students and teachers, as well as the physical setup, making its impact on

learning unpredictable. Therefore this plays an important role in the successful implementation of any educational model. This can be considered as an indeterminate factor for this study.

Many researchers have emphasized the assessments happening in institutions for the successful implementation of educational models. Black and Wiliam analyzed various assessment techniques and their effects on learning [5]. They highlighted that the effectiveness of assessments depends on their design, implementation, and student´s perceptions. The impact of assessment techniques is indeterminate due to differences in assessment types, teacher practices, and student responses to assessments. Therefore, all these factors are termed as indeterminate or uncertain throughout the study. Below we mention five such factors I1-I5.

- Student Motivation (I1) [1]
- Teaching Methods (I2) [2]
- Technological Tools (I3) [3]
- Classroom Environment (I4) [4]
- Assessment Techniques (I5) [5]

The indeterminate factors I1-I5 are mathematically represented by the neutrosophic sets and systems. Netrosophic theory is not limited to the field of mathematics but it is spreading its wings in various other fields. Researchers around the globe have employed neutrosophic techniques to solve several problems prevailing in the current scenario i.e. in [18] [19] it is being used to solve the problem in multi-criteria decision-making. Authors in [20] have used neutrosophic sets in understanding and enhancing the supply chain sustainability in the current scenario. The proposed approach claims to be efficient in solving decision-making problems while meeting the supply chain sustainability requirement. Authors in [21] used IoT and Fog computing to propose a healthcare system for the prediction and diagnosis of diseases. For this purpose, they introduced a neutrosophic multi-criteria decision-making technique. The above work by prominent researchers proved that the application of neutrosophic theory in various fields of research is the need of the hour and our research methodology for addressing the uncertainty in educational models is based on this.

3. Methodology

The proposed solution to indeterminacy uses the concept of a Neutrosophic Cognitive Map (NCM). It is a technique in Neutrosophy introduced by W. B. Vasantha Kandasamy [23]. The concept of Neutrosophic logic was introduced by Florentine Smarandache [22] [23], which is a merger of fuzzy logic together with the inclusion of indeterminacy.

When data under scrutiny contains indeterminate concepts, we are not able to formulate mathematical expressions. Presentation of Neutrosophic logic by Florentine Smarandache [24] [25] has put forward a panacea to this problem. It is the reason Neutrosophy has been introduced as an additional notion for the evaluation of educational models. Fuzzy theory evaluates the existence or non-existence of associateship but it has been less efficient to attribute the indeterminate relations among concepts but most data collected in the educational setup has many indeterminate and uncertain concepts. Therefore we have employed Neutrosophic Cognitive Maps (NCMs) in place of Fuzzy Cognitive Maps (FCMs) to evaluate the TEC21 model. Earlier research for evaluating the TEC21 model has not included indeterminacy which is a part and parcel of real life. Hence when it comes to assessing the development of transverse competencies and critical thinking by educational models, indeterminacy needs to be considered. Contemplating the importance of indeterminacy we propose to use NCM in evaluating the TEC21 model [8]. Now indeterminacy has been introduced in Fuzzy Cognitive Maps (FCMs) and the generalized structure so obtained is referred to as Neutrosophic Cognitive Maps (NCMs) by W. B. Vasantha Kandasamy [23]. NCM is a neutrosophic directed graph (a directed graph with a dotted edge representing indeterminacy) with concepts represented as nodes of the directed graph and relationship or indeterminacy as the edge of the

graph. Let us suppose C_1, C_2,……, and C_n are n nodes from the Neutrosophic vector space V. The edges of the graph are represented by (x_1,x_2,…..,x_n) where x_i's can be'0' or '1' or 'I' (I shows indeterminacy) where $x_i = 1$ indicates the ON state of the node whereas $x_i = 0$ indicates the OFF state and x_i = I indicates the indeterminate state of a node in that situation. Suppose C_i and C_j are two nodes in this model (NCM), a directed edge from C_i to C_j represents the relationship of C_i and C_j. The edges of the directed graph in NCM are weighted having value in set {-1, 0, 1, I}. When e_{ij} is the weight assigned to the directed edge from C_i to C_j then if the value of e_{ij} is '0' it shows C_i does not affect C_j, it is '1' representing an increase (or decrease) of C_i leads to an increase (or decrease) of C_j, when it is '-1' representing increase (or decrease) of C_i leads decrease (or increase) of C_j and when the value is 'I' it shows effect of C_i on C_j is indeterminate. These NCMs are called simple NCMs. Let N (E) be a matrix defined as N (E) = (e_{ij}) then N (E) is called a Neutrosophic adjacency matrix.

3.1. Neutrosophic Concepts

3.1.1. Neutrosophic Sets:

A neutrosophic set is a generalization of a classical set in which the membership of an element is characterized by three functions: truth membership (T), indeterminacy membership (I), and falsehood membership (F) [25] [26].

It can be said that a neutrosophic set A is defined as:

$$A=\{x|\mu_A(x)=(T(x), I(x), F(x))\},$$

where x is an element of the set and $\mu_A(x)$ represents the membership values of x in set A. It's used to model uncertain survey responses which very much occurs while carrying out surveys for educational research.

3.1.2. Neutrosophic Membership:

Neutrosophic membership values indicate the degree to which an element belongs to a neutrosophic set, taking into account truth, indeterminacy, and falsehood. For an element x in a neutrosophic set A, the neutrosophic membership value is represented as $\mu_A(x)=(T(x), I(x), F(x))$ where $T(x)+I(x)+F(x)\leq 1$ and $T(x)$, $I(x)$, and $F(x)$ are the degrees of truth, indeterminacy, and falsehood, respectively [25]. This can be understood from the following diagram. For example Neutrosophic membership values for a single day with a condition very hot with a bit of uncertainty, membership values = [0.8, 0.1, 0.1] is represented in Figure 1.

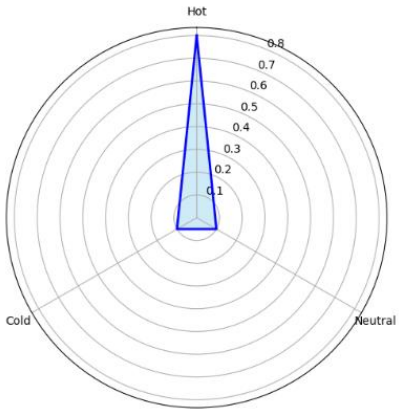

Figure 1 Neutrosophic Membership

3.1.3. Neutrosophic Cognitive Maps (NCMs):

An extension of fuzzy cognitive maps incorporating neutrosophic logic to model complex systems with uncertainties. It can be used to represent and analyze relationships between factors influencing critical thinking. This helps in modeling not only the determinate factors but also indeterminate factors that are also taken into consideration [23]. We can say that in the educational context, the nodes of NCM represent factors like "Innovative Teaching Methods," "Student Engagement," etc., with weighted edges reflecting the strength and uncertainty of their influence.

4. Results

4.1. FCM Adjacency Matrix

The FCM adjacency matrix captures the causal relationships between the determinate factors. Values are assigned based on literature and expert opinions, typically ranging between 0 and 1, where 0 means no influence and 1 means maximum influence. Relationships among Factors in FCM can be understood like C1 (Critical Thinking Skills) has a strong influence on C2 (Problem-Solving Ability) (0.4) and C7 (Reflective Thinking) (0.3). But on the other hand has a moderate influence on C3 (Creativity) (0.3), C4 (Collaboration) (0.2), and C6 (Self-Regulation) (0.2). Likewise, all values are assigned by taking into consideration the expert's opinions and extensive literature survey. These can be seen in the Table 1 and corresponding FCM is shown in Figure 2.

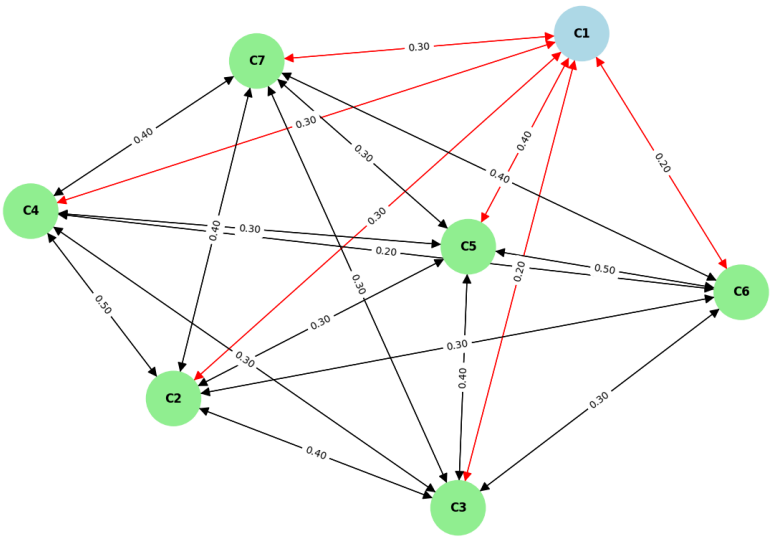

Figure 2 FCM: Factors Relationship to Critical Thinking (C1)

Table 1 Fuzzy Adjacency Matrix: Showing Relationship Among Determinate Factors

Determinate Factors	C1	C2	C3	C4	C5	C6	C7
C1	0	0.4	0.3	0.2	0.1	0.2	0.3
C2	0.3	0	0.5	0.4	0.2	0.3	0.4
C3	0.2	0.4	0	0.3	0.4	0.3	0.2
C4	0.3	0.5	0.3	0	0.4	0.2	0.3
C5	0.4	0.3	0.4	0.3	0	0.5	0.3
C6	0.2	0.3	0.3	0.2	0.5	0	0.4
C7	0.3	0.4	0.3	0.4	0.3	0.4	0

4.2. NCM Adjacency Matrix

The NCM adjacency matrix includes both determinate and indeterminate factors. Indeterminate factors are denoted as 'I' (values between 0 and 1) in the matrix. Relationships among factors in NCM can be understood as C1 (Critical Thinking Skills), Influences C2 (Problem-Solving Ability) (0.4), C3 (Creativity) (0.3), C4 (Collaboration) (0.2), C5 (Communication Skills) (0.1), C6 (Self-Regulation) (0.2), C7 (Reflective Thinking) (0.3), and indeterminate factors I1 (Student Motivation) (0.2**),** I2 (Teaching Methods) (0.1), I3 (Technological Tools)(0.2), I4 (Classroom Environment) (0.1), I5 (Assessment Techniques) (0.2). The NCM graph shown in Figure 3 shows all the nodes whether determinate or indeterminate. All the nodes are in the same color except C1 as it is the primary node that represents critical thinking in the system. This is done to emphasize its central role. In the graphs, each node represents a factor, and the edges represent the relationships between these factors, with the weight of the edges indicating the strength of these relationships. The directed edges indicate the influence from one factor to another. For example, an edge from C1 to C2 with a weight of 0.4 means that C1 influences C2 with a strength of 0.4. There are some thicker edges representing stronger relationships among nodes. For instance, C2 has a strong influence on C3 (weight 0.5) compared to its influence on C1 (weight 0.3). In a uniform color scheme, the emphasis can be placed on the edges connecting to and from C1 to show its influence. As per NCM in Figure 3 the corresponding neutrosophic adjacency matrix is formed in Table 2.

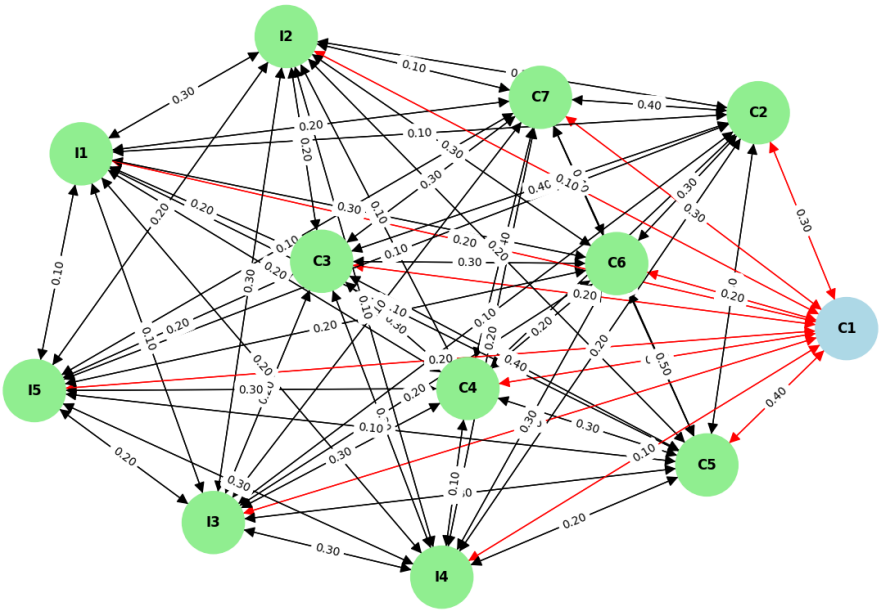

Figure 3 NCM: Factors Relationship to Critical Thinking (C1)

Table 2 Neutrosophy Adjacency Matrix: Showing Relationships Among Determinate and Indeterminate Factors

Determinate & Indeterminate Factors	C1	C2	C3	C4	C5	C6	C7	I1	I2	I3	I4	I5
C1	0	0.4	0.3	0.2	0.1	0.2	0.3	0.2	0.1	0.2	0.1	0.2
C2	0.3	0	0.5	0.4	0.2	0.3	0.4	0.1	0.3	0.1	0.2	0.1
C3	0.2	0.4	0	0.3	0.4	0.3	0.2	0.3	0.2	0.2	0.3	0.2

C4	0.3	0.5	0.3	0	0.4	0.2	0.3	0.2	0.1	0.3	0.1	0.3
C5	0.4	0.3	0.4	0.3	0	0.5	0.3	0.1	0.2	0.3	0.2	0.1
C6	0.2	0.3	0.3	0.2	0.5	0	0.3	0.3	0.3	0.2	0.3	0.2
C7	0.3	0.4	0.3	0.4	0.3	0.4	0	0.2	0.2	0.1	0.2	0.1
I1	0.2	0.1	0.2	0.2	0.1	0.3	0.2	0	0.3	0.1	0.2	0.1
I2	0.1	0.3	0.2	0.1	0.2	0.3	0.1	0.3	0	0.3	0.1	0.2
I3	0.2	0.1	0.2	0.3	0.3	0.2	0.1	0.1	0.3	0	0.3	0.2
I4	0.1	0.2	0.3	0.1	0.2	0.3	0.2	0.2	0.2	0.3	0	0.3
I5	0.2	0.1	0.2	0.3	0.1	0.2	0.1	0.1	0.1	0.2	0.3	0

|

4.3. Iterative Process

The first iteration starts by keeping the critical thinking state as ON rest all other factors are considered null at this time. So the Initial State becomes S(0)=[1,0,0,0,0,0,0,0,0,0,0,0] . The iterations for both FCM and NCM are done separately. The further iterations are carried out using the formula mentioned below. Iterations for FCM:

Iteration 1: S(1)=S(0)×WS(1)=S(0)×W
Iteration 2: S(2)=S(1)×WS(2)=S(1)×W
Iteration 1: S(1)=S(0)×WFCMS(1)=S(0)×WFCM
Iteration 2: S(2)=S(1)×WFCMS(2)=S(1)×WFCM

4.3.1. Iterations Results for FCM:

Initial State Vector V=(1,0,0,0,0,0,01,0,0,0,0,0,0)

1. **State after iteration 1:** (0.4,0.3,0.2,0.3,0.4,0.2,0.30.4,0.3,0.2,0.3,0.4,0.2,0.3)
2. **State after iteration 2:** (0.56,0.63,0.65,0.61,0.57,0.65,0.610.56,0.63,0.65,0.61,0.57,0.65,0.61)
3. **State after iteration 3:** (0.803,0.748,0.767,0.783,0.778,0.766,0.7830.803,0.748,0.767,0.783,0.778,0.766,0.783)
4. **State after iteration 4:** (0.924,0.925,0.93,0.926,0.929,0.93,0.9260.924,0.925,0.93,0.926,0.929,0.93,0.926)
5. **State after iteration 5:** (0.977,0.974,0.976,0.975,0.975,0.976,0.9750.977,0.974,0.976,0.975,0.975,0.976,0.975)
6. **State after iteration 6:** (0.994,0.991,0.992,0.992,0.992,0.992,0.9920.994,0.991,0.992,0.992,0.992,0.992,0.992)
7. **State after iteration 7:** (0.998,0.997,0.997,0.997,0.997,0.997,0.9970.998,0.997,0.997,0.997,0.997,0.997,0.997)
8. **State after iteration 8:** (0.999,0.999,0.999,0.999,0.999,0.999,0.9990.999,0.999,0.999,0.999,0.999,0.999,0.999)
9. **State after iteration 9:** (1.000,0.999,0.999,0.999,0.999,0.999,0.9991.000,0.999,0.999,0.999,0.999,0.999,0.999)

4.3.2. Iterations Results for NCM:

Initial State Vector V=(1,0,0,0,0,0,0,0,0,0,0,01,0,0,0,0,0,0,0,0,0,0,0)

Iterations:

1. **State after iteration**
   1: (0.4,0.3,0.2,0.3,0.4,0.2,0.3,0.2,0.1,0.2,0.1,0.20.4,0.3,0.2,0.3,0.4,0.2,0.3,0.2,0.1,0.2,0.1,0.2)
2. **State after iteration**
   2: (0.8793,0.7827,0.7821,0.7982,0.7686,0.7951,0.8003,0.7824,0.7488,0.7821,0.7488,0.78210.8793, 0.7827,0.7821,0.7982,0.7686,0.7951,0.8003,0.7824,0.7488,0.7821,0.7488,0.7821)
3. **State after iteration**
   3: (0.9352,0.8685,0.8675,0.8836,0.8541,0.8801,0.8856,0.8679,0.8324,0.8675,0.8324,0.86750.9352, 0.8685,0.8675,0.8836,0.8541,0.8801,0.8856,0.8679,0.8324,0.8675,0.8324,0.8675)
4. **State after iteration**
   4: (0.9518,0.8926,0.8910,0.9071,0.8776,0.9042,0.9090,0.8913,0.8578,0.8910,0.8578,0.89100.9518, 0.8926,0.8910,0.9071,0.8776,0.9042,0.9090,0.8913,0.8578,0.8910,0.8578,0.8910)
5. **State after iteration**
   5: (0.9661,0.9161,0.9154,0.9256,0.9066,0.9227,0.9271,0.9155,0.8870,0.9154,0.8870,0.91540.9661, 0.9161,0.9154,0.9256,0.9066,0.9227,0.9271,0.9155,0.8870,0.9154,0.8870,0.9154)
6. **State after iteration**
   6: (0.9743,0.9299,0.9295,0.9358,0.9216,0.9332,0.9372,0.9296,0.9020,0.9295,0.9020,0.92950.9743, 0.9299,0.9295,0.9358,0.9216,0.9332,0.9372,0.9296,0.9020,0.9295,0.9020,0.9295)
7. **State after iteration**
   7: (0.9791,0.9375,0.9372,0.9417,0.9302,0.9395,0.9433,0.9373,0.9115,0.9372,0.9115,0.93720.9791, 0.9375,0.9372,0.9417,0.9302,0.9395,0.9433,0.9373,0.9115,0.9372,0.9115,0.9372)
8. **State after iteration**
   8: (0.9820,0.9419,0.9417,0.9453,0.9352,0.9433,0.9469,0.9418,0.9170,0.9417,0.9170,0.94170.9820, 0.9419,0.9417,0.9453,0.9352,0.9433,0.9469,0.9418,0.9170,0.9417,0.9170,0.9417)
9. **State after iteration**
   9: (0.9839,0.9447,0.9445,0.9474,0.9383,0.9456,0.9491,0.9446,0.9205,0.9445,0.9205,0.94450.9839, 0.9447,0.9445,0.9474,0.9383,0.9456,0.9491,0.9446,0.9205,0.9445,0.9205,0.9445)
10. **State after iteration**
    10: (0.9852,0.9464,0.9463,0.9489,0.9403,0.9472,0.9506,0.9463,0.9226,0.9463,0.9226,0.94630.9852 ,0.9464,0.9463,0.9489,0.9403,0.9472,0.9506,0.9463,0.9226,0.9463,0.9226,0.9463)

The results are stabilized after the 9^{th} iteration in the case of FCM and after the 10^{th} in the case of NCM. When we compare the results obtained we can compare and contrast the values on four very important factors convergence patterns, degree of variability, interpretation of influence, and implications for educational analysis. If we talk about convergence patterns we can say that in the case of FCM, the results after iteration 9 show that all values have converged very closely to 1 (either 1.000 or 0.999).

This indicates a high level of agreement or certainty in the relationships between the factors while in the case of NCM, the results after iteration 10 show values ranging from approximately 0.9226 to 0.9852 demonstrating that while the system has reached a steady state, there remains a greater diversity in the influence values, reflecting the inclusion of indeterminate components. On grounds of the degree of variability, we can conclude that FCM shows a lack of variability (values very close to 1) suggesting that the factors are strongly and uniformly interrelated, with minimal uncertainty while in the case of NCM, the presence of values less than 1 and the variation among them (0.9226 to 0.9852) indicate that the factors have varying degrees of influence on one another, capturing the inherent uncertainties and indeterminate relationships.

That is the main reason that forms the basis for the use of neutrosophy for educational data analysis purposes. If we talk about the interpretation of influence we can notice that FCM suggests a highly deterministic model where each factor strongly influences the others. This deterministic

nature may overlook the nuanced, real-world complexities of educational data. On the other hand, by incorporating indeterminate factors, the NCM results provide a more realistic representation of the educational environment. The varied influence values reflect the complex and uncertain nature of the relationships between the factors. The comparison based on implications for educational analysis forms the basis of conducting this research. It says that close-to-unity values indicate that FCM may be suitable for contexts where relationships are well-defined and less subject to variability. But in the case of educational data, we have a lot of variability. Therefore, the broader range of values suggests that NCM is better suited for analyzing educational models like TEC21 for the development of transverse competencies or critical thinking, where indeterminacies and uncertainties are prevalent. This makes NCM a more robust tool for understanding and managing the complexities inherent in educational data and can be utilized to assess the development of critical thinking skills of the student using TEC21 model.

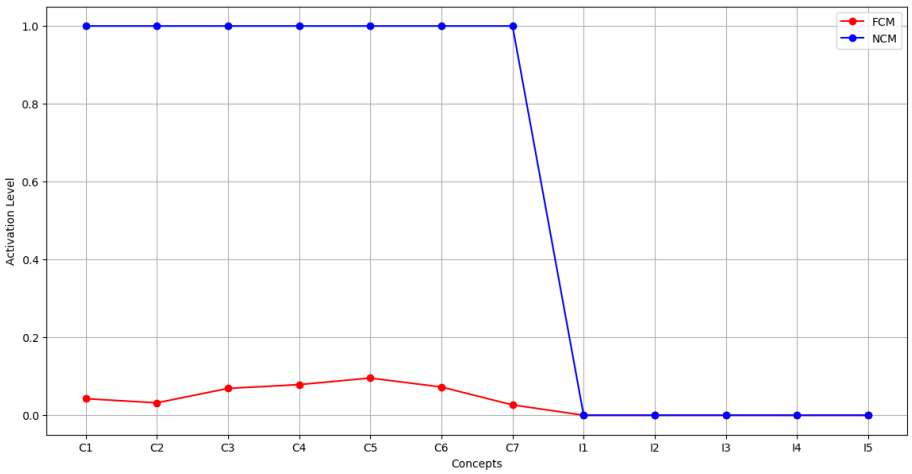

Figure 4 FCM vs NCM: Iterative Results Analysis

The graph in Figure 4 illustrates the activation levels of various concepts after iterative processes in both Fuzzy Cognitive Maps (FCM) and Neutrosophic Cognitive Maps (NCM). The red line represents the FCM results, which show a consistent convergence of activation levels close to 1, indicating stable and deterministic relationships between concepts. The blue line represents the NCM results, which exhibit higher variability and slower convergence, reflecting the inclusion of indeterminate factors and capturing the uncertainties within the system. This comparison highlights that while FCM provides a stable but simplistic view, NCM offers a more nuanced and realistic representation of complex, uncertain environments for assessing the critical thinking skills of the students in the TEC21 model.

Conclusion

In conclusion, the TEC21 model implemented by TEC de Monterrey represents a significant advancement in fostering critical thinking skills among students. By utilizing real-time data collected through the eOpen instrument, this research highlights the importance of addressing the inherent uncertainties and indeterminacies in educational data for determining the critical thinking skills of the students. The application of Neutrosophic Logic in analyzing the TEC21 model has proven to be highly effective, offering a more nuanced and comprehensive representation of the data. The comparative analysis between Neutrosophic Cognitive Maps (NCM) and Fuzzy Cognitive Maps (FCM) further demonstrates the superior capability of NCM in capturing and managing the indeterminate factors that influence educational outcomes. This pioneering work underscores the potential of neutrosophy in enhancing the analysis of educational models also to assess critical

thinking skills, particularly in environments characterized by significant uncertainty. By using this innovative approach, educational institutions can better prepare students with the critical thinking skills necessary for their future success.

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A Generalized Hybrid Distance Measure for SVNS with an Application in Medical Diagnosis

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Abstract: A generalized hybrid distance measure of a single value neutrosophic set (SVNS) is proposed to analyze the patient's factors and diseases. Since Neutrosophic Set (NS) may communicate contradictory and ambiguous information, it is a crucial and useful tool for modelling uncertainty information. A distance measure for NS information is a key tool that is always employed in many approaches including medical diagnostics. Several distance measures in NS can be found in the literature, however, only a few of them proposed hybrid techniques. This paper aims to develop a new distance measure by considering exponential function and to define new hybrid distance measures for NS. The work also includes the formulation of the properties and application in the medical diagnosis for the validity phase. This study examines the possibility of cardiac problems in pregnant women. The eight factors considered in this study are age, obesity, smoking, family pathological history, personal pathological history, electrocardiogram, ultrasound, and functional class. The generalized hybrid distance measure has been used to discuss the factors of six different diseases, present consistent results.

Keywords: Neutrosophic set; distance measure; hybrid distance measure; medical diagnosis

1. Introduction

A neutrosophic set (NS) is a generalization of the concept of a fuzzy set (FS) and intuitionistic fuzzy set (IFS), which allows for the inclusion of uncertainty, indeterminacy, and vagueness. This effective mathematical model was proposed by [1]. An extension of the neutrosophic set is the single-valued neutrosophic set (SVNS)[2]. The SVNS is a generalization of the classic set, FS, and IFS [3]. The IFS has difficulties when it comes to communicating decision-related information. It can be viewed as a level of membership and a level of non-membership that is unable to deal with all kinds of doubts and contradictory information. In order to manage uncertain and inconsistent information,

SVNS includes three terms: truth-membership, indeterminacy-membership, and falsity-membership. Over the years, a lot of information regarding the model measure for the SVNS model has been introduced and discussed. For instance, correlation coefficients, similarity measures, entropy measures, distance measures, and inclusion measures.

The similarity measure can be described as a crucial method in determining the degree of similarity between two objects. The numerous similarity measures can be represented into two categories which are crisp and neutrosophic similarity measures. The crisp similarity measures can be said to be real valued functions and neutrosophic similarity measures can be considered as the level of similarity, considering the similarity among the three membership values one by one. Because of that, the neutrosophic similarity acts towards a single valued neutrosophic number as a measure of the similarity. In multiple criteria decision making, to determine the differences between the criteria, similarity measure of neutrosophic set can be used [4 - 7]. According to [8], similarity measures are very important in many applications with various fields of decision making such as pattern recognition, image thresholding, and multicriteria decision making.

A hybrid similarity measure is a metric that combines two or more similarity measures to produce a single similarity value. A generalized hybrid similarity measure of a neutrosophic set is a metric that considers the different aspects of uncertainty, indeterminacy, and vagueness inherent in a neutrosophic set, and combines them to produce a single similarity value. New hybrid distance-based similarity measures are studied by [9] and [10]. Mondal et al [9] proposed hybrid binary logarithm similarity measure for dealing indeterminacy in decision making situations. Meanwhile, Ulucay et al [10] introduced a new hybrid distance-based similarity measure for refined rough neutrosophic set and the method is applied in medical diagnosis for some diseases.

Beside similarity measure, the distance measure also can be used to perform a similar task. Distance value represents the degree of dissimilarity between two sets. [11] developed a distance measure that improves decision-making flexibility under uncertainty by integrating soft expert systems with SVNSs. Zeng et al [12] developed a new distance measure between IFSs and applied it in pattern recognition. For SVNS, [13] produced a novel distance measure for SVNS and applied to multicriteria group decision-making problem. [14] integrated the distance measure with entropy weight measure for application in medical diagnosis.

The SVNS is important in modelling theory. Thus, it can be used in real scientific and technical applications. Due to the growing amount of information accessible from contemporary medical equipment, medical diagnosis contains a lot of incomplete, uncertain, imprecise, and inconsistent information, which is essential information about medical diagnosis problems [15]. Symptoms usually involves a lot of incomplete, ambiguous, and inconsistent information for a disease. According to [16], in medical diagnosis problems, symptoms and data inspection of diseases may be changed in different time intervals. This leads to the question of whether the use of a single inspection period can make it possible to conclude that a particular patient has a particular disease or not. Sometimes symptoms of different illnesses can appear in a person being treated. [17 – 18] applied the theory of SVNS to medical diagnosis. Studies using clustering approaches have indicated that the hybrid technique has enhanced the precision of disease diagnosis based on patient symptoms [18]. Besides SVNS, [19] applied the theory of rough neutrosophic set to medical diagnosis. [20] investigated a unique multi-parameter similarity measure for interval-valued neutrosophic sets (IVNSs) and it provides stable results and improves diagnostic accuracy by taking symptom interactions into account. In medical diagnostics, a bi-parametric discriminant measure addresses decision-making uncertainty and provides numerical examples to demonstrate its usefulness, allowing for effective symptomatic diagnosis [21].

The purposes of this study are to formulate a novelty distance measure of SVNS, which is the extension of [12] and to construct a new hybrid distance measure. The work starts by extending an existing distance measure from [12] to create a new one for SVNS. To increase the precision of the distance calculation between two NS, a hybrid distance measure is created by combining the recently created distance measure with the several existing distance measures for NS. The objective of the

hybrid distance measure would be to better capture the complexities and nuances of the relationship between two NSs. The details of the hybrid distance measure would depend on the specific requirements and applications for which it is being developed. Several factors were taken into consideration when developing the hybrid distance measure, such as the importance of imprecision and uncertainty as well as the particular needs of the applications it would be used for. Eventually, by exploiting the increased accuracy in processing neutrosophic data, the generalized hybrid distance measure will be used to address issues related to medical diagnosis.

The remainder of the paper is structured as follows. Section two contains preliminary definition for some key terms, while section three introduces new definitions of a novel distance measure, and the hybrid distance measure is introduced in section four. Section five focuses on the application of new distance measures in medical diagnosis. Finally, section six is the conclusion of the present study.

2. Preliminaries

2.1. Neutrosophic Set

Definition 2.1.1: Neutrosophic Set [1]

Consider a space of points (objects) denoted by X, with each individual element in X represented by x. Within this context, the neutrosophic set $NS(A)$ can be understood as an object that takes a specific form $A = \left[\left(x, T_A(x), I_A(x), F_A(x)\right) : x \in X\right]$, where the functions $T, I, F : X \rightarrow]^{-}0, 1^{+}[$ define respectively the truth-membership function, an indeterminacy membership function and a falsity-membership function of the element $x \in X$ to the set A with the condition :

$$^{-}0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^{+}.$$

The function $T_A(x), I_A(x)$ and $F_A(x)$ are real standard or nonstandard subsets of $]^{-}0, 1^{+}[$.

Example: Truth (*T*), Indeterminacy (*I*), and Falsity (*F*) values in a NS are usually expressed as ranges, providing flexibility in expressing uncertainty. Let's look at a NS A, which stands for a collection of tall students. The ranges for each student *x* are the Truth (*T*(x)), Indeterminacy (*I*(x)), and Falsity (*F*(x)) of being tall.

For a student x:
$T(x)$ = [0.7,0.9] (The student's height ranges from 70% to 90%).
$I(x)$ = [0.1, 0.2] (There is a 10% to 20% margin of error regarding the student's height),
$F(x)$ = [0.1, 0.3] (The student is not tall by 10% to 30%).
This indicates that although there is some uncertainty and a small amount of falsity, student x is generally regarded as tall.

Definition 2.1.2: Single Value Neutrosophic Set [2]

Consider a space of points (objects) denoted by X, with each individual element in X represented by x. A single-valued neutrosophic set $A\left(SVNS(A)\right)$ is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. For each point x in $X, T_A(x), I_A(x), F_A(x) \in [0,1]$. A SVNS can be written as

$$A = \{\langle x\ ;\ T_A(x), I_A(x), F_A(x)\rangle : x \in X\} \quad (1)$$

Example: A SVNS simplifies the neutrosophic set by giving truth, indeterminacy, and falsity fixed values (rather than ranges) within the interval [0,1]. Let's look at a SVNS A, which represents the percentage of exam takers who fall into the pass category.

For a student y:
$T(y)$ = 0.8 (A student is 80% likely to have passed)
$I(y)$ = 0.1 (There is 10% uncertainty over the student's passing)
$F(y)$ = 0.2 (The student is 20% likely to have failed)
In this instance, the fixed numbers give a clear picture of the student's exam result's validity, uncertainty, and falsity. As a result, the student is passed the exam

2.2 Distance Measure

One mathematical tool for calculating the distance between two neutrosophic sets is the distance measure for that sets. Depending on the intended use, a variety of distance measures, including Hamming distance, Hausdroff distance, Euclidean distance, and more generic distance functions, can be defined.

Definition 2.2.1: Several Distance Measures for SVNS
Let $X = \{x_1, x_2, \ldots, x_n\}$ be the universe of discourse. Let $A = \{x_i, T_A(x_i), I_A(x_i), F_A(x_i)) : x_i \in X\}$ and $B = \{x_i, T_B(x_i), I_B(x_i), F_B(x_i)) : x_i \in X\}$ be two neutrosophic sets. Several distance measures between two SVNS, A and B can be defined as:

1. Normalized Hamming distance measure [5]:

$$d_M^N(A,B) = \frac{1}{3n}\sum_{i=1}^{n}(|T_A(x_i) - T_B(x_i)| + |I_A(x_i) - I_B(x_i)| + |F_A(x_i) - F_B(x_i)|) \quad (2)$$

2. Normalized Euclidean distance measure [5]:

$$d_E^N(A,B) = \sqrt{\frac{1}{3n}\sum_{i=1}^{n}((T_A(x_i) - T_B(x_i))^2 + (I_A(x_i) - I_B(x_i))^2 + (F_A(x_i) - F_B(x_i))^2)} \quad (3)$$

3. Extended Hausdroff distance metric [5]:

$$d_S^N(A,B) = \frac{1}{n}\sum_{i=1}^{n} max(|T_A(x_i) - T_B(x_i)|, |I_A(x_i) - I_B(x_i)|, |F_A(x_i) - F_B(x_i)|) \quad (4)$$

4. Mustapha et al [17]:

$$d_{NM}^N(A,B) = \frac{2}{n}\sum_{i=1}^{n}\frac{sin\{\frac{\pi}{10}|T_A(x_i) - T_B(x_i)|\} + sin\{\frac{\pi}{10}|I_A(x_i) - I_B(x_i)|\} + sin\{\frac{\pi}{10}|F_A(x_i) - F_B(x_i)|\};}{1 + sin\{\frac{\pi}{10}|T_A(x_i) - T_B(x_i)|\} + sin\{\frac{\pi}{10}|I_A(x_i) - I_B(x_i)|\} + sin\{\frac{\pi}{10}|F_A(x_i) - F_B(x_i)|\}} \quad (5)$$

5. Ye and Zhang [6]:

$$d_{YZ}^N(A,B) = 1 - \frac{1}{3n}\sum_{i=1}^{n}\left(\frac{\min(T_A(x_i), T_B(x_i))}{\max(T_A(x_i), T_B(x_i))} + \frac{\min(I_A(x_i), I_B(x_i))}{\max(I_A(x_i), I_B(x_i))} + \frac{\min(F_A(x_i), F_B(x_i))}{\max(F_A(x_i), F_B(x_i))}\right) \quad (6)$$

6. Ren et al [7]:

$$d_{Ren}^{N}(A,B)=\frac{1}{2n}\sum_{i=1}^{n}\left(\frac{(T_A(x_i)-T_B(x_i))^2}{2+T_A(x_i)+T_B(x_i)}+\frac{(I_A(x_i)-I_B(x_i))^2}{2+I_A(x_i)+I_B(x_i)}+\frac{(F_A(x_i)-F_B(x_i))^2}{2+F_A(x_i)+F_B(x_i)}+|m_A(x_i)-m_B(x_i)|\right), \quad (7)$$

where $m_j(x_i)=\frac{1+T_j(x_i)-F_j(x_i)}{2}, j=A,B$

7. Binary Logarithm [9]:

$$BL(A,B)=1-\frac{1}{n}\sum_{i=1}^{n}log_2\left(2-\left(\frac{1}{3}\left(T_A(x_i)-T_B(x_i)\right)+\left(I_A(x_i)-I_B(x_i)\right)+\left(F_A(x_i)-F_B(x_i)\right)\right)\right) \quad (8)$$

3. New Distance Measure for Single Value Neutrosophic Set

In this section, a novel approach for determining the distance measure between SVNS is introduced. It is extended from the work by [12], which incorporates exponential distance to minimize any potential loss of information.

Definition 3.1: Let $X=\{x_1,x_2,\ldots,x_n\}$ be the universe of discourse. Let $A=\{x_i, T_A(x_i),I_A(x_i),F_A(x_i)):x_i\in X\}$ and $B=\{x_i, T_B(x_i),I_B(x_i),F_B(x_i)): x_i\in X\}$ be two neutrosophic sets. A distance measure between two SVNS, A and B can be defined as:

$$d_{New}^{N}(A,B)=\frac{1}{n}\sum_{i=1}^{n}\left(d_{exp}(A,B)\times d_{tif}(A,B)\right) \quad (9)$$

where $d_{exp}(A,B)=e^{|T_A(x_i)-T_B(x_i)|-1}$, $d_{tif}(A,B)=\frac{1}{3}\sum_{i=1}^{n}(|T_A(x_i)-T_B(x_i)|+|I_A(x_i)-I_B(x_i)|+|F_A(x_i)-F_B(x_i)|)$, $d_{exp}(A,B)$ denotes the exponential distance measure, which is determined by the difference of the ith membership degree between A and B. and $d_{tif}(A,B)$ denotes the average including absolute value of ith membership degree, intermediate degree and false degree between A and B.

Theorem 3.1: $d_{New}^{N}(A,B)$ is the distance measure between SVNS A and B.

Proposition:

$(S1)\ 0\le d_{New}^{N}(A,B)\le 1$

$(S2)\ d_{New}^{N}(A,B)=d_{New}^{N}(B,A)$

$(S3)\ d_{New}^{N}(A,B)=0$ if and only if $A=B$

$(S4)$ For $A\subseteq B\subseteq C$, then we have $d_{New}^{N}(A,C)\ge d_{New}^{N}(A,B)$ and $d_{New}^{N}(A,C)\ge d_{New}^{N}(B,C)$

Proof(S1):

Suppose $T_A(x_i), T_B(x_i), I_A(x_i), I_B(x_i), F_A(x_i)$ and $F_B(x_i)\in[0,1]$, thus, we have

$$|T_A(x_i)-T_B(x_i)|\in[0,1]\Rightarrow|T_A(x_i)-T_B(x_i)|-1\in[-1,0]$$

$$\Rightarrow e^{|T_A(x_i)-T_B(x_i)|-1}\in[e^{-1},e^{0}]\subseteq[0,1]$$

$\therefore d_{exp}(A,B) \in [0,1]$.

$$|T_A(x_i) - T_B(x_i)| + |I_A(x_i) - I_B(x_i)| + |F_A(x_i) - F_B(x_i)| \in [0,3]$$

$$\Longrightarrow \frac{|T_A(x_i) - T_B(x_i)| + |I_A(x_i) - I_B(x_i)| + |F_A(x_i) - F_B(x_i)|}{3} \in [0,1]$$

$\therefore d_{tif}(A,B) \in [0,1]$.

Hence,

$d_{exp}(A,B) \times d_{tif}(A,B) \in [0,1] \Longrightarrow d_{New}^{N}(A,B) \in [0,1]$.

Proof(*S2*):

$d_{New}^{N}(A,B) = d_{New}^{N}(B,A)$ is obvious.

Proof(*S3*):

$d_{New}^{N}(A,B) = 0$ if and only if $A = B$;

Sufficiency: If $d_{New}^{N}(A,B) = 0$, then for every x_i, we have $d_{exp}(A,B) = 0$, which means

$$\frac{|T_A(x_i) - T_B(x_i)| + |I_A(x_i) - I_B(x_i)| + |F_A(x_i) - F_B(x_i)|}{3} = 0$$

$$\Longrightarrow |T_A(x_i) - T_B(x_i)| + |I_A(x_i) - I_B(x_i)| + |F_A(x_i) - F_B(x_i)| = 0$$

Then,

$$|T_A(x_i) - T_B(x_i)| = 0$$
$$|I_A(x_i) - I_B(x_i)| = 0$$
$$|F_A(x_i) - F_B(x_i)| = 0$$

Hence, we can have $T_A(x_i) = T_B(x_i), I_A(x_i) = I_B(x_i)$ *and* $F_A(x_i) = F_B(x_i)$ for every x_i. That means *A=B*.

Necessity: If *A=B*, then it means for every x_i, we have $T_A(x_i) = T_B(x_i), I_A(x_i) = I_B(x_i)$ *and* $F_A(x_i) = F_B(x_i)$, thus

$$d_{tif}(A,B) = \frac{|T_A(x_i) - T_B(x_i)| + |I_A(x_i) - I_B(x_i)| + |F_A(x_i) - F_B(x_i)|}{3} = 0$$

$$\Longrightarrow d_{New}^{N}(A,B) = \frac{1}{n}\sum_{i=1}^{n}\left(d_{exp}(A,B) \times d_{tif}(A,B)\right) = 0$$

Proof(*S4*):

For $A \subseteq B \subseteq C$, then we have $d_{New}^{N}(A,C) \geq d_{New}^{N}(A,B)$ and $d_{New}^{N}(A,C) \geq d_{New}^{N}(B,C)$.

If $A \subseteq B \subseteq C$, we have

$$T_A(x_i) \leq T_B(x_i) \leq T_C(x_i)$$
$$I_A(x_i) \geq I_B(x_i) \geq I_C(x_i)$$
$$F_A(x_i) \geq F_B(x_i) \geq F_C(x_i)$$

Then, we get

$$|T_A(x_i) - T_C(x_i)| \geq |T_A(x_i) - T_B(x_i)|$$
$$|T_A(x_i) - T_C(x_i)| \geq |T_B(x_i) - T_A(x_i)|$$
$$|I_A(x_i) - I_C(x_i)| \geq |I_A(x_i) - I_B(x_i)|$$
$$|I_A(x_i) - I_C(x_i)| \geq |I_B(x_i) - I_A(x_i)|$$
$$|F_A(x_i) - F_C(x_i)| \geq |F_A(x_i) - F_B(x_i)|$$
$$|F_A(x_i) - F_C(x_i)| \geq |F_B(x_i) - F_A(x_i)|$$

Further, we can get

$$e^{|T_A(x_i) - T_C(x_i)| - 1} \geq e^{|T_A(x_i) - T_B(x_i)| - 1}$$

$$\frac{|T_A(x_i) - T_C(x_i)| + |I_A(x_i) - I_C(x_i)| + |F_A(x_i) - F_C(x_i)|}{3} \geq \frac{|T_A(x_i) - T_B(x_i)| + |I_A(x_i) - I_B(x_i)| + |F_A(x_i) - F_B(x_i)|}{3}$$

That is

$$d_{tif}(A, C) \geq d_{tif}(A, B)$$
$$d_{exp}(A, C) \geq d_{exp}(A, B)$$

Thus, we have $d_{New}^N(A, C) \geq d_{New}^N(A, B)$.
Similarly, we have $d_{New}^N(A, C) \geq d_{New}^N(B, C)$.
The theorem proved completely.

4. New Hybrid Distance Measure for SVNS

Hybrid formulas combine many methods or techniques, often coming from multiple disciplines or areas of expertise. Hybrid approaches utilize the benefits of each technique to get results that are more accurate than those obtained by using only one strategy. Assessing persistent similarities with greater accuracy should be the outcome of incorporating different similarity measures into a hybrid formula. Moreover, hybrid formulas are often more adaptable and dependable in a variety of datasets and situations. Because they can handle a wider range of input data types and attributes, they are suitable for a multitude of applications.

Definition 4.1: Hybrid vector distance measures of SVNS
Let $A = [T_A(x_i), I_A(x_i), F_A(x_i)]$ and $B = [T_B(x_i), I_B(x_i), F_B(x_i)]$ be two SVNSs in a universe of discourse $X = \{x_1, x_2, \dots, x_n\}$. Then, the hybrid distance measure of SVNSs in the vector space is defined as follows:

$$Hyb(A, B) = \tau\big(d(A, B)\big) + (1 - \tau) d_{New}^N(A, B)) \tag{10}$$

where $0 \leq \tau \leq 1$ and $d(A, B)$ is replaced with $BL(A, B), d_{NM}^N(A, B), d_M^N(A, B), d_E^N(A, B), d_S^N(A, B), d_{YZ}^N(A, B)$ and $d_{Ren}^N(A, B)$ in Equations (2-8) for various hybrid distance measures.

Theorem 4.1 The formulation of properties for new hybrid distance measure of SVNS
The generalized hybrid distance measure $Hyb(A, B)$ for neutrosophic set A and B satisfies the following properties. For proving purpose, let $d(A, B)$ is defined as binary logarithm distance(Equation (8)), then Equation (10) becomes

$$Hyb(A, B) = \tau(BL(A, B) + (1 - \tau) d_{New}^N(A, B)) \text{ with } 0 \leq \tau \leq 1$$

Proposition:

$(S1)\ 0 \le Hyb(A,B) \le 1$

$(S2)\ Hyb(A,B) = Hyb(B,A)$

$(S3)\ Hyb(A,B) = 0$ if and only if $A = B$

$(S4)$ For $A \subseteq B \subseteq C$, then we have $Hyb(A,C) \ge Hyb(A,B)$ and $Hyb(A,C) \ge Hyb(B,C)$

Proof$(S1)$**:**

Suppose $BL(A,B)$ and $d^N_{New}(A,B) \in [0,1]$, thus, we have

$$0 \le BL(A,B) + d^N_{New}(A,B) \le 2.$$

$With\ 0 \le \tau \le 1$,

$$0 \le \tau(BL(A,B) + (1-\tau)d^N_{New}(A,B)) \le 1$$

Hence, the hybrid distance measure of SVNS is within [0,1]. Thus $0 \le Hyb(A,B) \le 1$.

Proof$(S2)$:

$Hyb(A,B) = Hyb(B,A)$ is obvious.

Proof$(S3)$**:**

For any two distances $BL(A,B)$ and $d^N_{New}(A,B)$, if $A = B$, then we have $BL(A,A)$ and $d^N_{New}(A,A)$. According to the proof of $d^N_{New}(A,B)$ shown in Section 3.1 and the proof of $BL(A,B)$ in [9], we have

$$BL(A,A) = 0 \text{ and } d^N_{New}(A,A) = 0,$$

$with\ 0 \le \tau \le 1$,

$$\tau(BL(A,A) + (1-\tau)d^N_{New}(A,A)) = 0 + 0 = 0$$

Similarly, if $A = B$

Hence, $Hyb(A,B) = 0$ if and only if $A = B$.

Proof$(S4)$**:**

For $A \subseteq B \subseteq C$, according to the proof of $d^N_{New}(A,B)$ shown in Section 3.1 and the proof of $BL(A,B)$ in [6], we have $d^N_{New}(A,C) \ge d^N_{New}(A,B)$, $d^N_{New}(A,C) \ge d^N_{New}(B,C)$, $BL(A,C) \ge BL(A,B)$ and $BL(A,C) \ge BL(B,C)$. Then, we have

$$BL(A,C) + d^N_{New}(A,C) \ge BL(A,B) + d^N_{New}(A,B).$$

With $0 \le \tau \le 1$,

$$\tau BL(A,C) + (1-\tau)d^N_{New}(A,C)) \ge \tau BL(A,B) + (1-\tau)d^N_{New}(A,B).$$

Thus, we have $Hyb(A,C) \ge Hyb(A,B)$.

Similarly, we have $Hyb(A,C) \ge Hyb(B,C)$.

The theorem was proved completely. The other hybrid distance measures can be proven similarly.

5. Application of new hybrid distance measure in medical diagnosis

Since the physical and hormonal changes that take place during pregnancy can put the heart under additional stress, pregnancy can be a high-risk time for women with heart disease. Heart failure, irregular heart rhythm, and blood clots are just a few of the pregnancy issues that may be more likely to occur in women who already have heart disease. In extreme situations, a pregnant woman with heart problems may potentially have a heart attack. All things considered, it's critical for pregnant women who have heart disease to be aware of the potential hazards involved and to seek medical advice from a specialist to decide the best course of action.

In this study, the data is collected from [22]. The following eight symptoms are commonly used to assess the level of risk for a pregnant woman with heart disease by doctors or medical officers.

Age: Older women are at higher risk of developing heart disease, and pregnancy can increase this risk.
Obesity: Women who are overweight or obese have an increased risk of developing heart disease, and this risk is further increased during pregnancy.
Smoking: Smoking increases the risk of heart disease and should be avoided by pregnant women with heart disease.
Family Pathological History: A family history of heart disease can indicate an increased risk of heart disease in pregnant women.
Personal Pathological History: A personal history of heart disease or heart-related conditions can increase the risk of complications during pregnancy.
Electrocardiogram (ECG): An ECG can detect heart disease and help determine the risk to the mother and fetus during pregnancy.
Ultrasound: An ultrasound can help detect heart problems and monitor the fetus during pregnancy.
Functional Class: The functional class of a woman's heart disease can help determine the risk of complications during pregnancy and guide management decisions.

Table 1. The relation between a woman and factors

Factor	(T,I,F)
A	(0.6,0.50,0.37)
OB	(1.0,0.25,0.60)
TAB	(1.0,0.00,0.00)
FPH	(1.0,0.00,0.00)
PPH	(1.0,0.00,0.00)
ECG	(0.5,0.30,0.40)
ECO	(0.5,0.30,0.40)
FH	(0.7,0.15,1.00)

The risks of pregnancy can be increased in a woman *W*, with major factor I={age(A), obesity (OB), smoking (TAB), personal pathological history PPH), family pathological history (FPH),

electrocardiogram (ECG), ultrasound (ECO), functional class (FH)}. The diagnoses listed in the set are D = {obstruction at exit (OEX), obstruction at entry (OEN), rhythm disorders (RD), conduction disorder (CDS), congenital disease (CD), genetic disease (GD)} can affect pregnant women. These diseases can complicate pregnancy, delivery, and the postpartum period and have negative effects on both the mother and the fetus. OEX and OEN can make it more difficult for the fetus to be born and may necessitate a caesarean section. In addition to increasing the risk of difficulties during pregnancy and delivery, RD and CDS can lead to irregular heartbeats. CD and GD can cause congenital anomalies or genetic abnormalities in the fetus, which can impact its development and survival.

Pregnant women need to be examined and monitored by a healthcare provider if they have any of these conditions, to minimize the risk of complications and ensure the best possible outcome for both the mother and the fetus. The data from [22] are shown in Tables 1 and 2 which represent the relation between women and symptoms and the relation between symptoms and diagnoses, respectively.

Table 2. The relation between factors and diseases

	A	**OB**	**TAB**	**FPH**
OEX	(0.9,0.2,0.3)	(0.2,0.6,0.4)	(0.9,0.1,0.3)	(0.2,0.2,0.3)
OEN	(0.2,0.2,0.4)	(0.3,0.5,0.6)	(0.8,0.1,0.2)	(0.2,0.4,0.1)
RD	(0.8,0.3,0.2)	(0.9,0.2,0.3)	(0.9,0.2,0.3)	(0.2,0.1,0.1)
CDS	(0.9,0.2,0.3)	(0.8,0.2,0.1)	(0.9,0.2,0.3)	(0.3,0.6,0.5)
CD	(0.9,0.1,0.3)	(0.9,0.2,0.2)	(0.8,0.1,0.2)	(0.2,0.8,0.1)
GD	(0.9,0.2,0.3)	(0.8,0.2,0.3)	(0.2,0.1,0.4)	(0.8,0.2,0.3)
	PPH	**ECG**	**ECO**	**FH**
OEX	(0.9,0.1,0.2)	(0.8,0.2,0.3)	(0.7,0.2,0.2)	(0.9,0.2,0.1)
OEN	(0.8,0.2,0.3)	(0.7,0.2,0.3)	(0.8,0.2,0.3)	(0.9,0.2,0.3)
RD	(0.9,0.2,0.3)	(0.2,0.2,0.4)	(0.2,0.8,0.5)	(0.9,0.2,0.3)
CDS	(0.9,0.2,0.2)	(0.2,0.5,0.6)	(0.9,0.2,0.2)	(0.9,0.1,0.3)
CD	(0.2,0.5,0.5)	(0.9,0.2,0.2)	(0.8,0.2,0.3)	(0.9,0.2,0.2)
GD	(0.9,0.2,0.3)	(0.3,0.8,0.6)	(0.2,0.1,0.5)	(0.8,0.2,0.3)

The generalized hybrid distance measure is defined according to different values of τ as stated below:

i) If $\tau = 0$, the hybrid distance measure reduced to the new formulated distance measure (Equation 9).

ii) If $\tau = 1$, the hybrid distance measure is generalized to either Hamming, Hausdroff, Euclidean, Mustapha et al [17], Ye & Zhang [6], Ren et al [7] or Binary Logarithm (Mondal et al [9]) measure (Equations (2-8)).

iii) If $\tau = 0.5$, the hybrid distance measure is the mean distance measure between two distance measures.

iv) If $\tau = 0.1$ and $\tau = 0.9$, the hybrid distance measure is defined.

Table 3. The new distance measure for a woman W with diseases ($\tau = 0$)

Diseases	New Distance Measure
OEX	0.167808
OEN	0.157324
RD	0.088902
CDS	0.127549
CD	0.22461
GD	0.107783

Table 4. The new hybrid distance measures for a woman W with diseases when $\tau = 1$

	Hamming	Hausdorff	Euclidean	Mustapha et al (2021)	Ye & Zhang (2014)	Binary Logarithm (Mondal et al (2018))	Ren et al (2019)
OEX	0.260833	0.475	0.349929	0.379077	0.545701	0.205562	0.201525
OEN	0.254167	0.45875	0.331461	0.377367	0.532337	0.19821	0.198151
RD	0.225	0.425	0.296564	0.343536	0.514688	0.173466	0.163107
CDS	0.275	0.425	0.33434	0.397885	0.543531	0.217922	0.198624
CD	0.3125	0.5125	0.403774	0.432617	0.563323	0.252555	0.244957
GD	0.254167	0.4375	0.314431	0.380204	0.539364	0.197605	0.178489

The results of the hybrid distance measures, which were applied to analyze pregnant women's symptoms and forecast their propensity to develop specific diseases while pregnant, are shown in Tables 3–7. The results of the analysis using various parameter values are shown in the tables. The results from the hybrid distance measures are considered adequate for medical diagnostics despite the variances in parameter values. This shows that the suggested strategy is solid and capable of offering trustworthy information on the patient's condition.

Table 5. The new hybrid distance measures for a woman W with diseases when τ=0.5

	Hamming	Hausdorff	Euclidean	Mustapha et al (2021)	Ye & Zhang (2014)	Binary Logarithm (Mondal et al (2018))	Ren et al (2019)
OEX	0.214321	0.321404	0.258868	0.273443	0.356755	0.186685	0.184666
OEN	0.205745	0.308037	0.244393	0.267346	0.344831	0.177767	0.177737
RD	0.156951	0.256951	0.192733	0.216219	0.301795	0.131184	0.126004
CDS	0.201274	0.276274	0.230944	0.262717	0.33554	0.172735	0.163086
CD	0.268555	0.368555	0.314192	0.328613	0.393966	0.238583	0.234784
GD	0.180975	0.272642	0.211107	0.243993	0.323574	0.152694	0.143136

It is clear from looking at the results shown in the Tables 3-7 that the pregnant woman has a very high likelihood of developing RD throughout her pregnancy. Her symptoms exhibit important signs and patterns that are consistent with RD according to the hybrid distance measurements. Therefore, doctors could begin treating this patient early in order to prevent difficulties for the mother and the unborn child. Pre-existing heart problems, hypertension, and obesity all raise a woman's risk of getting an RD during pregnancy.

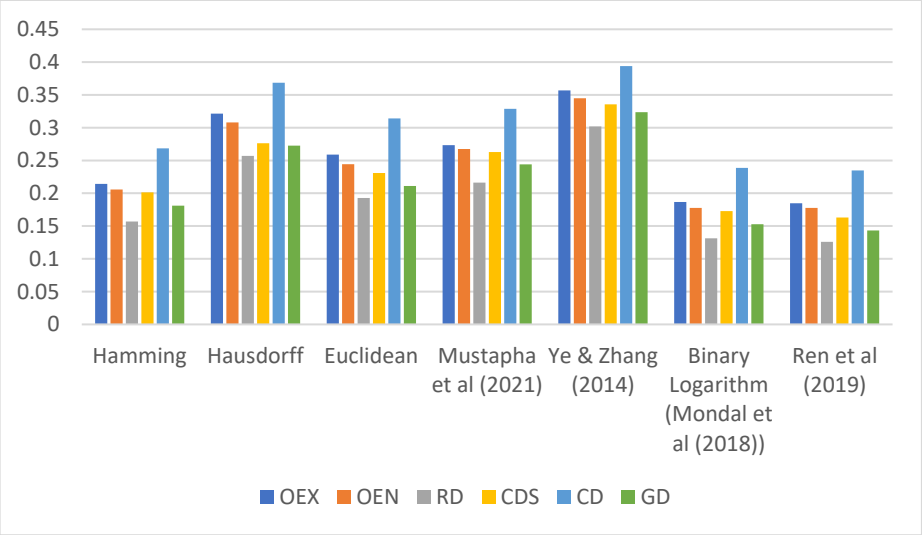

Figure 1. The new hybrid distance measures for a woman *W* with diseases when τ=0.5

Table 6. The new hybrid distance measures for a woman *W* with diseases when τ=0.9

	Hamming	**Hausdorff**	**Euclidean**	**Mustapha et al (2021)**	**Ye & Zhang (2014)**	**Binary Logarithm (Mondal et al (2018))**	**Ren et al (2019)**
OEX	0.251531	0.444281	0.331717	0.35795	0.507912	0.171584	0.198153
OEN	0.244482	0.428607	0.314048	0.355363	0.494836	0.161412	0.194068
RD	0.21139	0.39139	0.275797	0.318072	0.472109	0.097358	0.155687
CDS	0.260255	0.395255	0.313661	0.370852	0.501933	0.136586	0.191516
CD	0.303711	0.483711	0.385857	0.411816	0.529451	0.227404	0.242923
GD	0.239528	0.404528	0.293766	0.352961	0.496206	0.116765	0.171419

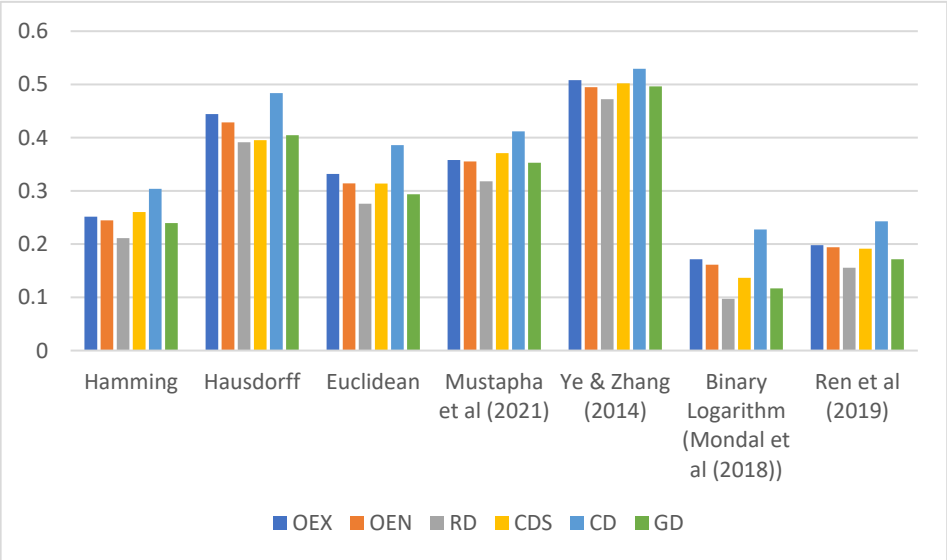

Figure 2. The new hybrid distance measures for a woman *W* with diseases when τ=0.9

Table 7. The new hybrid distance measures for a woman *W* with diseases when τ=0.1

	Hamming	Hausdorff	Euclidean	Mustapha et al (2021)	Ye & Zhang (2014)	Binary Logarithm (Mondal et al (2018))	Ren et al (2019)
OEX	0.177111	0.198527	0.18602	0.188935	0.205598	0.201787	0.17118
OEN	0.167008	0.187466	0.174738	0.179328	0.194825	0.194121	0.161406
RD	0.102511	0.122511	0.109668	0.114365	0.13148	0.165009	0.096322
CDS	0.142294	0.157294	0.148228	0.154582	0.169147	0.208885	0.134656
CD	0.233399	0.253399	0.242526	0.245411	0.258481	0.249761	0.226645
GD	0.122421	0.140755	0.128448	0.135025	0.150941	0.188623	0.114854

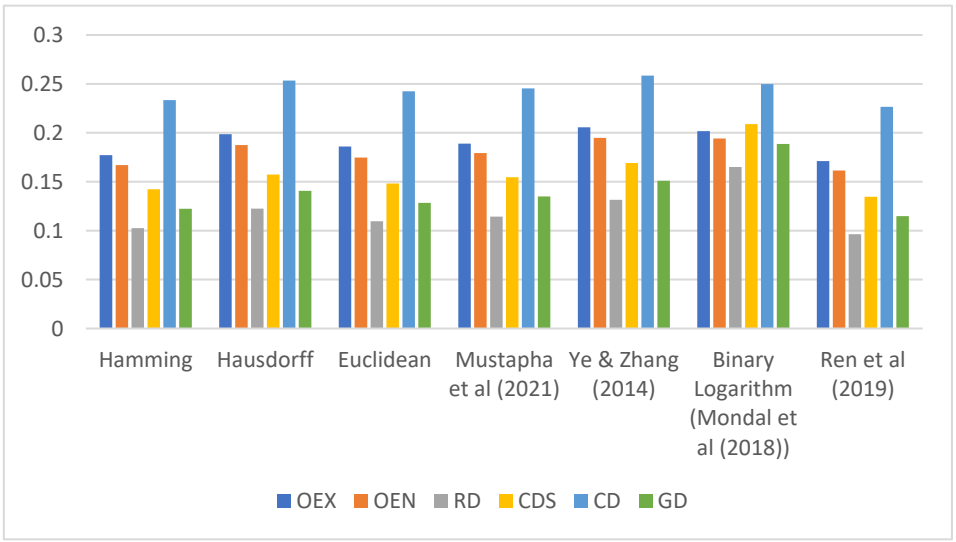

Figure 3. The new hybrid distance measures for a woman *W* with diseases when τ=0.1

As shown in Tables 5-7, new distance measure proposed in our study demonstrates its contribution to the analysis of RD compared to other distance measures, even when considering various values of the parameter, τ. The results obtained also presented clearly in Figures 1-3. This indicates that our new distance measure is particularly effective in capturing the relevant characteristics and patterns associated with RD, making it a valuable tool for diagnosing and predicting this condition.

In the analysis, we observed that the shortest distance measure is highly influenced by the hybridization between the new distance measure and [7] when τ is set to 0.1. This finding suggests that the combination of our new distance measure with Ren et al [7] yields the most accurate representation of RD symptoms, resulting in a distance measure that is closest to zero and indicating a higher likelihood of RD.

Furthermore, the second-lowest distance measure is predominantly influenced by the hybridization between the new distance measure and Binary Logarithm with a τ value of 0.1. This indicates that this particular combination captures important aspects of RD symptoms, contributing to a relatively low distance measure and suggesting a significant association with RD. In addition to the above observations, the results presented in Tables 3-7 and Figures 1-3 also highlight the relatively lower significance of CD compared to other diseases for the pregnant woman under consideration.

6. Conclusions

In conclusion, the proposed generalized hybrid distance measure of a NS provides a valuable tool for the analysis of patients's symptoms and diseases. This approach considers the inherent uncertainty and indeterminacy associated with medical data, allowing for a more comprehensive and accurate assessment of the patient's condition. By incorporating both the distance measure and the neutrosophic set theory, this hybrid approach addresses the limitations of traditional distance measures and offers a more nuanced representation of the patient's symptoms.

The utilization of this hybrid distance measure contributes significantly to the field of medical diagnosis and treatment. It enables healthcare professionals to make decisions based on a more comprehensive understanding of the patient's symptoms and diseases, leading to improved accuracy in diagnoses and tailored treatment plans.

Furthermore, the proposed approach has the potential for broader applications beyond medical analysis. Its ability to handle uncertainty and indeterminacy makes it suitable for decision-making processes in various domains where imprecise or incomplete information is common.

In conclusion, the generalized hybrid distance measure of a neutrosophic set offers a promising approach for analyzing patient symptoms and diseases, providing a robust framework for medical practitioners and researchers to improve diagnostic accuracy and optimize treatment outcomes.

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Bipolar Neutrosophic Frank Aggregation Operator and its Application in Multi Criteria Decision Making Problem

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Abstract: Aggregation operators can be used to combine and synthesise a finite number of numerical values into a single numerical value. Many areas, including decision-making, expert systems, risk analysis, and image processing, rely heavily on aggregating functions. In real-world situations, the neutrosophic set can manage the uncertainties associated with information from any decision-making challenge, whereas the fuzzy set and intuitionistic set cannot. The term "bipolarity" refers to the propensity of the human mind to weigh pros and drawbacks when thinking through decisions. Triangular norms are aggregation operators in a variety of fields, including fuzzy set theory, probability and statistics, and decision sciences. Thus, the individual assessments in this paper's study of and approach to multi-criteria decision-making (MCDM) problems that use bipolar neutrosophic numbers as the individual evaluations. Frank operational laws of bipolar neutrosophic numbers, bipolar neutrosophic Frank weighted geometric aggregation (BNFWGA) and the bipolar neutrosophic frank ordered weighted geometric aggregation (BNFOWGA) operators have been developed with its desirable properties. Additionally, the suggested aggregation operators have been used in the selection of bridges. The outcomes demonstrate the applicability and validity of the suggested approach. Comparative analysis has been performed using the current approach.

Abbreviation:

NS	Neutrosophic Set
IFS	Intuitionistic fuzzy set
IVIFS	Interval-valued intuitionistic fuzzy set
INS	Interval neutrosophic set
SVNS	Single valued neutrosophic set
MCDM	Multi criteria decision making
BNS	Bipolar neutrosophic set
MAGDM	Multi attribute group decision making
BNNs	Bipolar neutrosophic numbers
BNFWGA	Bipolar neutrosophic frank aggregation weighted geometric aggregation
BNFOWGA	Bipolar neutrosophic ordered weighted geometric aggregation

Keywords: Bipolar neutrosophic set, Frank Triangular Norms, Operational Laws, Aggregation Operators, Decision Making

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1. Introduction

A newly established model typically fixes the flaws of prior models in fuzzy theory. Because ambiguity and uncertainty present challenges in many real-world situations, routine mathematics is not always available. Many methods, including statistical hypothesis, probability, and fuzzy set hypothesis, have been presented as alternatives to traditional models and to guard against weaknesses in order to handle such difficulties. The majority of these mathematical alternatives, regrettably, have drawbacks and shortcomings of their own. Most words are in fact ambiguous and cannot be quantified, for instance, authentic and best-known. The authors of [1] started thinking about the chance based on the participation function that awards a membership grade in [0, 1] to handle such muddled and ambiguous information. Fuzzy sets are unable to handle the difficult issue since they only have one membership degree. Presented the intuitionistic fuzzy set (IFS) concept in [2]. IFS is used to provide an extremely flexible description of uncertain information. IFS offers degrees that are both membership- and non-membership-based. Introduced the idea of an intuitive fuzzy set with interval values in [3]. [4] developed the idea of a neutrosophic set (NS). NS includes membership, non-membership, and indeterminacy membership functions to define incompletes, inconsistent, and uncertain information. In order to adapt NS to real-world decision-making scenarios, [5, 6] introduced the interval Neutrosophic set (INS) and single valued Neutrosophic set (SVNS) concepts. Bipolar fuzzy sets are a generalization of fuzzy sets that were created by [7, 8]. The bipolar fuzzy relations study, in which each tuple is connected to a pair of

satisfaction degrees, was first presented in [9]. Two applications for bipolar fuzzy sets in groups called the bipolar fuzzy groups and the norm have been introduced in [10].

A popular area of research in fuzzy theory of decision analysis is the study of fuzzy multi-attribute group decision-making. A sequence of judgments made in a fuzzy environment, which is usually ambiguous or uncertain, give decision information in the process of choosing the best possible options in terms of several criteria. Gaining a comprehensive understanding of the data is crucial for information fusion, particularly when making difficult decisions. Aggregation functions are one of the most efficient and simple methods for obtaining the aggregated result, although there are other methods as well. An n-tuple of data can be condensed into a single output using an aggregate function, which uses non-decreasing functions and keeps the output in the same set as the input. [11] In situations where all attribute values were defined as intuitionistic fuzzy numbers or interval-valued intuitionistic fuzzy numbers, aggregation functions were employed to handle dynamic multi-attribute decision-making in [11].

On the premise that the decision makers' criteria or preferences are unrelated and that the aggregating operators—defined by the independence axiom—are linear operators based on additive measures, multiple aggregation processes have been implemented in [12,13]. According to [14], real decision-making issues show the occurrence of unique dependencies or interactions between criteria. Decision-makers are typically invited from the same or related fields for a choice dilemma. They have a comparable social status, a similar level of knowledge, and similar tastes. Their arbitrary preferences can be demonstrated to exhibit nonlinearity. As a result, both the mutually preferred independence of these criteria and the independence of decision-makers are compromised. Advanced neutrosophic planar graph concepts and their applications were introduced in [15]. In [16], it was suggested to utilize a neutrosophic graph to predict linkages in social networks. A novel method of link prediction using the rsm index was developed by the authors of [17]. Radio fuzzy graphs and used radio k-colouring graphs to assign frequencies in radio stations introduced in [18]. [19] investigated the edge colouring of fuzzy graphs; chromatic index and the strong chromatic index have been proposed with its related properties in [19]. The colouring of directed fuzzy graphs based on the influence of relationship was proposed in [20]. Bipolar Neutrosophic TOPSIS was introduced in [21] as a method for resolving Multi Attribute Decision-Making (MADM) issues in a bipolar Neutrosophic fuzzy environment. In [22], methods based on Frank Choquet Bonferroni Mean Operators were developed to address MADM difficulties in a bipolar Neutrosophic fuzzy environment. [23, 24] discussed a few aggregation operators on different models.

The lattice of closed interval-valued fuzzy sets has been extended using Frank t-norms-based extension operations, which were proposed in [25]. These operations have been given the necessary

and sufficient conditions to form a complete algebraic structure. [26] developed an analytical hierarchy process method for multi-attribute decision-making issues based on a logarithmic regression function and presented the idea of a triangular interval type-2 fuzzy set. [27] used arithmetic procedures like union and intersection between interval fuzzy linguistic numbers and Multi Attribute Group Decision Making problems to create a probabilistic linguistic framework. [28] a new signed distances-based linear assignment technique for MAGDM issues with fuzzy set information was created, and it was then applied to locate a landfill. [29] suggested a MAGDM and the idea of a trapezoidal interval type-2 fuzzy soft set. According to the aforementioned data, the research contribution based on bipolarity is relatively minimal, which is why we applied the concept in our research. MATLAB can be used to lessen the time complexity. The rest of the essay is organized as follows. Section 2 presents the fundamental antecedents. Section 3 established the Frank Aggregation operators' operating laws for bipolar neutrosophic numbers (BNNs). We go into great detail on BNFWGA and BNFOWGA, as well as their attributes, in Section 4. On the basis of bipolar neutrosophic Frank aggregation operations, we propose a number of comprehensive MCDM techniques in Section 5. The presented principles are applied to extend comprehensive techniques in Section 6. The proposed aggregation operators are used in section 7 to resolve the decision-making problem for choosing the best bridge. The comparative analysis and current methods have been discussed in Section 8. Section 9 provides the conclusion of the current study along with future directions.

2. Basic Concepts

For a better understanding, basic definitions pertaining to the current work are provided in this section.

Definition 2.1: Bipolar Neutrosophic Set (BNS) [30]

Let U be a fixed set. Then BNS can be defined as follows.

$$N(u) = < \chi_N^+(u), \xi_N^+(u), \Lambda_N^+(u), \chi_N^-(u), \xi_N^-(u), \Lambda_N^-(u) > / u \in U\},$$

Where $\chi_N^+(u), \xi_N^+(u), \Lambda_N^+(u) : U \to [0,1]$ and $\chi_N^-(u), \xi_N^-(u), \Lambda_N^-(u) : U \to [-1,0]$. The positive membership degrees $\chi_N^+(u), \xi_N^+(u), \Lambda_N^+(u)$ are the truth membership, indeterminacy membership degree and falsity membership degree of an element $u \in U$ corresponding to BNSN and the negative membership degrees $\chi_N^-(u), \xi_N^-(u), \Lambda_N^-(u)$ denote the truth membership degree, indeterminacy membership degree and falsity membership degree of an element $u \in U$ to some implicit counter property corresponding to a BNSN.

In particular, if 'U' has only one element, then

$N(u) =< \chi_N^+(u), \xi_N^+(u), \Lambda_N^+(u), \chi_N^-(u), \xi_N^-(u), \Lambda_N^-(u) >\}$, is called bipolar neutrosophic numbers (BNN).

Definition 2.2 : Algebraic operations of BNNs [30]

Let $N_1 =< \chi_1^+, \xi_1^+, \Lambda_1^+, \chi_1^-, \xi_1^-, \Lambda_1^- >$ and $N_2 =< \chi_2^+, \xi_2^+, \Lambda_2^+, \chi_2^-, \xi_2^-, \Lambda_2^- >$ be two BNNs. Then algebraic operations are defined as follows:

$$(1) N_1 \oplus N_2 =< \chi_1^+ + \chi_2^+ - \chi_1^+\chi_2^+, \xi_1^+\xi_2^+, \Lambda_1^+\Lambda_2^-, -\chi_1^+\chi_2^+, -(-\xi_1^- - \xi_2^- - \xi_1^-\xi_2^-), -(-\Lambda_1^- - \Lambda_2^- - \Lambda_1^-\Lambda_2^-) >$$

$$(2) N_1 \otimes N_2 =< \chi_1^+\chi_2^+, \xi_1^+ + \xi_2^+ - \xi_1^+\xi_2^+, \Lambda_1^+ + \Lambda_2^+ - \Lambda_1^+\Lambda_2^+, -(-\chi_1^- - \chi_2^- - \chi_1^-\chi_2^-), -\xi_1^-\xi_2^-, -\Lambda_1^-\Lambda_2^- >$$

$$(3) l.N_1 =< 1-(1-\chi_1^+)^l, \left(\xi_1^+\right)^l, \left(\Lambda_1^+\right)^l, \left(-\chi_1^-\right)^l, -\left(1-\left(1-\left(-\xi_1^-\right)^l\right)\right), -\left(1-\left(1-\left(-\Lambda_1^-\right)^l\right)\right) > (l>0),$$

$$(4) \mathrm{N}_1^l =< \left(\chi_1^+\right)^l, 1-\left(1-\xi_1^+\right)^l, 1-\left(1-\Lambda_1^+\right)^l, -\left(1-\left(1-\left(-\chi_1^-\right)^l\right)\right), -\left(-\xi_1^-\right)^l, -\left(-\Lambda_1^-\right)^l > (l>0)$$

Definition 2.3: Score and accuracy function of BNNs [26]

Let $N_1 =< \chi_1^+, \xi_1^+, \Lambda_1^+, \chi_1^-, \xi_1^-, \Lambda_1^- >$ be BNN. Then the score function s(aleph1),accuracy function $a\left(N_1\right)$ and certainty function

c(aleph1) are defined as:

$$s\left(N_1\right) = \frac{\chi_1^+ + 1 - \xi_1^+ + 1 - \Lambda_1^+ + 1 + \chi_1^- - \xi_1^- - \Lambda_1^-}{6} \quad \text{……..(1)}$$

$$a\left(N_1\right) = \left(\chi_1^+ - \Lambda_1^+\right) + \left(\chi_1^- - \Lambda_1^-\right) \quad \text{……..(2)}$$

$$c\left(N_1\right) = \chi_1^+ - \Lambda_1^- \quad \text{…….(3)}$$

Definition 2.4: Properties on Bipolar Neutrosophic Sets [30]

Let $N_1 =< \chi_1^+, \xi_1^+, \Lambda_1^+, \chi_1^-, \xi_1^-, \Lambda_1^- >$

And $N_2 =< \chi_2^+, \xi_2^+, \Lambda_2^+, \chi_2^-, \xi_2^-, \Lambda_2^- >$

Be two BNNs therefore

(1) If $s\left(N_1\right) > s\left(N_2\right), then N_1 > N_2$.

(2) If $s\left(N_1\right)=s\left(N_2\right)\&\, a\left(N_1\right)=a\left(N_2\right), then N_1>N_2$.

(3) If $s\left(N_1\right)=s\left(N_2\right)\&\, a\left(N_1\right)=a\left(N_2\right)\&\, c\left(N_1\right)=c\left(N_2\right), then N_1>N_2$

(4) If $s\left(N_1\right)=s\left(N_2\right)\&\, a\left(N_1\right)=a\left(N_2\right)\&\, c\left(N_1\right)=c\left(N_2\right), then N_1 \square N_2$

Definition 2.5: Frank Triangular Norms [27]

The sum and product of Frank triangular norms are defined as follows.

$$\mathrm{N}_1 \oplus_F \mathrm{N}_2 = 1-\log_\lambda\left(1+\frac{\left(\lambda^{1-\mathrm{N}_1}-1\right)\left(\lambda^{1-\mathrm{N}_2}-1\right)}{\lambda-1}\right)\lambda>1\forall\left(\mathrm{N}_1,\mathrm{N}_2\right)\in[0,1]^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots(4)$$

$$\mathrm{N}_1 \otimes_F \mathrm{N}_2 = \log_\lambda\left(1+\frac{\left(\lambda^{\mathrm{N}_1}-1\right)\left(\lambda^{\mathrm{N}_2}-1\right)}{\lambda-1}\right)\lambda>1\forall\left(\mathrm{N}_1,\mathrm{N}_2\right)\in[0,1]^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots(5)$$

3. Operational Laws of Frank Triangular Norms for Bipolar Neutrosophic Numbers:

In this section, Operational laws are proposed using Frank triangular norms for BNNs.

Definition 3.1: Let $\mathrm{N}_1=\left\langle \chi_1^+,\xi_1^+,\Lambda_1^+,\chi_1^-,\xi_1^-,\Lambda_1^-\right\rangle$ and $\mathrm{N}_2=\left\langle \chi_2^+,\xi_2^+,\Lambda_2^+,\chi_2^-,\xi_2^-,\Lambda_2^-\right\rangle$ be two BNNs and $\lambda\geq 1$. Then the operational laws are as follows.

(i). Addition:

$$N_1 \oplus_F N_2 = \begin{cases} < 1 - \log_\lambda \left(1 + \dfrac{\left(\lambda^{1-\chi_1^+} - 1\right)\left(\lambda^{1-\chi_2^+} - 1\right)}{\lambda - 1} \right) - \log_\lambda \left(1 + \dfrac{\left(\lambda^{\chi_1^+} - 1\right)\left(\lambda^{\chi_2^+} - 1\right)}{\lambda - 1} \right), \log_\lambda \left(1 + \dfrac{\left(\lambda^{\xi_1^+} - 1\right)\left(\lambda^{\xi_2^+} - 1\right)}{\lambda - 1} \right), \\ \log_\lambda \left(1 + \dfrac{\left(\lambda^{\Lambda_1^+} - 1\right)\left(\lambda^{\Lambda_1^-} - 1\right)}{\lambda - 1} \right), - \log_\lambda \left(1 + \dfrac{\left(\lambda^{\chi_1^+} - 1\right)\left(\lambda^{\chi_2^+} - 1\right)}{\lambda - 1} \right), \\ \log_\lambda \left(1 + \dfrac{\left(\lambda^{1+\xi_1^-} - 1\right)\left(\lambda^{1+\xi_2^-} - 1\right)}{\lambda - 1} \right) - 1 + \log_\lambda \left(1 + \dfrac{\left(\lambda^{-\xi_1^-} - 1\right)\left(\lambda^{-\xi_2^-} - 1\right)}{\lambda - 1} \right), \\ \log_\lambda \left(1 + \dfrac{\left(\lambda^{1+\Lambda_1^-} - 1\right)\left(\lambda^{1+\Lambda_2^-} - 1\right)}{\lambda - 1} \right) - 1 + \log_\lambda \left(1 + \dfrac{\left(\lambda^{-\Lambda_1^-} - 1\right)\left(\lambda^{-\Lambda_2^-} - 1\right)}{\lambda - 1} \right) > \end{cases}$$

(ii). Multiplication:

$$N_1 \otimes_F N_2 = \begin{cases} < \log_\lambda \left(1 + \dfrac{\left(\lambda^{\chi_1^+} - 1\right)\left(\lambda^{\chi_2^+} - 1\right)}{\lambda - 1} \right), \\ 1 - \log_\lambda \left(\dfrac{\lambda + \lambda^{2-\xi_1^+ - \xi_2^+} - \lambda^{1-\xi_1^+} - \lambda^{1-\xi_2^+}}{\lambda - 1} \right) - \log_\lambda \left(\dfrac{\lambda + \lambda^{\xi_1^+ + \xi_2^+} - \lambda^{\xi_1^+} - \lambda^{\xi_2^+}}{\lambda - 1} \right), \\ 1 - \log_\lambda \left(\dfrac{\lambda + \lambda^{2-\Lambda_1^+ - \Lambda_2^+} - \lambda^{1-\Lambda_1^+} - \lambda^{1-\Lambda_2^+}}{\lambda - 1} \right) - \log_\lambda \left(\dfrac{\lambda + \lambda^{\Lambda_1^+ + \Lambda_2^+} - \lambda^{\Lambda_1^+} - \lambda^{\Lambda_2^+}}{\lambda - 1} \right). \\ \log_\lambda \left(1 - \dfrac{\left(\lambda^{1+\chi_1^-} - 1\right)\left(\lambda^{1+\chi_2^-} - 1\right)}{\left(\lambda^{-\chi_1^-} - 1\right)\left(\lambda^{-\chi_2^-} - 1\right)} \right) - 1, - \log_\lambda \left(1 + \dfrac{\left(\lambda^{\xi_1^-} - 1\right)\left(\lambda^{\xi_2^-} - 1\right)}{\lambda - 1} \right), \\ - \log_\lambda \left(1 + \dfrac{\left(\lambda^{\Lambda_1^-} - 1\right)\left(\lambda^{\Lambda_2^-} - 1\right)}{\lambda - 1} \right) > \end{cases}$$

(iii). Multiplication by an ordinary number:

$$l._F N_1 = \begin{cases} < 1 - \log_\lambda \left(1 + \dfrac{\left(\lambda^{1-\chi_1^+} - 1\right)^k}{\left(\lambda - 1\right)^{k-1}} \right), \log_\lambda \left(1 + \dfrac{\left(\lambda^{\xi_1^+} - 1\right)^k}{\left(\lambda - 1\right)^{k-1}} \right) \\ \log_\lambda \left(1 + \dfrac{\left(\lambda^{\Omega_1^+} - 1\right)^k}{\left(\lambda - 1\right)^{k-1}} \right), - \log_\lambda \left(1 + \dfrac{\left(\lambda^{\Lambda_1^-} - 1\right)^k}{\left(\lambda - 1\right)^{k-1}} \right), \\ \log_\lambda \left(1 + \dfrac{\left(\lambda^{1-\xi_1^-} - 1\right)^k}{\left(\lambda - 1\right)^{k-1}} \right) - 1, \log_\lambda \left(1 + \dfrac{\left(\lambda^{1-\Lambda_1^-} - 1\right)^k}{\left(\lambda - 1\right)^{k-1}} \right) - 1 > \end{cases}$$

(iv) Power Operation:

$$N_1^{\wedge_F^k} = \begin{cases} < \log_\lambda \left(1 + \frac{\left(\lambda^{\chi_1^+} - 1\right)^k}{(\lambda-1)^{k-1}} \right), 1 - \log_\lambda \left(1 + \frac{\left(\lambda^{1-\xi_1^+} - 1\right)^k}{(\lambda-1)^{k-1}} \right) \\ 1 - \log_\lambda \left(1 + \frac{\left(\lambda^{1-\Lambda_1^+} - 1\right)^k}{(\lambda-1)^{k-1}} \right), \log_\lambda \left(1 + \frac{\left(\lambda^{1-\chi_1^-} - 1\right)^k}{(\lambda-1)^{k-1}} \right) - 1, \\ -\log_\lambda \left(1 + \frac{\left(\lambda^{\xi_1^-} - 1\right)^k}{(\lambda-1)^{k-1}} \right), -\log_\lambda \left(1 + \frac{\left(\lambda^{\Lambda_1^-} - 1\right)^k}{(\lambda-1)^{k-1}} \right) > \end{cases}$$

Theorem 3.2:

Let $N_1 = \left\langle \chi_1^+, \xi_1^+, \Lambda_1^+, \chi_1^-, \xi_1^-, \Lambda_1^- \right\rangle$ and $N_2 = \left\langle \chi_2^+, \xi_2^+, \Lambda_2^+, \chi_2^-, \xi_2^-, \Lambda_2^- \right\rangle$ be two BNNs and

$l, l_1, l_2 > 0$. Then the following properties can be proven easily

(a) $N_1 \oplus_F N_2 = N_2 \oplus_F N_1$

(b) $N_1 \otimes_F N_2 = N_2 \otimes_F N_1$

(c) $l .\!_F \left(N_1 \oplus_F N_2 \right) = l .\!_F N_2 \oplus_F l .\!_F N_1$

(d) $\left(N_1 \oplus_F N_2 \right)^{\wedge_F^l} = N_1^{\wedge_F^l} \oplus_F N_2^{\wedge_F^l}$

(e) $\left(l_1 + l_2 \right) .\!_F N_1 = l_1 .\!_F N_2 \oplus_F l_2 .\!_F N_1$

(f) $N_1^{\wedge_F^{(l_1+l_2)}} = N_2^{\wedge_F^{l_1}} \otimes N_2^{\wedge_F^{l_2}}$

4. Bipolar Neutrosophic Frank Weighted Geometric Aggregation Operator

In this section, we proposed bipolar neutrosophic Frank weighted geometric aggregation (BNFWGA) and the bipolar neutrosophic Frank ordered weighted geometric aggregation (BNFOWGA) operators and discussed different properties.

Definition 4.1:

Let $N_j = \left\langle \chi_j^+, \xi_j^+, \Lambda_j^+, \chi_j^-, \xi_j^-, \Lambda_j^- \right\rangle \left(j = 1, 2, 3 \ldots\ldots n \right)$ be a family of BNN's.

A mapping BNFWGA: $U^{'} = U$ is called BNFWGA operator, if it satisfies

$$BNFWGA\left(N_1, N_2, N_3.....N_n\right) = \overset{n}{\underset{j=1}{\otimes}} N_n^{\wedge_F^{\omega_n}}$$

$$= N_1^{\wedge_F^{\omega_1}} \otimes_F N_2^{\wedge_F^{\omega_2}} \otimes_F .....\otimes_F N_n^{\wedge_F^{\omega_n}}$$

and $\omega = \left(\omega_1, \omega_2, \omega_3.....\omega_n\right)^T$ be the weight vector of $\mathrm{N}_j, \omega_j \in [0,1] \& \sum_{j=1} \omega_j = 1.$

Theorem 4.1 :

Let $\mathrm{N}_j = \left\langle \chi_j^+, \xi_j^+, \Lambda_j^+, \chi_j^-, \xi_j^-, \Lambda_j^- \right\rangle \left(j = 1,2,3.....n\right)$ be a family of BNN's and $\omega = \left(\omega_1, \omega_2, \omega_3.....\omega_n\right)^T$ be the weight vector of $\mathrm{N}_j, \omega_j \in [0,1] \& \sum_{j=1} \omega_j = 1.$ Then, the value aggregated using BNFWGA operator is still a BNN i.e).

$$BNFWGA\left(N_1, N_2, N_3.....N_n\right) = N_1^{\wedge_F^{\omega_j}} \otimes_F N_2^{\wedge_F^{\omega_j}} \otimes_F .....\otimes_F N_n^{\wedge_F^{\omega_j}}$$

$$= \begin{cases} < \log_\lambda \left(1 + \prod_{j=1}^{n} \left(\lambda^{\chi_j^+} - 1\right)^{\omega_j}\right), 1 - \log\left(1 + \prod_{j=1}^{n} \left(\lambda^{1-\xi_j^+} - 1\right)^{\omega_j}\right) \\ 1 - \log \prod_{j=1}^{n} \left(\lambda^{1-\Lambda_j^+} - 1\right)^{\omega_j}, \log_\lambda \left(1 + \prod_{j=1}^{n} \left(\lambda^{1-(-\chi_j^-)} - 1\right)^{\omega_j}\right) - 1, \\ -\log_\lambda \left(1 + \prod_{j=1}^{n} \left(\lambda^{-\xi_j^-} - 1\right)^{\omega_j}\right), -\log_\lambda \left(1 + \prod_{j=1}^{n} \left(\lambda^{-\Lambda_j^-} - 1\right)^{\omega_j}\right), > (\omega > 0) \end{cases} \quad \text{……..(6)}$$

Proof:

By mathematical induction we prove the result.

Case (1): When n=2

Based on the Definition 4.1, the following result can be obtained

$$BNFWGA\left(N_1, N_2\right) = N_1^{\wedge_F^{\omega_1}} \otimes_F N_2^{\wedge_F^{\omega_2}}$$

$$N_1^{\wedge_F^{\omega}} = \begin{cases} < \log_\lambda \left(1 + \left(\lambda^{\chi_1^+} - 1\right)^{\omega}\right), 1 - \log_\lambda \left(1 + \left(\lambda^{1-\xi_1^+} - 1\right)^{\omega}\right), \\ 1 - \log_\lambda \left(1 + \left(\lambda^{1-\Lambda_1^+} - 1\right)^{\omega}\right), \log_\lambda \left(1 + \left(\lambda^{1-(-\chi_1^-)} - 1\right)^{\omega}\right) - 1, \\ -\log_\lambda \left(1 + \left(\lambda^{-\xi_1^-} - 1\right)^{\omega}\right), -\log_\lambda \left(1 + \left(\lambda^{-\Lambda_1^-} - 1\right)^{\omega}\right) > \end{cases}$$

$$N_2^{\wedge_F^{\omega_1}} = \begin{cases} < \log_\lambda \left(1 + \left(\lambda^{\chi_2^+} - 1 \right)^{\omega_2} \right), 1 - \log_\lambda \left(1 + \left(\lambda^{1-\xi_2^+} - 1 \right)^{\omega_2} \right), \\ 1 - \log_\lambda \left(1 + \left(\lambda^{1-\Lambda_2^+} - 1 \right)^{\omega_2} \right), \log_\lambda \left(1 + \left(\lambda^{1-(-\chi_2^-)} - 1 \right)^{\omega_2} \right) - 1, \\ - \log_\lambda \left(1 + \left(\lambda^{-\xi_2^-} - 1 \right)^{\omega_2} \right), - \log_\lambda \left(1 + \left(\lambda^{-\Lambda_2^-} - 1 \right)^{\omega_2} \right) > \end{cases}$$

$$N_1^{\wedge_F^{\omega_1}} \otimes_F N_2^{\wedge_F^{\omega_2}} == \begin{cases} < \log_\lambda \left(1 + \prod_{j=1}^{2} \left(\lambda^{\chi_j^+} - 1 \right)^{\omega_j} \right), 1 - \log \prod_{j=1}^{2} \left(\lambda^{1-\xi_j^+} - 1 \right)^{\omega_j}, \\ 1 - \log \prod_{j=1}^{2} \left(\lambda^{1-\Lambda_j^+} - 1 \right)^{\omega_j}, \log_\lambda \left(1 + \prod_{j=1}^{2} \left(\lambda^{1-(-\chi_j^-)} - 1 \right)^{\omega_j} \right) - 1, \\ - \log_\lambda \left(1 + \prod_{j=1}^{2} \left(\lambda^{-\xi_j^-} - 1 \right)^{\omega_j} \right), - \log_\lambda \left(1 + \prod_{j=1}^{2} \left(\lambda^{-\Lambda_j^-} - 1 \right)^{\omega_j} \right), > (\omega > 0) \end{cases}$$

Case (ii): When n=s

Using equation (6), the following result can be obtained.

$$BNFWGA\left(N_1, N_2 ..... N_s \right) = \otimes_{j=1}^{s} N_j^{\wedge^{\omega_j}}$$

$$= \begin{cases} < \log_\lambda \left(1 + \prod_{j=1}^{s} \left(\lambda^{\chi_j^+} - 1 \right)^{\omega_j} \right), 1 - \log \left(1 + \prod_{j=1}^{s} \left(\lambda^{1-\xi_j^+} - 1 \right)^{\omega_j} \right), \\ 1 - \log \prod_{j=1}^{s} \left(\lambda^{1-\Lambda_j^+} - 1 \right)^{\omega_j}, \log_\lambda \left(1 + \prod_{j=1}^{s} \left(\lambda^{1-(-\chi_j^-)} - 1 \right)^{\omega_j} \right) - 1, \\ - \log_\lambda \left(1 + \prod_{j=1}^{s} \left(\lambda^{-\xi_j^-} - 1 \right)^{\omega_j} \right), - \log_\lambda \left(1 + \prod_{j=1}^{s} \left(\lambda^{-\Lambda_j^-} - 1 \right)^{\omega_j} \right), > (\omega > 0) \end{cases}$$

Case (iii) When n=s+1 then following result can be obtained:

$$BNFWGA\left(N_1, N_2 .....N_s, N_{s+1}\right) = \otimes_{j=1}^{s} N_j^{\wedge^{\omega_j}} \otimes N_j^{\wedge^{\omega_{s+1}}}$$

$$= \begin{cases} < \log_\lambda \left(1+\prod_{j=1}^{s}\left(\lambda^{\chi_j^+}-1\right)^{\omega_j}\right), 1-\log\left(1+\prod_{j=1}^{s}\left(\lambda^{1-\xi_j^+}-1\right)^{\omega_j}\right), \\ 1-\log\left(1+\prod_{j=1}^{s}\left(\lambda^{1-\Lambda_j^+}-1\right)^{\omega_j}\right), \log_\lambda\left(1+\prod_{j=1}^{s}\left(\lambda^{1-(-\chi_j^-)}-1\right)^{\omega_j}\right)-1, \\ -\log_\lambda\left(1+\prod_{j=1}^{s}\left(\lambda^{-\xi_j^-}-1\right)^{\omega_j}\right), -\log_\lambda\left(1+\prod_{j=1}^{s}\left(\lambda^{-\Lambda_j^-}-1\right)^{\omega_j}\right), > (\omega>0) \end{cases}$$

$$\otimes_F \begin{cases} < \log_\lambda \left(1+\prod_{j=s+1}\left(\lambda^{\chi_j^+}-1\right)^{\omega_j}\right), 1-\log\left(1+\prod_{j=s+1}\left(\lambda^{1-\xi_j^+}-1\right)^{\omega_j}\right), \\ 1-\log\left(1+\prod_{j=s+1}\left(\lambda^{1-\Lambda_j^+}-1\right)^{\omega_j}\right), \log_\lambda\left(1+\prod_{j=s+1}\left(\lambda^{1-(-\chi_j^-)}-1\right)^{\omega_j}\right)-1, \\ -\log_\lambda\left(1+\prod_{j=s+1}\left(\lambda^{-\xi_j^-}-1\right)^{\omega_j}\right), -\log_\lambda\left(1+\prod_{j=s+1}\left(\lambda^{-\Lambda_j^-}-1\right)^{\omega_j}\right), > (\omega>0) \end{cases}$$

Thus the following result can be obtained.

$$BNFWGA\left(N_1, N_2, N_3 .....N_n\right) = N_1^{\wedge_F^{\omega_j}} \otimes_F N_2^{\wedge_F^{\omega_j}} \otimes_F .....\otimes_F N_n^{\wedge_F^{\omega_j}}$$

$$= \begin{cases} < \log_\lambda \left(1+\prod_{j=1}^{n}\left(\lambda^{\chi_j^+}-1\right)^{\omega_j}\right), 1-\log\left(1+\prod_{j=1}^{n}\left(\lambda^{1-\xi_j^+}-1\right)^{\omega_j}\right), \\ 1-\log\prod_{j=1}^{n}\left(\lambda^{1-\Lambda_j^+}-1\right)^{\omega_j}, \log_\lambda\left(1+\prod_{j=1}^{n}\left(\lambda^{1-(-\chi_j^-)}-1\right)^{\omega_j}\right)-1, \\ -\log_\lambda\left(1+\prod_{j=1}^{n}\left(\lambda^{-\xi_j^-}-1\right)^{\omega_j}\right), -\log_\lambda\left(1+\prod_{j=1}^{n}\left(\lambda^{-\Lambda_j^-}-1\right)^{\omega_j}\right), > (\omega>0) \end{cases}$$

Therefore, the theorem is true for n=s+1.

Hence the theorem.

Example 4.1: Let $N_1 =< 0.5, 0.7, 0.3, -0.6, -0.2, -0.6 >$

$N_2 =< 0.2, 0.5, 0.5, -0.8, -0.4, -0.3 >$

$N_3 =< 0.3, 0.6, 0.4, -0.7 - 0.3, -0.1 >$

be three BNNs and let weight vector of BNNs $N_j \ (j=1,2,3)$ be

$\omega = \left(\frac{1}{8}, \frac{3}{8}, \frac{1}{2}\right)^T$, $\omega_1 = \frac{1}{8}, \omega_2 = \frac{3}{8}, \omega_3 = \frac{1}{2}$ are the weight of $N_j \ (j=1,2,3)$ such that $\prod_{j=1} \omega_j = 1$

Then by above theorem

$$BNFWGA\left(N_1,N_2,N_3\right)=\begin{cases}\log_3\left(1+\left(3^{0.5}-1\right)^{\frac{1}{8}}+\left(3^{0.2}-1\right)^{\frac{3}{8}}+\left(3^{0.3}-1\right)^{\frac{1}{2}}\right),\\ 1-\log_3\left(1+\left(3^{1-0.7}-1\right)^{\frac{1}{8}}+\left(3^{1-0.5}-1\right)^{\frac{3}{8}}+\left(3^{1-0.6}-1\right)^{\frac{1}{2}}\right),\\ 1-\log_3\left(1+\left(3^{1-0.3}-1\right)^{\frac{1}{8}}+\left(3^{1-0.5}-1\right)^{\frac{3}{8}}+\left(3^{1-0.4}-1\right)^{\frac{1}{2}}\right),\\ \log_3\left(1+\left(3^{1-0.6}-1\right)^{\frac{1}{8}}+\left(3^{1-0.8}-1\right)^{\frac{3}{8}}+\left(3^{1-0.7}-1\right)^{\frac{1}{2}}\right)-1,\\ \log_3\left(1+\left(3^{0.2}-1\right)^{\frac{1}{8}}+\left(3^{0.4}-1\right)^{\frac{3}{8}}+\left(3^{0.3}-1\right)^{\frac{1}{2}}\right),\\ l\log_3\left(1+\left(3^{0.6}-1\right)^{\frac{1}{8}}+\left(3^{0.3}-1\right)^{\frac{3}{8}}+\left(3^{0.1}-1\right)^{\frac{1}{2}}\right)\end{cases}$$

$$BNFWGA\left(N_1,N_2,N_3\right)=\left(1.0,0.0391,0.2474,-0.3072,-1.0,-1.0\right)$$

The BNFWGA operator has the following properties:

(1) Idomopotency:

Let all the BNN's be $N_j=<\chi_j^+,\xi_j^+,\Lambda_j^+,\chi_j^-,\xi_j^-,\Lambda_j^->=N\left(j=1,2,3....n\right)$ where $\omega=\left(\omega_1,\omega_2,\omega_3.....\omega_n\right)^T$ be the weight vector of $N_j.\omega_j\in\left[0,1\right]$ and $\sum_{j=1}^{3}\omega_j=1$ then

$$BNFWGA\left(N_1,N_2,N_3.....N_j\right)=N$$

(2) Monotonicity: Let $N_j\ \left(j=1,2,3....n\right)$ and $N_j^{'}\ \left(j=1,2,3....n\right)$ be two families of BNNs, where $\omega=\left(\omega_1,\omega_2,\omega_3.....\omega_n\right)^T$ be the weight vector of N_j and $N_j^{'}$, $\omega_j\in\left[0,1\right]$ and $\prod_{j=1}\omega_j=1$. For all 'j' if $N_j\geq N_j^{'}$ then

$$BNFWGA\left(N_1,N_2,N_3.....N_j\right)\geq BNFWGA\left(N_1^{'},N_2^{'},N_3^{'}.....N_j^{'}\right)$$

(3) Boundedness: Let $N_j =< \chi_j^+, \xi_j^+, \Lambda_j^+, \chi_j^-, \xi_j^-, \Lambda_j^- >= N\left(j = 1,2,3....n\right)$ be a family of BNNs. Where $\omega = \left(\omega_1, \omega_2, \omega_3.....\omega_n\right)^T$ be the weight vector of N_j , $\omega_j \in [0,1]$ and $\sum_{j=1}^{3} \omega_j = 1$, Therefore we have

$$BNFWGA\left(N^-, N^-, ...N^-\right) \le BNFWGA\left(N_1, N_2, N_3.....N_j\right) \le BNFWGA\left(N^+, N^+, ...N^+\right)$$

where $N^- =< \chi_{N^-}^+, \xi_{N^-}^+, \Lambda_{N^-}^+, \chi_{N^-}^-, \xi_{N^-}^-, \Lambda_{N^-}^- >$

$$= \begin{cases} \min\left(\chi_1^+, \chi_2^+, \chi_3^+, ...\chi_n^+\right), \max\left(\xi_1^+, \xi_2^+, \xi_3^+, ...\xi_n^+\right), \\ \max\left(\Lambda_1^+, \Lambda_2^+, \Lambda_3^+, ...\Lambda_n^+\right), \max\left(\chi_1^-, \chi_2^-, \chi_3^-, ...\chi_n^-\right) \\ \min\left(\xi_1^-, \xi_2^-, \xi_3^-, ...\xi_n^-\right), \min\left(\Lambda_1^-, \Lambda_2^-, \Lambda_3^-, ...\Lambda_n^-\right) > \end{cases}$$

And $N^+ =< \chi_{N^+}^+, \xi_{N^+}^+, \Lambda_{N^+}^+, \chi_{N^+}^-, \xi_{N^+}^-, \Lambda_{N^+}^- >$

$$= \begin{cases} \max\left(\chi_1^+, \chi_2^+, \chi_3^+, ...\chi_n^+\right), \min\left(\xi_1^+, \xi_2^+, \xi_3^+, ...\xi_n^+\right), \\ \min\left(\Lambda_1^+, \Lambda_2^+, \Lambda_3^+, ...\Lambda_n^+\right), \min\left(\chi_1^-, \chi_2^-, \chi_3^-, ...\chi_n^-\right) \\ \max\left(\xi_1^-, \xi_2^-, \xi_3^-, ...\xi_n^-\right), \max\left(\Lambda_1^-, \Lambda_2^-, \Lambda_3^-, ...\Lambda_n^-\right) > \end{cases}$$

Proof: Since $N_j =< \chi_j^+, \xi_j^+, \Lambda_j^+, \chi_j^-, \xi_j^-, \Lambda_j^- >= N\left(j = 1,2,3....n\right)$ then, the following result can be obtained by using Equation (6). The following result can be obtained

$$BNFWGA\left(N_1, N_2, N_3.....N_n\right) = \begin{cases} < \log_\lambda\left(1 + \prod_{j=1}^{n}\left(\lambda^{\chi_j^+} - 1\right)^{\omega_j}\right), 1 - \log\prod_{j=1}^{n}\left(\lambda^{1-\xi_j^+} - 1\right)^{\omega_j}, \\ 1 - \log\prod_{j=1}^{n}\left(\lambda^{1-\Lambda_j^+} - 1\right)^{\omega_j}, \log_\lambda\left(1 + \prod_{j=1}^{n}\left(\lambda^{1-(-\chi_j^-)} - 1\right)^{\omega_j}\right) - 1, \\ -\log_\lambda\left(1 + \prod_{j=1}^{n}\left(\lambda^{-\xi_j^-} - 1\right)^{\omega_j}\right), -\log_\lambda\left(1 + \prod_{j=1}^{n}\left(\lambda^{-\Lambda_j^-} - 1\right)^{\omega_j}\right), > (\omega > 0) \end{cases}$$

$$= \begin{cases} < \log_\lambda\left(1 + \left(\lambda^{\chi^+} - 1\right)\right), 1 - \log_\lambda\left(1 + \left(\lambda^{1-\xi^+} - 1\right)\right), \\ 1 - \log_\lambda\left(1 + \left(\lambda^{1-\Lambda^+} - 1\right)\right), \log_\lambda\left(1 + \left(\lambda^{1-(-\chi^-)} - 1\right)\right) - 1, \\ -\log_\lambda\left(1 + \left(\lambda^{-\xi^-} - 1\right)\right), -\log_\lambda\left(1 + \left(\lambda^{-\Lambda^-} - 1\right)\right) > \end{cases}$$

$=< \chi^+, \xi^+, \Lambda^+, \chi^-, \xi^-, \Lambda^- >= N$ holds

(1) The property is obvious based on the equation (6).

(2) Let $N^{-} =< \chi_{N^{-}}^{+}, \xi_{N^{-}}^{+}, \Lambda_{N^{-}}^{+}, \chi_{N^{-}}^{-}, \xi_{N^{-}}^{-}, \Lambda_{N^{-}}^{-} > and\, N^{+} =< \chi_{N^{+}}^{+}, \xi_{N^{+}}^{+}, \Lambda_{N^{+}}^{+}, \chi_{N^{+}}^{-}, \xi_{N^{+}}^{-}, \Lambda_{N^{+}}^{-} >$

There are following inequalities:

$$\begin{cases} < \log_{\lambda}\left(1+\prod_{j=1}^{n}\left(\lambda^{\chi_{N^{-}}^{+}}-1\right)\right) \le \log_{\lambda}\left(1+\prod_{j=1}^{n}\left(\lambda^{\chi_{j}^{+}}-1\right)\right) \le \log_{\lambda}\left(1+\prod_{j=1}^{n}\left(\lambda^{\chi_{N^{+}}^{+}}-1\right)\right), \\ 1-\log\left(1+\left(\prod_{j=1}^{n}\left(\lambda^{1-\xi_{N^{-}}^{+}}-1\right)\right)\right) \le 1-\log\left(1+\left(\prod_{j=1}^{n}\left(\lambda^{1-\xi_{N^{-}}^{+}}-1\right)\right)\right) \le 1-\left(1+\left(\prod_{j=1}^{n}\left(\lambda^{1-\xi_{N^{-}}^{+}}-1\right)\right)\right) \\ 1-\log\prod_{j=1}^{n}\left(\lambda^{1-\Lambda_{N^{-}}^{+}}-1\right) \le 1-\log\prod_{j=1}^{n}\left(\lambda^{1-\Lambda_{j}^{+}}-1\right) \le 1-\log\prod_{j=1}^{n}\left(\lambda^{1-\Lambda_{N^{+}}^{+}}-1\right), \\ \log_{\lambda}\left(1+\prod_{j=1}^{n}\left(\lambda^{1-(-\chi_{N^{-}}^{-})}-1\right)\right)-1 \le \log_{\lambda}\left(1+\prod_{j=1}^{n}\left(\lambda^{1-(-\chi_{N^{-}}^{-})}-1\right)\right)-1 \le \log_{\lambda}\left(1+\prod_{j=1}^{n}\left(\lambda^{1-(-\chi_{N^{-}}^{-})}-1\right)\right)-1 \\ -\log_{\lambda}\left(1+\prod_{j=1}^{n}\left(\lambda^{-\xi_{N^{-}}^{-}}-1\right)\right) \le -\log_{\lambda}\left(1+\prod_{j=1}^{n}\left(\lambda^{-\xi_{j}^{-}}-1\right)\right) \le -\log_{\lambda}\left(1+\prod_{j=1}^{n}\left(\lambda^{-\xi_{N^{+}}^{-}}-1\right)\right), \\ -\log_{\lambda}\left(1+\prod_{j=1}^{n}\left(\lambda^{-\Lambda_{N^{-}}^{-}}-1\right)\right), \le -\log_{\lambda}\left(1+\prod_{j=1}^{n}\left(\lambda^{-\Lambda_{j}^{-}}-1\right)\right)-\log_{\lambda}\left(1+\prod_{j=1}^{n}\left(\lambda^{-\Lambda_{N^{+}}^{-}}-1\right)\right) > \end{cases}$$

Hence

$$BNFWGA\left(N^{-}, N^{-}, ...N^{-}\right) \le BNFWGA\left(N_1, N_2, N_3.....N_j\right) \le BNFWGA\left(N^{+}, N^{+}, ...N^{+}\right)$$

holds.

5. Bipolar Neutrosophic Frank Ordered Weighted Geometric Aggregation (BNFOWGA) Operator

This section proposes the BNFOWGA operator and describes its properties in detail.

Definition 5.1:

Let $\mathrm{N}_j = \left\langle \chi_j^{+}, \xi_j^{+}, \Lambda_j^{+}, \chi_j^{-}, \xi_j^{-}, \Lambda_j^{-} \right\rangle \left(j = 1,2,3.....n\right)$ be a family of BNNs.

A mapping BNFOWGA: $U^{'} = U$ is called BNDWGA operator, if it satisfies,

$$BNFWGA\left(N_1, N_2, N_3.....N_n\right) = \overset{n}{\underset{k=1}{\otimes}} N_{\rho(k)}^{\wedge_F^{\omega_n}}$$

$$= N_{\rho(1)}^{\wedge_F^{\omega_1}} \otimes_F N_{\rho(2)}^{\wedge_F^{\omega_2}} \otimes_F .....\otimes_F N_{\rho(n)}^{\wedge_F^{\omega_n}}$$

Where ρ is permutation that orders the elements

$N_{\rho(1)} \geq N_{\rho(2)} \geq N_{\rho(3)} \geq .....N_{\rho(n)} \geq$

and $\omega = (\omega_1, \omega_2, \omega_3.....\omega_n)^T$ be the weight vector of $N_j, \omega_j \in [0,1] \& \sum_{j=1} \omega_j = 1.$

Theorem 5.1:

Let $N_j = \langle \chi_j^+, \xi_j^+, \Lambda_j^+, \chi_j^-, \xi_j^-, \Lambda_j^- \rangle (j = 1,2,3.....n)$ be a family of BNN's and $\omega = (\omega_1, \omega_2, \omega_3.....\omega_n)^T$ be the weight vector of $N_j, \omega_j \in [0,1] \& \sum_{j=1} \omega_j = 1.$ Then, the value aggregated by using bipolar neutrosophic Frank ordered weighted geometric average operator is still a BNN

$$BNFWGA(N_1, N_2, N_3.....N_n) = \overset{n}{\underset{k=1}{\otimes}} N_{\rho(k)}^{\wedge_F^{\omega_n}}$$

$$= N_{\rho(1)}^{\wedge_F^{\omega_1}} \otimes_F N_{\rho(2)}^{\wedge_F^{\omega_2}} \otimes_F .....\otimes_F N_{\rho(n)}^{\wedge_F^{\omega_n}}$$

$$= \begin{cases} < \log_\lambda \left(1 + \prod_{k=1}^{n} \left(\lambda^{\chi_j^+} - 1\right)^{\omega_k}\right), 1 - \left(\log\left(1 + \prod_{k=1}^{n} \left(\lambda^{1-\xi_j^+} - 1\right)^{\omega_k}\right)\right) \\ 1 - \log \prod_{k=1}^{n} \left(\lambda^{1-\Lambda_j^+} - 1\right)^{\omega_k}, \log_\lambda \left(1 + \prod_{k=1}^{n} \left(\lambda^{1-(-\chi_j^-)} - 1\right)^{\omega_k}\right) - 1, \\ -\log_\lambda \left(1 + \prod_{k=1}^{n} \left(\lambda^{-\xi_j^-} - 1\right)^{\omega_k}\right), -\log_\lambda \left(1 + \prod_{k=1}^{n} \left(\lambda^{-\Lambda_j^-} - 1\right)^{\omega_k}\right), > (\omega > 0) \end{cases} \quad \text{..... (1)}$$

Proof:

If n=2, using Frank operations for bipolar neutrosophic numbers, the following result can be obtained $BNFWGA(N_1, N_2) = N_{\rho(1)}^{\wedge_F^{\omega_1}} \otimes_F N_{\rho(2)}^{\wedge_F^{\omega_2}}$

$$N_1^{\wedge_F^{\omega_1}} = \begin{cases} < \log_\lambda \left(1 + \left(\lambda^{\chi_1^+} - 1\right)^{\omega_1}\right), 1 - \log_\lambda \left(1 + \left(\lambda^{1-\xi_1^+} - 1\right)^{\omega_1}\right), \\ 1 - \log_\lambda \left(1 + \left(\lambda^{1-\Lambda_1^+} - 1\right)^{\omega_1}\right), \log_\lambda \left(1 + \left(\lambda^{1-(-\chi_1^-)} - 1\right)^{\omega_1}\right) - 1, \\ -\log_\lambda \left(1 + \left(\lambda^{-\xi_1^-} - 1\right)^{\omega_1}\right), -\log_\lambda \left(1 + \left(\lambda^{-\Lambda_1^-} - 1\right)^{\omega_1}\right) > \end{cases}$$

$$N_2^{\wedge_F^{\omega_1}} = \begin{cases} < \log_\lambda\left(1+\left(\lambda^{\chi_2^+}-1\right)^{\omega_2}\right), 1-\log_\lambda\left(1+\left(\lambda^{1-\xi_2^+}-1\right)^{\omega_2}\right), \\ 1-\log_\lambda\left(1+\left(\lambda^{1-\Lambda_2^+}-1\right)^{\omega_2}\right), \log_\lambda\left(1+\left(\lambda^{1-(-\chi_2^-)}-1\right)^{\omega_2}\right)-1, \\ -\log_\lambda\left(1+\left(\lambda^{-\xi_2^-}-1\right)^{\omega_2}\right), -\log_\lambda\left(1+\left(\lambda^{-\Lambda_2^-}-1\right)^{\omega_2}\right) > \end{cases}$$

$$N_1^{\wedge_F^{\omega_1}} \otimes_F N_2^{\wedge_F^{\omega_2}} == \begin{cases} < \log_\lambda\left(1+\prod_{j=1}^{2}\left(\lambda^{\chi_j^+}-1\right)^{\omega_j}\right), 1-\log\left(1+\prod_{j=1}^{2}\left(\lambda^{1-\xi_j^+}-1\right)^{\omega_j}\right), \\ 1-\log\prod_{j=1}^{2}\left(\lambda^{1-\Lambda_j^+}-1\right)^{\omega_j}, \log_\lambda\left(1+\prod_{j=1}^{2}\left(\lambda^{1-(-\chi_j^-)}-1\right)^{\omega_j}\right)-1, \\ -\log_\lambda\left(1+\prod_{j=1}^{2}\left(\lambda^{-\xi_j^-}-1\right)^{\omega_j}\right), -\log_\lambda\left(1+\prod_{j=1}^{2}\left(\lambda^{-\Lambda_j^-}-1\right)^{\omega_j}\right), > (\omega>0) \end{cases}$$

If k=s then,

$$BNFWGA\left(N_1, N_2 .....N_s\right) = \otimes_{k=1}^{s} N_{\rho(k)}^{\wedge^{\omega_k}}$$

$$= \begin{cases} < \log_\lambda\left(1+\prod_{k=1}^{s}\left(\lambda^{\chi_k^+}-1\right)^{\omega_k}\right), 1-\log\left(1+\prod_{j=1}^{s}\left(\lambda^{1-\xi_k^+}-1\right)^{\omega_k}\right), \\ 1-\log\prod_{j=1}^{s}\left(\lambda^{1-\Lambda_k^+}-1\right)^{\omega_k}, \log_\lambda\left(1+\prod_{k=1}^{s}\left(\lambda^{1-(-\chi_k^-)}-1\right)^{\omega_k}\right)-1, \\ -\log_\lambda\left(1+\prod_{k=1}^{s}\left(\lambda^{-\xi_k^-}-1\right)^{\omega_k}\right), -\log_\lambda\left(1+\prod_{k=1}^{s}\left(\lambda^{-\Lambda_k^-}-1\right)^{\omega_k}\right), > (\omega>0) \end{cases}$$

If k=s+1 then there is following result:

$$BNFOWGA\left(N_1, N_2 .....N_s, N_{s+1}\right) = \otimes_{k=1}^{s} N_{\rho(s)}^{\wedge^{\omega_s}} \otimes N_{\rho(s+1)}^{\wedge^{\omega_{s+1}}}$$

$$= \begin{cases} < \log_\lambda\left(1+\prod_{k=1}^{s}\left(\lambda^{\chi_k^+}-1\right)^{\omega_k}\right), 1-\log\left(1+\prod_{k=1}^{n}\left(\lambda^{1-\xi_k^+}-1\right)^{\omega_k}\right) \\ 1-\log\left(1+\prod_{k=1}^{s}\left(\lambda^{1-\Lambda_k^+}-1\right)^{\omega_k}\right), \log_\lambda\left(1+\prod_{k=1}^{s}\left(\lambda^{1-(-\chi_k^-)}-1\right)^{\omega_k}\right)-1, \\ -\log_\lambda\left(1+\prod_{k=1}^{s}\left(\lambda^{-\xi_k^-}-1\right)^{\omega_k}\right), -\log_\lambda\left(1+\prod_{k=1}^{s}\left(\lambda^{-\Lambda_k^-}-1\right)^{\omega_k}\right), > (\omega>0), \end{cases}$$

Hence the theorem true for r=s+1. Thus the result

$$\begin{cases} < \log_\lambda \left(1+\prod_{k=1}^{n}\left(\lambda^{\chi_k^+}-1\right)^{\omega_k}\right), 1-\log\left(1+\prod_{k=1}^{n}\left(\lambda^{1-\xi_k^+}-1\right)^{\omega_k}\right) \\ 1-\log\prod_{k=1}^{n}\left(\lambda^{1-\Lambda_k^+}-1\right)^{\omega_k}, \log_\lambda\left(1+\prod_{k=1}^{n}\left(\lambda^{1-(-\chi_k^-)}-1\right)^{\omega_k}\right)-1, \\ -\log_\lambda\left(1+\prod_{k=1}^{n}\left(\lambda^{-\xi_k^-}-1\right)^{\omega_k}\right), -\log_\lambda\left(1+\prod_{k=1}^{n}\left(\lambda^{-\Lambda_k^-}-1\right)^{\omega_k}\right), > (\omega>0) \end{cases}$$

holds for all n.

$$N_1 =< 0.5, 0.7, 0.3, -0.6, -0.2, -0.6 >$$

Example 5.1: Let $N_2 =< 0.2, 0.5, 0.5, -0.8, -0.4, -0.3 >$ be three BNNs and let weight vector of

$$N_3 =< 0.3, 0.6, 0.4, -0.7 - 0.3, -0.1 >$$

BNNs $N_k \ (k=1,2,3)$ be $\omega = \left(\frac{1}{8}, \frac{3}{8}, \frac{1}{2}\right)^T, \omega_1 = \frac{1}{8}, \omega_2 = \frac{3}{8}, \omega_3 = \frac{1}{2}$ are the weight of N_k $(k=1,2,3)$ such that $\prod_{k=1}\omega_k = 1$. Then by above theorem

$$BNFOWGA\left(N_1, N_2, N_3\right) = \begin{cases} \log_3\left(1+\left(3^{0.3}-1\right)^{\frac{1}{8}}+\left(3^{0.2}-1\right)^{\frac{3}{8}}+\left(3^{0.5}-1\right)^{\frac{1}{2}}\right), \\ 1-\log_3\left(1+\left(3^{1-0.6}-1\right)^{\frac{1}{8}}+\left(3^{1-0.5}-1\right)^{\frac{3}{8}}+\left(3^{1-0.7}-1\right)^{\frac{1}{2}}\right), \\ 1-\log_3\left(1+\left(3^{1-0.4}-1\right)^{\frac{1}{8}}+\left(3^{1-0.5}-1\right)^{\frac{3}{8}}+\left(3^{1-0.3}-1\right)^{\frac{1}{2}}\right), \\ \log_3\left(1+\left(3^{1-0.7}-1\right)^{\frac{1}{8}}+\left(3^{1-0.8}-1\right)^{\frac{3}{8}}+\left(3^{1-0.6}-1\right)^{\frac{1}{2}}\right)-1, \\ \log_3\left(1+\left(3^{0.3}-1\right)^{\frac{1}{8}}+\left(3^{0.4}-1\right)^{\frac{3}{8}}+\left(3^{0.2}-1\right)^{\frac{1}{2}}\right), \\ l\log_3\left(1+\left(3^{0.1}-1\right)^{\frac{1}{8}}+\left(3^{0.3}-1\right)^{\frac{3}{8}}+\left(3^{0.6}-1\right)^{\frac{1}{2}}\right) \end{cases}$$

$$BNFOWGA\left(N_1, N_2, N_3\right) = \left(1.0, 0.125, 0.2520, -0.0652, -1.0, -1.0\right).$$

The BNFOWGA operator has the following properties:

(4) Idomopotency:

Let all the BNN′s be $N_n =< \chi_k^+, \xi_k^+, \Lambda_k^+, \chi_k^-, \xi_k^-, \Lambda_k^- >= N\left(k=1,2,3....n\right)$ where $\omega = \left(\omega_1, \omega_2, \omega_3.....\omega_n\right)^T$ be the weight vector of $N_k.\omega_k \in [0,1]$ and $\prod_{k=1}\omega_k = 1$ then

$BNFWGA\left(N_1, N_2, N_3.....N_k\right) = N$.

(5) Monotonicity: Let $N_k \ \left(k = 1,2,3....n\right)$ and $N_j^{'} \ \left(j = 1,2,3....n\right)$ be two families of BNNs, where $\omega = \left(\omega_1, \omega_2, \omega_3.....\omega_n\right)^T$ be the weight vector of N_k and $N_k^{'}$, $\omega_k \in \left[0,1\right]$ and $\prod_{k=1} \omega_k = 1$. For all 'j' if $N_j \geq N_j^{'}$ then

$$BNFWGA\left(N_1, N_2, N_3.....N_k\right) \geq BNFWGA\left(N_1^{'}, N_2^{'}, N_3^{'}.....N_k^{'}\right).$$

(6) Boundedness: Let $N_k =< \chi_k^+, \xi_k^+, \Lambda_k^+, \chi_k^-, \xi_k^-, \Lambda_j^- >= N\left(k = 1,2,3....n\right)$ be a family of BNNs. Where $\omega = \left(\omega_1, \omega_2, \omega_3.....\omega_n\right)^T$ be the weight vector of N_k, $\omega_k \in \left[0,1\right]$ and $\sum_{k=1}^{3} \omega_k = 1$, Therefore we have

$$BNFOWGA\left(N^-, N^-, ...N^-\right) \leq BNFOWGA\left(N_1, N_2, N_3.....N_j\right) \leq BNFOWGA\left(N^+, N^+, ...N^+\right)$$

where $N^- =< \chi_{N^-}^+, \xi_{N^-}^+, \Lambda_{N^-}^+, \chi_{N^-}^-, \xi_{N^-}^-, \Lambda_{N^-}^- >$

$$= \begin{cases} \min\left(\chi_1^+, \chi_2^+, \chi_3^+, ...\chi_n^+\right), \max\left(\xi_1^+, \xi_2^+, \xi_3^+, ...\xi_n^+\right), \\ \max\left(\Lambda_1^+, \Lambda_2^+, \Lambda_3^+, ...\Lambda_n^+\right), \max\left(\chi_1^-, \chi_2^-, \chi_3^-, ...\chi_n^-\right) \text{ And} \\ \min\left(\xi_1^-, \xi_2^-, \xi_3^-, ...\xi_n^-\right), \min\left(\Lambda_1^-, \Lambda_2^-, \Lambda_3^-, ...\Lambda_n^-\right) > \end{cases}$$

$N^+ =< \chi_{N^+}^+, \xi_{N^+}^+, \Lambda_{N^+}^+, \chi_{N^+}^-, \xi_{N^+}^-, \Lambda_{N^+}^- >$

$$= \begin{cases} \max\left(\chi_1^+, \chi_2^+, \chi_3^+, ...\chi_n^+\right), \min\left(\xi_1^+, \xi_2^+, \xi_3^+, ...\xi_n^+\right), \\ \min\left(\Lambda_1^+, \Lambda_2^+, \Lambda_3^+, ...\Lambda_n^+\right), \min\left(\chi_1^-, \chi_2^-, \chi_3^-, ...\chi_n^-\right) \\ \max\left(\xi_1^-, \xi_2^-, \xi_3^-, ...\xi_n^-\right), \max\left(\Lambda_1^-, \Lambda_2^-, \Lambda_3^-, ...\Lambda_n^-\right) > \end{cases}.$$

Proof:

(3) Since $N_j =< \chi_j^+, \xi_j^+, \Lambda_j^+, \chi_j^-, \xi_j^-, \Lambda_j^- >= N\left(j = 1,2,3....n\right)$ then, the following result can be obtained by using Equation (1)The following result can be obtained

$$BNFOWGA\left(N_1, N_2, N_3.....N_n\right) = \begin{cases} < \log_\lambda \left(1+\prod_{k=1}^{n}\left(\lambda^{\chi_k^+}-1\right)^{\omega_k}\right), 1-\log \prod_{k=1}^{n}\left(\lambda^{1-\xi_k^+}-1\right)^{\omega_k}, \\ 1-\log \prod_{k=1}^{n}\left(\lambda^{1-\Lambda_k^+}-1\right)^{\omega_k}, \log_\lambda \left(1+\prod_{k=1}^{n}\left(\lambda^{1-(-\chi_k^-)}-1\right)^{\omega_k}\right)-1, \\ -\log_\lambda \left(1+\prod_{k=1}^{n}\left(\lambda^{-\xi_k^-}-1\right)^{\omega_k}\right), -\log_\lambda \left(1+\prod_{k=1}^{n}\left(\lambda^{-\Lambda_k^-}-1\right)^{\omega_k}\right), > (\omega > 0) \end{cases}$$

$$= \begin{cases} < \log_\lambda \left(1+\left(\lambda^{\chi^+}-1\right)\right), 1-\log_\lambda \left(1+\left(\lambda^{1-\xi^+}-1\right)\right), \\ 1-\log_\lambda \left(1+\left(\lambda^{1-\Lambda^+}-1\right)\right), \log_\lambda \left(1+\left(\lambda^{1-(-\chi^-)}-1\right)\right)-1, \\ -\log_\lambda \left(1+\left(\lambda^{-\xi^-}-1\right)\right), -\log_\lambda \left(1+\left(\lambda^{-\Lambda^-}-1\right)\right) > \end{cases}$$

$$=< \chi^+, \xi^+, \Lambda^+, \chi^-, \xi^-, \Lambda^- >= N \text{ holds}$$

(4) The property is obvious based on the equation (6).

(5) Let $N^- =< \chi_{N^-}^+, \xi_{N^-}^+, \Lambda_{N^-}^+, \chi_{N^-}^-, \xi_{N^-}^-, \Lambda_{N^-}^- > and\, N^+ =< \chi_{N^+}^+, \xi_{N^+}^+, \Lambda_{N^+}^+, \chi_{N^+}^-, \xi_{N^+}^-, \Lambda_{N^+}^- >$.

There are the following inequalities:

$$\begin{cases} < \log_\lambda \left(1+\prod_{k=1}^{n}\left(\lambda^{\chi_{N^-}^+}-1\right)\right) \le \log_\lambda \left(1+\prod_{k=1}^{n}\left(\lambda^{\chi_k^+}-1\right)\right) \le \log_\lambda \left(1+\prod_{j=1}^{n}\left(\lambda^{\chi_{N^+}^+}-1\right)\right), \\ 1-\log \prod_{k=1}^{n}\left(\lambda^{1-\xi_{N^-}^+}-1\right) \le 1-\log \prod_{k=1}^{n}\left(\lambda^{1-\xi_k^+}-1\right) \le, 1-\log \prod_{k=1}^{n}\left(\lambda^{1-\xi_{N^+}^+}-1\right), \\ 1-\log \prod_{k=1}^{n}\left(\lambda^{1-\Lambda_{N^-}^+}-1\right) \le 1-\log \prod_{k=1}^{n}\left(\lambda^{1-\Lambda_k^+}-1\right) \le 1-\log \prod_{j=1}^{n}\left(\lambda^{1-\Lambda_{N^+}^+}-1\right), \\ \log_\lambda \left(1+\prod_{k=1}^{n}\left(\lambda^{1-(-\chi_{N^-}^-)}-1\right)\right)-1 \le \log_\lambda \left(1+\prod_{k=1}^{n}\left(\lambda^{1-(-\chi_{N^-}^-)}-1\right)\right)-1 \le \log_\lambda \left(1+\prod_{k=1}^{n}\left(\lambda^{1-(-\chi_{N^-}^-)}-1\right)\right)-1 \\ -\log_\lambda \left(1+\prod_{k=1}^{n}\left(\lambda^{-\xi_{N^-}^-}-1\right)\right) \le -\log_\lambda \left(1+\prod_{k=1}^{n}\left(\lambda^{-\xi_k^-}-1\right)\right) \le -\log_\lambda \left(1+\prod_{k=1}^{n}\left(\lambda^{-\xi_{N^+}^-}-1\right)\right), \\ -\log_\lambda \left(1+\prod_{k=1}^{n}\left(\lambda^{-\Lambda_{N^-}^-}-1\right)\right), \le -\log_\lambda \left(1+\prod_{k=1}^{n}\left(\lambda^{-\Lambda_k^-}-1\right)\right)-\log_\lambda \left(1+\prod_{k=1}^{n}\left(\lambda^{-\Lambda_{N^+}^-}-1\right)\right) > \end{cases}$$

.

Hence

$$BNFOWGA\left(N^-, N^-, ...N^-\right) \le BNFOWGA\left(N_1, N_2, N_3.....N_j\right) \le BNFOWGA\left(N^+, N^+, ...N^+\right)$$

holds for all 'n'.

6. Model for MCDM Using Bipolar Neutrosophic Information

Using the BNFWGA and BNFOWGA operators that are suggested, two comprehensive MCDM approaches are expanded in this section.

For MCDM model with bipolar neutrosophic fuzzy information, Let $A=\left(A_1,A_2,A_3.....A_n\right)$ be the set of alternatives and $C=\left(C_1,C_2,C_3.....C_n\right)$ be a set of attributes.

Let $\omega=\left(\omega_1,\omega_2,\omega_3.....\omega_n\right)^T$ be the weight to attribute C_K.

Suppose that $N=\left(N_{jk}\right)_{s\times r}=\left(\chi_{jk}^{+},\xi_{jk}^{+},\Lambda_{jk}^{+},\chi_{jk}^{-},\xi_{jk}^{-},\Lambda_{jk}^{-}\right)_{s\times r}\left(j=1,2,3.....s\right)\left(k=1,2,.....r\right)$ is BNN decision matrix, where $\chi_{jk}^{+},\xi_{jk}^{+},\Lambda_{jk}^{+}$ indicates the truth membership degree, indeterminacy membership degree and falsity membership degree of alternative A_j under attribute C_K with respect to positive preferences and $\chi_{jk}^{-},\xi_{jk}^{-},\Lambda_{jk}^{-}$ indicates the truth membership degree, indeterminacy membership degree and falsity membership degree of alternative A_j under attribute C_K with respect to negative preferences. We have conditions $\chi_{jk}^{+},\xi_{jk}^{+},\Lambda_{jk}^{+},\chi_{jk}^{-},\xi_{jk}^{-},\&\Lambda_{jk}^{-}\in\left[0,1\right]$ such that $0\le\chi_{jk}^{+},\xi_{jk}^{+},\Lambda_{jk}^{+},\chi_{jk}^{-},\xi_{jk}^{-},\Lambda_{jk}^{-}\le 6$ for $\left(j=1,2,3.....s\right)\left(k=1,2,.....r\right)$.

6.1 Proposed Algorithm using BNFOWGA operator to solve MCDM problem

Step 1 Collect information on the bipolar neutrosophic evaluation

Step 2 Calculate score and the accuracy values of collected information.

The score values $s\left(N_{jk}\right)$ and accuracy values $a\left(N_{jk}\right)$ of alternatives A_j can be calculated by using Equations (1) and (2).

Step 3 The compression method in Definition 4.1 to reorder information on evaluation under each attribute. The comparison method is used to reorder $\left(N_{jk}\right)$.

Step 4 Derive the collective BNN $N_j\left(j=1,2,.....s\right)$ for the alternative $A_j\left(j=1,2,.....s\right)$

Method (1) .

Utilize BNFWGA operator to calculate the collective BNN for each alternative, then $N_j = BNFWGA\left(N_{j1}, N_{j2}, N_{j3}.....N_{jn}\right) = \overset{n}{\underset{k=1}{\otimes}} N_{jk}^{\hat{}_F^{\omega_k}}$.

$$N_j = BNFWGA\left(N_1, N_2.....N_{jn}\right) = \otimes_{k=1}^{n} N_{\rho(jk)}^{\hat{}^{\omega_k}}$$

$$= \begin{cases} < \log_\lambda \left(1 + \prod_{k=1}^{n} \left(\lambda^{\chi_{jk}^{+}} - 1\right)^{\omega_k}\right), \\ 1 - \log_\lambda \left(1 + \prod_{k=1}^{n} \left(\lambda^{1-\xi_{jk}^{+}} - 1\right)^{\omega_k}\right), \\ 1 - \log_\lambda \left(1 + \prod_{k=1}^{n} \left(\lambda^{1-\Lambda_{jk}^{+}} - 1\right)^{\omega_k}\right), \\ \log_\lambda \left(1 + \prod_{k=1}^{n} \left(\lambda^{1-(-\chi_{jk}^{-})} - 1\right)^{\omega_k}\right) - 1, \\ -\log_\lambda \left(1 + \prod_{k=1}^{s} \left(\lambda^{-\xi_{jk}^{-}} - 1\right)^{\omega_k}\right), \\ -\log_\lambda \left(1 + \prod_{k=1}^{s} \left(\lambda^{-\Lambda_{jk}^{-}} - 1\right)^{\omega_k}\right), > (\omega > 0) \end{cases}$$

Where $\omega = \left(\omega_1, \omega_2, \omega_3.....\omega_n\right)^T$ is the weight vector such that $\omega_k \in [0,1]$ and $\sum_{k=1}^{n} \omega_k = 1$

Method (2).

Utilize BNFOWGA operator to calculate the collective BNN for each alternative, then

$$N_j = BNFOWGA\left(N_1, N_2.....N_{jl}\right) = \otimes_{k=1}^{n} N_{\rho(jl)}^{\hat{}^{\omega_k}}$$

$$= \begin{cases} < \log_\lambda \left(1 + \prod_{l=1}^{n} \left(\lambda^{\chi_{jl}^{+}} - 1\right)^{\omega_l}\right), 1 - \log_\lambda \left(1 + \prod_{k=1}^{n} \left(\lambda^{1-\xi_{jl}^{+}} - 1\right)^{\omega_l}\right), \\ 1 - \log_\lambda \left(1 + \prod_{k=1}^{n} \left(\lambda^{1-\Lambda_{jl}^{+}} - 1\right)^{\omega_l}\right), \log_\lambda \left(1 + \prod_{k=1}^{n} \left(\lambda^{1-(-\chi_{jl}^{-})} - 1\right)^{\omega_l}\right) - 1, \\ -\log_\lambda \left(1 + \prod_{k=1}^{s} \left(\lambda^{-\xi_{jl}^{-}} - 1\right)^{\omega_l}\right), -\log_\lambda \left(1 + \prod_{k=1}^{s} \left(\lambda^{-\Lambda_{jl}^{-}} - 1\right)^{\omega_l}\right), > (\omega > 0) \end{cases}$$

Where ρ is permutation that orders the elements: $N_{\rho(j1)} \geq N_{\rho(j2)} \geq N_{\rho(j3)} \geq .....N_{\rho(jn)}$

Where $\omega = \left(\omega_1, \omega_2, \omega_3.....\omega_n\right)^T$ is the weight vector such that $\omega_l \in [0,1]$ and $\sum_{l=1}^{n} \omega_l = 1$.

Step 5: Calculate the score values $s\left(N_j\right)\left(j = 1,2,3.....s\right)$ of BNNs $\left(N_j\right)\left(j = 1,2,3.....s\right)$ to rank all the alternatives $A_j\left(j = 1,2,3.....s\right)$ and then select favorable one. If score values of BNNs

N_j & N_k are equal, then we calculate accuracy values $a\left(N_j\right) \& a\left(N_k\right)$ of BNNs N_j & N_k respectively and then rank the alternatives A_j & A_i as accuracy values $a\left(N_j\right) \& a\left(N_k\right)$

Step 6 Rank all the alternatives $A_j\ (j = 1,2,3.....s)$ and select favorable one.

Step 7 End

7. Bridge Management to avoid traffic congestion using proposed Algorithm

The best solution to a bridge management problem from Amin Amini & Navid Nikraz [32] is discovered in this part in order to prevent traffic jams. To determine the optimum path that avoids traffic jams. Additionally, parametric analysis and comparison analysis are performed to confirm the adaptability and efficiency of the suggested algorithm to address the problem of decision-making.
People who reside in and travel through affected neighbourhoods, as well as on state routes, are greatly affected by traffic jams. The behaviour of road users, the safety of the road and bridge infrastructure conditions and characteristics, and vehicles interact continuously to form the traffic process. People that are delayed are late for key daily tasks including work, school, appointments, and other things. When clients and consumers have trouble contacting them, business suffers. When ambulances, rescue teams, and fire vehicles are unable to drive on their usual routes, routine incidents can quickly become life-threatening. Therefore, our programme was created to determine the optimum path while taking into consideration three factors: connectivity in a single lane, avoiding traffic incidents, and saving time for human resources. Consider three bridges A_1, A_2, A_3.

Based on the recommendations of the experts group in terms of three criteria, the roads departments chose to construct the bridge in order to reduce traffic namely multiple road connection in single lane $\left(C_1\right)$, avoid road accidents $\left(C_2\right)$, time saver for human resources $\left(C_3\right)$.

Using the proposed algorithm bridge selection has been done as follows:
Step1: Collect information on bipolar neutrosophic evaluation
The information collected from expert discussion on evaluation is given in Table1

Table 1: Bipolar-neutrosophic evaluation information under

	C_1	C_2	C_3
A_1	(0.5,0.7,0.2,-0.7,-0.3,-0.6)	(0.4,0.4,0.5,-0.7,-0.8,-0.4)	(0.7,0.7,0.5,-0.8,-0.7-0.6)

A_2	(0.9,0.7,0.5,-0.7,-0.7,-0.1)	(0.7,0.6,0.8,-0.7,-0.5,-0.1)	(0.9,0.4,0.6,-0.1,-0.7,-0.5)
A_3	(0.3,0.4,0.2,-0.6,-0.3,-0.7)	(0.2,0.2,0.2,-0.4,-0.7,-0.4)	(0.9,0.5,0.5,-0.6,-0.5,-0.2)

Step 2: Calculate Score and accuracy values of collected information.

For each alternative A_j the attribute C_k ,the score values $s\left(N_{jk}\right)$ and the accuracy values $a\left(N_{jk}\right)$ can be calculated based on Equation (1) and Equation (2). The Score values $s\left(N_{jk}\right)$ and the accuracy values $a\left(N_{jk}\right)$ are shown in Tables 2 and 3 respectively.

Table 2: Score values $s\left(N_{jk}\right)$

	C_1	C_2	C_3
A_1	0.4067	0.5000	0.5000
A_2	0.4667	0.3667	0.6667
A_3	0.5167	0.5833	0.5000

Table 2: Accuracy Values $a\left(N_{jk}\right)$

	C_1	C_2	C_3
A_1	0.2000	-0.4000	0
A_2	-0.2000	-0.7000	0.7000
A_3	0.2000	0	0

Step 3: Reorder information on evaluation under each attribute

The comparison method in Definition 4.1 is used to reorder N_{jk}

Table 4 reordering bipolar neutrosophic evaluation information by using comparison method based on Definition 4.1

	C_1	C_2	C_3
A_1	(0.7,0.7,0.5,-0.8,-0.7-0.6)	(0.4,0.4,0.5,-0.7,-0.8,-0.4)	(0.5,0.7,0.2,-0.7,-0.3,-0.6)
A_2	(0.9,0.4,0.6,-0.1,-0.7,-0.5)	0.5,0.2,0.7,-0.5,-0.1,-0.9)	(0.9,0.7,0.5,-0.7,-0.7,-0.1)
A_3	(0.2,0.2,0.2,-0.4,-0.7,-0.4)	(0.3,0.4,0.2,-0.6,-0.3,-0.7)	(0.9,0.5,0.5,-0.6,-0.5,-0.2)

Table 5: Score values $s\left(N_{jk}\right)$

	C_1	C_2	C_3
A_1	0.5000	0.5000	0.4067
A_2	0.6667	0.5167	0.4667
A_3	0.5833	0.5167	0.5000

Table 6: Accuracy Values $a\left(N_{jk}\right)$

	C_1	C_2	C_3
A_1	0	-0.4000	0.2000
A_2	0.7000	0.2000	-0.2000
A_3	0	0.2000	0

Step 4: Derive the collective BNN $N_j\left(j=1,2,3...s\right)$ for the alternative $A_j\left(j=1,2,3...s\right)$ Method 1 BNFWGA operator using Eqn(8) and supporting $\lambda=7$ to calculate the collective BNN for each alternative, then

$$N_1=\langle 0.1301,0.6857,0.6345,-0.7517,-0.2494,-0.4480\rangle$$
$$N_2=\langle 0.5735,0.6795,0.7665,-0.6913,-0.1301,-0.1038\rangle$$
$$N_3=\langle 0.1301,0.6857,0.6345,-0.7517,-0.2494,-0.4480\rangle$$

The value of BNN by power operation when $\lambda=7$

$$\langle 0.7168,0.2758,0.2858,-0.2968,-0.6981,-0.6484\rangle$$

Method (2) Utilize BNFOWGA operator using Eq (9) and supporting $\lambda = 7$ to calculate the collective BNN for each alternative, then

$$N_1 = \langle 0.1301, 0.6857, 0.6345, -0.78414, -0.2512, -0.4480 \rangle$$
$$N_2 = \langle 0.5493, 0.6357, 0.7496, -0.6569, -0.1193, -0.1193 \rangle$$
$$N_3 = \langle 0.2160, 0.4527, 0.4264, -0.5661, -0.2506, -0.2335 \rangle$$

The value is $\langle 0.7100, 0.2758, 0.2668, -0.2739, -0.6343, -0.6529 \rangle$

Step 5: Calculate the score values $s(N_j)\ A_j\ (j = 1,2,3)$ of BNNs $N_j\ (j = 1,2,3)$ for each alternatives $A_j\ (j = 1,2,3)\ A_k\ (k = 1,2,3)$. The score values $s(N_{jk})$ are calculated using Equation (2)

Method 1 The following score values are obtained by using the BNFWGA operation

$s(N_1) = 0.2875 : s(N_2) = 0.2783 : s(N_3) = 0.3751$

Method 2 The following score values are obtained by using the BNFOWGA operation

$s(N_1) = 0.2875 : s(N_2) = 0.2910 : s(N_3) = 0.3758$

Step 6: Rank all the alternatives $A_j\ (j = 1,2,3)$ and select favorable one.

The alternative can be ranked in descending order based on the comparison method, and favorable alternative can be selected.

Method 1 The following ranking order based on score values is obtained by using BNFWGA operator:

$A_3 > A_2 > A_1$ Thus A_3 is favorable.

Method 2 The following ranking order based on the score values is obtained by using BNFOWGA operator: $A_3 > A_2 > A_1$ Thus A_3 is favorable.

Step 7: End

8. Comparative Analysis

To demonstrate the soundness of the suggested work, this section contrasts and compares it to the current approaches.

Table: 7 Ranking Orders obtained by Different Methods.

Methods	Rankings

A_{ω} [22]	$A_3 > A_2 > A_1$
G_{ω} [22]	$A_3 > A_2 > A_1$
BNDGA Operator $\lambda = 7$ [33]	$A_3 > A_2 > A_1$
BNDOGA Operator $\lambda = 7$ [33]	$A_3 > A_2 > A_1$
FBNCGBM operator (s,t=1)[21]	$A_3 > A_2 > A_1$
FBNCOGBM operator (s,t=1)[21]	$A_3 > A_2 > A_1$
FATTT2FFAAOWA $\lambda = 7$ []	$A_2 > A_1 > A_3$
FATTT2FFAAFWA $\lambda = 7$ [33]	$A_2 > A_3 > A_1$
BNFWGA Operator $\lambda = 7$	$A_3 > A_2 > A_1$
BNFOWGA Operator $\lambda = 7$	$A_3 > A_2 > A_1$

Table: 8 Characteristic Comparison of different methods.

Methods	Flexible measure easier
A_{ω} [18]	No
G_{ω} [18]	No
BNDGA Operator $\lambda = 7$ [29]	No
BNDOGA Operator $\lambda = 7$ [29]	No
FBNCGBM operator (s,t=1)[22]	No
FBNCOGBM operator (s,t=1)[22]	No
FATTT2FFAAOWA $\lambda = 7$ [31]	Yes
FATTT2FFAAFWA $\lambda = 7$ [31]	Yes
BNFWGA Operator $\lambda = 7$	Yes
BNFOWGA Operator $\lambda = 7$	No

In G_ω , BNDGA and BNDOGA methods, the proposed method based on proposed BNFWGA operator does not consider the interaction and interrelation among attributes. In contrast to A_ω, G_ω, FATTT2FFAAOWA $(\lambda = 7)$ and FATTT2FFAAFWA $(\lambda = 7)$ methods, the proposed BNFWGA operator choose the appropriate parameters according to the preferences of Decision Makings. In similar to FBNCGBM operator (s,t=1) and FBNCOGBM operator (s,t=1), BNFWGA Operator for $\lambda = 7$, BNFOWGA Operator for $\lambda = 7$, the proposed BNFWGA operators can select the appropriate parameters according to preferences of the decision making.

In G_ω , BNDGA and BNDOGA methods, the proposed method based on proposed BNFOWGA operator does not consider the interaction and interrelation among attributes. In contrast to A_ω, G_ω, FATTT2FFAAOWA $(\lambda = 7)$ and FATTT2FFAAFWA $(\lambda = 7)$ methods, the proposed BNFOWGA operator selects the relevant settings in accordance with Decision Makings' preferences. In similar to FBNCGBM operator (s,t=1) and FBNCOGBM operator (s,t=1), BNFWGA Operator $\lambda = 7$, BNFOWGA Operator $\lambda = 7$ the proposed BNFWGA operators can select the appropriate parameters according to preferences of the decision makings. Therefore, based on the proposed BNFWGA operator, the proposed technique may be used for decision-making. As a result, the suggested operators are more dependable and adaptable. These suggested strategies can be used in practice MCDM situations for decision-making based on the suggested operators and their requirements.

9. Conclusion

The tendency of the human mind to think about both positive and negative impacts when making decisions is referred to as bipolarity. A generalization of fuzzy, intuitionistic, and neutrosophic sets, the bipolar neutrosophic set enables it to handle ambiguous information in the decision-making process with higher adaptability. As a result, the operational laws of the proposed aggregation operation for both BNFWGA and BNFOWGA have been investigated and given in this study utilising Frank triangular norms in a bipolar neutrosophic environment. The use of the BNFWGA and BNFOWGA operators' proposed method to the MCDM bridge selection problem demonstrated its viability and cogency, and the suggested principles were used to choose the best bridge for reducing traffic congestion. Additionally, a comparison of the new method with the old method has been conducted. In the future, aggregation operators may be created employing a variety of triangular norms in different neutrosophic contexts.

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Picture-Neutrosophic Trusted Safe Semi-Supervised Fuzzy Clustering for Noisy Data

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Abstract: Clustering is a crucial method for deciphering data structure and producing new information. Due to its significance in revealing fundamental connections between the human brain and events, it is essential to utilize clustering for cognitive research. Dealing with noisy data caused by inaccurate synthesis from several sources or misleading data production processes is one of the most intriguing clustering difficulties. Noisy data can lead to incorrect object recognition and inference. This research aims to innovate a novel clustering approach, named Picture-Neutrosophic Trusted Safe Semi-Supervised Fuzzy Clustering (PNTS3FCM), to solve the clustering problem with noisy data using neutral and refusal degrees in the definition of Picture Fuzzy Set (PFS) and Neutrosophic Set (NS). Our contribution is to propose a new optimization model with four essential components: clustering, outlier removal, safe semi-supervised fuzzy clustering and partitioning with labeled and unlabeled data. The effectiveness and flexibility of the proposed technique are estimated and compared with the state-of-art methods, standard Picture fuzzy clustering (FC-PFS) and Confidence-weighted safe semi-supervised clustering (CS3FCM) on benchmark UCI datasets. The experimental results show that our method is better at least 10/15 datasets than the compared methods in terms of clustering quality and computational time.

Keywords: Safe semi-supervised fuzzy clustering; picture fuzzy set; neutrosophic set; data partition with noises; fuzzy clustering

1 Introduction

The finding of underlying connections between the human brain and events has made the development of sophisticated clustering algorithms fashionable in cognitive research [1,2]. Dealing

with noisy data is one of the most intriguing clustering difficulties. Incorrect data with noises that affect the quality of results could be seen in many applications, such as satellite images [3], medical image processing [4,5], control systems [6], etc.

Semi-supervised fuzzy clustering techniques were introduced with additional information provided by users [7–9] to enhance the range of applications and the quality of clusters. The differences in incorporating various supplementary information forms were demonstrated in [10] which provided a summary of the semi-supervised fuzzy clustering technique. Accordingly, object segmentation using semi-supervised fuzzy clustering is effective as long as the proper supplementary information, also known as "safe information" and clean data are supplied. However, real-world data are frequently unreliable, noisy and inaccurate. These situations require more effective clustering methods.

The safe semi-supervised fuzzy clustering approach introduced in [11–13] is the typical method to deal with safe information in semi-supervised fuzzy clustering. There are two primary phases in their strategy after the core concept. The confidence weights for labeled data are calculated in the first phase. Then, the high confidence weights are used to generate and identify centers of clusters and fuzzy element values under the labeled data in the second phase. Safe semi-supervised Fuzzy C-Means clustering (S3FCM) approach was firstly presented in [11]. By balancing semi-supervised and unsupervised clustering, this technique investigated the incorrectly classified data. A local homogeneous graph was employed in the first phase [12]. The Local Homogeneous Consistent Safe SSFCM (LHC-S3FCM) method performed effectively on datasets with a large percentage of incorrectly categorized data by utilizing this graph. The CS3FCM, an enhanced safe semi-supervised clustering model, based on confidence weights, was put out in [13]. This approach provides good results in minimizing the negative impact of incorrectly labeled samples on the clustering process, assuming each data sample has its own safe confidence weight.

To establish the safe level of each sample in the data set, Guo et al. [14] have recently suggested a safe semi-supervised clustering with a safe degree. The model provides the essential procedures to reduce the adverse effects of risk in both labeled and unlabeled samples based on the safe degree value. Despite performing better than other approaches when dealing with "safe information", safe semi-supervised fuzzy clustering algorithms can still not solve the challenge of clustering inaccurate data with noises. Noisy data division can lead to incorrect object detection and inference. Data points, isolated or at the edge of some clusters, are considered to contain noisy data. It is a must to improve safe semi-supervised fuzzy clustering algorithms for dealing with noisy data.

This research aims to develop a new clustering method to remove the noise from data and increase the performance of the clustering method. This method integrates the semi-supervised clustering method and the picture fuzzy set [15]. There are four membership degrees in the PFS [3] with Neutrosophic set [16], including the positive degree, neutral degree, negative degree and rejection degree. Noisy data typically have a high rejection rate. Additionally, the neutral degree is used to determine the data points belonging to the boundary of clusters. It is clear that PFS could be used to identify noisy data in datasets.

Based on the original Fuzzy C-Means (FCM) model, a fuzzy clustering algorithm for images (a.k.a. FC-PFS) introduced in [17] outperforms the other fuzzy clustering techniques in terms of average clustering indices such as the mean accuracy and computational time. As an extension of collaborative distributed fuzzy clustering (CDFCM) [18] on PFS, a form of FC-PFS on distributed computing known as DPFCM was demonstrated in [19]. As stated in the paper, a strategy to reduce computational time and increase clustering quality is the idea of semi-supervised clustering using distributed and cloud computing. Wu and Chen presented an adaptive picture fuzzy clustering

technique based on entropy weight [20]. This approach improved accuracy, addressed noisy data in image segmentation and overcame the time-consuming limitation in existing picture fuzzy clustering algorithms. Two practical, robust picture fuzzy clustering techniques for decreasing computational time were also introduced [21,22]. Nonetheless, those fuzzy clustering algorithms struggle with managing both the "safe information" and the "noisy data" because if labeled data has noise, the clustering quality will be seriously affected.

To handle problems with enhancing "safe information" and reducing the effect of "noisy data", Picture-Neutrosophic Trusted Safe Semi-Supervised Fuzzy Clustering (PNTS3FCM) is introduced. This is a new technique to address the issue of data partition with noisy information. The PNTS3FCM approach includes picture fuzzy and neutrosophic set concepts in the semi-supervised fuzzy clustering with a safe information procedure. The research proposes a new optimization model consisting of four essential components: a clustering component, an outlier-solving component and a safe semi-supervised fuzzy clustering using labeled and unlabeled data. The first two parts employed FC-PFS and the last two are the new parts to enhance safe information and reduce noisy data. An iterative technique from the formulation is also provided to construct the cluster centers and memberships. In fact, the survey has revealed a new field of study: safe, semi-supervised clustering on the picture fuzzy set. To compare PNTS3FCM with other available methods on benchmark datasets, two similar algorithms-FC-PFS [17] and CS3FCM [13], are chosen.

The remaining paper is structured as follows: Section 2 offers the essential information underpinning our study. The proposed approach is introduced in Section 3 and the experimental results are presented in Section 4. Some conclusions are given in the last section.

2 Preliminaries

In this section, some fundamental concepts and methods of semi-supervised clustering are presented, including Safe semi-supervised clustering and Picture fuzzy set and picture fuzzy clustering.

2.1 Safe Semi-Supervised Clustering

Safe semi-supervised fuzzy clustering approaches, including S3FCM [11], LHC-S3FCM [12] and CS3FCM [13] are proposed by *Gan et al*. Herein, we present the fundamental knowledge of these approaches.

For S3FCM, consider the dataset $X = \{X_1, X_2, \ldots, X_k, \ldots, X_n\}$ where n is the number of data elements. C is denoted for the number of clusters. The cluster center V is defined by $\{V_1, V_2, \ldots, V_j, \ldots, V_C\}$. The membership degree of k^{th} element belonging to the i^{th} cluster is characterized by u_{ik} and m is the fuzzifier parameter. The value b_k expresses a label indicator; the value $b_k = 1$ if X_k is labeled and $b_k = 0$ otherwise. f_{ik} are the fuzzy degrees of labeled samples. The objective function of S3FCM [11] is as below:

$$J_{sa} = \sum_{k=1}^{n}\sum_{i=1}^{C} u_{ik}^{m} d_{ik}^{2} + \lambda_1 \sum_{k=1}^{n}\sum_{i=1}^{C} (u_{ik} - f_{ik} b_k)^2 d_{ik}^{2} + \lambda_2 \sum_{k=1}^{n}\sum_{i=1}^{C} \left(u_{ik} - \hat{u}_{ik} b_k\right)^2 d_{ik}^{2} \rightarrow Min \quad (1)$$

with: $\sum_{i=1}^{c} u_{ik} = 1, \forall k = \overline{1,n}, u_{ik} \in [0,1], \forall k = \overline{1,n}$. Where λ_1 and λ_2 are the regulatory factor in which $\hat{U} = \left[\hat{u}_{ik}\right]_{c\times n}$ is the partition matrix after using FCM on unlabeled data, d_{ik} is the distance between the

k^{th} element and i^{th} cluster. The final cluster labels are determined through the algorithm [11] and the value u_{ik} is specified as follows:

$$u_{ik} = \frac{1}{1+\lambda_1+\lambda_2}\left(\frac{1+\lambda_1+\lambda_2-\sum_{j=1}^{C}\Delta_{ik}}{\sum_{j=1}^{C}\frac{d_{ik}^2}{d_{jk}^2}}+\Delta_{ik}\right) \tag{2}$$

where $\Delta_{ik} = \lambda_1 f_{ik} b_k + \lambda_2 \hat{u}_{ik} b_k$.

The below function calculates the center v_i:

$$v_i = \frac{\sum_{k=1}^{n} u_{ik}^2 x_k + \lambda_1 \sum_{k=1}^{n} (u_{ik} - f_{ik} b_k)^2 x_k + \lambda_2 \sum_{k=1}^{n} \left(u_{ik} - \hat{u}_{ik} b_k\right)^2 x_k}{\sum_{k=1}^{n} u_{ik}^2 + \lambda_1 \sum_{k=1}^{n} (u_{ik} - f_{ik} b_k)^2 + \lambda_2 \sum_{k=1}^{n} \left(u_{ik} - \hat{u}_{ik} b_k\right)^2} \tag{3}$$

On the other hand, the LHC-S3FCM [12] is expected to deal with wrong labels from additional information. The objective function is defined as follows:

$$J_{sa} = \sum_{k=1}^{n}\sum_{i=1}^{C} u_{ik}^m d_{ik}^2 + \lambda_1 \sum_{k=1}^{l}\sum_{i=1}^{C} (u_{ik} - f_{ik})^m d_{ik}^2 + \lambda_2 \sum_{k=1}^{l}\sum_{r=l+1}^{n} w_{kr} \sum_{i=1}^{C} (u_{ik} - u_{ir})^2 \to Min \tag{4}$$

with the constraints: $\sum_{i=1}^{c} u_{ik} = 1, \forall k = \overline{1,n}$

Therefore, the cluster centers v_i, the value u_{ik} for labeled data x_k and the value u_{ir} for unlabeled data x_r correspond to the below functions:

$$\mathrm{v}_i = \frac{\sum_{k=1}^{n} u_{ik}^2 x_k + \lambda_1 \sum_{k=1}^{l} (u_{ik} - f_{ik})^2 x_k}{\sum_{k=1}^{n} u_{ik}^2 + \lambda_1 \sum_{k=1}^{l} (u_{ik} - f_{ik})^2}; \ \mathrm{u}_{ik} = \frac{p_{ik} + \frac{1-\sum_{j=1}^{C}\frac{p_{jk}}{q_{jk}}}{\sum_{j=1}^{C}\frac{1}{q_{jk}}}}{q_{ik}} \text{ and } \mathrm{u}_{ir} = \frac{S_{ir} + \frac{1-\sum_{j=1}^{C}\frac{s_{jr}}{t_{jr}}}{\sum_{j=1}^{C}\frac{1}{s_{jr}}}}{t_{ir}} \tag{5}$$

Another approach of FCM is Confidence-weighted Safe Semi-supervised Clustering (CS3FCM) [13] by using confidence weights. The confidence weights show various effects of samples on performance degradation. The following is the goal:

$$J_c = \sum_{k=1}^{n}\sum_{i=1}^{C} u_{ik}^m d_{ik}^2 + \lambda_1 \sum_{k=1}^{l} s_k \sum_{i=1}^{C} (u_{ik} - f_{ik})^2 d_{ik}^2 + \lambda_2 \sum_{k=1}^{l} \frac{1}{s_k} \sum_{r=l+1}^{n} w_{kr} \sum_{i=1}^{C} (u_{ik} - u_{ir})^2 \to Min \tag{6}$$

with $\sum_{i=1}^{c} u_{ik} = 1, \forall k = \overline{1,n}$; λ_1 and λ_2 are the regulatory factors. Therefore, the value of v_i, u_{ik} and u_{ir} are determined by the following functions:

$$\mathrm{v}_i = \frac{\sum_{k=1}^{n} u_{ik}^2 x_k + \lambda_1 \sum_{k=1}^{l} s_k (u_{ik} - f_{ik})^2 x_k}{\sum_{k=1}^{n} u_{ik}^2 + \lambda_1 \sum_{k=1}^{l} s_k (u_{ik} - f_{ik})^2}; \ \mathrm{u}_{ik} = \frac{p_{ik} + \frac{1-\sum_{j=1}^{C}\frac{p_{jk}}{q_{jk}}}{\sum_{j=1}^{C}\frac{1}{q_{jk}}}}{q_{ik}} \text{ and } \mathrm{u}_{ir} = \frac{z_{ir} + \frac{1-\sum_{j=1}^{C}\frac{z_{ir}}{t_{ir}}}{\sum_{j=1}^{C}\frac{1}{t_{ir}}}}{t_{ir}} \tag{7}$$

The methods of Gan [11–13] (S3FCM, LHC-S3FCM, CS3FCM) achieved good clustering accuracy. However, if there may be data outliers, they would affect the determination of the final clusters.

2.2 Picture Fuzzy Set and Picture Fuzzy Clustering

By generalizing the fuzzy set in [9] and the intuitionistic fuzzy set [23], Cuong et al. introduced a definition of the picture fuzzy set [15] in 2014 and have the form as follows:

$$S = \{(x, \mu_S(x), \eta_S(x), \gamma_S(x)) \mid x \in X\} \tag{8}$$

where $\mu_S(x)$, $\eta_S(x)$ and $\gamma_S(x)$ correspond to the positive degree, the neutral degree and the negative degree of each element. And these degrees satisfy the following conditions:

$$0 \le \mu_S(x), \eta_S(x), \gamma_S(x) \le 1; 0 \le \mu_S(x) + \eta_S(x) + \gamma_S(x) \le 1 \tag{9}$$

Then, the refusal degree is computed by function:

$$\xi_S(x) = 1 - (\mu_S(x) + \eta_S(x) + \gamma_S(x)) \tag{10}$$

The objective of FC-PFS [17] aims to group the data in clusters and reduce the outliers through the concept of entropy as follows:

$$J_m(U, \eta, \xi, V) = \sum_{i=1}^{n}\sum_{j=1}^{C}\left(\mu_{ij}\left(2 - \xi_{ij}\right)\right)^m \left\|x_i - v_j\right\|^2 + \sum_{i=1}^{n}\sum_{j=1}^{C}\eta_{ij}\left(\log\eta_{ij} + \xi_{ij}\right) \to Min \tag{11}$$

with the constraints:

$$\mu_{ij}, \eta_{ij}, \xi_{ij} \in [0,1],\ \mu_{ij} + \eta_{ij} + \xi_{ij} \in [0,1], \sum_{j=1}^{C}\mu_{ij}\left(2 - \xi_{ij}\right) = 1, \sum_{j=1}^{C}\left(\eta_{ij} + \frac{1}{C}\xi_{ij}\right) = 1, i = \overline{1,n} \text{ and } j = \overline{1,C} \tag{12}$$

The values of $\mu_S(x)$, $\eta_S(x)$ and $\gamma_S(x)$ correspond to the positive, neutral and negative degrees of PFS [15]. The vector of the cluster centers denotes V.

For the above objective function, the cluster centers v_j, the membership degrees μ_{ij} and non-membership degrees η_{ij} are computed using the following formulas:

$$v_j = \frac{\sum_{i=1}^{n}\left(\mu_{ij}\left(2 - \xi_{ij}\right)\right)^m x_i}{\sum_{i=1}^{n}\left(\mu_{ij}\left(2 - \xi_{ij}\right)\right)^m} \tag{13}$$

$$\mu_{ij} = \frac{1}{\left(2 - \xi_{ij}\right)\sum_{k=1}^{C}\left(\frac{\left\|x_i - v_j\right\|^2}{\left\|x_i - v_k\right\|^2}\right)^{1/(m-1)}} \tag{14}$$

$$\eta_{ij} = \frac{\exp\left(-\xi_{ij}\right)}{\sum_{k=1}^{C}\exp\left(-\xi_{ik}\right)}\left(1 - \frac{1}{C}\sum_{k=1}^{C}\xi_{ik}\right) \tag{15}$$

In [15], the refusal degree ξ_{ij} is calculated using the Yager complement operator as follows:

$$\xi_{ij} = 1 - \left(\mu_{ij} + \eta_{ij}\right) - \left(1 - \left(\mu_{ij} + \eta_{ij}\right)^{\alpha}\right)^{1/\alpha} \quad (16)$$

where $\alpha \in (0, 1)$ is a regulatory factor and it is often chosen within [0.6–0.8].

The detailed steps for the FC-PFS algorithm are shown below.

Algorithm 1: The main steps of the FC-PFS algorithm

Input: Data set X with n elements in R^d; threshold ε; the number of clusters C; fuzzifier m; exponent $\alpha \in (0, 1)$ and the maximal number of iterations $Maxsteps > 0$.
Output: Membership matrices μ, η, ξ and cluster centers V.
1: Initialize the iteration: $t = 0$
2: ${\mu_{kj}}^t \leftarrow random$; ${\eta_{kj}}^t \leftarrow random$; ${\xi_{kj}}^t \leftarrow random$ $\left(k = \overline{1,n};\ j = \overline{1,C}\right)$ satisfy Eqs. (11)–(11)
3: Repeat
4: $t = t + 1$
5: Calculate V_j $(j = 1, \ldots, C)$ by Eq. (13)
6: Calculate $\mu_{kj}^{(t)}$ for labeled data $\left(k = \overline{1,n};\ j = \overline{1,C}\right)$ by Eq. (14)
7: Calculate $\eta_{kj}^{(t)}$ $\left(k = \overline{1,n};\ j = \overline{1,C}\right)$ by Eq. (15)
8: Calculate $\xi_{kj}^{(t)}$ $\left(k = \overline{1,n};\ j = \overline{1,C}\right)$ by Eq. (16)
9: Until the matrices μ, η, ξ satisfy the condition $\left\|\mu^t - \mu^{t-1}\right\| + \left\|\eta^t - \eta^{t-1}\right\| + \left\|\xi^t - \xi^{t-1}\right\| \leq \varepsilon$ or the number of iterations reaches $Maxsteps$.

3 The Proposed Picture-Neutrosophic Trusted Safe Semi-Supervised Fuzzy Clustering

3.1 Main Ideas

The idea behind the proposed method (PNTS3FCM) is the combination between PFS and safe semi-supervised fuzzy clustering by introducing a novel objective function with four primary components. The first and the second stages are employed from the original picture fuzzy clustering method [17]. The two last stages are the semi-supervised component used to orient the clustering process by labeled and unlabeled data. The main idea is represented in Fig. 1 and the detailed steps are described in Fig. 2.

Fig. 1 illustrates the method and concept in which the input data are provided to the block of PNTS3FCM. Through the use of picture fuzzy degrees, the first step of PNTS3FCM is to reduce the distance between data components and cluster centers. The picture fuzzy set model's second step involves processing the "noisy data" by integrating the entropy quantity between the neutral and refuse degrees. The refusal degree plays an important role in reducing the effect of noise data in the objective function because of its higher value relating to noise data following [17].

To deal with "safe information", the two last stages coordinate the safe semi-supervised fuzzy clustering using both labeled and unlabeled data. PNTS3FCM has two phases: Firstly, FC-PFS is used to partition all data to get the clustering result with positive, neutral and refusal values. The second phase uses all data with these values to partition data to archive better clustering quality by enhancing safe data information and reducing noisy data.

The technique produces final clusters that are reliable and confident. We will discuss the formulation and algorithm for this concept in the next section.

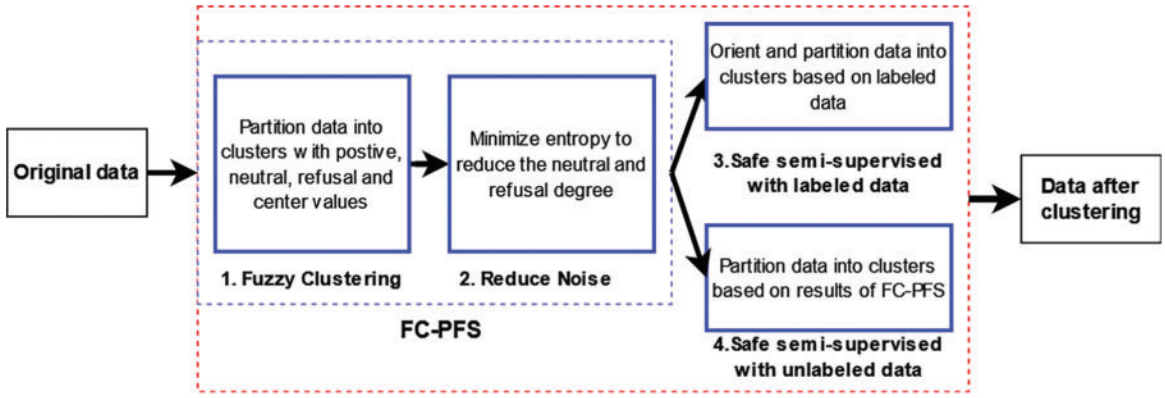
Original data
Partition data into clusters w ith postive, neutral, refusal and center values
Minimize entropy to reduce the neutral and refusal degree
1. Fuzzy Clustering
2. Reduce Noise
FC-PFS
Orient and partition data into clusters based on labeled data
3.Safe semi-supervised with labeled data
Partition data into clusters based on results of FC-PFS
4.Safe semi-supervised with unlabeled data
Data after clustering

Figure 1: The main idea of the PNTS3FCM

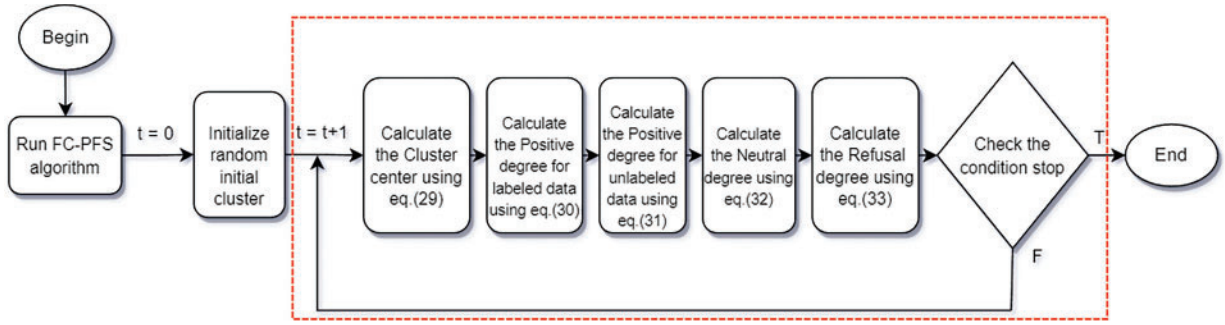
Begin
Run FC-PFS algorithm
t = 0
Initialize random initial cluster
t = t+1
Calculate the Cluster center using eq.(29)
Calculate the Positive degree for labeled data using eq.(30)
Calculate the Positive degree for unlabeled data using eq.(31)
Calculate the Neutral degree using eq.(32)
Calculate the Refusal degree using eq.(33)
Check the condition stop
T
F
End

Figure 2: The details of the PNTS3FCM

3.2 Details of PNTS3FCM

As illustrated by the main idea above, this section will describe the details of the proposed model. The objective function is stated by the following formula:

$$J = \sum_{k=1}^{n}\sum_{j=1}^{C}\left(\mu_{kj}\left(2-\xi_{kj}\right)\right)^2\left\|X_k - V_j\right\|^2 + \sum_{k=1}^{n}\sum_{j=1}^{C}\eta_{kj}\left(\log\eta_{kj} + \xi_{kj}\right)$$
$$+\sum_{k=1}^{L}\sum_{j=1}^{C}\frac{\left(\mu_{kj}\left(2-\xi_{kj}\right)-f_{kj}\right)^2}{1+\left(\overline{\mu}_{kj}\left(2-\overline{\xi}_{kj}\right)-f_{kj}\right)^2}\left\|X_k - V_j\right\|^2 + \sum_{k=L+1}^{n}\sum_{j=1}^{C}\frac{\left(\mu_{kj}\left(2-\xi_{kj}\right)\right)^2}{1+\overline{\xi_{kj}}}\left\|X_k - V_j\right\|^2 \to Min \quad (17)$$

With the constraints $\left(k=\overline{1,n};\ j=\overline{1,C}\right)$:

$$\mu_{kj},\ \eta_{kj},\ \xi_{kj} \le 1, \sum_{j=1}^{C}\left(\eta_{kj} + \frac{\xi_{kj}}{C}\right) = 1, \text{and} \sum_{j=1}^{C}\left(\mu_{kj}\left(2-\xi_{kj}\right)\right) = 1 \quad (18)$$

where data set $X = \{X_1, X_2, \ldots, X_n\}$ having n elements, the number of labeled data in X: $L < n$; the number of clusters C; the values of positive, neutral and refusal degrees of element X_k belong to cluster j: μ_{kj}, η_{kj} and ξ_{kj}. Each part of the objective function has its own meaning. The first two parts of Eq. (17), as shown, are those of the original picture fuzzy clustering (FC-PFS) [14]. The safe semi-supervised fuzzy clustering on the picture fuzzy set is covered in the last two parts.

- The first part represents fuzzy clustering on the PFS.

- The second part represents entropy information which helps to reduce noisy data through the neutral and refusal degrees of a data point.
- The third part is the component for labeled data elements, where $k = 1..L$ and L is the number of labeled data elements. The numerator $\left(\mu_{kj}\left(2-\xi_{kj}\right)-f_{kj}\right)^2$ describes semi-supervised fuzzy clustering, in which f_{kj} is a given constant that has a value of 1 or 0.

$$f_{kj}=\begin{cases}1 & \text{if the element k is in cluster j}\\ 0 & \text{if the element k isn't in cluster j}\end{cases} \tag{19}$$

The denominator $1+\left(\overline{\mu}_{kj}\left(2-\overline{\xi}_{kj}\right)-f_{kj}\right)^2$ describes the safe semi-supervised clustering. The meaning of this component is as follows: After clustering, if any data point is assigned to the correct label, the weight will be increased; otherwise, the weight will be decreased.

- Finally, the fourth part is the component of the unlabeled data elements, where the numerator is the same as the first part and the denominator $\left(1+\overline{\xi}_{kj}\right)$ is added to the component $\overline{\xi}_{kj}$. The meaning of this value is that after applying clustering to all data points, the denominator $\left(1+\overline{\xi}_{kj}\right)$ will be greater than 1 for unlabeled data elements with high refusal value of $\overline{\xi}_{kj}$. Indeed, the weights of these data elements are reduced.
- The additional information for semi-supervised fuzzy clustering is the prior picture membership degrees. We use the original FC-PFS algorithm to cluster all data, including labeled and unlabeled data. From that, we calculate four values $\left(\overline{\mu}_{kj},\overline{\eta}_{kj},\overline{\xi}_{kj},\overline{V}\right)$ that guide the calculation for all data elements.

Using the Lagrangian method, the optimal solutions to the stated problem are presented in Eqs. (20)–(24) below.

$$V_j=\frac{\sum_{k=1}^{n}\left(\mu_{kj}\left(2-\xi_{kj}\right)\right)^2 X_k+\sum_{k=1}^{L}\frac{\left(\mu_{kj}\left(2-\xi_{kj}\right)-f_{kj}\right)^2}{1+\left(\overline{\mu}_{kj}\left(2-\overline{\xi}_{kj}\right)-f_{kj}\right)^2}X_k+\sum_{k=L+1}^{n}\frac{\left(\mu_{kj}\left(2-\xi_{kj}\right)\right)^2}{\left(1+\overline{\xi}_{kj}\right)^2}X_k}{\sum_{k=1}^{n}\left(\mu_{kj}\left(2-\xi_{kj}\right)\right)^2+\sum_{k=1}^{L}\frac{\left(\mu_{kj}\left(2-\xi_{kj}\right)-f_{kj}\right)^2}{1+\left(\overline{\mu}_{kj}\left(2-\overline{\xi}_{kj}\right)-f_{kj}\right)^2}+\sum_{k=L+1}^{n}\frac{\left(\mu_{kj}\left(2-\xi_{kj}\right)\right)^2}{\left(1+\overline{\xi}_{kj}\right)^2}} \tag{20}$$

The positive degree u of the labeled data elements is

$$\mu_{kj}=\frac{f_{kj}}{\left(2-\xi_{kj}\right)\left(2+\left(\overline{\mu}_{kj}\left(2-\overline{\xi}_{kj}\right)-f_{kj}\right)^2\right)}+ \\ +\frac{1-\sum_{i=1}^{C}\frac{f_{ki}}{2+\left(\overline{\mu}_{ki}\left(2-\overline{\xi}_{ki}\right)-f_{ki}\right)^2}}{\left(2-\xi_{kj}\right)\frac{\left(2+\left(\overline{\mu}_{kj}\left(2-\overline{\xi}_{kj}\right)-f_{kj}\right)^2\right)}{\left(1+\left(\overline{\mu}_{kj}\left(2-\overline{\xi}_{kj}\right)-f_{kj}\right)^2\right)}\sum_{i=1}^{C}\frac{\left\|X_k-V_j\right\|^2\left(1+\left(\overline{\mu}_{ki}\left(2-\overline{\xi}_{ki}\right)-f_{ki}\right)^2\right)}{\left\|X_k-V_i\right\|^2\left(2+\left(\overline{\mu}_{ki}\left(2-\overline{\xi}_{ki}\right)-f_{ki}\right)^2\right)}} \tag{21}$$

The positive degree u of the unlabeled data elements is

$$\mu_{kj}=\frac{1}{\left(2-\xi_{kj}\right)\sum_{i=1}^{C}\left(\frac{1+\frac{1}{\overline{\xi}_{kj}}}{1+\frac{1}{\overline{\xi}_{ki}}}\right)\frac{\left\|X_k-V_j\right\|^2}{\left\|X_k-V_i\right\|^2}} \tag{22}$$

Other degrees are shown below:

$$\eta_{kj}=\left(1-\frac{1}{C}\sum_{i=1}^{C}\xi_{ki}\right)\frac{e^{-\xi_{kj}}}{\sum_{i=1}^{C}e^{-\xi_{ki}}} \tag{23}$$

$$\xi_{kj} = \frac{1}{1 + e^{3-(\mu_{kj}+\eta_{kj})-(3-(\mu_{kj}+\eta_{kj})^{\alpha})^{\frac{1}{\alpha}}}} \tag{24}$$

Details of the FPNTS3FCM algorithm are below.

Algorithm 2: The PNTS3FCM algorithm

Input: Data set X with the number of data elements n in d dimensions, the number of labeled data in X: $L < n$; threshold ε; the number of clusters(C); fuzzifier m; exponent $\alpha \in (0, 1]$ and the maximal number of iteration $Maxsteps > 0$

Output: Membership matrices μ, η, ξ and cluster centers V.

1: Execute the FC-PFS algorithm with all data elements to get $(\overline{\mu}_{kj}, \overline{\eta}_{kj}, \overline{\xi}_{kj}, \overline{V})$

2: Initialize the iteration: $t = 0$

3: ${\mu_{kj}}^{t} \leftarrow random$; ${\eta_{kj}}^{t} \leftarrow random$; ${\xi_{kj}}^{t} \leftarrow random$ $\left(k = \overline{1,n};\ j = \overline{1,C}\right)$ satisfy Eq. (18)

4: Repeat

5: $t = t + 1$

6: Calculate V_j $(j = 1, \ldots, C)$ by Eq. (20)

7: Calculate $\mu_{kj}^{(t)}$ for labeled data $\left(k = \overline{1,n};\ j = \overline{1,C}\right)$ by Eq. (21)

8: Calculate $\mu_{kj}^{(t)}$ for unlabeled data $\left(k = \overline{1,n};\ j = \overline{1,C}\right)$ by Eq. (22)

9: Calculate $\eta_{kj}^{(t)}$ $\left\langle\left(k = \overline{1,n};\ j = \overline{1,C}\right)\right\rangle$ by Eq. (23)

10: Calculate $\xi_{kj}^{(t)}$ $\left(k = \overline{1,n};\ j = \overline{1,C}\right)$ by Eq. (24)

11: Until the matrices μ, η, ξ satisfy the condition $\left\|\mu^{t} - \mu^{t-1}\right\| + \left\|\eta^{t} - \eta^{t-1}\right\| + \left\|\xi^{t} - \xi^{t-1}\right\| \leq \varepsilon$ or the number of iterations reaches $Maxsteps$.

3.3 Remarks

Advantages of the PNTS3FCM algorithm:

a) PNTS3FCM has better clustering quality than the related methods, such as FC-PFS and CS3FCM algorithm, due to the capability to handle noisy data.

b) PNTS3FCM produces more information about the clusters, such as the cluster centers and the picture fuzzy degrees (positive, neutral, negative, refusal). It deals with both "safe information" and "noisy data".

c) PNTS3FCM is the combination of three major concepts: SAFE, SEMI Clustering and PICTURE Fuzzy Set. The combination is the first trial in the literature toward practical problems.

Disadvantages of the PNTS3FCM algorithm

a) PNTS3FCM takes more computational time than the other algorithms due to the calculation of two additional parts in the objective function (24).

b) The model contains many parameters which need to be tuned in some real-world applications.

4 Experimental Results

4.1 Environmental Configuration

The experiments are performed on a Core i5-powered HP laptop using the C programming language. The selected benchmark UCI datasets [24] are described in Table 1. Outlier Detection DataSets (ODDS) [25] are given in Table 2.

Table 1: Datasets without outliers

Dataset	No. of records	No. of attributes	No. of clusters
Australian	690	14	2
Balance-scale	625	4	3
Dermatology	366	34	6
Heart	270	13	2
Iris	150	4	3
Spambase	4601	57	2
Tae	151	5	3
Waweform	5000	40	3
WDBC	569	30	2

Table 2: Datasets with outliers

Dataset	No. of samples	No. of features	No. of clusters	No. of coutlier (%)
Ecoli	336	7	8	2.6
Glass	214	9	6	4.2
Yeast	1364	8	10	4.7
Wine	178	13	3	7.7
Vertebral	310	6	3	12.5
Ionosphere	351	34	2	36

Experiments are executed to compare the proposed PNTS3FCM approach and the state-of-art methods, CS3FCM [13] and FC-PFS [17]. The classification accuracy (CA), computing time (CT) and clustering quality indicators, including DB, PBM and ASWC [26], are the criteria for evaluation. The CT is the amount of time needed to complete the computation. Value CT is computed as in (25).

$$CT = T_2 - T_1 \tag{25}$$

where T_1, T_2 is the starting time and ending time of the algorithm, respectively. The smaller value of CT reaches, the better performance of the method is. The calculation of CA [13] is given by the below equation.

$$CA = \frac{\sum_{k=1}^{n} \delta\left(y_k, map\left(\tilde{y}_k\right)\right)}{n} \tag{26}$$

where $map\left(\tilde{y}_k\right)$ is the function that determines the equivalent label for $\tilde{y}_k$ using the Kuhn–Munkres algorithm [12]. The function $\delta(x, y)$ gets two values (0 if $x \neq y$ and 1 if $x = y$). The performance of the CA index is better when it has a higher value.

The value of ASWC is computed by Eq. (27).

$$s_{xj} = \frac{b_{p,j}}{a_{p,j} + \varepsilon} \tag{27}$$

where $a_{p,j}$ is the average distance from i^{th} element to all other parts in p^{th} cluster; $b_{p,j}$ is the average distance from i^{th} element to all other elements in p^{th}cluster. ε is a tiny constant. It is added to make the denominator differ from zero (when $a_{p,j} = 0$). The higher value of the ASWC index leads to better performance.

The value of PBM [26] is determined by:

$$PBM = \left(\frac{1}{C}\frac{E_1}{E_K}D_K\right)^2 \tag{28}$$

where $E_1 = \sum_{i=1}^{n}\left\|X_i - \overline{X}\right\|$, $E_K = \sum_{j=1}^{C}\sum_{X_i \in clusterj}\left\|X_i - \overline{X}_j\right\|$, $D_K = \max_{j,l=1,\ldots,C}\left\|\overline{X}_j - \overline{X}_l\right\|$ *with* $\overline{X}_j$*is the average value of all elements in the* j^{th} *cluster*, $j = \overline{1, C}$. The higher value of the PBM index has, the better performance is.

The DB [27] is determined by (29)

$$DB = \frac{1}{C}\sum_{i=1}^{C}\left(\max_{j:\, j\neq i}\left\{\frac{S_i + S_j}{M_{ij}}\right\}\right) \tag{29}$$

where T_i is the size of i^{th} cluster. In which S_i and M_{ij} are computed by

$$S_i = \sqrt{\frac{1}{T_i}\sum_{j=1}^{T_i}\left|X_j - V_i\right|^2}; M_{ij} = \left\|V_i - V_j\right\| \text{ with } i,j = \overline{1, C},\ \ i \neq j \tag{30}$$

The average value and standard deviation value in experimental results are denoted as Ave and STD Dev, respectively.

4.2 Experimental Results

4.2.1 Classification Accuracy

Herein, the proposed method is assessed by classification accuracy in two situations, including on all data and labeled data. Herein, the experimental results are presented following two of these cases.

Evaluation by classification accuracy on all data

Using all the data elements of 15 datasets, the classification accuracy of PNTS3FCM, FC-PFS and CS3FCM are calculated and presented as follows. Table 3 shows the classification accuracy of all data without outliers.

As shown in Table 3, PNTS3FCM gets the best results of CA on 7/9 datasets (except Australian and WDBC). FC-PFS has not achieved the highest CA on all datasets. CS3FCM is the best model on 2/9 datasets (Australian, WDBC).

From the results in Table 4, it is clear that PNTS3FCM gives correct classification results in 4 out of 6 datasets (Glass, Yeast, Vertebral, Ionosphere). The other FC-PFS is only better on the Wine dataset and CS3FCM only gives good results on the Ecoli dataset.

Table 3: Classification accuracy on all data without outliers (Bold values indicate the best results)

METHOD	PNTS3FCM		FC-PFS		CS3FCM	
	Ave	STD Dev	Ave	STD Dev	Ave	STD Dev
Australian	0.69002	0.00786	0.61856	0.00063	0.69739	0.00254
Balance-scale	0.62383	0.00556	0.51412	0.01218	0.51685	0.01939
Dermatology	0.75452	0.01209	0.55878	0.01647	0.64483	0.01245
Heart	0.75298	0.00790	0.6552	0.00195	0.7421	0.00232
Iris	0.93085	0.00392	0.92299	0.01076	0.89076	0.01963
Spambase	0.84633	0.01458	0.77682	0.00924	0.75396	0.0088
Tae	0.48949	0.03453	0.47875	0.00337	0.45421	0.00663
Waweform	0.65930	0.01140	0.55104	0.0074	0.52295	0.0076
WDBC	0.77269	0.00921	0.70639	0.0024	0.7836	0.0026

Table 4: Classification accuracy values on all data with outliers (Bold values indicate the best results)

METHOD	PNTS3FCM		FC-PFS		CS3FCM	
	Ave	STD Dev	Ave	STD Dev	Ave	STD Dev
Ecoli	0.51301	0.00936	0.51399	0.00509	0.5661	0.00974
Glass	0.42946	0.00741	0.42114	0.0035	0.42905	0.00625
Yeast	0.48018	0.02388	0.32437	0.00401	0.32909	0.0101
Wine	0.89228	0.01452	0.95722	0.0015	0.92186	0.00484
Vertebral	0.51908	0.00830	0.48733	0.00227	0.51600	0.00655
Ionosphere	0.53823	0.00102	0.52571	0.00037	0.53613	0.00053

Summary: During the evaluation by classification accuracy on all data, including outlier and non-outlier (15 datasets), PNTS3FCM is the best on 11 datasets (Balance-scale, Dermatology, Heart, Iris, Spambase, Tae, Waveform, Glass, Yeast, Vertebral, Ionosphere). FC-PFS is the best model on the Wine dataset. CS3FCM is the best model on three datasets (Australian, WDBC, Ecoli).

Evaluation by classification accuracy on labeled data

By using the labeled data elements of 15 datasets, the classification accuracy (CA) of PNTS3FCM, FC-PFS and CS3FCM are calculated and presented as follows. Table 5 shows the classification accuracy of labeled data without outliers.

In Table 5, PNTS3FCM gets the best results of CA on 7/9 datasets (except Iris and WDBC). FC-PFS has no highest value on all datasets. CS3FCM is the best model on 2/9 datasets (Iris, WDBC). As shown in Table 6, PNTS3FCM shows the highest values on 4/6 datasets (Ecoli, Glass, Yeast, Vertebral). FC-PFS has no highest CA on all datasets. CS3FCM is the best model on 2/6 datasets (Wine, Ionosphere).

Table 5: Classification accuracy on labeled data without outliers (Bold values indicate the best results)

METHOD	PNTS3FCM		FC-PFS		CS3FCM	
	Ave	STD Dev	Ave	STD Dev	Ave	STD Dev
Australian	0.73497	0.03576	0.59368	0.02729	0.73395	0.00452
Balance-scale	0.88029	0.07930	0.47638	0.02352	0.53585	0.01231
Dermatology	0.69973	0.05447	0.44638	0.04177	0.47918	0.03509
Heart	0.76174	0.04002	0.61693	0.03477	0.74145	0.00235
Iris	0.83296	0.02356	0.77855	0.09295	0.85016	0.01784
Spambase	0.92198	0.06753	0.69296	0.05677	0.70896	0.02728
Tae	0.72589	0.06706	0.53238	0.03506	0.55857	0.00782
Waweform	0.70632	0.05428	0.50336	0.03123	0.52359	0.0094
WDBC	0.70137	0.05464	0.6598	0.05484	0.81087	0.00795

Table 6: Classification accuracy on labeled data with outliers (Bold values indicate the best results)

METHOD	PNTS3FCM		FC-PFS		CS3FCM	
	Ave	STD Dev	Ave	STD Dev	Ave	STD Dev
Ecoli	0.75089	0.104192044	0.55551	0.05475	0.51386	0.01736
Glass	0.64169	0.068188815	0.43231	0.02221	0.44376	0.00876
Yeast	0.86177	0.10076	0.29044	0.02543	0.35129	0.01674
Wine	0.76932	0.072052299	0.80023	0.09646	0.82272	0.02488
Vertebral	0.67821	0.058126313	0.49862	0.03127	0.58464	0.01392
Ionosphere	0.52633	0.001123518	0.53435	0.01013	0.55682	0.00212

Summary: During the evaluation by classification accuracy on labeled data, including outlier and non-outlier (15 datasets), PNTS3FCM has better results on 11 datasets (Australian, Balance-scale, Dermatology, Heart, Spambase, Tae, Waveform, Ecoli, Glass, Yeast, Vertebral). FC-PFS has not had the highest CA on all datasets. CS3FCM is the best model on four datasets (Iris, WDBC, Wine, Ionosphere).

4.2.2 Evaluation by Clustering Quality

Summary: As in Table 7, During the evaluation clustering quality by DB index on all data, including outlier and non-outlier (15 datasets), PNTS3FCM gets the best results on ten datasets (Australian, Dermatology, Heart, Tae, Waveform, WDBC, Ecoli, Glass, Yeast, Wine). FC-PFS is the best model on three datasets (Iris, Vertebral, Ionosphere). CS3FCM is the best model on 3 datasets (Balance-scale, Spambase). This pointed out that the proposed method was better in clustering quality in not only outlier not also non-outlier data compared to others.

Table 7: The results of the DB index on all datasets (bold values indicate the best results)

METHOD	PNTS3FCM		FC-PFS		CS3FCM	
	Ave	STD Dev	Ave	STD Dev	Ave	STD Dev
Australian	3.29466	0.07284	3.59062	0.29431	3.80124	0.24324
Balance-scale	6.54266	0.69000	52.46293	4.3444	5.54124	0.22229
Dermatology	10.50804	0.76929	15.64693	7.57343	18.65225	3.36386
Heart	3.12663	0.08336	5.1217	4.26367	3.94698	0.19342
Iris	2.95875	0.02369	2.85474	0.0293	3.57822	0.21908
Spambase	31.26761	10.24416	33.89317	0.26046	25.57641	0.16618
Tae	3.44788	0.01591	3.8189	0.01948	3.70213	0.01689
Waweform	14.42766	2.92785	15.15752	3.34036	16.15447	2.87025
WDBC	2.35134	0.02410	2.41815	0.00587	2.83812	0.05349
Ecoli	5.97278	0.28630	6.49862	0.16336	8.88006	0.38829
Glass	5.37032	0.16483	6.62155	0.32037	6.4242	0.52104
Yeast	10.61662	0.58698	28.00517	2.14368	12.03848	1.11802
Wine	2.82494	0.01133	2.91238	0.00254	4.10052	0.10899
Vertebral	3.11417	0.02677	3.07404	0.0136	3.82931	0.09941
Ionosphere	2.98972	0.01699	2.95349	0.01212	3.3906	0.06444

4.2.3 Evaluation by Computational Time (in seconds)

We compare PNTS3FCM and CS3FCM on 15 datasets using computational time. Table 8 shows the results of evaluation clustering quality by computational time on data without outlier datasets.

Table 8: The computational time on all datasets (Bold values indicate the best results)

METHOD	PNTS3FCM		CS3FCM	
	Ave	STD Dev	Ave	STD Dev
Australian	0.16610	0.01027	0.32286	0.02052
Balance-scale	0.11395	0.00931	1.20235	0.00396
Dermatology	0.91512	0.11784	1.38068	0.05733
Heart	0.06533	0.00100	0.06518	0.00315
Iris	0.02343	0.02343	0.02542	0.00312
Spambase	2.53490	0.18252	3.16697	0.87895
Tae	0.04673	0.00202	0.03258	0.00480
Waweform	3.54067	0.09443	8.08507	4.26694
WDBC	0.20360	0.00334	0.39131	0.00786
Ecoli	1.71820	0.14015	0.92510	0.06711
Glass	0.57863	0.02675	0.41492	0.01662
Yeast	11.35089	0.41673	6.10282	0.80203

(Continued)

Table 8: Continued

METHOD	PNTS3FCM		CS3FCM	
	Ave	STD Dev	Ave	STD Dev
Wine	0.05700	0.00268	0.05651	0.00182
Vertebral	0.11455	0.00255	0.18393	0.00592
Ionosphere	0.20543	0.00606	0.45496	0.01373

Summary: During the evaluation of computational time on all data, including outlier and non-outlier (15 datasets), PNTS3FCM has better results on nine datasets. CS3FCM is the best model on six datasets. The proposed method seems to be better with a more significant number of data clusters. To get better results, the proposed method is firstly based on Picture fuzzy set that has more information to reduce the noise or hesitation in partitioning data. Secondly, PNTS3PFCM has a safe semi-supervised part for labeled and unlabeled data that can cope with the doubt labeled data, then reduce their effectiveness in the clustering process.

5 Conclusion

This research suggested a novel technique called Picture-Neutrosophic Trusted Safe Semi-Supervised Fuzzy Clustering (PNTS3FCM) to address the issue of data clustering with high confidence and noisy information. PNTS3FCM is constructed based on combining Picture Fuzzy Sets, Neutrosophic Sets and safe fuzzy semi-supervised clustering (PFS). This method consists of 4 critical parts: the clustering portion, the outlier solution part and the safe semi-supervised fuzzy clustering with labeled and unlabeled data. Through the use of PFS, the first stage of PNTS3FCM aims to reduce the distance between data components and cluster centers. The model's second step involves processing the "noisy data" by integrating the entropy quantity between the neutral and refuse degrees. The third and fourth stages coordinate the safe semi-supervised fuzzy clustering using both labeled and unlabeled data to solve the safety information. We also provide an iterative technique from the formulation to construct the cluster centers and memberships. The method produces final clusters that are reliable and confident.

PNTS3FCM has illustrated its effectiveness by comparing it with two related methods, including FC-PFS and CS3FCM algorithm. The experiment results show that PNTS3FCM is better than the others in terms of computational time and clustering quality. Even though the proposed PNTS3FCM mainly focuses on eliminating or reducing noisy data elements, this method still has some limitations. First of all, PNTS3FCM takes a long time to compute. Secondly, it needs an increased number of parameters. In the future, an effective optimization algorithm will be studied and introduced to overcome these limitations.

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On Single Valued Neutrosophic Regularity Spaces

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ABSTRACT

This article aims to present new terms of single-valued neutrosophic notions in the Šostak sense, known as single-valued neutrosophic regularity spaces. Concepts such as *r*-single-valued neutrosophic semi £-open, *r*-single-valued neutrosophic pre-£-open, *r*-single valued neutrosophic regular-£-open and *r*-single valued neutrosophic α£-open are defined and their properties are studied as well as the relationship between them. Moreover, we introduce the concept of *r*-single valued neutrosophic θ£-cluster point and *r*-single-valued neutrosophic γ£-cluster point, *r*-θ£-closed, and θ£-closure operators and study some of their properties. Also, we present and investigate the notions of *r*-single-valued neutrosophic θ£-connectedness and *r*-single valued neutrosophic δ£-connectedness and investigate relationship with single-valued neutrosophic almost £-regular. We compare all these forms of connectedness and investigate their properties in single-valued neutrosophic semiregular and single-valued neutrosophic almost regular in neutrosophic ideal topological spaces in Šostak sense. The usefulness of these concepts are incorporated to multiple attribute groups of comparison within the connectedness and separateness of θ£ and δ£.

KEYWORDS

Single valued neutrosophic θ£-closed; single valued neutrosophic θ£-separated; single valued neutrosophic δ£-separated; single-valued neutrosophic δ£-connected; single valued neutrosophic δ£-connected; single valued neutrosophic almost £-egular

1 Introduction

A neutrosophic set can be practical in addressing problems with indeterminate, imperfect, and inconsistent materials. The concept of neutrosophic set theory was introduced by Smarandache [1] as a new mathematical method that corresponds to the indeterminacy degree (uncertainty, etc.). Bakbak et al. [2] and Mishra et al. [3] applied the soft set theory successfully applied in several

areas, such as the smoothness of functions, as well as architecture-based, neuro-linguistic programming. Wang et al. [4] proposed single-valued neutrosophic sets (SVNSs). Meanwhile, Kim et al. [5,6] inspected the single valued neutrosophic relations (SVNRs) and symmetric closure of SVNR, respectively. Recently, Saber et al. [7–9] introduced the concepts of single-valued neutrosophic ideal open local function and single-valued neutrosophic topological space. Many of their applications appear in the studies of Das et al. [10]. Alsharari et al. [11–13]. Riaz et al. [14]. Salama et al. [15–17]. Hur et al. [18,19]. Yang et al. [20]. El-Gayyar [21], AL-Nafee et al. [22]. Muhiuddin et al. [23,24] and Mukherjee et al. [25].

First, we define single-valued neutrosophic θ£-closed and single-valued neutrosophic δ£-closed sets as well as some of their core properties. We also present and explore the properties and characterizations of single valued neutrosophic operators namely θ£-closure ($\mathrm{CI}^{\theta\pounds}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}$) and δ£-closure ($\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}$) in the single valued neutrosophic ideal topological space ($\tilde{\mathcal{F}}, \tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, \pounds^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}$). We then define the concept of single valued neutrosophic regularity spaces. Next, we study single-valued neutrosophic θ£-separated and single-valued neutrosophic δ£-separated with giving some definitions and theorems. Furthermore, we also introduce single-valued neutrosophic θ£-connected and single valued neutrosophic δ£-connected relying on the single valued neutrosophic θ£-closure and δ£-closure operators.

We define a fixed universe $\tilde{\mathcal{F}}$ to be a finite set of objects and ζ a closed unit interval [0, 1]. Additionally, we denote $\zeta^{\mathcal{F}}$ as the set of all single-valued neutrosophic subsets of $\tilde{\mathcal{F}}$.

2 Preliminaries

This section provides a complete survey, some previous studies, and concepts associated with this study.

Definition 1. [1] Let $\tilde{\mathcal{F}}$ be a non-empty set. A neutrosophic set (briefly, $\mathcal{NS}$) in $\tilde{\mathcal{F}}$ is an object having the form $\alpha_n = \{\langle \upsilon, \tilde{\varrho}_{\alpha_n}(\upsilon), \tilde{\sigma}_{\sigma_n}(\omega), \tilde{\varsigma}_{\alpha_n}(\upsilon)\rangle : \upsilon \in \tilde{\mathcal{F}}\}$ where

$$\tilde{\varrho}: \tilde{\mathcal{F}} \to \rfloor^{-}0, 1^{+}\lfloor, \tilde{\sigma}: \tilde{\mathcal{F}} \to \rfloor^{-}0, 1^{+}\lfloor, \tilde{\varsigma}: \tilde{\mathcal{F}} \to \rfloor^{-}0, 1^{+}\lfloor \text{ and } {}^{-}0 \leq \tilde{\varrho}_{\alpha_n}(\upsilon) + \tilde{\sigma}_{\alpha_n}(\upsilon) + \tilde{\varsigma}_{\alpha_n}(\upsilon) \leq 3^{+} \quad (1)$$

Represent the degree of membership ($\tilde{\varrho}_{\alpha_n}$), the degree of indeterminacy ($\tilde{\sigma}_{\alpha_n}$), and the degree of non-membership ($\tilde{\varsigma}_{\alpha_n}$) respectively of any $\upsilon \in \tilde{\mathcal{F}}$ to the set α_n.

Definition 2. [4] Suppose that $\tilde{\mathcal{F}}$ is a universal set a space of points (objects), with a generic element in $\tilde{\mathcal{F}}$ denoted by υ. Then α_n is called a single valued neutrosophic set (briefly, $\mathcal{SVNS}$) in $\tilde{\mathcal{F}}$, if α_n has the form $\alpha_n = \{\langle \upsilon, \tilde{\varrho}_{\alpha_n}(\upsilon), \tilde{\sigma}_{\alpha_n}(\upsilon), \tilde{\varsigma}_{\alpha_n}(\upsilon)\rangle : \upsilon \in \tilde{\mathcal{F}}\}$. Now, $\tilde{\varrho}_{\alpha_n}, \tilde{\sigma}_{\sigma_n}, \tilde{\varsigma}_{\alpha_n}$ indicate the degree of non-membership, the degree of indeterminacy, and the degree of membership, respectively of any $\upsilon \in \tilde{\mathcal{F}}$ to the set α_n.

Definition 3. [4] Let $\alpha_n = \{\langle \upsilon, \tilde{\varrho}_{\alpha_n}(\upsilon), \tilde{\sigma}_{\sigma_n}(\upsilon), \tilde{\varsigma}_{\alpha_n}(\upsilon)\rangle : \upsilon \in \tilde{\mathcal{F}}\}$ be an SVNS on $\tilde{\mathcal{F}}$. The complement of the set α_n (briefly, α_n^c) defined as follows: $\tilde{\varrho}_{\alpha_n^c}(\upsilon) = \tilde{\varsigma}_{\alpha_n}(\upsilon), \tilde{\sigma}_{\alpha_n}(\upsilon) = [\tilde{\sigma}_{\alpha_n}]^c(\upsilon), \tilde{\varsigma}_{\alpha_n^c}(\upsilon) = \tilde{\varrho}_{\alpha_n}(\upsilon)$.

Definition 4. [26] Let $\tilde{\mathcal{F}}$ be a non-empty set and $\alpha_n, \varepsilon_n \in \zeta^{\tilde{\mathcal{F}}}$ be in the form: $\alpha_n = \{\langle \upsilon, \tilde{\varrho}_{\alpha_n}(\upsilon), \tilde{\sigma}_{\alpha_n}(\upsilon), \tilde{\varsigma}_{\alpha_n}(\upsilon)\rangle : \upsilon \in \tilde{\mathcal{F}}\}$ and $\varepsilon_n = \{\langle \upsilon, \tilde{\varrho}_{\varepsilon_n}(\upsilon), \tilde{\sigma}_{\varepsilon_n}(\upsilon), \tilde{\varsigma}_{\varepsilon_n}(\upsilon)\rangle : \upsilon \in \tilde{\mathcal{F}}\}$ on $\tilde{\mathcal{F}}$ then,

(a) $\alpha_n \subseteq \varepsilon_n$ for every $\upsilon \in \tilde{\mathcal{F}}$; $\tilde{\varrho}_{\alpha_n}(\upsilon) \leq \tilde{\varrho}_{\varepsilon_n}(\upsilon), \tilde{\sigma}_{\alpha_n}(\upsilon) \geq \tilde{\sigma}_{\varepsilon_n}(\upsilon), \tilde{\varsigma}_{\alpha_n}(\upsilon) \geq \tilde{\varsigma}_{\varepsilon_n}(\upsilon)$.
(b) $\alpha_n = \varepsilon_n$ iff $\sigma_n \subseteq \varepsilon_n$ and $\sigma_n \supseteq \varepsilon_n$.
(c) $\tilde{0} = \langle 0, 1, 1\rangle$ and $\tilde{1} = \langle 1, 0, 0\rangle$.

Definition 5. [20] Let $\alpha_n, \varepsilon_n \in \zeta^{\tilde{\mathcal{F}}}$. Then,

(a) $\alpha_n \cap \varepsilon_n$ is an SVNS, if for every $\upsilon \in \tilde{\mathcal{F}}$,

$$\boldsymbol{\alpha_n \cap \varepsilon_n} = \left\langle \left(\tilde{\varrho}_{\alpha_n} \cap \tilde{\varrho}_{\varepsilon_n}\right)(\upsilon), \left(\tilde{\sigma}_{\alpha_n} \cup \tilde{\sigma}_{\varepsilon_n}\right)(\upsilon), \left(\tilde{\zeta}_{\alpha_n} \cup \tilde{\zeta}_{\varepsilon_n}\right)(\upsilon) \right\rangle, \tag{2}$$

where, $(\tilde{\varrho}_{\alpha_n} \cap \tilde{\varrho}_{\varepsilon_n})(\upsilon) = \tilde{\varrho}_{\alpha_n}(\upsilon) \cap \tilde{\varrho}_{\varepsilon_n}(\upsilon)$ and $(\tilde{\zeta}_{\alpha_n} \cup \tilde{\zeta}_{\varepsilon_n})(\upsilon) = \tilde{\zeta}_{\alpha_n}(\upsilon) \cup \tilde{\zeta}_{\varepsilon_n}(\upsilon)$, for all $\upsilon \in \tilde{\mathcal{F}}$,

(b) $\alpha_n \cup \varepsilon_n$ is an SVNS, if for every $\upsilon \in \tilde{\mathcal{F}}$,

$$\boldsymbol{\alpha_n \cup \varepsilon_n} = \langle \left(\tilde{\varrho}_{\alpha_n} \cup \tilde{\varrho}_{\varepsilon_n}\right)(\upsilon), \left(\tilde{\sigma}_{\alpha_n} \cap \tilde{\sigma}_{\varepsilon_n}\right)(\upsilon), \left(\tilde{\zeta}_{\alpha_n} \cap \tilde{\zeta}_{\varepsilon_n}\right)(\upsilon) \rangle. \tag{3}$$

Definition 6. [15] For an any arbitrary family $\{\alpha_n\}_{i\in j} \in \zeta^{\tilde{\mathcal{F}}}$ of SVNS the union and intersection are given by

(a) $\bigcap_{i\in j}[\alpha_n]_i = \langle \cap_{i\in j}\tilde{\varrho}_{[\alpha_n]_i}(\upsilon), \cup_{i\in j}\tilde{\sigma}_{[\alpha_n]_i}(\upsilon), \cup_{i\in j}\tilde{\zeta}_{[\alpha_n]_i}(\upsilon)\rangle$,

(b) $\bigcup_{i\in j}[\alpha_n]_i = \langle \cup_{i\in j}\tilde{\varrho}_{[\alpha_n]_i}(\upsilon), \cap_{i\in j}\tilde{\sigma}_{[\alpha_n]_i}(\upsilon), \cap_{i\in j}\tilde{\zeta}_{[\alpha_n]_i}(\upsilon)\rangle$.

Definition 7. [21] A single-valued neutrosophic topological spaces is an ordered $(\tilde{\mathcal{F}}, \tilde{\tau}^{\tilde{\varrho}}, \tilde{\tau}^{\tilde{\sigma}}, \tilde{\tau}^{\tilde{\zeta}})$ where $\tilde{\tau}^{\tilde{\varrho}}, \tilde{\tau}^{\tilde{\sigma}}, \tilde{\tau}^{\tilde{\zeta}} : \zeta^{\tilde{\mathcal{F}}} \to \zeta$ is a mapping satisfying the following axioms:

(SV NT1) $\tilde{\tau}^{\tilde{\varrho}}(\tilde{0}) = \tilde{\tau}^{\tilde{\varrho}}(\tilde{1}) = \tilde{\tau}^{\tilde{\sigma}}(\tilde{0}) = \tilde{\tau}^{\tilde{\sigma}}(\tilde{1}) = 0$ and $\tilde{\tau}^{\tilde{\zeta}}(\tilde{0}) = \tilde{\tau}^{\tilde{\zeta}}(\tilde{1}) = 1$.

(SV NT2) $\tilde{\tau}^{\tilde{\varrho}}(\alpha_n \cap \varepsilon_n) \geq \tilde{\tau}^{\tilde{\varrho}}(\alpha_n) \cap \tilde{\tau}^{\tilde{\varrho}}(\varepsilon_n), \tilde{\tau}^{\tilde{\sigma}}(\alpha_n \cap \varepsilon_n) \leq \tau^{\tilde{\sigma}}(\alpha_n) \cup \tilde{\tau}^{\tilde{\sigma}}(\varepsilon_n), \tilde{\tau}^{\tilde{\zeta}}(\alpha_n \cap \varepsilon_n) \leq \tilde{\tau}^{\tilde{\zeta}}(\alpha_n) \cup \tilde{\tau}^{\tilde{\zeta}}(\varepsilon_n)$ for every, $\alpha_n, \varepsilon_n \in \zeta^{\tilde{\mathcal{F}}}$

(SV NT3) $\tilde{\tau}^{\tilde{\varrho}}(\cup_{j\in\Gamma}[\alpha_n]_j) \geq \cap_{j\in\Gamma}\tilde{\tau}^{\tilde{\varrho}}([\alpha_n]_j), \tilde{\tau}^{\tilde{\sigma}}(\cup_{i\in\Gamma}[\alpha_n]_j) \leq \cup_{j\in\Gamma}\tilde{\tau}^{\tilde{\sigma}}([\alpha_n]_j), \tilde{\tau}^{\tilde{\zeta}}(\cup_{j\in\Gamma}[\alpha_n]_j) \leq \cup_{j\in\Gamma}\tilde{\tau}^{\tilde{\zeta}}([\alpha_n]_j)$, for every $[\alpha_n]_j \in \zeta^{\tilde{\mathcal{F}}}$.

The quadruple $(\tilde{\mathcal{F}}, \tilde{\tau}^{\tilde{\varrho}}, \tilde{\tau}^{\tilde{\sigma}}, \tilde{\tau}^{\tilde{\zeta}})$ is called a single-valued neutrosophic topological spaces (briefly, SVNT, for short). Occasionally write $\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\zeta}}$ for $(\tilde{\tau}^{\tilde{\varrho}}, \tilde{\tau}^{\tilde{\sigma}}, \tilde{\tau}^{\tilde{\zeta}})$ and it will cause no ambiguity.

Definition 8. [7] Let $(\tilde{\mathcal{F}}, \tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\zeta}})$ be an SV NTS. Then, for every $\alpha_n \in \zeta^{\tilde{\mathcal{F}}}$ and $r \in \zeta_0$. Then the single valued neutrosophic closure and single valued neutrosophic interior of α_n are define by:

$$C_{\boldsymbol{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\zeta}}}(\boldsymbol{\alpha_n}, \boldsymbol{r}) = \bigcap \{\boldsymbol{\varepsilon_n} \in \boldsymbol{\zeta}^{\tilde{\mathcal{F}}} : \boldsymbol{\alpha_n} \leq \boldsymbol{\varepsilon_n}, \boldsymbol{\tau}^{\tilde{\varrho}}([\boldsymbol{\varepsilon_n}]^c) \geq \boldsymbol{r}, \boldsymbol{\tau}^{\tilde{\sigma}}([\boldsymbol{\varepsilon_n}]^c) \leq 1 - \boldsymbol{r}, \boldsymbol{\tau}^{\tilde{\zeta}}([\boldsymbol{\varepsilon_n}]^c) \leq 1 - \boldsymbol{r}\} \tag{4}$$

$$\text{int}_{\boldsymbol{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\zeta}}}(\boldsymbol{\alpha_n}, \boldsymbol{r}) = \bigcup \{\boldsymbol{\varepsilon_n} \in \boldsymbol{\zeta}^{\tilde{\mathcal{F}}} : \boldsymbol{\alpha_n} \geq \boldsymbol{\varepsilon_n}, \boldsymbol{\tau}^{\tilde{\varrho}}(\boldsymbol{\varepsilon_n}) \geq \boldsymbol{r}, \boldsymbol{\tau}^{\tilde{\sigma}}(\boldsymbol{\varepsilon_n}) \leq 1 - \boldsymbol{r}, \boldsymbol{\tau}^{\tilde{\zeta}}(\boldsymbol{\varepsilon_n}) \leq 1 - \boldsymbol{r}\} \tag{5}$$

Definition 9. [7] Let $(\tilde{\mathcal{F}})$ be a nonempty set and $\upsilon \in \tilde{\mathcal{F}}$, let $s \in (0, 1]$, $t \in [0, 1)$ and $k \in [0, 1)$, then the single-valued neutrosophic point $x_{s,t,k}$ in $\tilde{\mathcal{F}}$ given by

$$x_{s,t,k}(\upsilon) = \begin{cases} (s, t, k), & \text{if } x = \upsilon \\ (0, 1, 1), & \text{otherwise.} \end{cases}$$

We define that, $x_{s,t,p} \in \alpha_n$ iff $s < \tilde{\varrho}_{\alpha_n}(\upsilon)$, $t \geq \tilde{\sigma}_{\alpha_n}(\upsilon)$ and $k \geq \tilde{\tilde{\zeta}}_{\alpha_n}(\upsilon)$. We indicate the set of all single-valued neutrosophic points in $\tilde{\mathcal{F}}$ as $P_{x_{s,t,k}}(\tilde{\mathcal{F}})$. A single-valued neutrosophic set α_n is said to be quasi-coincident with another single-valued neutrosophic set ε_n, denoted by $\alpha_n q \varepsilon_n$, if there exists an element $\upsilon \in \tilde{\mathcal{F}}$ such that $\tilde{\varrho}_{\alpha_n}(\upsilon) + \tilde{\varrho}_{\varepsilon_n}(\upsilon) > 1, \tilde{\sigma}_{\alpha_n}(\upsilon) + \tilde{\sigma}_{\varepsilon_n}(\upsilon) \leq 1, \tilde{\zeta}_{\alpha_n}(\upsilon) + \tilde{\zeta}_{\varepsilon_n}(\upsilon) \leq 1$.

Definition 10. [7] A mapping $\pounds^{\tilde{\varrho}}, \pounds^{\tilde{\sigma}}, \pounds^{\tilde{\varsigma}}: \zeta^{\tilde{\mathcal{F}}} \to \zeta$ is called single-valued neutrosophic ideal (SVNI) on $\tilde{\mathcal{F}}$ if, it satisfies the following conditions:

($\pounds_1$) $\pounds^{\tilde{\varrho}}(\tilde{0}) = 1$ and $\pounds^{\tilde{\sigma}}(\tilde{0}) = \pounds^{\tilde{\varsigma}}(\tilde{0}) = 0$.

($\pounds_2$) If $\sigma_n \leq \gamma_n$, then $\pounds^{\tilde{\varrho}}(\varepsilon_n) \leq \pounds^{\tilde{\varrho}}(\alpha_n)$, $\pounds^{\tilde{\sigma}}(\varepsilon_n) \geq \pounds^{\tilde{\sigma}}(\alpha_n)$ and $\pounds^{\tilde{\varsigma}}(\varepsilon_n) \geq \pounds^{\tilde{\varsigma}}(\alpha_n)$, for $\varepsilon_n, \alpha_n \in \zeta^{\tilde{\mathcal{F}}}$.

($\pounds_3$) $\pounds^{\tilde{\varrho}}(\alpha_n \cup \varepsilon_n) \geq \pounds^{\tilde{\varrho}}(\alpha_n) \cap \pounds^{\tilde{\varrho}}(\varepsilon_n)$, $\pounds^{\tilde{\sigma}}(\alpha_n \cup \varepsilon_n) \leq \pounds^{\tilde{\sigma}}(\alpha_n) \cup \pounds^{\tilde{\sigma}}(\varepsilon_n)$ and $\pounds^{\tilde{\varsigma}}(\alpha_n \cup \varepsilon_n) \leq \pounds^{\tilde{\varsigma}}(\alpha_n) \cup \pounds^{\tilde{\varsigma}}(\varepsilon_n)$, for $\alpha_n, \varepsilon_n \in \zeta^{\tilde{\mathcal{F}}}$.

The tribal $(\tilde{\mathcal{F}}, \tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, \pounds^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$ is called a single valued neutrosophic ideal topological space in Šostak sense (briefly, SVNITS).

Definition 11. [7] Let $(\tilde{\mathcal{F}}, \tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, \pounds^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$ be an SVNITS for each $\alpha_n \in \zeta^{\tilde{\mathcal{F}}}$. Then, the single valued neutrosophic ideal open local function $[\alpha_n]_r^{\odot}(\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, \pounds^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$ of α_n is the union of all single-valued neutrosophic points $x_{s,t,k}$ such that if $\varepsilon_n \in Q_{\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(x_{s,t,k}, r)$ and $\pounds^{\tilde{\varrho}}(\omega_n) \geq r$, $\pounds^{\tilde{\sigma}}(\omega_n) \leq 1 - r$, $\pounds^{\tilde{\varsigma}}(\omega_n) \leq 1 - r$, then there is at least one $\upsilon \in \tilde{\mathcal{F}}$ for which

$$\tilde{\varrho}_{\alpha_n}(\upsilon) + \tilde{\varrho}_{\varepsilon_n}(\upsilon) - 1 > \tilde{\varrho}_{\omega_n}(\upsilon), \quad \tilde{\sigma}_{\alpha_n}(\upsilon) + \tilde{\sigma}_{\varepsilon_n}(\upsilon) - 1 \leq \tilde{\sigma}_{\omega_n}(\upsilon), \quad \tilde{\varsigma}_{\alpha_n}(\upsilon) + \tilde{\varsigma}_{\varepsilon_n}(\upsilon) - 1 \leq \tilde{\varsigma}_{\omega_n}(\upsilon) \quad (6)$$

Occasionally, we will write $[\alpha_n]_r^{\odot}$ for $[\alpha_n]_r^{\odot}(\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, \pounds^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$ herein to avoid ambiguity.

Remark 1. [7] Let $(\tilde{\mathcal{F}}, \tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, \pounds^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$ be an SVNITS and $\alpha_n \in \zeta^{\tilde{\mathcal{F}}}$. Hence, we can write

$$\mathrm{CI}^{\odot}_{\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\alpha_n, r) = \alpha_n \cup [\alpha_n]_r^{\odot}, \quad \mathrm{int}^{\odot}_{\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\alpha_n, r) = \alpha_n \cap [(\alpha_n^c)_r^{\odot}]^c \quad (7)$$

Clearly, $\mathrm{CI}^{\odot}_{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}$ is a single-valued neutrosophic closure operator and $(\tau^{\tilde{\varrho}\odot}(\pounds), \tau^{\tilde{\sigma}\odot}(\pounds), \tau^{\tilde{\varsigma}\odot}(\pounds))$ is the single-valued neutrosophic topology generated by $\mathrm{CI}^{\odot}_{\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}$, i.e., $\tau^{\odot}(\mathcal{J})(\alpha_n) = \bigcup\{r | \mathrm{CI}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\alpha_n^c, r) = \alpha_n^c\}$.

Theorem 1. [7] Let $\{[\alpha_n]_i\}_{i \in J} \subset \zeta^{\tilde{\mathcal{F}}}$ be a family of single-valued neutrosophic sets on $\tilde{\mathcal{F}}$ and $(\tilde{\mathcal{F}}, \tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, \pounds^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$ be a SVNITS. Then,

(a) $(\cup([\alpha_n]_i)_r^{\odot} : i \in J) \leq (\cup[\alpha_n]_i : i \in j)_r^{\odot}$,
(b) $(\cap([\alpha_n]_i) : i \in j)_r^{\odot} \geq (\cap([\alpha_n]_i)_r^{\odot} : i \in J)$.

Theorem 2. [7] Let $(\tilde{\mathcal{F}}, \tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, \pounds^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$ be an SVNITS and $r \in \zeta, \alpha_n, \varepsilon_n \in \zeta^{\tilde{\mathcal{F}}}$. Then,

(a) $\mathrm{int}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\alpha_n \vee \varepsilon_n, r) \leq \mathrm{int}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\alpha_n, r) \vee \mathrm{int}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\varepsilon_n, r)$,
(b) $\mathrm{int}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\alpha_n, r) \leq \mathrm{int}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\alpha_n, r) \leq \alpha_n \leq \mathrm{CI}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\alpha_n, r) \leq \mathrm{C}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\alpha_n, r)$,
(c) $\mathrm{CI}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}([\alpha_n]^c, r) = [\mathrm{int}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\alpha_n, r)]^c$,
(d) $[\mathrm{CI}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\alpha_n, r)]^c = \mathrm{int}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}([\alpha_n]^c, r)$,
(e) $\mathrm{int}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\alpha_n \wedge \varepsilon_n, r) = \mathrm{int}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\alpha_n, r) \wedge \mathrm{int}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\varepsilon_n, r)$.

Definition 12. [8] Let $(\tilde{\mathcal{F}}, \tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$ be an SVNITS. For every $\alpha_n, \varepsilon_n, \omega_n \in \zeta^{\tilde{\mathcal{F}}}$, α_n and ε_n are called r-single-valued neutrosophic separated if for $r \in \zeta_0$,

$$\mathrm{CI}_{\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\alpha_n, r) \cap \varepsilon_n = \mathrm{CI}_{\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\varepsilon_n, r) \cap \alpha_n = \tilde{0} \quad (8)$$

An $\mathcal{SVNS}$, ω_n is called r-single-valued neutrosophic connected if r-$\mathcal{SVNSEP}$ $\alpha_n, \varepsilon_n \in \zeta^{\tilde{\mathcal{F}}} - \{\tilde{0}\}$ such that $\omega_n = \alpha_n \cup \varepsilon_n$ does not exist. A $\mathcal{SVNS}$ α_n is said to be r-single-valued neutrosophic connected if it is r-single-valued neutrosophic connected for any $r \in \zeta_0$. A $(\tilde{\mathcal{F}}, \tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$ is said to be r-single-valued neutrosophic connected if $\tilde{1}$ is r-single-valued neutrosophic connected.

3 Single Valued Neutrosophic δ£-Cluster Point and Single Valued Neutrosophic θ£-Cluster Point

In this section, we introduce the r-single-valued neutrosophic δ£-cluster point (abbreviated $SVN\delta$£-cluster point) and r-single-valued neutrosophic £-closed set (abbreviated SVN£C). Furthermore, we analyze the single-valued neutrosophic δ£-closure operator (δ£-closure operator for brevity) and single-valued neutrosophic θ£-closure operator (θ£-closure operator for brevity).

Definition 13. Let $(\tilde{\mathcal{F}}, \tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, £^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$ be an SVNITS and $\alpha_n \in \zeta^{\tilde{\mathcal{F}}}$, $r \in \zeta_0$. Then,

(a) α_n is said to be r-single valued neutrosophic £-open (briefly, r-$SVN£O$), if and only if $\alpha_n \leq \text{int}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}([\alpha_n]_r^{\odot}, r)$,

(b) α_n is said to be r-single valued neutrosophic semi-£-open (briefly, r-$SVNS£O$) if and only if $\alpha_n \leq \text{CI}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\text{int}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}([\alpha_n]_r^{\odot}, r), r)$,

(c) α_n is called r-single valued neutrosophic pre-£-open (briefly, r-$SVNP£O$) if and only if $\alpha_n \leq \text{int}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\text{CI}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}([\alpha_n]_r^{\odot}, r), r)$,

(d) α_n is called r-single valued neutrosophic regular-£-open (briefly, r-$SVNR£O$) if and only if $\alpha_n = \text{int}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\text{CI}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}([\alpha_n]_r^{\odot}, r), r)$,

(e) α_n is said to be r-single valued neutrosophic α£-open (briefly, r-$SVN\alpha£O$) if and only if $\alpha_n \leq \text{int}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\text{CI}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\text{int}_{\tilde{\tau}^{\tilde{\varrho}}}([\alpha_n]_r^{\odot}, r)), r)$,

(f) α_n is said to be r-single valued neutrosophic $\star$-open set (briefly, r-$SVN\star O$) if and only if $\alpha_n = \text{CI}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\alpha_n, r)$.

The complement of an r−SVN£O (resp, r-SVNS£O, r-SVNP£O, r-SVNR£O, r-SVNα£O, r-SVN$\star$O) is said to be an r−SVN£C (resp, r-SVNS£C, r-SVNP£C, r-SVNR£C, r-SVNα£C, r-SVN$\star$C) respectively.

Remark 2. r-single valued neutrosophic open set (r$-\mathcal{SVN}$O) and r-SVN£O are independent notions as shown by the following example.

Example 1. Let $\tilde{\mathcal{F}} = \{a, b, c\}$ be a set. Define $\varepsilon_n, \pi_n, \omega_n \in \zeta^{\tilde{\mathcal{F}}}$ as follows:

$\varepsilon_n = \langle (0.3, 0.3, 0.3), (0.3, 0.3, 0.3), (0.3, 0.3, 0.3) \rangle$; $\pi_n = \langle (0.4, 0.4, 0.4), (0.4, 0.4, 0.4), (0.4, 0.4, 0.4) \rangle$,

$\omega_n = \langle (0.5, 0.5, 0.5), (0.2, 0.2, 0.2), (0.1, 0.1, 0.1) \rangle$.

We define an SVNITS$(\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, £^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$ on $\tilde{\mathcal{F}}$ as follows: for each $\alpha_n \in \zeta^{\tilde{\mathcal{F}}}$,

$$\tilde{\tau}^{\tilde{\varrho}}(\alpha_n) = \begin{cases} 1, & \text{if } \alpha_n = \left\{\tilde{0}, \tilde{1}\right\}, \\ \frac{2}{3}, & \text{if } \alpha_n = \{\varepsilon_n, \pi_n\}, \\ 0, & \text{otherwise}, \end{cases} \quad £^{\tilde{\varrho}}(\alpha_n) = \begin{cases} 1, & \text{if } \alpha_n = \tilde{0}, \\ \frac{2}{3}, & \text{if } 0 < \alpha_n \leq \omega_n \\ 0, & \text{otherwise}, \end{cases}$$

$$\tilde{\tau}^{\tilde{\sigma}}(\alpha_n)=\begin{cases}0, & \text{if } \alpha_n=\left\{\tilde{0},\tilde{1}\right\},\\ \frac{1}{3}, & \text{if } \alpha_n=\{\varepsilon_n,\pi_n\},\\ 1, & \text{otherwise},\end{cases}\quad \pounds^{\tilde{\sigma}}(\alpha_n)=\begin{cases}0, & \text{if } \alpha_n=\tilde{0},\\ \frac{1}{3}, & \text{if } 0<\alpha_n\leq\omega_n,\\ 1, & \text{otherwise},\end{cases}$$

$$\tilde{\tau}^{\tilde{\varsigma}}(\alpha_n)=\begin{cases}0, & \text{if } \alpha_n=\left\{\tilde{0},\tilde{1}\right\},\\ \frac{1}{3}, & \text{if } \alpha_n=\{\varepsilon_n,\pi_n\},\\ 1, & \text{otherwise},\end{cases}\quad \pounds^{\tilde{\varsigma}}(\alpha_n)=\begin{cases}0, & \text{if } \alpha_n=\tilde{0},\\ \frac{1}{3}, & \text{if } 0<\alpha_n\leq\omega_n,\\ 1, & \text{otherwise}.\end{cases}$$

Based on $\varepsilon_n=\langle(0.3,0.3,0.3),(0.3,0.3,0.3),(0.3,0.3,0.3)\rangle$, it's clear that, $\frac{2}{3}-\boldsymbol{\mathcal{SVNO}}$ is set because $\tau^{\tilde{\varrho}}(\langle(0.3,0.3,0.3),(0.3,0.3,0.3),(0.3,0.3,0.3)\rangle)\geq\frac{2}{3},\tau^{\tilde{\sigma}}(\langle(0.3,0.3,0.3),(0.3,0.3,0.3),(0.3,0.3,0.3)\rangle)\leq\frac{1}{3},\tau^{\tilde{\varsigma}}(\langle(0.3,0.3,0.3),(0.3,0.3,0.3),(0.3,0.3,0.3)\rangle)\leq\frac{1}{3}$.

However ε_n is not an r-SVN£O set, and for that, we must prove that $\varepsilon_n\not\leq \mathrm{int}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}([\varepsilon_n]^{\odot}_{\frac{2}{3}},\frac{2}{3})$. So, we must first obtain $[\varepsilon_n]^{\odot}_{\frac{2}{3}}$. Based on Eq. (11), $\tilde{1},\varepsilon_n,\pi_n\in Q_{\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(x_{s,t,k},\frac{2}{3})$ and $\pounds^{\tilde{\varrho}}(\langle(0.5,0.5,0.5),(0.2,0.2,0.2),(0.1,0.1,0.1)\rangle)\geq\frac{2}{3}$, $\pounds^{\tilde{\sigma}}(\langle(0.5,0.5,0.5),(0.2,0.2,0.2),(0.1,0.1,0.1)\rangle)\leq\frac{1}{3}$, $\pounds^{\tilde{\varsigma}}(\langle(0.5,0.5,0.5),(0.2,0.2,0.2),(0.1,0.1,0.1)\rangle)\leq\frac{1}{3}$,
such that by using Eqs. (2), (3) and (6) we obtain,

$$\tilde{\varrho}_{\varepsilon_n}(\upsilon)+\tilde{\varrho}_{\tilde{1}}(\nu)-1>\tilde{\varrho}_{\omega_n}(\upsilon),\tilde{\sigma}_{\varepsilon_n}(\upsilon)+\tilde{\sigma}_{\tilde{1}}(\upsilon)-1\leq\tilde{\sigma}_{\omega_n}(\upsilon),\tilde{\varsigma}_{\varepsilon_n}(\upsilon)+\tilde{\varsigma}_{\tilde{1}}(\upsilon)-1\leq\tilde{\varsigma}_{\omega_n}(\upsilon).$$

$$(0.3,0.3,0.3)(\upsilon)+(1,1,1)(\nu)-1\not>(0.5,0.5,0.5)(\upsilon),$$

$$(0.3,0.3,0.3)(\upsilon)+(0,0,0)(\upsilon)-1\leq(0.2,0.2,0.2)(\upsilon),$$

$$(0.3,0.3,0.3)(\upsilon)+(0,0,0)(\upsilon)-1\leq(0.1,0.1,0.1)(\upsilon),$$

$$\tilde{\varrho}_{\varepsilon_n}(\upsilon)+\tilde{\varrho}_{\pi_n}(\nu)-1>\tilde{\varrho}_{\omega_n}(\upsilon),\tilde{\sigma}_{\varepsilon_n}(\upsilon)+\tilde{\sigma}_{\pi_n}(\upsilon)-1\leq\tilde{\sigma}_{\omega_n}(\upsilon),\tilde{\varsigma}_{\varepsilon_n}(\upsilon)+\tilde{\varsigma}_{\pi_n}(\upsilon)-1\leq\tilde{\varsigma}_{\omega_n}(\upsilon).$$

$$(0.3,0.3,0.3)(\upsilon)+(0.4,0.4,0.4)(\nu)-1\not>(0.5,0.5,0.5)(\upsilon),$$

$$(0.3,0.3,0.3)(\upsilon)+(0.4,0.4,0.4)(\upsilon)-1\leq(0.2,0.2,0.2)(\upsilon),$$

$$(0.3,0.3,0.3)(\upsilon)+(0.4,0.4,0.4)(\upsilon)-1\leq(0.1,0.1,0.1)(\upsilon)$$

$$\tilde{\varrho}_{\varepsilon_n}(\upsilon)+\tilde{\varrho}_{\varepsilon_n}(\nu)-1>\tilde{\varrho}_{\omega_n}(\upsilon),\tilde{\sigma}_{\varepsilon_n}(\upsilon)+\tilde{\sigma}_{\varepsilon_n}(\upsilon)-1\leq\tilde{\sigma}_{\omega_n}(\upsilon),\tilde{\varsigma}_{\varepsilon_n}(\upsilon)+\tilde{\varsigma}_{\varepsilon_n}(\upsilon)-1\leq\tilde{\varsigma}_{\omega_n}(\upsilon).$$

$$(0.3,0.3,0.3)(\upsilon)+(0.3,0.3,0.3)(\nu)-1\not>(0.5,0.5,0.5)(\upsilon),$$

$$(0.3,0.3,0.3)(\upsilon)+(0.3,0.3,0.3)(\upsilon)-1\leq(0.2,0.2,0.2)(\upsilon),$$

$$(0.3,0.3,0.3)(\upsilon)+(0.3,0.3,0.3)(\upsilon)-1\leq(0.1,0.1,0.1)(\upsilon)$$

Therefore, $[\varepsilon_n]^{\odot}_{\frac{2}{3}}=\tilde{0}$. Subsequently, using Eq. (7) we obtain $\mathrm{int}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}([\varepsilon_n]^{\odot}_{\frac{2}{3}},\frac{2}{3})=int_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\tilde{0},\frac{2}{3})=\tilde{0}$, which implies that

$$\langle(0.3,0.3,0.3),(0.3,0.3,0.3),(0.3,0.3,0.3)\rangle=\varepsilon_n\not\leq\mathrm{int}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}\left([\varepsilon_n]^{\odot}_{\frac{2}{3}},\frac{2}{3}\right)=\tilde{0}.$$

Hence, ε_n is not an r-SVN£O set.

Definition 14. Let $(\tilde{\boldsymbol{\mathcal{F}}}, \tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, £^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$ be an SVNITS, $\alpha_n \in \zeta^{\tilde{\boldsymbol{\mathcal{F}}}}$, $x_{s,t,k} \in \mathrm{P}_{s,t,k}(\tilde{\boldsymbol{\mathcal{F}}})$ and $r \in \zeta_0$. Then,

(a) α_n is an r-single valued neutrosophic $Q_{\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}$-neighborhood of $x_{s,t,k}$ if $x_{s,t,k} q \alpha_n$ with $\tau^{\tilde{\varrho}}(\alpha_n) \geq r, \tau^{\tilde{\sigma}}(\alpha_n) \leq 1-r, \tau^{\tilde{\varsigma}}(\alpha_n) \leq 1-r$;

(b) $x_{s,t,k}$ is an r-single valued neutrosophic θ£-cluster point (r-δ£-*cluster point*) of α_n if for every $\varepsilon_n \in Q_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(x_{s,t,k}, r)$, we have $\alpha_n q \mathrm{int}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\mathrm{CI}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\varepsilon_n, r), r)$;

(c) δ£-closure operator is the mapping of $\mathrm{CI}^{\delta£}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}} : \zeta^{\tilde{\boldsymbol{\mathcal{F}}}} \times \zeta_0 \to \zeta^{\tilde{\boldsymbol{\mathcal{F}}}}$ defined as

$$\mathrm{CI}^{\delta£}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\alpha_n, r) = \cup \left\{ x_{s,t,k} \in \mathrm{P}_{s,t,k}\left(\tilde{\boldsymbol{\mathcal{F}}}\right) : x_{s,t,k} \textit{ is } r\text{–}\delta£\text{–}\textit{cluster point of } \alpha_n \right\}.$$

Definition 15. Let $(\tilde{\boldsymbol{\mathcal{F}}}, \tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, £^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$ be an SVNITS, $\alpha_n \in \zeta^{\tilde{\boldsymbol{\mathcal{F}}}}$, $x_{s,t,k} \in \mathrm{P}_{s,t,k}(\tilde{\boldsymbol{\mathcal{F}}})$ and $r \in \zeta_0$. Then,

(a) α_n is called r-Single valued neutrosophic $\mathfrak{R}^{£}_{\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}$-neighborhood of $x_{s,t,k}$ if $x_{s,t,k} q \alpha_n$ and αn is *r-SVNRIO*. We denote $\mathfrak{R}^{£}_{\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}} = \left\{ \alpha_n \in \zeta^{\tilde{\boldsymbol{\mathcal{F}}}} | x_{s,t,k} q \alpha_n, \alpha_n \textit{ is } \mathrm{r-SVNRIO} \right\}$,

(b) $x_{s,t,k}$ is called r-single valued neutrosophic θ£-cluster point (r-θ£-*cluster point*) of α_n if for any $\varepsilon_n \in Q_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}\left(x_{s,t,k}, r\right)$, we have $\alpha_n q \mathrm{CI}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\varepsilon_n, r)$,

(c) θ£-closure operator is mapping $\mathrm{CI}^{\theta£}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}} : \zeta^{\tilde{\boldsymbol{\mathcal{F}}}} \times \zeta_0 \to \zeta^{\tilde{\boldsymbol{\mathcal{F}}}}$ defined as

$$\mathrm{CI}^{\boldsymbol{\theta£}}_{\tilde{\boldsymbol{\tau}}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\boldsymbol{\alpha_n}, \boldsymbol{r}) = \cup \left\{ \boldsymbol{x_{s,t,k}} \in \boldsymbol{P_{s,t,k}}\left(\tilde{\boldsymbol{\mathcal{F}}}\right) : \boldsymbol{x_{s,t,k}} \textit{ is } \boldsymbol{r\text{–}\theta£\text{–}\textit{cluster point of }} \boldsymbol{\alpha_n} \right\} \tag{9}$$

Example 2. Let $\tilde{\boldsymbol{\mathcal{F}}} = \{a, b, c\}$ be a set. Define $\varepsilon_n, \pi_n \in \zeta^{\tilde{\boldsymbol{\mathcal{F}}}}$ as follows:

$\varepsilon_n = \langle (0.4, 0.4, 0.4), (0.4, 0.4, 0.4), (0.4, 0.4, 0.4) \rangle ; \pi_n = \langle (0.2, 0.2, 0.2), (0.2, 0.2, 0.2), (0.2, 0.2, 0.2) \rangle .$

We define an SVNITS$(\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, £^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$ on $\tilde{\boldsymbol{\mathcal{F}}}$ as follows: for each $\alpha_n \in \zeta^{\tilde{\boldsymbol{\mathcal{F}}}}$,

$$\tilde{\tau}^{\tilde{\varrho}}(\alpha_n) = \begin{cases} 1, & \textit{if } \alpha_n = \tilde{0}, \\ 1, & \textit{if } \alpha_n = \tilde{1}, \\ \dfrac{2}{3}, & \textit{if } \alpha_n = \varepsilon_n, \\ 0, & \textit{otherwise}, \end{cases} \qquad £^{\tilde{\varrho}}(\alpha_n) = \begin{cases} 1, & \textit{if } \alpha_n = \tilde{0}, \\ \dfrac{1}{3}, & \textit{if } \pi_n = \varepsilon_n \\ \dfrac{2}{3}, & \textit{if } 0 < \alpha_n < \pi_n \\ 0, & \textit{otherwise}, \end{cases}$$

$$\tilde{\tau}^{\tilde{\sigma}}(\alpha_n) = \begin{cases} 0, & \textit{if } \alpha_n = \tilde{0}, \\ 0, & \textit{if } \alpha_n = \tilde{1}, \\ \dfrac{1}{3}, & \textit{if } \alpha_n = \varepsilon_n, \\ 1, & \textit{otherwise}, \end{cases} \qquad £^{\tilde{\sigma}}(\alpha_n) = \begin{cases} 0, & \textit{if } \alpha_n = \tilde{0}, \\ \dfrac{2}{3}, & \textit{if } \pi_n = \varepsilon_n \\ \dfrac{1}{3}, & \textit{if } 0 < \alpha_n < \pi_n \\ 1, & \textit{otherwise}, \end{cases}$$

$$\tilde{\tau}^{\tilde{\varsigma}}(\alpha_n)=\begin{cases}0, & if\ \alpha_n=\tilde{0},\\ 0, & if\ \alpha_n=\tilde{1},\\ \dfrac{1}{3}, & if\ \alpha_n=\varepsilon_n,\\ 1, & otherwise,\end{cases}\qquad \pounds^{\tilde{\varsigma}}(\alpha_n)=\begin{cases}0, & if\ \alpha_n=\tilde{0},\\ \dfrac{2}{3}, & if\ \pi_n=\varepsilon_n\\ \dfrac{1}{3}, & if\ 0<\alpha_n<\pi_n\\ 1, & otherwise,\end{cases}$$

From using (9) we get, we obtain

$$\mathrm{CI}^{\theta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n,r)=\begin{cases}\tilde{0}, & if\ \alpha_n=\tilde{0},\\ \varepsilon_n^c, & if\ \tilde{0}\neq\alpha_n\leq\varepsilon_n^c, r\leq\dfrac{1}{3}, 1-r\geq\dfrac{2}{3},\\ 1, & otherwise.\end{cases}$$

Theorem 3. Let $(\tilde{\mathcal{F}},\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}},\pounds^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$ be an SVNITS, $r\in\zeta_0$ and $\alpha_n,\varepsilon_n\in\zeta^{\tilde{\mathcal{F}}}$. Then the following properties are holds:

(a) $\alpha_n\leq\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n,r)$,

(b) If $\alpha_n\leq\varepsilon_n$, then $\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n,r)\leq\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n,r)$,

(c) $\mathrm{int}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\mathrm{CI}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n,r),r)$ is r-*SVNRIO*,

(d) $\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n,r)=\cap\{\varepsilon_n\in\zeta^{\tilde{\mathcal{F}}}|\alpha_n\leq\varepsilon_n,\varepsilon_n$ is r-*SVNRIC*$\}$,

(e) $\mathrm{CI}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n,r)\leq\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n,r)$.

Proof. (a) and (b) are easily proved from (9).

(c) Let $\varepsilon_n\in\zeta^{\tilde{\mathcal{F}}}$ and $\varepsilon_n=\mathrm{int}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\mathrm{CI}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n,r),r)$. Then, we have

$$\begin{aligned}\mathrm{int}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}\left(\mathrm{CI}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n,r),r\right)&=\mathrm{int}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}\left(\mathrm{CI}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}\left(\mathrm{int}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}\left(\mathrm{CI}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n,r),r\right),r\right),r\right)\\&\leq\mathrm{int}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\mathrm{CI}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\mathrm{CI}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n,r),r),r)\\&=\mathrm{int}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}\left(\mathrm{CI}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n,r),r\right)=\varepsilon_n.\end{aligned}$$

Since $\varepsilon_n=\mathrm{int}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n,r)\leq\mathrm{int}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\mathrm{CI}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n,r),r)$, we have $\mathrm{int}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\mathrm{CI}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n,r),r)=\varepsilon_n$.

(d) Based on $\mathcal{P}=\cap\{\varepsilon_n\in\zeta^{\tilde{\mathcal{F}}}|\alpha_n\leq\varepsilon_n,\varepsilon_n$ is *r-SVNRIC*$\}$, let $\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n,r)\not\geq\mathcal{P}$; therefore, $\upsilon\in\tilde{\mathcal{F}}$ and $s\in(0,1]$, $t\in[0,1),k\in[0,1)]$ exist such that

$$\left.\begin{aligned}&\tilde{\varrho}_{\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}}(\alpha_n,r)}(\upsilon)<s<\tilde{\varrho}_{\mathcal{P}}(\upsilon)\\&\tilde{\sigma}_{CI^{\delta\pounds}_{\tilde{\tau}\tilde{\sigma}}(\alpha_n,r)}(\upsilon)\geq t\geq\tilde{\sigma}_{\mathcal{P}}(\upsilon)\\&\tilde{\varsigma}_{CI^{\delta\pounds}_{\tilde{\tau}\tilde{\varsigma}}(\alpha_n,r)}(\upsilon)\geq k\geq\tilde{\varsigma}_{\mathcal{P}}(\upsilon)\end{aligned}\right\}\tag{10}$$

Therefore, $x_{s,t,k}$ is not an *r-δ£-cluster point* of α_n. As such, $\varepsilon_n \in Q_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}\left(x_{s,t,k}, r\right)$ and $\alpha_n \leq [\mathrm{int}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r)]^c$. Consequently, $\alpha_n \leq [\mathrm{int}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\mathrm{CI}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r), r)]^c = \mathrm{Cl}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\mathrm{int}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}([\varepsilon_n]^c, r), r)$.

Since $\mathrm{Cl}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\mathrm{int}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}([\varepsilon_n]^c, r), r)$ is *r-SVNRIC*, we have $\tilde{\varrho}_{\boldsymbol{\mathcal{P}}}(\upsilon) \leq \tilde{\varrho}_{\mathrm{Cl}_{\tilde{\tau}\tilde{\varrho}}(\mathrm{int}^{\odot}_{\tilde{\tau}\tilde{\varrho}}([\varepsilon_n]^c, r), r)}(\upsilon) < s, \tilde{\sigma}_{\boldsymbol{\mathcal{P}}}(\upsilon) \geq \tilde{\sigma}_{Cl_{\tilde{\tau}\tilde{\sigma}}(int^{\odot}_{\tilde{\tau}\tilde{\sigma}}([\varepsilon_n]^c, r), r)}(\upsilon) > t$ and $\tilde{\varsigma}_{\boldsymbol{\mathcal{P}}}(\upsilon) \geq \tilde{\varsigma}_{Cl_{\tilde{\tau}\tilde{\varsigma}}(int^{\odot}_{\tilde{\tau}\tilde{\varsigma}}([\varepsilon_n]^c, r), r)}(\upsilon) > k$. This is a contradiction to Eq. (10). Therefore, $\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r) \geq \boldsymbol{\mathcal{P}}$.

Meanwhile, by setting $\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r) \not\leq \boldsymbol{\mathcal{P}}$, then an *r-δ£-cluster point* of $y_{s_1,t_1,k_1} \in P_{s,t,k}(\tilde{\boldsymbol{\mathcal{F}}})$ of α_n exists such that

$$\left.\begin{aligned} &\tilde{\boldsymbol{\varrho}}_{\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}}(\boldsymbol{\alpha_n}, \boldsymbol{r})}(\boldsymbol{y}) > \boldsymbol{s}_1 > \tilde{\boldsymbol{\varrho}}_{\boldsymbol{\mathcal{P}}}(\boldsymbol{y}) \\ &\tilde{\boldsymbol{\sigma}}_{\boldsymbol{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\sigma}}(\boldsymbol{\alpha_n}, \boldsymbol{r})}(\boldsymbol{y}) \leq \boldsymbol{t}_1 \leq \tilde{\boldsymbol{\sigma}}_{\boldsymbol{\mathcal{P}}}(\boldsymbol{y}) \\ &\tilde{\boldsymbol{\varsigma}}_{\boldsymbol{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varsigma}}(\boldsymbol{\alpha_n}, \boldsymbol{r})}(\boldsymbol{y}) \leq \boldsymbol{k}_1 \leq \tilde{\boldsymbol{\varsigma}}_{\boldsymbol{\mathcal{P}}}(\boldsymbol{y}) \end{aligned}\right\} \tag{11}$$

Owing to $\boldsymbol{\mathcal{P}}$, there exists *r-SVNRIC* $\varepsilon_n \in \zeta^{\tilde{\boldsymbol{\mathcal{F}}}}$ *with* $\alpha_n \leq \varepsilon_n$ such that $\tilde{\varrho}_{\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}}(\alpha_n, r)}(y) > s_1 > \tilde{\varrho}_{\varepsilon_n} \geq \tilde{\varrho}_{\boldsymbol{\mathcal{P}}}(y)$, $\tilde{\sigma}_{CI^{\delta\pounds}_{\tilde{\tau}\tilde{\sigma}}(\alpha_n, r)}(y) \leq t_1 \leq \tilde{\varrho}_{\varepsilon_n} \leq \tilde{\sigma}_{\boldsymbol{\mathcal{P}}}(y)$ and $\tilde{\varsigma}_{CI^{\delta\pounds}_{\tilde{\tau}\tilde{\varsigma}}(\alpha_n, r)}(y) \leq k_1 \leq \tilde{\varrho}_{\varepsilon_n} \leq \tilde{\varsigma}_{\boldsymbol{\mathcal{P}}}(y)$. Therefore, $[\varepsilon_n]^c \in Q_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(y_{s_1,t_1}, k_1)$. So, $\alpha_n \leq \varepsilon_n = [\mathrm{int}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}([\varepsilon_n]^c, r), r)]^c$. Hence, $\alpha_n \overline{q} \mathrm{int}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}([\varepsilon_n]^c, r), r)$.

Additionally, y_{s_1,t_1,k_1} is not an *r-δ£-cluster point* of α_n, that is, $\tilde{\varrho}_{\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}}(\alpha_n, r)}(y) < s_1, \tilde{\sigma}_{CI^{\delta\pounds}_{\tilde{\tau}\tilde{\sigma}}(\alpha_n, r)}(y) \geq t_1, \tilde{\varsigma}_{CI^{\delta\pounds}_{\tilde{\tau}\tilde{\varsigma}}(\alpha_n, r)}(y) \geq k_1$. This is a contradiction to Eq. (11). Therefore, $\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r) \leq \boldsymbol{\mathcal{P}}$,

(e) Suppose that $\mathrm{Cl}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r) \not\leq \mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r)$; therefore, $\upsilon \in \tilde{\boldsymbol{\mathcal{F}}}$ and $[s \in (0, 1], t \in [0, 1), k \in [0, 1)]$ exist such that

$$\left.\begin{aligned} &\tilde{\boldsymbol{\varrho}}_{\mathrm{CI}_{\tilde{\tau}\tilde{\varrho}}(\boldsymbol{\alpha_n}, \boldsymbol{r})}(\boldsymbol{\upsilon}) > \boldsymbol{s} > \tilde{\boldsymbol{\varrho}}_{\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}}(\boldsymbol{\alpha_n}, \boldsymbol{r})}(\boldsymbol{\upsilon}) \\ &\tilde{\boldsymbol{\sigma}}_{\boldsymbol{CI}_{\tilde{\tau}\tilde{\sigma}}(\boldsymbol{\alpha_n}, \boldsymbol{r})}(\boldsymbol{\upsilon}) \leq \boldsymbol{t} \leq \tilde{\boldsymbol{\sigma}}_{\boldsymbol{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\sigma}}(\boldsymbol{\alpha_n}, \boldsymbol{r})}(\boldsymbol{\upsilon}) \\ &\tilde{\boldsymbol{\varsigma}}_{\boldsymbol{CI}_{\tilde{\tau}\tilde{\sigma}}(\boldsymbol{\alpha_n}, \boldsymbol{r})}(\boldsymbol{\upsilon}) \leq \boldsymbol{k} \leq \tilde{\boldsymbol{\varsigma}}_{\boldsymbol{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varsigma}}(\boldsymbol{\alpha_n}, \boldsymbol{r})}(\boldsymbol{\upsilon}), \end{aligned}\right\} \tag{12}$$

Since, $\tilde{\varrho}_{\mathrm{CI}_{\tilde{\tau}\tilde{\varrho}}(\alpha_n, r)}(\upsilon) < s, \tilde{\sigma}_{\mathrm{CI}_{\tilde{\tau}\tilde{\sigma}}(\alpha_n, r)}(\upsilon) \geq t, \tilde{\varsigma}_{\mathrm{CI}_{\tilde{\tau}\tilde{\sigma}}(\alpha_n, r)}(\upsilon) \geq k$, we have $x_{s,t,k}$ not *r-δ£-cluster point* of α_n. Therefore, there exists $\varepsilon_n \in Q_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(x_{s,t,k}, r)$ and $\alpha_n \leq [\mathrm{int}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r), r)]^c$. Hence, $\tilde{\varrho}_{\mathrm{Cl}_{\tilde{\tau}\tilde{\varrho}}(\alpha_n, r)}(\upsilon) \leq \tilde{\varrho}_{[\mathrm{int}_{\tilde{\tau}\tilde{\varrho}}(\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}}(\varepsilon_n, r), r)]^c}(\upsilon) < s$, $\tilde{\sigma}_{Cl_{\tilde{\tau}\tilde{\sigma}}(\alpha_n, r)}(\upsilon) \leq \tilde{\varrho}_{[int_{\tilde{\tau}\tilde{\sigma}}(Cl^{\odot}_{\tilde{\tau}\tilde{\sigma}}(\varepsilon_n, r), r)]^c}(\upsilon) \geq t$ and $\tilde{\varsigma}_{Cl_{\tilde{\tau}\tilde{\varsigma}}(\alpha_n, r)}(\upsilon) \leq \tilde{\varrho}_{[int_{\tilde{\tau}\tilde{\varsigma}}(Cl^{\odot}_{\tilde{\tau}\tilde{\varsigma}}(\varepsilon_n, r), r)]^c}(\upsilon) \geq k$. It is a contradiction for Eq. (12). Thus $\mathrm{Cl}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r) \leq \mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r)$.

Theorem 4. Let $(\tilde{\boldsymbol{\mathcal{F}}}, \tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, \pounds^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$ be an SVNITS, for each $r \in \zeta_0$ and $\alpha_n, \varepsilon_n \in \zeta^{\tilde{\boldsymbol{\mathcal{F}}}}$. Then the following properties hold:

(a) $\alpha_{\mathrm{n}} \leq \mathrm{CI}^{\theta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_{\mathrm{n}}, \mathrm{r})$,

(b) If $\alpha_{\mathrm{n}} \leq \varepsilon_{\mathrm{n}}$, then $\mathrm{CI}^{\theta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_{\mathrm{n}}, \mathrm{r}) \leq \mathrm{CI}^{\theta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_{\mathrm{n}}, \mathrm{r})$,

(c) $\mathrm{CI}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_{\mathrm{n}}, \mathrm{r}) \leq \cup\{\mathrm{x}_{\mathrm{s,t,k}} \in \mathrm{P}_{\mathrm{s,t,k}}(\tilde{\boldsymbol{\mathcal{F}}}) | \mathrm{x}_{\mathrm{s,t,k}}$ is r-δ£-cluster point of $\alpha_{\mathrm{n}}\}$,

(d) $\mathrm{CI}^{\theta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r) = \cap\{\varepsilon_n \in \zeta^{\tilde{\mathcal{F}}} | \alpha_n \leq \mathrm{int}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r),\ \tau^{\tilde{\varrho}}([\varepsilon_n]^c) \geq r,\ \tau^{\tilde{\sigma}}([\varepsilon_n]^c) \leq 1-r,\ \tau^{\tilde{\varsigma}}([\varepsilon_n]^c) \leq 1-r\}$,

(e) $\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r) = \cap\{\varepsilon_n \in \zeta^{\tilde{\mathcal{F}}} | \alpha_n \leq \varepsilon_n, \varepsilon_n \text{ is r-}\delta\pounds\text{-cluster point of } \alpha_n\}$

(f) $x_{s,t,k}$ is r-$\theta\pounds$-cluster point of α_n iff $x_{s,t,k} \in \mathrm{CI}^{\theta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r)$,

(g) $x_{s,t,k}$ is r-$\delta\pounds$-cluster point of α_n iff $x_{s,t,k} \in \mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r)$,

(h) If $\alpha_n = \mathrm{Cl}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\mathrm{intl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r), r)$, then $\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r) = \alpha_n$,

(i) $\alpha_n \leq \mathrm{CI}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r) \leq \mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r) \leq \mathrm{CI}^{\theta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r)$,

(j) $\mathrm{W}(\alpha_n \vee \varepsilon_n, r) = \mathrm{W}(\alpha_n, r) \vee \mathrm{W}(\varepsilon_n, r)$ for each $\mathrm{W} = \left\{ CI^{\theta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, CI^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}} \right\}$,

(k) $\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r), r) = \mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r)$.

Proof. (a) and (b) are easily proved from Definition 14.

(c) Set $\mathcal{P} = \cup\{x_{s,t,k} \in P_{s,t,k}(\tilde{\mathcal{F}}) | x_{s,t,k} \text{ as an } r\text{-}\delta\pounds\text{-cluster point of } \alpha_n\}$. Suppose that $\mathrm{CI}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r) \not\leq \mathcal{P}$. Then there exists $\upsilon \in \tilde{\mathcal{F}}$, and $[s \in (0,1],\ t \in [0,1),\ k \in [0,1)]$ such that

$$\left.\begin{array}{l} \tilde{\varrho}_{\mathrm{CI}_{\tilde{\tau}\tilde{\varrho}}(\alpha_n, r)}(\upsilon) > s > \tilde{\varrho}_{\mathcal{P}}(\upsilon) \\ \tilde{\sigma}_{CI_{\tilde{\tau}\tilde{\sigma}}(\alpha_n, r)}(\upsilon) \leq t \leq \tilde{\sigma}_{\mathcal{P}}(\upsilon) \\ \tilde{\varsigma}_{CI_{\tilde{\tau}\tilde{\varsigma}}(\alpha_n, r)}(\upsilon) \leq k \leq \tilde{\varsigma}_{\mathcal{P}}(\upsilon) \end{array}\right\} \tag{13}$$

Consequently, $x_{s,t,k}$ is not r-$\delta\pounds$-cluster point of α_n. So, there exists $\varepsilon_n \in Q_{\tau\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(x_{s,t,k}, r)$ and $\alpha_n \leq \left[\mathrm{int}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}\left(\mathrm{CI}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r), r\right)\right]^c \leq [\varepsilon_n]^c$

Based on Eq. (4), $\tilde{\varrho}_{\mathrm{CI}_{\tilde{\tau}\tilde{\varrho}}(\alpha_n, r)}(\upsilon) \leq \tilde{\varrho}_{[\varepsilon_n]^c}(\upsilon) < s, \tilde{\sigma}_{CI_{\tilde{\tau}\tilde{\sigma}}(\alpha_n, r)}(\upsilon) \geq \tilde{\sigma}_{[\varepsilon_n]^c}(\upsilon) \geq t$ and $\tilde{\varsigma}_{CI_{\tilde{\tau}\tilde{\varsigma}}(\alpha_n, r)}(\upsilon) \geq \tilde{\varsigma}_{[\varepsilon_n]^c}(\upsilon) \geq k$.

It is a contradiction for Eq. (13). Thus $\mathrm{CI}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r) \leq \mathcal{P}$.

(d) $\gamma = \cap\{\varepsilon_n \in \zeta^{\tilde{\mathcal{F}}} | \alpha_n \leq \mathrm{int}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r),\ \tau^{\tilde{\varrho}}([\varepsilon_n]^c) \geq r,\ \tau^{\tilde{\sigma}}([\varepsilon_n]^c) \leq 1-r,\ \tau^{\tilde{\varsigma}}([\varepsilon_n]^c) \leq 1-r\}$.

Suppose that $\mathrm{CI}^{\theta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r) \not\geq \gamma$, then there exists $\upsilon \in \tilde{\mathcal{F}}$ and $[s \in (0,1],\ t \in [0,1),\ k \in [0,1)]$ such that

$$\left.\begin{array}{l} \tilde{\varrho}_{\mathrm{CI}^{\theta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r)}(\upsilon) < s \leq \tilde{\varrho}_{\gamma}(\upsilon) \\ \tilde{\sigma}_{CI^{\theta\pounds}_{\tilde{\tau}\tilde{\sigma}}(\alpha_n, r)}(\upsilon) > t \geq \tilde{\sigma}_{\gamma}(\upsilon) \\ \tilde{\varsigma}_{CI^{\theta\pounds}_{\tilde{\tau}\tilde{\varsigma}}(\alpha_n, r)}(\upsilon) > k \geq \tilde{\varsigma}_{\gamma}(\upsilon) \end{array}\right\} \tag{14}$$

Consequently, $x_{s,t,k}$ is not r-$\theta\pounds$-cluster point of α_n. So, there exists $\varepsilon_n \in Q_{\tau\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(x_{s,t,k}, r)$, $\alpha_n \leq [(\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r), r)]^c$. Thus, $\alpha_n \leq [(\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r), r)]^c = (\mathrm{int}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}([\varepsilon_n]^c, r), r)$, $\tau^{\tilde{\varrho}}(\varepsilon_n) \geq r,\ \tau^{\tilde{\sigma}}(\varepsilon_n) \leq 1-r,\ \tau^{\tilde{\varsigma}}(\varepsilon_n) \leq 1-r\}$. Hence, $\tilde{\varrho}_{\gamma}(\upsilon) \leq \tilde{\varrho}_{[\varepsilon_n]^c}(\upsilon) < s, \tilde{\sigma}_{\gamma}(\upsilon) \leq \tilde{\sigma}_{[\varepsilon_n]^c}(\upsilon) < t, \tilde{\varsigma}_{\gamma}(\upsilon) \leq \tilde{\varsigma}_{[\varepsilon_n]^c}(\upsilon) < k$.

It is a contradiction to Eq. (14). Thus $\mathrm{CI}^{\theta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r) \geq \gamma$.

Suppose that $\mathrm{CI}^{\theta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r) \not\leq \gamma$, then there exists r-$\theta\pounds$-cluster point of α_n. $y_{s_1,t_1,k_1} \in P_{s,t,k}(\tilde{\mathcal{F}})$ of α_n, such that

$$\left.\begin{array}{l} \tilde{\varrho}_{\mathrm{CI}^{\theta\pounds}_{\tilde{\tau}\tilde{\varrho}}(\alpha_n,r)}(y) > s_1 > \tilde{\varrho}_{\gamma}(y) \\ \tilde{\sigma}_{CI^{\theta\pounds}_{\tilde{\tau}\tilde{\sigma}}(\alpha_n,r)}(y) < t_1 \leq \tilde{\sigma}_{\gamma}(y) \\ \tilde{\varsigma}_{CI^{\theta\pounds}_{\tilde{\tau}\tilde{\varsigma}}(\alpha_n,r)}(y) < k_1 \leq \tilde{\varsigma}_{\gamma}(y) \end{array}\right\} \tag{15}$$

By the definition of γ, there exists $\varepsilon_n \in \zeta^{\tilde{\mathcal{F}}}$ with $\tau^{\tilde{\varrho}}(\varepsilon_n) \geq r$, $\tau^{\tilde{\sigma}}(\varepsilon_n) \leq 1-r, \tau^{\tilde{\varsigma}}(\varepsilon_n) \leq 1-r$ and $\alpha_n \leq \mathrm{int}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r)$, s.t $\tilde{\varrho}_{\mathrm{CI}^{\theta\pounds}_{\tilde{\tau}\tilde{\varrho}}(\alpha_n,r)}(y) > s_1 > \tilde{\varrho}_{\varepsilon_n}(y) \geq \tilde{\varrho}_{\gamma}(y), \tilde{\sigma}_{CI^{\theta\pounds}_{\tilde{\tau}\tilde{\sigma}}(\alpha_n,r)}(y) < t_1 \leq \tilde{\sigma}_{\varepsilon_n}(y) \leq \tilde{\sigma}_{\gamma}(y)$ and $\tilde{\varsigma}_{CI^{\theta\pounds}_{\tilde{\tau}\tilde{\varsigma}}(\alpha_n,r)}(y) < k_1 \leq \tilde{\varsigma}_{\varepsilon_n}(y) \leq \tilde{\varsigma}_{\gamma}(y)$. Additionally, $[\varepsilon_n]^c \in Q_{\tau\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(y_{s_1,t_1,k_1}, r)$. $\alpha_n \leq \mathrm{int}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r) = [\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}([\varepsilon_n]^c, r)]^c$, implies $\alpha_n \overline{q} \mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}([\varepsilon_n]^c, r)$. Hence y_{s_1,t_1,k_1} is not an r-$\theta\pounds$-cluster point of α_n. It is a contradiction for Eq. (15). Thus $\mathrm{CI}^{\theta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r) \leq \gamma$.

(e) Similar results are shown in (c) and (d).

(f) ($\Rightarrow$), clear.

($\Leftarrow$) Suppose that $x_{s,t,k}$ is not an r-$\theta\pounds$-cluster point of α_n. There exists $\varepsilon_n \in Q_{\tau\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(x_{s,t,k}, r)$ such that $\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r) \leq \alpha_n$. Thus $\alpha_n \leq [\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r)]^c = \mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}([\varepsilon_n]^c, r)$. By (d), $\tilde{\varrho}_{\mathrm{CI}^{\theta\pounds}_{\tilde{\tau}\tilde{\varrho}}(\alpha_n,r)}(\upsilon) \leq \tilde{\varrho}_{[\varepsilon_n]^c}(\upsilon) < s, \tilde{\sigma}_{CI^{\theta\pounds}_{\tilde{\tau}\tilde{\sigma}}(\alpha_n,r)}(\upsilon) \geq \tilde{\sigma}_{[\varepsilon_n]^c}(\upsilon) > t$ and $\tilde{\varsigma}_{CI^{\theta\pounds}_{\tilde{\tau}\tilde{\varsigma}}(\alpha_n,r)}(\upsilon) \geq \tilde{\varsigma}_{[\varepsilon_n]^c}(\upsilon) > t$. Hence $x_{s,t,k} \notin \mathrm{CI}^{\theta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r)$.

(g) is similarly proved as in (f).

(h) The validity of this axiom is obvious from Theorem 3 (4).

(i) Based on Theorem 3(e), we show that $\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r) \leq \mathrm{CI}^{\theta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r)$. Suppose that $\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r) \not\leq \mathrm{CI}^{\theta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r)$, then there exists $\upsilon \in \zeta$ and $[s \in (0,1],\ t \in [0,1),\ k \in [0,1)]$ such that

$$\left.\begin{array}{l} \tilde{\varrho}_{\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}}(\alpha_n,r)}(\upsilon) > s > \tilde{\varrho}_{\mathrm{CI}^{\theta\pounds}_{\tilde{\tau}\tilde{\varrho}}(\alpha_n,r)}(\upsilon) \\ \tilde{\sigma}_{CI^{\delta\pounds}_{\tilde{\tau}\tilde{\sigma}}(\alpha_n,r)}(\upsilon) > t \geq \tilde{\sigma}_{CI^{\theta\pounds}_{\tilde{\tau}\tilde{\sigma}}(\alpha_n,r)}(\upsilon) \\ \tilde{\varsigma}_{CI^{\delta\pounds}_{\tilde{\varsigma}}(\alpha_n,r)}(\upsilon) > k \geq \tilde{\varsigma}_{CI^{\theta\pounds}_{\tilde{\tau}\tilde{\varsigma}}(\alpha_n,r)}(\upsilon) \end{array}\right\} \tag{16}$$

Since $\tilde{\varrho}_{\mathrm{CI}^{\theta\pounds}_{\tilde{\tau}\tilde{\varrho}}(\alpha_n,r)}(\upsilon) < s, \tilde{\sigma}_{\mathrm{CI}^{\theta\pounds}_{\tilde{\tau}\tilde{\sigma}}(\alpha_n,r)}(\upsilon) \leq t$ and $\tilde{\varsigma}_{\mathrm{CI}^{\theta\pounds}_{\tilde{\tau}\tilde{\varsigma}}(\alpha_n,r)}(\upsilon) \leq k$, then we have $x_{s,t,k}$ is not r-$\theta\pounds$-cluster point of α_n So, there exists $\varepsilon_n \in Q_{\tau\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(y_{s_1,t_1,k_1}, r)$, $\alpha_n \leq [\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r)]^c$, implies $A\overline{q}\mathrm{int}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r), r)$. Hence, $x_{s,t,k}$ is not r-$\delta\pounds$-cluster point of α_n, by (7), we can get than, $\tilde{\varrho}_{\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}}(\alpha_n,r)}(\upsilon) < s, \sigma_{\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\sigma}}(\alpha_n,r)}(\upsilon) \geq t, \tilde{\varsigma}_{\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varsigma}}(\alpha_n,r)}(\upsilon) \geq k$. It is a contradiction for Eq. (16). Thus, $\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r) \leq \mathrm{CI}^{\theta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r)$.

(j) Let $\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n,r)\vee\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n,r)\not\geq\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n\vee\varepsilon_n,r)$. Then there exists $\upsilon\in\tilde{\mathcal{F}}$ such that

$$\left.\begin{array}{l}\tilde{\varrho}_{\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}}(\boldsymbol{\varepsilon_n},\boldsymbol{r})}(\boldsymbol{\upsilon})\vee\tilde{\varrho}_{\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}}(\boldsymbol{\alpha_n},\boldsymbol{r})}(\boldsymbol{\upsilon})<\boldsymbol{s}<\tilde{\varrho}_{\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}}(\boldsymbol{\alpha_n}\vee\boldsymbol{\varepsilon_n},\boldsymbol{r})}(\boldsymbol{\upsilon})\\ \tilde{\sigma}_{\boldsymbol{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\sigma}}(\boldsymbol{\varepsilon_n},\boldsymbol{r})}(\boldsymbol{\upsilon})\vee\tilde{\sigma}_{\boldsymbol{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\sigma}}(\boldsymbol{\alpha_n},\boldsymbol{r})}(\boldsymbol{\upsilon})>\boldsymbol{t}>\tilde{\sigma}_{\boldsymbol{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}}(\boldsymbol{\alpha_n}\vee\boldsymbol{\varepsilon_n},\boldsymbol{r})}(\boldsymbol{\upsilon})\\ \tilde{\varsigma}_{\boldsymbol{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varsigma}}(\boldsymbol{\varepsilon_n},\boldsymbol{r})}(\boldsymbol{\upsilon})\vee\tilde{\varsigma}_{\boldsymbol{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\sigma}}(\boldsymbol{\alpha_n},\boldsymbol{r})}(\boldsymbol{\upsilon})>\boldsymbol{t}>\tilde{\varsigma}_{\boldsymbol{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varsigma}}(\boldsymbol{\alpha_n}\vee\boldsymbol{\varepsilon_n},\boldsymbol{r})}(\boldsymbol{\upsilon})\end{array}\right\}\tag{17}$$

Since $\tilde{\varrho}_{\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}}(\alpha_n,r)}(\upsilon)<s,\tilde{\sigma}_{CI^{\delta\pounds}_{\tilde{\tau}\tilde{\sigma}}(\alpha_n,r)}(\upsilon)>t$, $\tilde{\varsigma}_{CI^{\delta\pounds}_{\tilde{\tau}\tilde{\varsigma}}(\alpha_n,r)}(\upsilon)>k$ and $\tilde{\varrho}_{\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}}(\varepsilon_n,r)}(\upsilon)<s,\tilde{\sigma}_{CI^{\delta\pounds}_{\tilde{\tau}\tilde{\sigma}}(\varepsilon_n,r)}(\upsilon)>t$, $\tilde{\varsigma}_{CI^{\delta\pounds}_{\tilde{\tau}\tilde{\varsigma}}(\varepsilon_n,r)}(\upsilon)>k$. We obtain, $x_{s,t,k}$ is not r-$\delta\pounds$-cluster point of α_n and ε_n So, there exists $[\alpha_n]_1,[\varepsilon_n]_1\in Q_{\tau\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(x_{s,t,k},r)$, and $\alpha_n\leq[\mathrm{int}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}([\alpha_n]_1,r),r)]^c,\varepsilon_n\leq[int_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(Cl^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}([\varepsilon_n]_1,r),r)]^c$. Thus, $[\alpha_n]_1\wedge[\varepsilon_n]_1\in Q_{\tau\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}\left(x_{s,t,k},r\right)$.

Using Eqs. (4) and (5) we obtain,

$$\begin{aligned}\alpha_n\vee\varepsilon_n&\leq\left[\mathrm{int}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}\left(\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}\left([\alpha_n]_1,r\right),r\right)\wedge int_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}\left(Cl^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}\left([\varepsilon_n]_1,r\right),r\right)\right]^c\\&\leq[\mathrm{int}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}([\alpha_n]_1,r)\wedge\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}([\varepsilon_n]_1,r),r)]^c\\&\leq\left[\mathrm{int}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}\left(\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}\left([\alpha_n]_1\wedge[\varepsilon_n]_1,r\right),r\right)\right]^c.\end{aligned}$$

Therefore, $\alpha_n\vee\varepsilon_n\overline{q}\ \mathrm{int}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}([\alpha_n]_1\wedge[\varepsilon_n]_1,r),r)$. Hence, $x_{s,t,k}$ is not r-$\delta\pounds$-cluster point of $\alpha_n\vee\varepsilon_n$, by (g), $\tilde{\varrho}_{\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}}(\alpha_n\vee\varepsilon_n,r)}(\upsilon)<s,\tilde{\sigma}_{Cl^{\odot}_{\tilde{\tau}\tilde{\sigma}}\alpha_n\vee\varepsilon_n(,r)}(\upsilon)>t$ and $\tilde{\varsigma}_{Cl^{\odot}_{\tilde{\tau}\tilde{\varsigma}}(\alpha_n\vee\varepsilon_n,r)}(\upsilon)>k$. It is a contradiction for Eq. (17), and hence, $\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n\vee\varepsilon_n,r)\leq\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n,r)\vee\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n,r)$.

Meanwhile, $\alpha_n\vee\varepsilon_n\geq\alpha_n$ and $\alpha_n\vee\varepsilon_n\geq\varepsilon_n$. Hence $\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n\vee\varepsilon_n,r)\geq\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n,r)\vee\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n,r)$. Therefore, $\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n,r)\vee\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n,r)=\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n\vee\varepsilon_n,r)$.

(k) Since $\alpha_n\leq\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n,r)$, we have $\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n,r)\leq\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n,r),r)$. On the other hand, suppose that $\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n,r)\not\geq\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n,r),r)$. Then there exists $\upsilon\in\tilde{\mathcal{F}}$ and $[s\in(0,1],t\in[0,1),k\in[0,1)]$ such that

$$\left.\begin{array}{l}\tilde{\varrho}_{\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}}(\boldsymbol{\alpha_n},\boldsymbol{r})}(\boldsymbol{\upsilon})<\boldsymbol{s}<\tilde{\varrho}_{\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}}\left(\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}}(\boldsymbol{\alpha_n},\boldsymbol{r}),\boldsymbol{r}\right)}(\boldsymbol{\upsilon})\\ \tilde{\sigma}_{\boldsymbol{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\sigma}}(\boldsymbol{\alpha_n},\boldsymbol{r})}(\boldsymbol{\upsilon})>\boldsymbol{t}\geq\tilde{\sigma}_{\boldsymbol{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}}\left(\boldsymbol{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\sigma}}(\boldsymbol{\alpha_n},\boldsymbol{r}),\boldsymbol{r}\right)}(\boldsymbol{\upsilon})\\ \tilde{\varsigma}_{\boldsymbol{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varsigma}}(\boldsymbol{\alpha_n},\boldsymbol{r})}(\boldsymbol{\upsilon})>\boldsymbol{k}\geq\tilde{\varsigma}_{\boldsymbol{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}}(\boldsymbol{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varsigma}}(\boldsymbol{\alpha_n},\boldsymbol{r}),\boldsymbol{r})}(\boldsymbol{\upsilon})\end{array}\right\}\tag{18}$$

Since $\tilde{\varrho}_{\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}}(\alpha_n,r)}(\upsilon)<s,\tilde{\sigma}_{\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\sigma}}(\alpha_n,r)}(\upsilon)>t,\tilde{\varsigma}_{\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varsigma}}(\alpha_n,r)}(\upsilon)>k$, we have $x_{s,t,k}$ is not an r-$\delta\pounds$-cluster point of α_n. So, there exists $\varepsilon_n\in Q_{\tau\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(x_{s,t,k},r)$ such that $\alpha_n\leq[\mathrm{int}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_{n,r}),r)]^c=\mathrm{Cl}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\mathrm{int}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n,r,r)$, since, $\mathrm{Cl}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\mathrm{int}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n,r,r)$ is r−SVNRIC. Then by Theorem 3(d), $\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n,r)\leq\mathrm{Cl}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\mathrm{int}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n,r,r)$.

Similarly, $\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n,r),r)\leq\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\mathrm{Cl}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\mathrm{int}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n,r),r),r)=\mathrm{Cl}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\mathrm{int}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n,r),r)$. Hence,

$\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n,r),r) \leq \mathrm{Cl}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\mathrm{int}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n,r),r) < x_{s,t,k}$. It is a contradiction for Eq. (18).

Theorem 5. Let $(\tilde{\mathcal{F}}, \tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, \pounds^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$ be an SVNITS, for $r \in \zeta_0$ and $\alpha_n, \varepsilon_n \in \zeta^{\tilde{\mathcal{F}}}$. Then the following properties hold:

(a) α_n is r-*SVNPIC* iff $\mathrm{CI}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n,r) = \mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n,r)$,

(b) α_n is r-*SVNSIC* iff $\mathrm{CI}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n,r) = \mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n,r)$,

(c) α_n is r-*SVNαIO* iff $\mathrm{CI}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n,r) = \mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n,r) = \mathrm{CI}^{\theta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n,r)$.

Proof. (a) Let α_n be an *r-SVNPIC*. Then $\alpha_n \leq \mathrm{CI}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n,r)$, and by Theorem 3 (3) and (4), we have

$$\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n,r) \leq \mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}\left(\mathrm{CI}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}\left(\mathrm{int}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n,r),r\right),r\right) = \mathrm{Cl}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}\left(\mathrm{int}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n,r),r\right) \leq \mathrm{Cl}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n,r)$$
$$\leq \mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n,r).$$

Conversely, suppose that there exist $\upsilon \in \tilde{\mathcal{F}}$ and $[s \in (0,1],\ t \in [0,1),\ k \in [0,1)]$ such that $\tilde{\varrho}_{\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}}(\alpha_n,r)}(\upsilon) > s > \tilde{\varrho}_{\mathrm{Cl}_{\tilde{\tau}\tilde{\varrho}}(\alpha_n,r)}(\upsilon), \tilde{\sigma}_{CI^{\delta\pounds}_{\tilde{\tau}\tilde{\sigma}}(\alpha_n,r)}(\upsilon) < t \leq \tilde{\sigma}_{Cl_{\tilde{\tau}\tilde{\sigma}}(\alpha_n,r)}(\upsilon)$ and $\tilde{\varsigma}_{CI^{\delta\pounds}_{\tilde{\tau}\tilde{\varsigma}}(\alpha_n,r)}(\upsilon) < k \leq \tilde{\varsigma}_{Cl_{\tilde{\tau}\tilde{\varsigma}}(\alpha_n,r)}(\upsilon)$. Then $x_{s,t,k}$ is not r-δ-cluster point of α_n. So, there exists $\varepsilon_n \in Q_{\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(x_{s,t,k},r)$, with $\alpha_n \leq [\varepsilon_n]^c$ Since $x_{s,t,k}$ is r-$\delta\pounds$-cluster point of α_n, for $\varepsilon_n \in Q_{\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(x_{s,t,k},r)$, we have $\mathrm{int}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n,r),r) q \alpha_n$. Since,

$$\mathrm{int}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}\left(\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n,r),r\right) \leq \mathrm{int}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}\left(\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}([\alpha_n]^c,r),r\right),$$

we obtain, $\alpha_n \geq [\mathrm{int}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n,r),r)]^c \geq [\mathrm{int}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}([\alpha_n]^c,r),r)]^c = \mathrm{Cl}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\mathrm{int}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}([\alpha_n],r),r)$.

Hence, α_n is not *r-SVNIC* set.

(b) Let α_n is an *r-SVNSIC* set. Then, $\alpha_n \leq \mathrm{int}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\mathrm{Cl}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}([\alpha_n]^c,r),r)$ and $\tau^{\tilde{\varrho}}([\mathrm{Cl}_{\tilde{\tau}\tilde{\varrho}}([\alpha_n,r)]^c \geq r, \tau^{\tilde{\sigma}}([\mathrm{Cl}_{\tilde{\tau}\tilde{\sigma}}([\alpha_n,r)]^c \leq r, \tau^{\tilde{\varsigma}}([\mathrm{Cl}_{\tilde{\tau}\tilde{\varsigma}}([\alpha_n,r)]^c \leq r$. By Theorem 4(d), we have $\mathrm{CI}^{\theta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n,r) \leq \mathrm{CI}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n,r)$,

Conversely, suppose that there exist $\alpha_n \in \zeta^{\tilde{\mathcal{F}}}, r \in \zeta_0, \upsilon \in \tilde{\mathcal{F}}$ and $[s \in (0,1], t \in [0,1),\ k \in [0,1)]$ such that $\tilde{\varrho}_{\mathrm{CI}^{\theta\pounds}_{\tilde{\tau}\tilde{\varrho}}(\alpha_n,r)}(\upsilon) > t > \tilde{\varrho}_{\mathrm{Cl}_{\tilde{\tau}\tilde{\varrho}}(\alpha_n,r)}(\upsilon), \tilde{\sigma}_{CI^{\theta\pounds}_{\tilde{\tau}\tilde{\sigma}}(\alpha_n,r)}(\upsilon) < t \leq \tilde{\sigma}_{Cl_{\tilde{\tau}\tilde{\sigma}}(\alpha_n,r)}(\upsilon)$ and $\tilde{\varsigma}_{CI^{\theta\pounds}_{\tilde{\tau}\tilde{\varsigma}}(\alpha_n,r)}(\upsilon) < t \leq \tilde{\varsigma}_{Cl_{\tilde{\tau}\tilde{\varsigma}}(\alpha_n,r)}(\upsilon)$. Then $[\mathrm{Cl}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n,r]^c) = \mathrm{int}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}([\alpha_n]^c,r) \in Q_{\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(x_{s,t,k},r)$ Since $x_{s,t,k}$ is r-$\theta\pounds$-cluster point of α_n, we have $\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\mathrm{int}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}([\alpha_n]^c,r),r) q \alpha_n$. It implies $\alpha_n \nleq [\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\mathrm{int}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}([\alpha_n]^c,r),r)]^c = \mathrm{int}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\mathrm{Cl}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n,r),r)$. Thus, α_n is not an *r-SVNSIC*.

(c) Similar results are shown in (a) and (b).

4 *r*-δ£-Closed and *r*-θ£-Closed

In this section, we firstly introduce and analyze the *r*-δ£-closed and *r*-θ£-closed of an SVNITS($\tilde{\mathcal{F}}, \tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, £^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}$). Subsequently, we define and analyze the single-valued neutrosophic £-regular and the single-valued neutrosophic almost £-regular of $\tilde{\mathcal{F}}$. The findings have resulted in many theorems.

Definition 16. Let $(\tilde{\mathcal{F}}, \tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, £^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$ be an SVNITS. For $r \in \zeta_0$ and $\alpha_n, \varepsilon_n \in \zeta^{\tilde{\mathcal{F}}}$. Therefore,

(a) α_n is said to be *r*-δ£-closed ($[\alpha_n]_{\delta£}$) [resp. *r*-θ£-closed $[\alpha_n]_{\theta£}$] iff $\mathrm{CI}^{\delta£}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r) = \alpha_n$ (resp. $\mathrm{CI}^{\theta£}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r) = \alpha_n$). We define

$$\Delta^{\delta£}_{\tilde{\boldsymbol{\tau}}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\boldsymbol{\alpha_n}, \boldsymbol{r}) = \cap\{\boldsymbol{\varepsilon_n} | \boldsymbol{\alpha_n} \leq \boldsymbol{\varepsilon_n}, \boldsymbol{\varepsilon_n} = \mathrm{CI}^{\delta£}_{\tilde{\boldsymbol{\tau}}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\boldsymbol{\varepsilon_n}, \boldsymbol{r})\} \tag{19}$$

$$\Theta^{\boldsymbol{\theta£}}_{\tilde{\boldsymbol{\tau}}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\boldsymbol{\alpha_n}, \boldsymbol{r}) = \cap\{\boldsymbol{\varepsilon_n} | \boldsymbol{\alpha_n} \leq \boldsymbol{\varepsilon_n}, \boldsymbol{\varepsilon_n} = \mathrm{CI}^{\boldsymbol{\theta£}}_{\tilde{\boldsymbol{\tau}}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\boldsymbol{\varepsilon_n}, \boldsymbol{r})\} \tag{20}$$

(b) The complement of *r*-δ£-closed (resp. *r*-θ£-closed) set is called *r*-δ£-open (resp. *r*-θ£-open).

Theorem 6. Let $(\tilde{\mathcal{F}}, \tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, £^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$ be an SVNITS. For $r \in \zeta_0$ and $\alpha_n \in \zeta^{\tilde{\mathcal{F}}}$. Then the following properties are holds:

(c). $\Delta^{\delta£}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r) = \mathrm{CI}^{\delta£}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r)$,

(d). $\Delta^{\delta£}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r)$ is r-δ£-closed,

(e). $\Theta^{\theta£}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r) = \mathrm{CI}^{\theta£}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\Theta^{\delta£}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r), r)$,

(f). $\Theta^{\theta£}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r)$ is r-θ£-closed,

(g). $\mathrm{CI}^{\theta£}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r) \leq \Theta^{\theta£}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r)$.

Proof. (1) Based on Theorem 4(i,j), $\alpha_n \leq \mathrm{CI}^{\delta£}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r) = \mathrm{CI}^{\delta£}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\mathrm{CI}^{\delta£}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r), r)$, which implies $\Delta^{\delta£}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r) \leq \mathrm{CI}^{\delta£}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r)$. Suppose that $\Delta^{\delta£}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r) \not\geq \mathrm{CI}^{\delta£}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r)$. Then there exist $\upsilon \in \tilde{\mathcal{F}}$ and $[s \in (0,1], t \in [0,1),\ k \in [0,1)]$ such that $\tilde{\varrho}_{\Delta^{\delta£}_{\tilde{\tau}\tilde{\varrho}}(\alpha_n, r)}(\upsilon) < s < \tilde{\varrho}_{\mathrm{CI}^{\delta£}_{\tilde{\tau}\tilde{\varrho}}(\alpha_n, r)}(\upsilon), \tilde{\sigma}_{\Delta^{\delta£}_{\tilde{\tau}\tilde{\sigma}}(\alpha_n, r)}(\upsilon) > t > \tilde{\sigma}_{CI^{\delta£}_{\tilde{\tau}\tilde{\sigma}}(\alpha_n, r)}(\upsilon)$ and $\tilde{\varsigma}_{\Delta^{\delta£}_{\tilde{\tau}\tilde{\varsigma}}(\alpha_n, r)}(\upsilon) > k > \tilde{\varsigma}_{CI^{\delta£}_{\tilde{\tau}\tilde{\varsigma}}(\alpha_n, r)}(\upsilon)$. Based on Eq. (19), there exist $\varepsilon_n \in \zeta^{\tilde{\mathcal{F}}}$ and $\alpha_n \leq \varepsilon_n = \mathrm{CI}^{\delta£}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r)$ such that $\tilde{\varrho}_{\Delta^{\delta£}_{\tilde{\tau}\tilde{\varrho}}(\alpha_n, r)}(\upsilon) \leq \tilde{\varrho}_{\varepsilon_n}(\upsilon) < s < \tilde{\varrho}_{\mathrm{CI}^{\delta£}_{\tilde{\tau}\tilde{\varrho}}(\alpha_n, r)}(\upsilon), \tilde{\sigma}_{\Delta^{\delta£}_{\tilde{\tau}\tilde{\sigma}}(\alpha_n, r)}(\upsilon) \geq \tilde{\varrho}_{\varepsilon_n}(\upsilon) > t > \tilde{\sigma}_{CI^{\delta£}_{\tilde{\tau}\tilde{\sigma}}(\alpha_n, r)}(\upsilon)$ and $\tilde{\varsigma}_{\Delta^{\delta£}_{\tilde{\tau}\tilde{\varsigma}}(\alpha_n, r)}(\upsilon) \geq \tilde{\varrho}_{\varepsilon_n}(\upsilon) > k > \tilde{\varsigma}_{CI^{\delta£}_{\tilde{\tau}\tilde{\varsigma}}(\alpha_n, r)}(\upsilon)$.

Meanwhile, $\mathrm{CI}^{\delta£}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r) \leq \mathrm{CI}^{\delta£}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r) = \varepsilon_n$, which is a contradiction. Hence, $\Delta^{\delta£}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r) \geq \mathrm{CI}^{\delta£}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r)$.

(b) is similar to Theorem 4 (k).

(c) Let $\alpha_n \leq [\varepsilon_n]_i = \mathrm{CI}^{\theta£}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}([\varepsilon_n]_i, r)$. Therefore, $\underset{i\in\Gamma}{\wedge}[\varepsilon_n]_i \leq \mathrm{CI}^{\theta£}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\underset{i\in\Gamma}{\wedge}[\varepsilon_n]_i, r) \leq \mathrm{CI}^{\theta£}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}([\varepsilon_n]_i, r) = [\varepsilon_n]_i$. Consequently, $\wedge_{i\in\Gamma}[\varepsilon_n]_i \leq C_{\theta\mathcal{J}\tau}(\wedge_{i\in\Gamma}[\varepsilon_n]_i, r)$. Hence, $\Theta^{\theta£}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r) = \mathrm{CI}^{\theta£}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\Theta^{\theta£}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r), r)$.

(d) It is directly obtained from (c).

(e) Since $\alpha_n \leq \Theta^{\theta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r)$, by (c) and Eq. (19), $\mathrm{CI}^{\theta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r) \leq \mathrm{CI}^{\theta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\Theta^{\theta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r), r) = \Theta^{\theta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r)$.

Definition 17. Let $(\tilde{\mathcal{F}}, \tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, \pounds^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$ be an SVNITS, $\alpha_n, \varepsilon_n \in \zeta^{\tilde{\mathcal{F}}}$, and $r \in \zeta_0$. Then $\widetilde{\tilde{\mathcal{F}}}$ is called,

(a) single valued neutrosophic £-regular (SVN£-regular) if for any $\alpha_n \in Q_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(x_{s,t,k}, r)$, there exists $\varepsilon_n \in Q_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(x_{s,t,k}, r)$ such that $\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r) \leq \alpha_n$,

(b) single valued neutrosophic almost £-regular ($SVNA$£-regular), if for any $\alpha_n \in \mathfrak{R}^{\pounds}_{\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(x_{s,t,k}, r)$, then there exists $\varepsilon_n \in \mathfrak{R}^{\pounds}_{\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(x_{s,t,k}, r)$ such that $\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r) \leq \alpha_n$.

Theorem 7. Let $(\tilde{\mathcal{F}}, \tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, \pounds^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$ be an SVNITS, $\alpha_n, \varepsilon_n \in \zeta^{\tilde{\mathcal{F}}}$ and $r \in \zeta_0$. Then the following statements are equivalent:

(a) $(\tilde{\mathcal{F}}, \tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, \pounds^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$ is called SVN£-regular,

(b) For each $x_{s,t,k} \in P_{s,t,k}\left(\tilde{\mathcal{F}}\right)$ and $\alpha_n \in Q_{\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}\left(x_{s,t,k}, r\right)$, there exists $\varepsilon_n \in \mathfrak{R}^{\pounds}_{\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}\left(x_{s,t,k}, r\right)$ such that $\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r) \leq \mathrm{int}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}\left(\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r), r\right)$,

(c) For each $x_{s,t,k} \in P_{s,t,k}\left(\tilde{\mathcal{F}}\right)$ and each $\alpha_n \in Q_{\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}\left(x_{s,t,k}, r\right)$, there exists $\varepsilon_n \in Q_{\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}\left(x_{s,t,k}, r\right)$ such that $\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r) \leq \mathrm{int}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}\left(\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r), r\right)$,

(d) For each $x_{s,t,k} \in P_{s,t,k}(\tilde{\mathcal{F}})$ and r-$SVNRIC$ set $\omega_n \in \zeta^{\tilde{\mathcal{F}}}$ with $x_{s,t,k} \notin \omega_n$, there exists $\varepsilon_n \in Q_{\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(x_{s,t,k}, r)$ and α_n is r-$SVN\star$-open set such that $\omega_n \leq \alpha_n$ and $\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r)\overline{q}\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r)$,

(e) For each $x_{s,t,k} \in P_{s,t,k}\left(\tilde{\mathcal{F}}\right)$ and r-$SVNRIC$ set $\omega_n \in \zeta^{\tilde{\mathcal{F}}}$ with $x_{s,t,k} \notin \omega_n$, there exists $\varepsilon_n \in Q_{\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}\left(x_{s,t,k}, r\right)$ and α_n is r-$SVN\star$-open set such that $\omega_n \leq \alpha_n$ and $\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r)\,\overline{q}\alpha_n$,

(f) For each r-$SVNRIO$ set $\alpha_n \in \zeta^{\tilde{\mathcal{F}}}$ with $\omega_n q \alpha_n$, there exists r-$SVNRIO$ set $\varepsilon_n \in \zeta^{\tilde{\mathcal{F}}}$ such that $\omega_n q \varepsilon_n \leq \mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r) \leq \alpha_n$.

(g) For each r-$SVNRIC$ set $\alpha_n \in \zeta^{\tilde{\mathcal{F}}}$ with $\omega_n \not\leq \alpha_n$, there exists r-$SVNRIO$ set $\varepsilon_n \in \zeta^{\tilde{\mathcal{F}}}$ and is r-$SVN\star$-open set $\pi_n \in \zeta^{\tilde{\mathcal{F}}}$ such that $\omega_n q \varepsilon_n$, $\alpha_n \leq \pi_n$ and $\varepsilon_n \overline{q} \pi_n$.

Proof. The proof of (a)⇒(b) and (b)⇒(c) are clear.

(c)⇒(a): $x_{s,t,k} \in P_{s,t,k}(\tilde{\mathcal{F}})$ and $\alpha_n \in \mathfrak{R}^{\pounds}_{\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(x_{s,t,k}, r)$. Then, by (c), there exists $\varepsilon_n \in Q_{\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(x_{s,t,k}, r)$ such that $\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r) \leq \mathrm{int}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}((\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r), r) = \alpha_n$. since, $\varepsilon_n \in Q_{\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(x_{s,t,k}, r)$ we have $\mathrm{int}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r), r) \in \mathfrak{R}^{\pounds}_{\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(x_{s,t,k}, r)$.

Moreover, since, $\omega_n = \mathrm{int}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r), r) \leq \mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r)$, we have $\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\omega_n, r) \leq \mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r)$, and hence $x_{s,t,k} q \omega_n \leq \mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\omega_n, r) \leq \mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r) \leq \alpha_n$ where $\omega_n \in \mathfrak{R}^{\pounds}_{\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(x_{s,t,k}, r)$.

(c)⇒(d): Let ω_n be an r-$SVNRIC$ set in $\tilde{\mathcal{F}}$ and $x_t \in \mathrm{P}_{s,t,k}(\tilde{\mathcal{F}})$ with $x_{s,t,k} \notin \omega_n$. Then $x_{s,t,k} q [\omega_n]^c$ and $[\omega_n]^c \in \mathfrak{R}^{\pounds}_{\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(x_{s,t,k}, r) \subset Q_{\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(x_{s,t,k}, r)$. By (c), there exists $\pi_n \in Q_{\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(x_{s,t,k}, r)$ such that

$$\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\pi_n, r) \leq \mathrm{int}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}\left(\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}\left([\omega_n]^c, r\right), r\right) = [\omega_n]^c.$$

Next, $x_{s,t,k}q\text{int}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}\left(\text{Cl}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\pi_n,r),r\right)$, then $\text{int}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}\left(\text{Cl}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\pi_n,r),r\right)\in Q_{\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}\left(x_{s,t,k},r\right)$, and hence by hypothesis, there exists $\varepsilon_n\in Q_{\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(x_{s,t,k},r)$ such that $\text{Cl}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\varepsilon_n,r)\leq\text{int}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\text{Cl}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\pi_n,r),r)$. Then, $\omega_n\leq[\text{Cl}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\pi_n,r),r)]^c$. Put $\alpha_n=[\text{Cl}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\pi_n,r),r)]^c$ then α_n is *r-SVN⋆O* set. Hence

$$\text{Cl}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\alpha_n,r)\leq\left[\text{int}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}\left(\text{Cl}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\pi_n,r),r\right)\right]^c\leq\left[\text{Cl}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\varepsilon_n,r)\right.$$

Therefore, $\text{Cl}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\varepsilon_n,r)\overline{q}\text{Cl}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\alpha_n,r)$.

(d)⇒(e): It is trivial.

(e)⇒(f): Suppose that α_n is an *r-SVNRIO* set with $\omega_nq\alpha_n$, then $\omega_n\not\leq[\alpha_n]^c$. Hence there exists $x_{s,t,k}\in\text{P}_{s,t,k}(\tilde{\mathcal{F}})$ such that $x_{s,t,k}\in\omega_n$ and $\omega_n\not\leq[\alpha_n]^c$ where $[\alpha_n]^c$ is *r-SVNRIC* set. By (e), there exists $\varepsilon_n\in Q_{\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(x_{s,t,k},r)$ and $\pi_n\in\zeta^{\tilde{\mathcal{F}}}$ is *r-SVN⋆O* set such that $[\alpha_n]^c\leq\pi_n$ and $\text{Cl}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\varepsilon_n,r)\overline{q}\pi_n$. From $\varepsilon_n\in Q_{\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(x_{s,t,k},r)$ we have $x_{s,t,k}q\varepsilon_n\leq\text{int}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\text{Cl}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\varepsilon_n,r),r)$.

By setting $[\varepsilon_n]_1=\text{int}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\text{Cl}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\varepsilon_n,r),r)$, we have $\omega_nq[\varepsilon_n]_1$ and $[\varepsilon_n]_1$ is *r-SVNRIO* set such that

$$\omega_nq\left[\varepsilon_n\right]_1\leq\text{Cl}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}\left(\left[\varepsilon_n\right]_1,r\right)\leq\text{Cl}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\varepsilon_n,r)\leq\underline{1}-\pi_n\leq\alpha_n$$

(f)⇒(g): Let α_n be an *r-SVNRIC* set $\alpha_n\in\zeta^{\tilde{\mathcal{F}}}$ with $\omega_n\not\leq\alpha_n$. Therefore, $\omega_nq[\alpha_n]^c$ and hence by, then there exists an *r-SVNRIO* set $\varepsilon_n\in\zeta^{\tilde{\mathcal{F}}}$ such that $\omega_nq\varepsilon_n\leq\text{Cl}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\varepsilon_n,r)\leq[\alpha_n]^c$. Then, ε_n is an *r-SVNRIO* set and $[\text{Cl}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\varepsilon_n,r)]^c$ is an *r-SVN⋆O* set such that $\omega_nq\varepsilon_n$, $\alpha_n\leq[\text{Cl}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\varepsilon_n,r)]^c$ and $\varepsilon_n\overline{q}[\text{Cl}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\varepsilon_n,r)]^c$.

(g)⇒(a): Let $\alpha_n\in\mathfrak{R}^{\text{£}}_{\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(x_{s,t,k},r)$ Then $x_{s,t,k}\not\leq[\alpha_n]^c$ and $[\alpha_n]^c$ is an *r-SVNRIC* set. By (g), there exist *r-SVNRIO* set $\varepsilon_n\in\zeta^{\tilde{\mathcal{F}}}$ and it is *r-SVN⋆O* set $\pi_n\in\zeta^{\tilde{\mathcal{F}}}$ such that $x_{s,t,k}q\varepsilon_n$, $[\alpha_n]^c\leq\pi_n$ and $\varepsilon_n\overline{q}\pi_n$. Then, $\varepsilon_n\in\mathfrak{R}^{\text{£}}_{\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(x_{s,t,k},r)$. Since, π_n is *r-SVN⋆O* set, $\text{Cl}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\varepsilon_n,r)\overline{q}\pi_n$. Therefore, $x_{s,t,k}q\varepsilon_n\leq\text{Cl}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\varepsilon_n,r)\leq[\pi_n]^c\leq\alpha_n$. Hence $(\tilde{\mathcal{F}},\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}},\text{£}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$ is *SVN*£-regular.

Theorem 8. Let $(\tilde{\mathcal{F}},\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}},\text{£}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$ be an SVNITS, $\alpha_n\in\zeta^{\tilde{\mathcal{F}}}$ and $r\in\zeta_0$. Then the following statements are equivalent:

(a) $(\tilde{\mathcal{F}},\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}},\text{£}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$ is called *SVN*£-regular,

(b) For each $x_{s,t,k}\in P_{s,t,k}(\tilde{\mathcal{F}})$, $\alpha_n\in\zeta^{\tilde{\mathcal{F}}}$ with $\tau^{\tilde{\varrho}}([\alpha_n]^c)\geq r,\tau^{\tilde{\sigma}}([\alpha_n]^c)\leq1-r,\tau^{\tilde{\varsigma}}([\alpha_n]^c)\leq1-r$, and $x_{s,t,k}\notin\alpha_n$, there exists $\varepsilon_n\in\zeta^{\tilde{\mathcal{F}}}$ with ε_n is r-*SVN⋆O* such that $x_{s,t,k}\notin\text{Cl}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\varepsilon_n,r)$ and $\alpha_n\leq\varepsilon_n$,

(c) For each $x_{s,t,k}\in P_{s,t,k}(\tilde{\mathcal{F}})$, $\alpha_n\in\zeta^{\tilde{\mathcal{F}}}$ with $\tau^{\tilde{\varrho}}([\alpha_n]^c)\geq r$, $\tau^{\tilde{\sigma}}([\alpha_n]^c)\leq1-r$, $\tau^{\tilde{\varsigma}}([\alpha_n]^c)\leq1-r$, and $x_{s,t,k}\notin\alpha_n$, there exists, $\varepsilon_n\in Q_{\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(x_{s,t,k},r)$ and $\pi_n\in\zeta^{\tilde{\mathcal{F}}}$ with π_n is r-*SVN⋆O* such that $\alpha_n\leq\varepsilon_n$ and $\varepsilon_n\overline{q}\pi_n$,

(d) For each $\omega_n,\alpha_n\in\zeta^{\tilde{\mathcal{F}}}$ with $\tau^{\tilde{\varrho}}([\alpha_n]^c)\geq r$, $\tau^{\tilde{\sigma}}([\alpha_n]^c)\leq1-r$, $\tau^{\tilde{\varsigma}}([\alpha_n]^c)\leq1-r$, and $\omega_n\not\leq\alpha_n$, then there exists $\varepsilon_n\in Q_{\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(x_{s,t,k},r)$ and $\varepsilon_n,\pi_n\in\zeta^{\tilde{\mathcal{F}}}$ with $\tau^{\tilde{\varrho}}(\varepsilon_n)\geq r,\tau^{\tilde{\sigma}}(\varepsilon_n)\leq1-r,\tau^{\tilde{\varsigma}}(\varepsilon_n)\leq1-r$ and π_n is r-*SVN⋆O* sets such that $\omega_nq\varepsilon_n$, $\alpha_n\leq\pi_n$ and $\varepsilon_n\overline{q}\pi_n$.

Proof. Similar to the proof of Theorem 7.

Theorem 9. An SVNITS $(\tilde{\mathcal{F}}, \tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, £^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$ is *SVNA*£-regular iff for each $\alpha_n \in \zeta^{\tilde{\mathcal{F}}}$ and $r \in \zeta_0, \mathrm{CI}^{\delta£}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\alpha_n, r) = \mathrm{CI}^{\theta£}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\alpha_n, r)$.

Proof. From Theorem 4(i), we only show that $\mathrm{CI}^{\delta£}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\alpha_n, r) \geq \mathrm{CI}^{\theta£}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\alpha_n, r)$.

Suppose that $\mathrm{CI}^{\delta£}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\alpha_n, r) \not\geq \mathrm{CI}^{\theta£}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\alpha_n, r)$. Then there exist $\upsilon \in \tilde{\mathcal{F}}$ and $[s \in (0,1], t \in [0,1), k \in [0,1)]$ such that

$$\left.\begin{array}{l} \tilde{\varrho}_{\mathrm{CI}^{\delta£}_{\tilde{\tau}^{\tilde{\varrho}}}(\alpha_n, r)}(\upsilon) < s < \tilde{\varrho}_{\mathrm{CI}^{\theta£}_{\tilde{\tau}^{\tilde{\varrho}}}(\alpha_n, r)}(\upsilon) \\ \tilde{\sigma}_{\mathrm{CI}^{\delta£}_{\tilde{\tau}^{\tilde{\sigma}}}(\alpha_n, r)}(\upsilon) > t > \tilde{\sigma}_{\mathrm{CI}^{\theta£}_{\tilde{\tau}^{\tilde{\sigma}}}(\alpha_n, r)}(\upsilon) \\ \tilde{\varsigma}_{\mathrm{CI}^{\delta£}_{\tilde{\tau}^{\tilde{\varsigma}}}(\alpha_n, r)}(\upsilon) > k > \tilde{\varsigma}_{\mathrm{CI}^{\theta£}_{\tilde{\tau}^{\tilde{\varsigma}}}(\alpha_n, r)}(\upsilon) \end{array}\right\} \tag{21}$$

Because $\tilde{\varrho}_{\mathrm{CI}^{\delta£}_{\tilde{\tau}^{\tilde{\varrho}}}(\alpha_n, r)}(\upsilon) < s, \tilde{\sigma}_{\mathrm{CI}^{\delta£}_{\tilde{\tau}^{\tilde{\sigma}}}(\alpha_n, r)}(\upsilon) > t, \tilde{\varsigma}_{\mathrm{CI}^{\delta£}_{\tilde{\tau}^{\tilde{\varsigma}}}(\alpha_n, r)}(\upsilon) > k$, and $x_{s,t,k}$ is not an r-$\delta£$-cluster point of α_n. So, there exists $\varepsilon_n \in Q_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(x_{s,t,k}, r)$ with $\alpha_n \leq [\mathrm{int}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\mathrm{Cl}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\varepsilon_n, r), r)]^c$ Since $\varepsilon_n \in Q_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(x_{s,t,k}, r)$ we have $\mathrm{int}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\mathrm{Cl}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\varepsilon_n, r), r) \in \mathfrak{R}^{£}_{\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(x_{s,t,k}, r)$. By *SVNA*£-regularity of $\tilde{\mathcal{F}}$, there exists $\omega_n \in \mathfrak{R}^{£}_{\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(x_{s,t,k}, r)$ such that $\mathrm{Cl}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\omega_n, r), r) \leq \mathrm{int}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\mathrm{Cl}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\varepsilon_n, r), r)$. Thus,

$$\alpha_n \leq \left[\mathrm{int}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}\left(\mathrm{Cl}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\varepsilon_n, r), r\right)\right]^c \leq \left[\mathrm{Cl}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\omega_n, r)\right]^c = \mathrm{int}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}\left([\omega_n]^c, r\right),$$

and $\tau^{\tilde{\varrho}}(\omega_n) \geq r, \tau^{\tilde{\sigma}}(\omega_n) \leq 1 - r, \tau^{\tilde{\varsigma}}(\omega_n) \leq 1 - r$. By Theorem 4(d), $\tilde{\varrho}_{\mathrm{CI}^{\theta£}_{\tilde{\tau}^{\tilde{\varrho}}}(\alpha_n)}(\upsilon) \leq \tilde{\varrho}_{[\omega_n]^c}(\upsilon) < s, \tilde{\sigma}_{CI^{\theta£}_{\tilde{\tau}^{\tilde{\sigma}}}(\alpha_n, r)}(\upsilon) \geq \tilde{\sigma}_{[\omega_n]^c}(\upsilon) > t$ and $\tilde{\varsigma}_{CI^{\theta£}_{\tilde{\tau}^{\tilde{\varsigma}}}(\alpha_n, r)}(\upsilon) \geq \tilde{\varsigma}_{[\omega_n]^c}(\upsilon) > k$. It is a contradiction for Eq. (21).

Conversely, let $\alpha_n \in \mathfrak{R}^{£}_{\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(x_{s,t,k}, r) \subset Q_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(x_{s,t,k})$. Then by Theorem 4(h), $\mathrm{s} > \tilde{\varrho}_{[\alpha_n]^n}(\upsilon) = \tilde{\varrho}_{\mathrm{CI}^{\delta£}_{\tilde{\tau}^{\tilde{\varrho}}}([\alpha_n]^c, r)}(\upsilon), \mathrm{s} > \tilde{\varrho}_{[\alpha_n]^n}(\upsilon) = \tilde{\varrho}_{\mathrm{CI}^{\delta£}_{\tilde{\tau}^{\tilde{\varrho}}}([\alpha_n]^c, r)}(\upsilon)$ and $k < \tilde{\sigma}_{[\alpha_n]^n}(\upsilon) = \tilde{\sigma}_{CI^{\delta£}_{\tilde{\tau}^{\tilde{\sigma}}}([\alpha_n]^c, r)}(\upsilon)$. Since, $\mathrm{CI}^{\delta£}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}([\alpha_n]^c, r) = \mathrm{CI}^{\theta£}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}([\alpha_n]^c, r)$, $x_{s,t,k}$ is not an r-$\theta\mathcal{J}$-cluster point of $[\alpha_n]^c$. Then there exists $\varepsilon_n \in Q_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(x_{s,t,k}, r)$ such that $[\alpha_n]^c \overline{q} \mathrm{Cl}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\varepsilon_n, r)$ implies $\mathrm{Cl}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\varepsilon_n, r) \leq \alpha_n = \mathrm{int}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\mathrm{Cl}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\alpha_n, r), r)$ and by Theorem 7(c), $(\tilde{\mathcal{F}}, \tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, £^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$ is *SVNA*£-regular.

Theorem 10. An SVNITS$(\tilde{\mathcal{F}}, \tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, £^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$ is *SVNA*£-regular iff for each r-*SVNRIC* set $\alpha_n \in \zeta^{\tilde{\mathcal{F}}}$ and $\in \zeta_0, \mathrm{CI}^{\theta£}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\alpha_n, r) = \alpha_n$.

Proof. The proof is similar to Theorem 9; additionally, r-*SVNRIC* set is r-$\delta£$-closed.

Conversely, let α_n be any r-FRIC set with $x_t \notin \alpha_n$. Then, $x_t \notin \mathrm{CI}^{\theta£}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\alpha_n, r)$ and hence, x_t is not r-$\theta£$-cluster point of α_n so, there there exists $\varepsilon_n \in Q_{\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(x_{s,t,k}, r)$ such that $\alpha_n \overline{q} \mathrm{Cl}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\varepsilon_n, r)$. Thus, $\alpha_n \leq [\mathrm{Cl}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\varepsilon_n, r)]^c = \omega_n$ and ω_n is r-*SVN⋆O* implies $\omega_n \overline{q} \mathrm{Cl}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\varepsilon_n, r)$. Hence, by Theorem 4(e), $(\tilde{\mathcal{F}}, \tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, £^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$ is SVNA£-regular.

Lemma 1. If $\alpha_n, \varepsilon_n \in \zeta^{\tilde{\mathcal{F}}}$, $r \in \zeta_0$ such that $\alpha_n \overline{q} \varepsilon_n$ where ε_n is r-$\delta£$-open, then $\mathrm{CI}^{\delta£}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\alpha_n, r) \overline{q} \varepsilon_n$.

Proof. Let $\alpha_n \overline{q} \varepsilon_n$ where ε_n is r-δ£-open. Then, $\alpha_n \leq [\varepsilon_n]^c = \mathrm{CI}^{\delta£}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}([\varepsilon_n]^c$, by Theorem 4(k), $\mathrm{CI}^{\delta£}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\alpha_n, r) \leq \mathrm{CI}^{\delta£}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\mathrm{CI}^{\delta£}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}([\varepsilon_n]^c, r), r) = \mathrm{CI}^{\delta£}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}([\varepsilon_n]^c, r) = [\varepsilon_n]^c$. Hence, $\mathrm{CI}^{\delta£}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\mathcal{A}, r) \overline{q} \varepsilon_n$.

Lemma 2. Let $(\tilde{\mathcal{F}}, \tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, £^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$ be an SVNITS and $\alpha_n \in \zeta^{(\tilde{\mathcal{F}}}$ is δ£-open iff for each $x_{x,t,k} \in Q_{\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(x_{s,t,k}, r)$ with $x_{s,t,k} q \alpha_n$, there exists r-*SVNRIO* set $\varepsilon_n \in \zeta^{\tilde{\mathcal{F}}}$ such that $x_{x,t,k} q \varepsilon_n \leq \alpha_n$.

Proof. Let $x_{x,t,k} \in \mathrm{P}_{s,t,k}((\tilde{\mathcal{F}})$ with $x_{x,t,k} q \alpha_n$ Then $x_{x,t,k} \notin \alpha_n]^c$. Since α_n is an r-δ£-open set, $x_{x,t,k} \notin [\alpha_n]^c = \mathrm{CI}^{\delta£}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}([\alpha_n]^c, r)$. Thus, $x_{x,t,k}$ is not r-δ£-cluster point of $[\alpha_n]^c$. So, there exists $\omega_n \in Q_{\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(x_{s,t,k}, r)$ such that $[\alpha_n]^c \overline{q} \mathrm{CI}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\mathrm{Cl}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\omega_n, r), r)$. Put $\varepsilon_n = \mathrm{int}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\mathrm{Cl}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\omega_n, r), r)$, so, ε_n is an r-*SVNRIO* set with $x_{x,t,k} q \varepsilon_n \leq \alpha_n$.

Conversely, let $[\alpha_n]^c \neq \mathrm{CI}^{\delta£}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}([\alpha_n]^c, r)$, then there exist $\upsilon \in \tilde{\mathcal{F}}$ and $s, t, k \in \zeta_0$ such that

$$\left.\begin{aligned} \tilde{\varrho}_{[\alpha_n]^c}(\upsilon) < s < \tilde{\varrho}_{\mathrm{CI}^{\delta£}_{\tilde{\tau}^{\tilde{\varrho}}}([\alpha_n]^c, r)}(\upsilon) \\ \tilde{\sigma}_{[\alpha_n]^c}(\upsilon) > t > \tilde{\sigma}_{CI^{\delta£}_{\tilde{\tau}^{\tilde{\sigma}}}([\alpha_n]^c, r)}(\upsilon) \\ \tilde{\varsigma}_{[\alpha_n]^c}(\upsilon) > k > \tilde{\varsigma}_{CI^{\delta£}_{\tilde{\tau}^{\tilde{\varsigma}}}([\alpha_n]^c, r)}(\upsilon). \end{aligned}\right\} \tag{22}$$

Because of $x_{x,t,k} q \alpha_n$, then there exists an r-*SVNRIO* set ε_n such that $x_{x,t,k} q \varepsilon_n \leq \alpha_n$. This implies $[\alpha_n]^c \leq [\varepsilon_n]^n = \mathrm{Cl}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\mathrm{int}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}([\varepsilon_n]^n, r), r)$. By Theorem 3(d), we have $\tilde{\varrho}_{\mathrm{CI}^{\delta£}_{\tilde{\tau}^{\tilde{\varrho}}}([\alpha_n]^c, r)}(\upsilon) \tilde{\varrho}_{\leq([\varepsilon_n]^n)}(\upsilon) < s, \tilde{\sigma}_{CI^{\delta£}_{\tilde{\tau}^{\tilde{\sigma}}}([\alpha_n]^c, r)}(\upsilon) \tilde{\sigma}_{\leq([\varepsilon_n]^n)}(\upsilon) > t$ and $\tilde{\varsigma}_{CI^{\delta£}_{\tilde{\tau}^{\tilde{\varsigma}}}([\alpha_n]^c, r)}(\upsilon) \tilde{\varsigma}_{\leq([\varepsilon_n]^n)}(\upsilon) > k$. It is a contradiction for Eq. (22). Hence, $[\alpha_n]^c = \mathrm{CI}^{\delta£}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}([\alpha_n]^c, r)$, i.e., α_n is an r-δ£-open set.

Lemma 3. If $\tau^{\tilde{\varrho}}(\alpha_n) \geq r, \tau^{\tilde{\sigma}}(\alpha_n) \leq 1 - r, \tau^{\tilde{\varsigma}}(\alpha_n) \leq 1 - r$, then $\mathrm{CI}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\alpha_n, r) = \mathrm{CI}^{\delta£}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\alpha_n, r)$.

Proof. Follows easily by virtue of Theorem 4.

Theorem 11. Let $(\tilde{\mathcal{F}}, \tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, £^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$ be an SVNITS. Then the following statements are equivalent:

(a) $(\tilde{\mathcal{F}}, \tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, £^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$ is *SVNA*£-regular,

(b) For each $r - \delta$£-open set $\alpha_n \in \zeta^{\tilde{\mathcal{F}}}$ and each $x_{x,t,k} \in \mathrm{P}_{s,t,k}(\tilde{\mathcal{F}})$ with $x_{s,t,k} q \mathcal{A}$, there exists r-δ£-open set $\varepsilon_n \in \zeta^{\tilde{\mathcal{F}}}$ such that $x_{x,t,k} q \varepsilon_n \leq \mathrm{Cl}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\varepsilon_n, r) \leq \alpha_n$.

Proof. (a)⇒(b): Let α_n be r-fuzzy $\delta\mathcal{J}$-open set such each $x_{s,t,k} q \alpha_n$. Then by Lemma 3, there exists an r-*SVNRIO* set $\pi_n \in \zeta^{\tilde{\mathcal{F}}}$ such that $x_{s,t,k} q \pi_n \leq \alpha_n$. By *SVNA*£-regularity of X, there exists an r-FR IO set ε_n (which is also r-δ£-open such that $x_{s,t,k} q \varepsilon_n \leq \mathrm{Cl}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\varepsilon_n, r) \leq \pi_n \leq \alpha_n$.

Therefore, (b) (a) is clear.

5 Single Valued Neutrosophic θ£-Connected

The aim of this section is to introduce the r-single-valued neutrosophic θ£-separated and r-single-valued neutrosophic δ£-separated. Moreover, we introduce r-single-valued neutrosophic θ£-connected and r-single valued neutrosophic δ£-connected related to the r-single valued neutrosophic operator θ and δ defined on the set $\tilde{\mathcal{F}}$.

Definition 18. Let $(\tilde{\mathcal{F}}, \tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, £^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$ be an SVNITS. For $r \in \zeta_0$ and $\alpha_n, \varepsilon_n \in \zeta^{\tilde{\mathcal{F}}}$. Then,

(a) Two non-null SVNSs $\alpha_n, \varepsilon_n \in \zeta^{\tilde{\mathcal{F}}}$ are said to be r-single-valued neutrosophic θ£-separated if $\alpha_n \overline{q} [\varepsilon_n]_{\theta£}$ and $\varepsilon_n \overline{q} [\alpha_n]_{\theta£}$,

(b) Two non-null SVNSs $\alpha_n, \varepsilon_n \in \zeta^{\tilde{\mathcal{F}}}$ are said to be r-single-valued neutrosophic δ£-separated if $\alpha_n \overline{q} [\varepsilon_n]_{\delta£}$ and $\varepsilon_n \overline{q} [\alpha_n]_{\delta£}$,

Remark 2. For any two non-null SVNSs$\alpha_n, \varepsilon_n \in \zeta^{\tilde{\mathcal{F}}}$, and by Eq. (8). The following implications hold: r-single-valued neutrosophic θ£-separated $\Rightarrow$ r-single-valued neutrosophic δ£-separated $\Rightarrow$ r-single-valued neutrosophic separated.

The following example shows that the concept of r-single-valued neutrosophic δ£-separated is weaker than that of r-single-valued neutrosophic θ£-separated.

Example 3. Let $\tilde{\mathcal{F}} = \{a, b, c\}$ be a set. Define $[\varepsilon_n]_1, [\varepsilon_n]_2 \in \zeta^{\tilde{\mathcal{F}}}$ as follows:

$[\varepsilon_n]_1 = \langle (1,1,0), (1,1,0), (1,1,0) \rangle ; [\varepsilon_n]_2 = \langle (0,0,1), (0,0,1), (0,0,1) \rangle .$

We define an SVNITS$(\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, £^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$ on $\tilde{\mathcal{F}}$ as follows: for each $\alpha_n \in \zeta^{\tilde{\mathcal{F}}}$,

$$\tilde{\tau}^{\tilde{\varrho}}(\alpha_n) = \begin{cases} 1, & if\ \alpha_n = \tilde{0}, \\ 1, & if\ \alpha_n = \tilde{1}, \\ \frac{1}{3}, & if\ \alpha_n = [\varepsilon_n]_1, \\ \frac{1}{2}, & if\ \alpha_n = [\varepsilon_n]_2, \\ 0, & otherwise, \end{cases} \quad £^{\tilde{\varrho}}(\alpha_n) = \begin{cases} 1, & if\ \alpha_n = \tilde{0}, \\ 0, & otherwise, \end{cases}$$

$$\tilde{\tau}^{\tilde{\sigma}}(\alpha_n) = \begin{cases} 0, & if\ \alpha_n = \tilde{0}, \\ 0, & if\ \alpha_n = \tilde{1}, \\ \frac{2}{3}, & if\ \alpha_n = [\varepsilon_n]_1, \\ \frac{1}{2}, & if\ \alpha_n = [\varepsilon_n]_2, \\ 1, & otherwise \end{cases} \quad £^{\tilde{\sigma}}(\alpha_n) = \begin{cases} 0, & if\ \alpha_n = \tilde{1}, \\ 1, & otherwise, \end{cases}$$

$$\tilde{\tau}^{\tilde{\varsigma}}(\alpha_n)=\begin{cases}0, & if\,\alpha_n=\tilde{0},\\ 0, & if\ \alpha_n=\tilde{0},\\ \frac{2}{3}, & if\ \alpha_n=[\varepsilon_n]_1,\\ \frac{1}{2}, & if\ \alpha_n=[\varepsilon_n]_2,\\ 1, & otherwise,\end{cases}\qquad \pounds^{\tilde{\varsigma}}(\alpha_n)=\begin{cases}0, & if\ \alpha_n=\tilde{1},\\ 1, & otherwise.\end{cases}$$

Therefore, we obtain

$$\mathrm{CI}^{\theta\pounds}_{\tilde{\tau}^{\tilde{\varsigma}}}(\alpha_n,r)=\begin{cases}\tilde{0}, & if\ \alpha_n=\tilde{0}, r\in\zeta_0,\\ \boldsymbol{\mathcal{E}}^c_2, & if\ \alpha_n\le[\varepsilon_n]_1, r\le\frac{1}{2}, 1-r\ge\frac{1}{2},\\ \boldsymbol{\mathcal{E}}^c_1, & if\ \alpha_n\le[\varepsilon_n]_2, r\le\frac{1}{3}, 1-r\ge\frac{2}{3},\\ \tilde{0}, & otherwise.\end{cases}$$

If $r\le\frac{1}{3}$ and $1-r\ge\frac{2}{3}$, then $[\varepsilon_n]^c_2$ and $[\varepsilon_n]_2$ are not r-single-valued neutrosophic $\theta\pounds$-separated for $r\le\frac{1}{3}$ and $1-r\ge\frac{2}{3}$. If $r>\frac{1}{3}$ and $1-r<\frac{2}{3}$, we have $[\varepsilon_n]^c_2$ and $[\varepsilon_n]_2$ are r-single-valued neutrosophic separated.

Theorem 12. Let $(\tilde{\boldsymbol{\mathcal{F}}},\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}},\pounds^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$ be an SVNITS. For $r\in\zeta_0$ and $\alpha_n,\varepsilon_n\in\zeta^{\tilde{\boldsymbol{\mathcal{F}}}}$.

(a) If α_n and ε_n are single-valued neutrosophic $\theta\pounds$-separated, and $[\alpha_n]_1,[\varepsilon_n]_1\in\zeta^{\tilde{\boldsymbol{\mathcal{F}}}}$ such that $[\alpha_n]_1\le\alpha_n[\varepsilon_n]_1\le\varepsilon_n$, then $[\alpha_n]_1$ and $[\varepsilon_n]_1$ are also single-valued neutrosophic $\theta\pounds$-separated,
(b) If $\alpha_n\overline{q}\varepsilon_n$ either both are r-$\theta\pounds$-open or r-$\delta\pounds$-closed, then α_n and ε_n are single-valued neutrosophic $\theta\pounds$-separated,
(c) If α_n and ε_n either both are r-$\theta\pounds$-open or r-$\delta\pounds$-closed and if $[\omega_n]_1=\alpha_n\cap[\varepsilon_n]^c$ and $\omega_2=\varepsilon_n\cap[\alpha_n]^c$, then $[\omega_n]_1$ and $[\omega_n]_1$ are single-valued neutrosophic $\theta\pounds$-separated.

Proof. (a) Since $[\alpha_n]_1\le\alpha_n$ we have $[[\alpha_n]_1]_{\theta\pounds}\le[\alpha_n]_{\theta\pounds}$. Then, $\varepsilon_n\le[\alpha_n]_{\theta\pounds}\Rightarrow[\varepsilon_n]_1\le[\alpha_n]_{\theta\pounds}\Rightarrow[\varepsilon_n]_1\le[[\alpha_n]_1]_{\theta\pounds}$. Similarly $[\alpha_n]_1\le[[\varepsilon_n]_1]_{\theta\pounds}$. Hence $[\alpha_n]_1$ and $[\varepsilon_n]_1$ are single-valued neutrosophic $\theta\pounds$-separated.

(b) When α_n and ε_n are r-$\delta\pounds$-closed, then $\alpha_n=[\alpha_n]_{\theta\pounds}$ and $\varepsilon_n=[\varepsilon_n]_{\theta\pounds}$. Since $\alpha_n\overline{q}\varepsilon_n$ we have $[\alpha_n]_{\theta\pounds}\overline{q}\varepsilon_n$ and $[\varepsilon_n]_{\theta\pounds}\overline{q}\alpha_n$.

When α_n and ε_n are r-$\theta\pounds$-open, $[\alpha_n]^c$ and $[\varepsilon_n]^c$ are r-$\theta\pounds$-closed. Then $\alpha_n\overline{q}\varepsilon_n\Rightarrow\alpha_n\le[\varepsilon_n]^c\Rightarrow[\alpha_n]_{\theta\pounds}\le[[\varepsilon_n]^c]_{\theta\pounds}=[\varepsilon_n]^c\Rightarrow[\alpha_n]_{\theta\pounds}\overline{q}\varepsilon_n$. Similarly, $[\varepsilon_n]_{\theta\pounds}\overline{q}\alpha_n$. Hence α_n and ε_n are single-valued neutrosophic $\theta\pounds$-separated.

(c) When α_n and ε_n are r-$\theta\pounds$-open, $[\alpha_n]^c$ and $[\varepsilon_n]^c$ are r-$\theta\pounds$-closed. Since $[\omega_n]_1\le[\varepsilon_n]^c$, $[[\omega_n]_1]_{\theta\pounds}\le[[\varepsilon_n]^c]_{\theta\pounds}=[\varepsilon_n]^c$ and so $[[\omega_n]_1]_{\theta\pounds}\overline{q}\varepsilon_n$. Thus $[\omega_n]_2\overline{q}[[\omega_n]_1]_{\theta\pounds}$. Similarly, $[\omega_n]_1\overline{q}[[\omega_n]_2]_{\theta\pounds}$. Hence $[\omega_n]_1$ and $[\omega_n]_1$ are single-valued neutrosophic $\theta\pounds$-separated.

When α_n and ε_n are r-$\theta\pounds$-closed, $\alpha_n = [\alpha_n]_{\theta\pounds}$ and $\varepsilon_n = [\varepsilon_n]_{\theta\pounds}$. Since $[\omega_n]_1 \leq [\varepsilon_n]^c$, $[\varepsilon_n]_{\theta\pounds}\overline{q}[\omega_n]_1$ and hence $[[\omega_n]_2]_{\theta\pounds}\overline{q}[\omega_n]_1$. Similarly, $[[\omega_n]_1]_{\theta\pounds}\overline{q}[\omega_n]_2$. Hence $[\omega_n]_1$ and $[\omega_n]_1$ are single-valued neutrosophic $\theta\pounds$-separated.

Theorem 13. Two non-null $\alpha_n, \varepsilon_n \in \zeta^{\tilde{\mathcal{F}}}$ are single-valued neutrosophic $\theta\pounds$-separated if and only if there exist two r-$\theta\pounds$-open sets ω_n and π_n such that $\alpha_n \leq \omega_n$, $\varepsilon_n \leq \pi_n$, $\alpha_n\overline{q}\pi_n$ and $\varepsilon_n\overline{q}\omega_n$.

Proof. Let α_n and ε_n be single-valued neutrosophic $\theta\pounds$-separated. Putting $\pi_n = [[\alpha_n]_{\theta\pounds}]^c$ and $\omega_n = [[\varepsilon_n]_{\theta\pounds}]^c$, then ω_n and π_n are r-$\theta\pounds$-open such that $\alpha_n \leq \omega_n$, $\varepsilon_n \leq \pi_n$, $\alpha_n\overline{q}\pi_n$ and $\varepsilon_n\overline{q}\omega_n$.

Conversely, let ω_n and π_n be r-$\theta\pounds$-open sets such that $\alpha_n \leq \omega_n$, $\varepsilon_n \leq \pi_n$, $\alpha_n\overline{q}\pi_n$ and $\varepsilon_n\overline{q}\omega_n$. Since $[\pi_n]^c$ and $[\omega_n]^c$ are r-$\theta\pounds$-closed, we have $[\alpha_n]_{\theta\pounds} \leq [\pi_n]^c \leq [\varepsilon_n]^c$ and $[\varepsilon_n]_{\theta\pounds} \leq [\omega_n]^c \leq [\alpha_n]^c$. Thus $[\alpha_n]_{\theta\pounds}\overline{q}\varepsilon_n$ and $[\varepsilon_n]_{\theta\pounds}\overline{q}\alpha_n$. Hence α_n and ε_n are single-valued neutrosophic $\theta\pounds$-separated.

Definition 19. An SVNS which cannot be expressed as the union of two single-valued neutrosophic $\theta\pounds$-separated is said to be single-valued neutrosophic $\theta\pounds$-connected.

Definition 20. An SVNS α_n in a SVNITS $(\tilde{\mathcal{F}}, \tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, \pounds^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$ is said to be single-valued neutrosophic $\delta\pounds$-connected if α_n cannot be expressed as the union of two single-valued neutrosophic $\delta\pounds$-separated.

For an SVNS α_n in a SVNITS$(\tilde{\mathcal{F}}, \tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, \pounds^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$, the following implications hold: single-valued neutrosophic connected $\Rightarrow$ single-valued neutrosophic $\delta\pounds$-connected $\Rightarrow$ single-valued neutrosophic $\theta\pounds$-connected. If $\tau^{\tilde{\varrho}}(\alpha_n) \geq r, \tau^{\tilde{\sigma}}(\alpha_n) \leq 1-r, \tau^{\tilde{\varsigma}}(\alpha_n) \leq 1-r$, then these three properties are equivalent.

Theorem 14. Let α_n be a non-null single-valued neutrosophic $\theta\pounds$-connected in a SVNITS $(\tilde{\mathcal{F}}, \tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, \pounds^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$. If α_n is contained in the union of two single-valued neutrosophic $\theta\pounds$-separated ε_n and ω_n, then exactly one of the following conditions (a) or (b) holds:

(a) $\alpha_n \leq \varepsilon_n$ and $\alpha_n \cap \omega_n = \tilde{0}$,
(b) $\alpha_n \leq \omega_n$ and $\alpha_n \cap \varepsilon_n = \tilde{0}$.

Proof. We first note that when $\alpha_n \cap \omega_n = \tilde{0}$, then $\alpha_n \leq \varepsilon_n$, since $\alpha_n \leq \varepsilon_n \cup \omega_n$. Similarly, when $\alpha_n \cap \varepsilon_n = \tilde{0}$, we have $\alpha_n \leq \omega_n$. Since $\alpha_n \leq \varepsilon_n \cup \omega_n$, both $\alpha_n \cap \varepsilon_n = \tilde{0}$ and $\alpha_n \cap \omega_n = \tilde{0}$ cannot hold simultaneously. Again, if $\alpha_n \cap \varepsilon_n \neq \tilde{0}$ and $\alpha_n \cap \omega_n \neq \tilde{0}$, then, by Theorem 12 (1), $\alpha_n \cap \omega_n$ and $\alpha_n \cap \varepsilon_n$ are single-valued neutrosophic $\theta\pounds$-separated such that $\alpha_n = (\alpha_n \cap \varepsilon_n) \cup (\alpha_n \cap \omega_n)$, contradicting the single-valued neutrosophic $\theta\pounds$-connectedness of α_n. Hence, exactly one of the conditions (1) or (2) above must hold.

Theorem 15. Let $\{[\alpha_n]_j | j \in J\}$ be a collection of single-valued neutrosophic $\theta\pounds$-connected in $(\tilde{\mathcal{F}}, \tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, \pounds^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$. If there exists $i \in J$ such that $[\alpha_n]_j \cap [\alpha_n]_i \neq \tilde{0}$ for each $j \in J$, then $\alpha_n = \cup\{[\alpha_n]_j | j \in J\}$ is single-valued neutrosophic $\theta\pounds$-connected.

Proof. Suppose that α_n is not single-valued neutrosophic $\theta\pounds$-connected. Then there exist single-valued neutrosophic $\theta\pounds$-separated ε_n and ω_n such that $\alpha_n = \varepsilon_n \cap \omega_n$. By Theorem 14, we have either (a) $[\alpha_n]_j \leq \varepsilon_n$ with $[\alpha_n]_j \cap \omega_n = \tilde{0}$ or (b) $[\alpha_n]_j \leq \omega_n$ with $[\alpha_n]_j \cap \varepsilon_n = \tilde{0}$ for each $j \in J$. Similarly, either (a′)$[\alpha_n]_i \leq \varepsilon_n$ with $[\alpha_n]_i \cap \omega_n = \tilde{0}$ or (b′)$[\alpha_n]_i \leq \omega_n$ with $[\alpha_n]_i \cap \varepsilon_n = \tilde{0}$ for each $i \in J$. We may assume, without loss of generality, that $[\alpha_n]_j$ is non-null for each $j \in J$, and hence exactly one of the conditions (a) and (b), and exactly one of (a′) and (b′) will hold.

Since $[\alpha_n]_j \cap [\alpha_n]_i \neq \tilde{0}$ for each $j \in J$, the conditions (a) and (b′) cannot happen, and similarly (b) and (1′) cannot hold simultaneously. If (a) and (a′) hold, then $[\alpha_n]_j \leq \varepsilon_n$ with $[\alpha_n]_j \cap \omega_n = \tilde{0}$.

Then $\alpha_n \leq \varepsilon_n$ with $\alpha_n \cap \omega_n = \tilde{0}$ and thus $\omega_n = \tilde{0}$ a contradiction. Similarly, if (b) and (b′) hold, then we have $\varepsilon_n = \tilde{0}$ again a contradiction.

Lemma 4. An SVNITS$(\tilde{\mathcal{F}}, \tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, £^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$ is SVNA£-regular iff $[\alpha_n]_{\delta£} = [\alpha_n]_{\theta£}$ for every $\alpha_n \in \zeta^{\tilde{\mathcal{F}}}$.

Proof. Obvious.

Theorem 16. Let $(\tilde{\mathcal{F}}, \tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, £^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$ be an SVNITS, $\alpha_n \in \zeta^{\tilde{\mathcal{F}}}$, $r \in \zeta_0$. If $(\tilde{\mathcal{F}}, \tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, £^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$ is SVNA£-regular and α_n is single-valued neutrosophic $\theta£$-connected set, then α_n is single-valued neutrosophic $\delta£$-connected set.

Proof. Follows easily by virtue of Lemma 4.

Corollary 1. For a $\alpha_n \in \zeta^{\tilde{\mathcal{F}}}$ of SVNA£-regular space $(\tilde{\mathcal{F}}, \tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, £^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$, the following are equivalent:

(a) α_n is r-single-valued neutrosophic connected,
(b) α_n is r-single-valued neutrosophic $\delta£$-connected,
(c) α_n is r-single-valued neutrosophic $\theta£$-connected.

Proof. Follows easily by virtue of Theorem 16.

6 Conclusion

The neutrosophic set theory has been established and applied extensively to many problems involving uncertainties. Herein, we provided clear definitions of single-valued neutrosophic operators $\mathrm{CI}^{\theta£}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}$ and $\mathrm{CI}^{\delta£}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}$ created from an SVNI topological space $\left(\tilde{\mathcal{F}}, \tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, £^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}\right)$ and we established that $\mathrm{CI}^{\delta£}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\alpha_n, r) = \mathrm{CI}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\alpha_n, r)$ when $£^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}} = £_0^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}$. In addition, we presented the idea of r-single-valued neutrosophic $\theta£$-connectedness based on a single-valued neutrosophic ideal $£^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}$ which has kindred with a preceding r-single-valued neutrosophic connectedness and the relationships among them are inspected. Moreover, we introduced an r-single-valued neutrosophic $\delta£$-connectedness connected to a single-valued neutrosophic δ on the set $\tilde{\mathcal{F}}$ and analyzed some of their properties. This study not only provides a hypothetical basis for additional requests in neutrosophic topology, but also for the expansion of other methodical aspects.

Discussion for further works:

The current concept can be extended by

- Investigating neutrosophic metric topological spaces;
- Investigating the products of connected and Hausdorff spaces for $(\tilde{\mathcal{F}}, \tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, £^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$.

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Analyzing the Sustainability of Industry Affected in COVID-19 Pandemic Scenario Using Cosine Similarity Measure under SVPNS and PNN Model

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Abstract

COVID-19 outbreak is a reminder of the fact that the pandemics have happened in the past and will also occur in the future. The COVID-19 not only has affected the economy; but also it has affected the livelihood, which leads to the changes in businesses. This study aims to identify the most significant indicator (or parameter) that impacts the sustainability of industries. The study should also develop a real-time monitoring system for the sustainability of industries affected by COVID 19. In this work, the Polynomial Neural Network (PNN) and cosine similarity measure under SVPNS (Single-Valued Pentapartitioned Neutrosophic Set) environment have found their use in analyzing the sustainability of the industry.

Keywords: COVID-19; SVPNS; cosine similarity measure; Polynomial Neural Network; sustainability.

1. Introduction

The Currently, a lethal virus of the name Corona Virus is engulfing the whole of the world. Both economic outcomes and social outcomes are jeopardised owing to the said virus [1]. WHO, i.e. World Health Organization says that it is a widespread disease [2]. As per the report rendered to this organization, a great many cases of pneumonia because of a cause which was not known were reported to have been in the city of Wuhan belonging to China on the 31st day of the month of December of the year of 2019 [3, 4]. In the month of January 2020, the virus which enabled the outbreak to appear was detected, and 2019 Novel Corona Virus is the name attributed to it. In the month of February of 2020, Corona Virus Disease 2019 is the name which was attributed to this virus by WHO because of its appearance in the year of 2019 [5]. Human beings‟ lungs are its first prey, and the symptoms pneumonia and influenza are impregnated with are had by it [6]. How COVID-19 transmits from one man to another is unknown, but its transmission method is like other respiratory diseases‟ transmission method [7, 8]. Environmental factors play a pivotal role in its movement [9]. Reports say that till 10th August 2020, the number of confirmed cases are 20,025,245 out of which 733,997 expired and 12,899,801 came round [10]. The health as well as livelihood, especially employment of people are put at stake by the COVID-19 [11]. Companies are precluded from smoothly running consequent upon lockdown‟s having been imposed by many a country. As UNESCO reports universities, colleges and, schools have been closed by more than 100 countries, and consequently the education of the majority of the students of the world has got adversely affected [12]. The environment has been permitted to restore because of this lockdown, which closes industries and reduces the of vehicles, thus improving the quality of the air of the world.

1.1. Motivation:

The International Labour Organization gauged that more than 400 million human beings will lose their full-time jobs due to this pandemic [13]. Many industries are unable to run smoothly in this pandemic situation. Many developed countries like G7 countries, sharing 65% of the manufacturing of the world, 60% of the world,,s demand and supply (GDP), and 41% of world exports are overdone by the pandemic same as other countries. Besides developed countries, many developing countries are also fighting to give wages to the factory workers of their nations. So, people are becoming jobless. There are various factors responsible that have emerged due to COVID-19 for the downfall of the industrial sector around the world. As a result, in this work, we have studied various parameters responsible for the risk in the sustainability of the industry. Although all the parameters are responsible, in reality, all these parameters are not equally responsible for the downfall of a company. So, our objective is to identify the most significant indicator (or parameter) which influences the sustainability of the industries, to suffer a lot. Also in this study, we develop a real-time system for monitoring the sustainability of any industry affected by COVID 19 as the impacts of corona are long term. Moreover, every parameter which influences the sustainability of the industries is not equally responsible for the changing of the performances of the industries. An automated framework is designed to estimate the value of the function continues to represent sustainability using PNN based GMDH training algorithm. To develop a continuous sustainability monitoring system in real-time by applying cosine similarity measure under SVPNS method and a PNN-based predictive model, we developed an approach to evaluating sustainability in real-time. Using such a method provides many advantages, including the consistency and accuracy of the predictive models developed based on the technique, in comparison to conventional linear or non-linear predictive models. In the predictive model, it is possible to assess sustainability unbiasedly since the indicator's priority is hidden and unchangeable.

1.2. Novelty:

The novelty of this work lies in three folds. Firstly, we have analyzed the problem dealing with the sustainability of any industry in the recent worst phase of COVID-19 in the light of the economy, efficiency, and location of the industry, which are immensely affected by this COVID-19. Secondly, for the first time, we have applied the cosine similarity measure under SVPNS-environment to analyze the sustainability of the industry. Lastly, we have introduced a framework for the continuous assessment of sustainability for any industry affected by COVID-19.

2. Literature Review

In 1965, Zadeh invented the fuzzy set (FS) to deal with real-world problems involving uncertainty [14]. The membership value of each element in an FS lies within the interval [0, 1]. As a consequence, Atanassov found that establishing the notion of the intuitionistic fuzzy set (IFS) by generalizing the notion of the fuzzy set (FS) is crucial for reducing the uncertainty in problems [15]. There are two types of membership, membership, and non-membership in every IFS, and both values lie in the interval [0, 1]. Numerous researchers around the world have applied FS, IFS, and their extensions in both theoretical and practical research. The idea of crisp sets, FS, and IFS cannot explain the indeterminacy part of uncertainty events. Consider the fact that Smarandache [16] generalized the concept of FS and IFS to deal with uncertain events having indeterminacy to formulate the neutrosophic set (NS). NS has three types of membership values: truth, indeterminacy, and false. Each value is between 0 and 1. A single-valued neutrosophic set (SVNS) was introduced by Wang et al. [17] in 2010 by extending the concept of NS. The SVNS concept is more effective for events with indeterminate information and uncertainty.

Mallick and Pramanik developed the concept of single-valued pentapartitioned neutrosophic set (SVPNS) by dividing the indeterminacy membership function into three independent membership functions, namely contradiction membership function, ignorance membership function, and unknown membership function [18].

Based on the cosine similarity measure under SVPNS-environment, Majumder et al. [19] identified the most significant COVID-19 risk factors in the economy in 2021. An analysis of COVID-19 medical diagnosis with a decision-making method in a single-valued neutrosophic soft set setting in 2022 was performed by Das et al. [20].

The use of new Plithogenic sub cognitive maps in the diagnostic model for COVID-19 with mediated effects of factors [21]. An intuitionistic fuzzy logic approach to the prediction of Covid-19 time series [22]. The use of fuzzy inference to make decisions regarding Corona's prognosis [23]. After the pandemic,

warehouse location decisions in the medical sector changed [24]. Modeling and predicting COVID-19 time series using a hybrid intelligent parameter tuning approach [25]. Using q-Rung Orthopair Fuzzy Soft Bonferroni Mean Operators to Minimize the Death Rate Due to COVID-19 [26]. The TOPSIS method is used to predict human behavior [27]. A model for pattern recognition using the Fermatean fuzzy modified composite relation [28].

3. Definitions of Relevant Terms:

Here is an overview of some of the results and definitions.

The discourse sphere is V. This is the definition of an SVPNS [1]:

$\square'' = \{(j'', \zeta_{\square''}(j''), \Xi_{\square''}(j''), \aleph_{\square''}(j''), \ell_{\square''}(j''), \hat{\lambda}_{\square''}(j'')) : j'' \in \psi'''\}$.

Here, $\zeta_{\square''}(j''), \Xi_{\square''}(j''), \aleph_{\square''}(j''), \ell_{\square''}(j'')$ and $\hat{\lambda}_{\square''}(j'')$

Are the truth, contradiction, ignorance, unknown and false membership functions from V to the unit interval [0, 1] respectively i.e., $\zeta_{\square''}(j''), \Xi_{\square''}(j''), \aleph_{\square''}(j''), \ell_{\square''}(j'')$ and $\hat{\lambda}_{\square''}(j'') \in [0,1]$, for each $j'' \in \psi'''$. So, $0 \le \zeta_{\square''}(j'') + \Xi_{\square''}(j'') + \aleph_{\square''}(j'') + \ell_{\square''}(j'') + \hat{\lambda}_{\square''}(j'') \le 1$, for each $j'' \in \Psi'''$.

Based on a fixed set AA, the following will be the norms for the null SVPNS (0PN) and the absolute SVPNS (1PN) [42].

(*a*) $1_{PN} = \{(j'', 1, 1, 0, 0, 0) : j'' \in \psi'''\}$,

(*b*) $0_{PN} = \{(j'', 0, 0, 1, 1, 1) : j'' \in \psi'''\}$.

Let

$\square'' = \{(j'', \zeta_{\square''}(j''), \Xi_{\square''}(j''), \aleph_{\square''}(j''), \ell_{\square''}(j''), \hat{\lambda}_{\square''}(j''))) : j'' \in \psi'''\}$,

$\square'' = \{(j'', \zeta_{\square''}(j''), \Xi_{\square''}(j''), \aleph_{\square''}(j''), \ell_{\square''}(j''), \hat{\lambda}_{\square''}(j''))) : j'' \in \psi'''\}$ be any two [42] SVPNSs over *V*. Then,

(*a*) $\square'' \subseteq \square''$ if and only if $\zeta_{\square''}(j'') \le \zeta_{\square''}(j'')$, $\Xi_{\square''}(j'') \le \Xi_{\square''}(j'')$, $\aleph_{\square''}(j'') \ge \aleph_{\square''}(j'')$, $\ell_{\square''}(j'') \ge \ell_{\square''}(j'')$, $\hat{\lambda}_{\square''}(j'') \ge \hat{\lambda}_{\square''}(j'')$, for all $j'' \in \psi'''$.

(*b*) $\square''^{C} = \{(j'', \hat{\lambda}_{\square''}(j''), \ell_{\square''}(j''), 1 - \aleph_{\square''}(j''), \Xi_{\square''}(j''), \zeta_{\square''}(j'')) : j'' \in \psi'''\}$;

(c)

$\square'' \cup \square'' = \left\{\left(j'', \max\{\zeta_{\square''}(j''), \zeta_{\square''}(j'')\}, \max\{\Xi_{\square''}(j''), \Xi_{\square''}(j'')\}, \min\{\aleph_{\square''}(j''), \aleph_{\square''}(j'')\}, \min\{\ell_{\square''}(j''), \ell_{\square''}(j'')\}, \min\{\hat{\lambda}_{\square''}(j''), \hat{\lambda}_{\square''}(j'')\}\right) : j'' \in \psi'''\right\}$

(d)

$\square'' \cap \square'' = \left\{\left(j'', \min\{\zeta_{\square''}(j''), \zeta_{\square''}(j'')\}, \min\{\Xi_{\square''}(j''), \Xi_{\square''}(j'')\}, \max\{\aleph_{\square''}(j''), \aleph_{\square''}(j'')\}, \max\{\ell_{\square''}(j''), \ell_{\square''}(j'')\}, \max\{\hat{\lambda}_{\square''}(j''), \hat{\lambda}_{\square''}(j'')\}\right) : j'' \in \psi'''\right\}$

Consider the case where $\square''$ = {(r‟, 0.71, 0.41, 0.51, 0.11, 0.31), (s‟, 0.61, 0.31, 0.41, 0.31, 0.21)} and $\square''$ = {(r‟, 0.81, 0.51, 0.61, 0.21, 0.51), (s‟, 0.81, 0.51, 0.61, 0.41, 0.41)} would correspond to two SVPNSs over a set of discourses V = {r‟, s‟}.

Then,

(*i*) $\square'' \subseteq \square''$;

(*ii*) $\square''^{C}$ = {(*r'*, 0.31, 0. 11, 0.51, 0.41, 0.71), (*s'*, 0.21, 0.31, 0.61, 0.31, 0.41)} and $\square''^{C}$ = {(*r'*, 0.51, 0.21, 0.41, 0.51, 0.21), (*s'*, 0.41, 0.41, 0.41, 0.51, 0.81)};

(*iii*) $\square'' \cup \square''$ = {(*r',* 0.81, 0.51, 0.51, 0.11, 0.31), (*s'*, 0.81, 0.51, 0.41, 0.31, 0.21)};

(*iv*) $\square'' \cap \square''$ = {(*r'*, 0.71, 0.41, 0.61, 0.21, 0.51), (*s'*, 0.61, 0.31, 0.61, 0.41, 0.41)}.

4. Method Used:

There are two parts to this section. The first sub-section 3.1 discusses decision-making process, and the second sub-section 3.2 discusses the PNN model.

4.1. MADM Strategy Using SVPNWESM under SVPNS Environment:

With the help of SVPNWESM, an MADM model is proposed to be made under SVPNS environment in this section.

Let us consider in a MADM problem two sets of alternatives and attributes $\Gamma = \{\square_1, \square_2, \square_3, ..., \square_n\}$ and $\rho = \{\varphi_1, \varphi_2, \varphi_3, ..., \varphi_m\}$. SVPNS can then be used by decision-makers in order to evaluate each alternat $\square_i (i = 1(1)m)$ compared to each attribute $\varphi_j (j = 1(1)k)$. A decision matrix (DM) can be created based on

the whole evaluation information provided by the decision-maker. The MADM Strategy based on SVPNS is depicted in Figure 1.

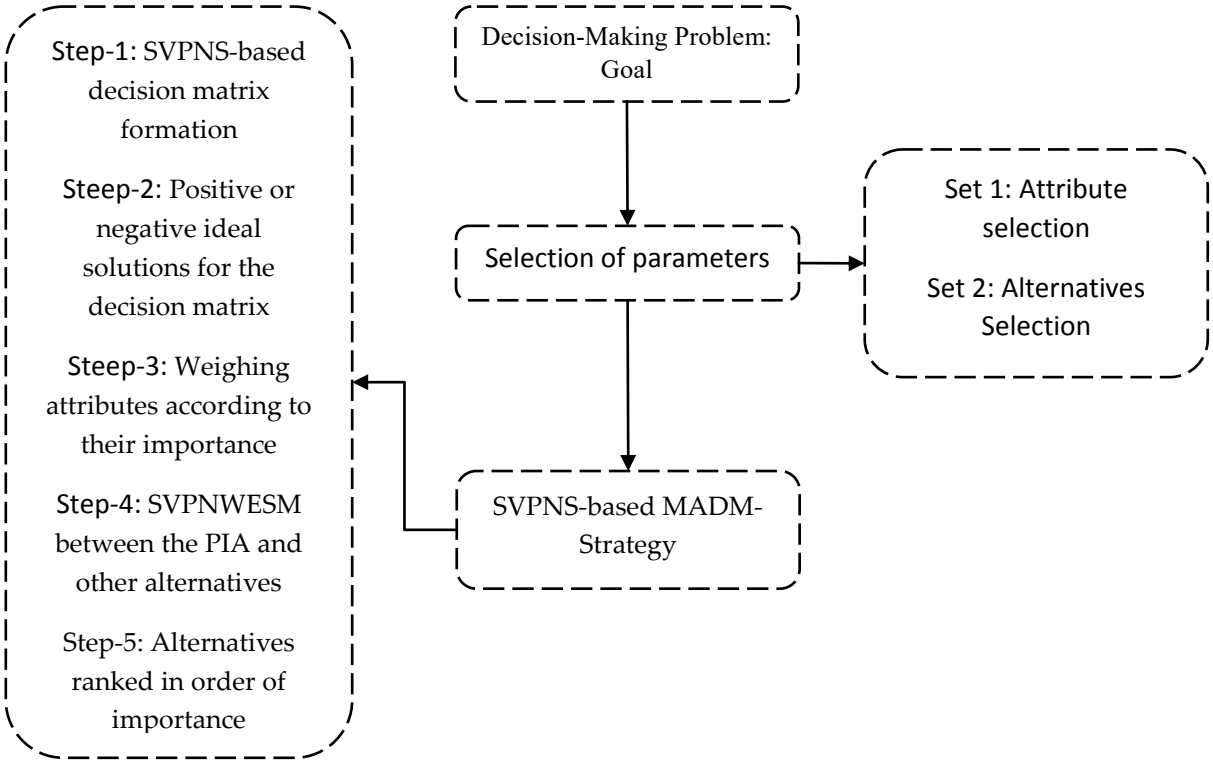
Step-1: SVPNS-based decision matrix formation
Steep-2: Positive or negative ideal solutions for the decision matrix
Steep-3: Weighing attributes according to their importance
Step-4: SVPNWESM between the PIA and other alternatives
Step-5: Alternatives ranked in order of importance
Decision-Making Problem: Goal
Selection of parameters
Set 1: Attribute selection
Set 2: Alternatives Selection
SVPNS-based MADM-Strategy

Figure 1: MADM-Strategy based on SVPNS

Step-1: Decision matrix (DM) formation using SVPNS.

The construction of a decision matrix through making use of the evaluation information which is $\mathrm{M}_{\square_i}=\left\{\left(\varphi_j,\alpha_{\varphi_{ij}}\left(\square_i,\varphi_j\right),\beta_{\varphi_{ij}}\left(\square_i,\varphi_j\right),\delta_{\varphi_{ij}}\left(\square_i,\varphi_j\right),\gamma_{\varphi_{ij}}\left(\square_i,\varphi_j\right),\lambda_{\varphi_{ij}}\left(\square_i,\varphi_j\right)\right):\varphi_j\in\rho\right\}$ for each alternative $\square_i\left(i=1(1)m\right)$ with respect to the attributes $\varphi_j\left(j=1(1)k\right)$, in which $\left(\varphi_j,\alpha_{\varphi_{ij}}\left(\square_i,\varphi_j\right),\beta_{\varphi_{ij}}\left(\square_i,\varphi_j\right),\delta_{\varphi_{ij}}\left(\square_i,\varphi_j\right),\gamma_{\varphi_{ij}}\left(\square_i,\varphi_j\right),\lambda_{\varphi_{ij}}\left(\square_i,\varphi_j\right)\right)=\left(\square_i,\varphi_j\right)$ (say) $\left(i=1(1)m \text{ and } j=1(1)k\right)$ refers to the decision maker's evaluation for alternatives $\square_i\left(i=1(1)m\right)$ with respect to the attribute $\varphi_j\left(j=1(1)k\right)$.

These are DM expressions:

$$\begin{array}{c} \\ \square_1 \\ \square_2 \\ \vdots \\ \square_m \end{array}\begin{array}{c} \begin{array}{cccc} \varphi_1 & \varphi_2 & \cdots & \varphi_k \end{array} \\ \begin{pmatrix} \left(\square_1,\varphi_1\right) & \left(\square_1,\varphi_2\right) & \cdots & \left(\square_1,\varphi_k\right) \\ \left(\square_2,\varphi_1\right) & \left(\square_2,\varphi_2\right) & \cdots & \left(\square_2,\varphi_k\right) \\ \vdots & \vdots & \vdots & \vdots \\ \left(\square_m,\varphi_1\right) & \left(\square_m,\varphi_2\right) & \cdots & \left(\square_m,\varphi_k\right) \end{pmatrix} \end{array}$$

Step-2. Determination of the Weights for Each Attribute.

It is important to determine the weights for every attribute in every MADM strategy. Using the compromise function, a decision-maker can determine the weights for each attribute provided that the

attributes" weights are not known. For ℓ_j and $\square_j$, the compromise function can be defined as follows:

$$\ell_j = \sum_{i=1}^{m} \frac{\left\lfloor 3 + \alpha_{\varphi_{ij}}\left(\square_i, \varphi_j\right) + \beta_{\varphi_{ij}}\left(\square_i, \varphi_j\right) - \delta_{\varphi_{ij}}\left(\square_i, \varphi_j\right) - \gamma_{\varphi_{ij}}\left(\square_i, \varphi_j\right) - \lambda_{\varphi_{ij}}\left(\square_i, \varphi_j\right)\right\rfloor}{5} \quad \text{.....(1)}$$

The j-th attribute's weight is defined by

$$p_j = \frac{\ell_j}{\sum_{j=1}^{k} \ell_j} \quad \text{.....(2)}$$

Here $\sum_{j=1}^{k} p_j = 1$.

Step-3. Positive Ideal Alternative (PIA) selection.

A decision-maker can use the maximum operator for all the attributes in this step to create a PIA.

The PIA ∂ consists of the following:

$$\partial = \left(\pi_1, \pi_2, ..., \pi_k\right) \quad \text{.....(3)}$$

$$\text{Where } \pi_j = \left(\max\left\{\alpha_{\varphi_{ij}}\left(\square_i, \varphi_j\right) : i = 1(1)m\right\}, \max\left\{\beta_{\varphi_{ij}}\left(\square_i, \varphi_j\right) : i = 1(1)m\right\}, \min\left\{\delta_{\varphi_{ij}}\left(\square_i, \varphi_j\right) : i = 1(1)m\right\},\right.$$
$$\left.\min\left\{\gamma_{\varphi_{ij}}\left(\square_i, \varphi_j\right) : i = 1(1)m\right\}, \min\left\{\lambda_{\varphi_{ij}}\left(\square_i, \varphi_j\right) : i = 1(1)m\right\}\right), \; j = 1(1)k \quad \text{.....(4)}$$

Step-4. Accumulated Measures Values Determination.

To aggregate the SVPNCSM for each alternative, use the accumulated measure function (AMF).

As defined by the AMF,

$$\mathrm{M}_{AMF}\left(\square_i\right) = \sum_{j=1}^{k} p_j \times P_{SVPNCSM}\left(\left(\square_i, \varphi_j\right), \pi_j\right) \quad \text{.....(5)}$$

Where

$$P_{SVPNCSM}\left(\square, \varphi\right) = 1 - \frac{1}{n}\sum_{\phi \in U} \cos\left\lfloor \frac{\pi}{10}\left(\begin{array}{l}\left|\alpha_{\square}(\phi) - \alpha_{\varphi}(\phi)\right| + \left|\beta_{\square}(\phi) - \beta_{\varphi}(\phi)\right| + \left|\delta_{\square}(\phi) - \delta_{\varphi}(\phi)\right| \\ + \left|\gamma_{\square}(\phi) - \gamma_{\varphi}(\phi)\right| + \left|\lambda_{\square}(\phi) - \lambda_{\varphi}(\phi)\right|\end{array}\right)\right\rfloor \text{ and}$$

$$\left(\square_i, \varphi_j\right) = \left(\varphi_j, \alpha_{\varphi_{ij}}\left(\square_i, \varphi_j\right), \beta_{\varphi_{ij}}\left(\square_i, \varphi_j\right), \delta_{\varphi_{ij}}\left(\square_i, \varphi_j\right), \gamma_{\varphi_{ij}}\left(\square_i, \varphi_j\right), \lambda_{\varphi_{ij}}\left(\square_i, \varphi_j\right)\right)$$

Step-5. Alternatives ranked.

The last step is to rank the alternatives descending in the accumulated measure value order. Alternatives which are linked to the lowest accumulated measure value are most apt.

4.2. Polynomial Neural Network (PNN):

Polynomial Neural Networks (PNN) is the most upgraded version of the Artificial Neural Network (ANN). The Group Method of Data Handling (GMDH) algorithm is used to train the PNN network which was firstly suggested by Ivakhnenko [29]. The merits of the technique are the subsequent addition of a new layer and this step will be repeated up to the meeting of the termination criteria. The demerit of GMDH lies in the complexity in the network that is required for the development of relatively simple systems. The governing equation of GMDH is

$$u_h = P\left(y_i, y_j\right) = a_1 + a_2 y_i + a_3 y_j + a_4 y_i y_j + a_5 {y_j}^2 + a_6 {y_j}^2 \ldots\ldots\ldots\ldots(6),\ h = 1,2,\ldots, p$$

where k is the total number of input. The data are divided into testing and training data sets. The testing data set finds its use to determine the estimated GMDH model and the training data set is utilized to design a GMDH model. Development of $p = \frac{k(k-1)}{2}$ new variables $\{u_1, u_2, \ldots, u_k\}$ in the training data set for all independent variables and selection of partial descriptions (PD) of the GMDH. Conventional GMDH is updated using the polynomial represented by the equation 9 as PD.

5. Methodology:

The most important aim of this study is the construction of a framework to assess stability, estimating the economic stability of an industry at a real-time for the reduction of the time in general required in the judgment of the stability of an arrangement in COVID-19 pandemic situation. In this field of study, a framework is developed using the features representing the performance stability of an industry as a mapping of weight of the important indicators.

Entire methodology is ordered in three stages. First stage includes selecting and arranging alternatives depending upon the importance of theirs and then contributing to represent the stability of any company. Stability is a term which is reciprocal of risk. The risk is the difference between a system‟s actual stability and intended stability. In a company there are some parameters which are specific and exert an influence on the system‟s performance. Similar parameters are trustworthy for deviation in the framework execution from the evaluated presentation albeit all the boundaries are not similarly predominant in affecting the framework execution. However upon all the alternatives is dependent is the performance‟s being evaluated, which not only consumes time but also expensive. RTM (Real-Time Monitoring) is, at any point in time able to monitor of the company‟s stability provided that in the said, system an automated model becomes designed and gets embedded in order to, via making use of the real time data of the alternatives which are included.

MCDM method‟s being applied in order to estimate the relative importance of the indicators select, which is made use of for sorting the indicators select that are included in the aggregation function is provided in section 4.1. The automated framework‟s progress through making use of what is known as PNN technique in order for the single function‟s being estimated becomes, in Section 4.2, discussed. Section 4.3 delineates prototype RTM‟s development.

5.1.Application of MCDM:

The focus of this study is the identification of the most significant indicator responsible for the sustainability of a company in the COVID-19 pandemic scenario. Therefore, three sub-sections in total are had by MCDM‟s being applied. Sub-section 5.1.1 depicts the process involved in selecting criteria and alternatives. The pairwise comparison between the criteria and alternatives is discussed in sub-section 5.1.2. Sub-section 5.1.3 provides the construction of aggregation function which is proportional to the stability of a company.

5.1.1. Selection of Parameter:

The parameters, mainly responsible for the sustainability of the company in the COVID-19 pandemic situation are chosen using an expert survey. Observations from the surveys are compared with the obtained results of the review of the related literature [30, 31].

What is shown by Figure 2 is the reason lying behind selecting the criteria for comparing the alternatives by making use of the decision-making goal attempted to be reached by the investigation on the tapis. Identified parameters with the prospective for the purpose of transmogrifying the indicators select become as the criteria in order to compare the alternatives. The company‟s stability gets changed by all of the parameters which are capable of transforming the indicators select potential to change the select.

Here, in this case, we consider three criteria namely cost, efficiency and location, and six alternatives which are manpower, state of the machine, raw materials, sales, assets, and liabilities [12, 30, 31, 32, 33]. Due to the widespread of corona the industries have to shut down and as a result, they are forced to fire their workers. Moreover, many workers come to their native places and many have died on the roads due to a shortage of food. Transport facilities are affected causing a massive problem in the sales and importing the raw materials. Due to the shortage of manpower and raw materials many machines are shut down and as a result, due to the lack of their operation, it is a matter of concern about their state. As the number of sales

comes down, the liabilities of the company will rise up which ultimately affects the assets of the company [30].

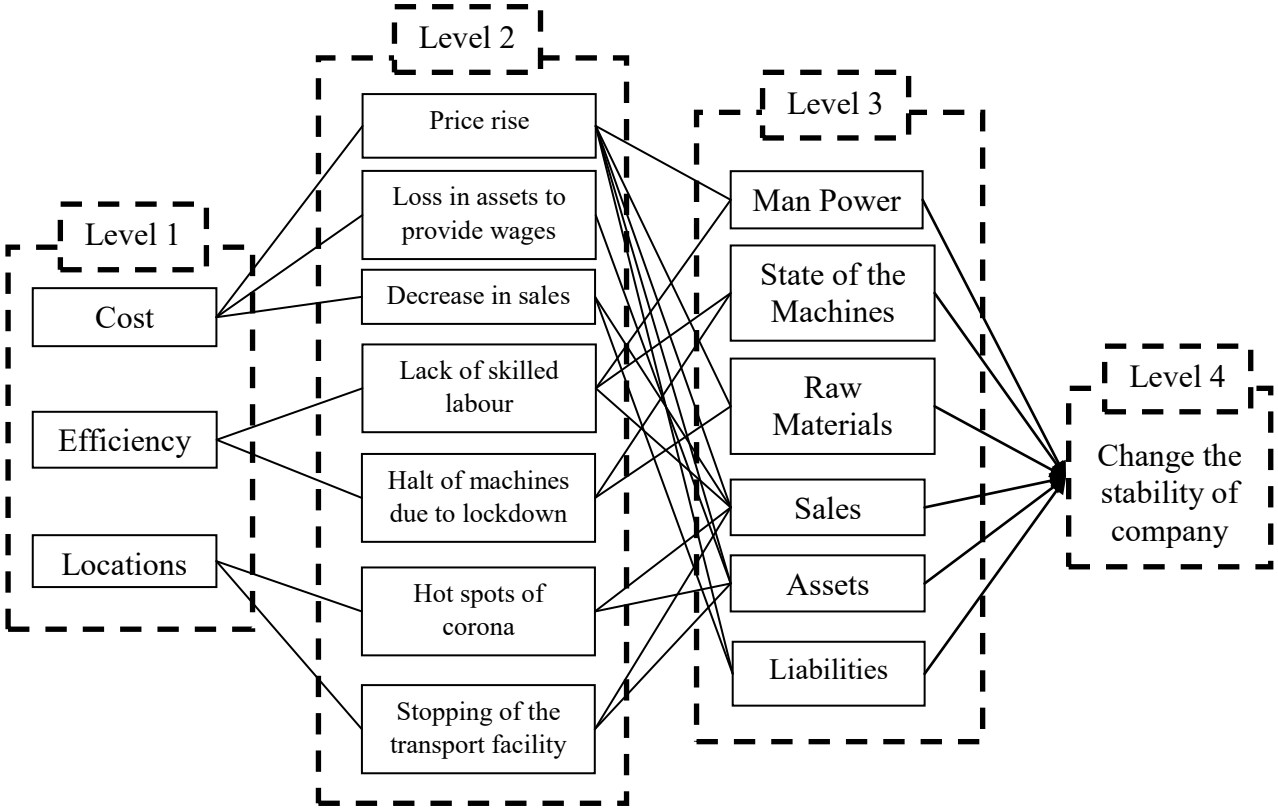

Figure 2: Justification of selection of criteria and alternatives in COVID-19 pandemic situation

5.1.2. Application of cosine similarity measure:

In the study on the tapis, as well as each indicator‟s local weight (or PV) according to each criterion [15] [19]. Present study considers three criteria (Cost (φ_1), Efficiency (φ_2) and Locations (φ_3)) and six alternative alternatives (Man Power ($\square_1$), State of the Machines ($\square_2$), Assets ($\square_3$), Raw Materials ($\square_4$), Liabilities ($\square_5$) and Sales ($\square_6$))[12, 30, 31, 32, 33]. MADM strategy which is proposed is as follows vouchsafed:

The decision matrix is prepared based on the decision-makers‟ evaluation information as has been shown in Table 1.

Table 1: Decision Matrix

	φ_1	φ_2	φ_3
$\square_1$	(1, 0.2, 0.2, 0, 0)	(0.8, 0.1, 0.1, 0.1, 0.1)	(0.9, 0, 0.2, 0.1, 0.1)
$\square_2$	(1, 0.1, 0.2, 0.1, 0.2)	(0.9, 0.1, 0.2, 0.2, 0.1)	(0.8, 0.1, 0, 0, 0.1)
$\square_3$	(1, 0.1, 0, 0, 0.1)	(0.9, 0, 0.1, 0.1, 0.2)	(0.9, 0.1, 0.1, 0.1, 0.1)
$\square_4$	(0.9, 0.2, 0.1, 0.1, 0)	(1, 0.1, 0, 0, 0.1)	(0.9, 0.1, 0.1, 0.1, 0.1)
$\square_5$	(1, 0.2, 0.1, 0.1, 0.1)	(0.8, 0.1, 0, 0, 0.1)	(0.7, 0.2, 0, 0.1, 0.1)
$\square_6$	(0.8, 0.1, 0.1, 0.2, 0.1)	(1, 0, 0.1, 0.2, 0.1)	(0.9, 0, 0.2, 0.1, 0.1)

5.1.3. Development of Aggregation Function:

After the completion of the estimation of the weight and the results were validated, the indicators‟ magnitude and weight had been made use of in order for the index representing a company‟s stability to be

estimated and equation 7 depicts it. A company"s stability is coherently represented by the way S-mapping had been made. The more the S-mapping"s value, the more company"s stability, which is implied by that.

$$S = \frac{e^{aI}}{1-e^{-aI}} \text{.....(7)}, a \in \square^{+}$$

$$\text{Where, } I = \frac{I_1}{I_2}, I_1 = \frac{\sum_B p_B x_B}{\sum_B p_B}, I_2 = \frac{\sum_{NB} p_{NB} x_{NB}}{\sum_{NB} p_{NB}}$$

Beneficial (B) as well as non-beneficial (NB) indicators" weighted average is here represented by I_1 and I_2 respectively. B as well as NB indicators" PV is represented by p_B and p_{NB} respectively, while the said indicators" magnitude value is represented in the normalized form by x_B and x_{NB}. φ_i's magnitude is here denoted by x_i. In the present study, the indicators, namely assets, sales, raw materials, and state of machines are beneficial, while the indicators liability and manpower are not beneficial.

5.2. Application of PNN:

An objective function is minimized during training or learning of optimum condition weights in PNN models. In the PNN model, the optimum condition weights are determined by minimizing an objective function. In order to develop a PNN model, a training dataset is required, which can be denoted as $T_{DATA} = \{x_i : x_i \in [0,1]\}$. Based on the design of the experiment approach, the training data was generated by generating 1000 different combinations of inputs, for which Equation 7 calculates the corresponding output. The weights of the connections of the neural network are randomly generated during training, and the estimated output is compared with the given result. Using a specific technique, the difference between the estimated and actual outputs was minimized. The model was trained using the Group Method of Data Handling. The training results were taken once the minimum difference was identified. In order to predict the output, the weight thus estimated was used as a model. The PNN will find a cognitive equation for estimating the sustainability of the industry. Equation (7) will be encoded in the control center of the interconnected sensors. An algorithm will be set to execute an if-then rule so that the RTM behaves as desired.

6. Result and Discussion:

This study investigates the most significant indicator of the sustainability of any company as well as the monitoring of the sustainability of any industry in real-time. In these aspects, the result of present study divided into four parts namely result from MCDM, parameter estimation results from PNN model and real-time monitoring of sustainability of industry and discussion which are described in section 5.1, 5.2 and 5.3 respectively.

6.1. Result from MCDM:

Now, by using equation (1) and (2), the weights $p_1 = 0.343$, $p_2 = 0.329$, and $p_3 = 0.328$ are gotten. Making use of equation (3) and (4) has formed PIA, which Table-2 shows as follows:

Table 2: PIA value

	φ_1	φ_2	φ_3
∂	(1.0,0.2,0.0,0.0,0.0)	(1.0,0.1,0.0,0.0,0.1)	(0.9,0.2,0.0,0.0,0.1)

Presently, by utilizing condition (5), we work out the collected measure upsides of every alternative $\square_1$, $\square_2$, $\square_3$, $\square_4$, $\square_5$ and $\square_6$ as follows:

$$P_{AMF}(\square_1) = 0.90141$$

$P_{AMF}(\square_2) = 0.901754$

$P_{AMF}(\square_3) = 0.901298$

$P_{AMF}(\square_4) = 0.90098$

$P_{AMF}(\square_5) = 0.901044$

$P_{AMF}(\square_6) = 0.902163$

The following is the accumulated measure values‟ order: $P_{AMF}(\square_6) > P_{AMF}(\square_2) > P_{AMF}(\square_1) > P_{AMF}(\square_3) > P_{AMF}(\square_5) > P_{AMF}(\square_4)$. According to the result, we can see that indicate "sales" is the more responsible for the sustainability of an industry.

6.2. Results of estimation of parameter obtained from PNN model:

PNN models are intended to predict S-functions (Equation 7), which indicate the manageability of an industry. Four factors are taken into account when planning the PNN model: hidden layers, enactment work from contribution to covering up to covering up to entry, and the loads of association among neural systems. As part of the preparation stage, boundaries for parameters are set based on a sample informational index or the preparation informational index, and the prepared model is tested with informational indexes that were not used during preparation and with the aid of the informational collection, the model is tested for accuracy. For building up the most precise model, PNN models can recognize the number of shrouded layers and the number of contributions with the assistance of a wellness work and the preparation informational collection. The preparation calculations, such as GMDH, are used to understand the association's load. The model was tested for accuracy once the association's loads were predetermined. The model was tested ion and testing stage, the Absolute Relative Error (ARE) was used as the wellness capacity of exactness for the created model. In Figure 3, real and anticipated S-functions are examined. Show that the created model has a normal exactness of 99.51% and 99.85% which portrays that the precision of the model is marginally more in the testing stage contrasted with the preparation stage. The model condition is shown by equation 8.

This condition was encoded in the control arrangement of the RTM. The advantage of this condition was it can cover up the weightage of significance allocated to the pointer and subsequently unprejudiced forecast of manageability can be conceivable.

S = 0.00194672 - N11*0.585287 + N3*1.58468.....(8)

N3 = 0.364709 - N465*0.117088 + N4*1.00387

N4 = 0.0113612 - N193*0.171125 + N8*1.1676

N8 = 0.0462697 - N305*0.0695867 + N16*1.05522

N16 = -0.467105 + N455*0.161786 + N32*0.98322

N455 = 9.92001 - "x6, cubert"*12.4749 + "x6, cubert"*N498*1.07894

N305 = 2.4685 - N370*0.550213 + N370*N443*0.24387 - N443*0.271623

N193 = 0.974792 + N328*N279*0.0108044 + N279*0.585922

N279 = 1.77231 + N337*N389*0.132478 - N389*0.212736

N389 = -0.24909 + N400*N498*0.338207

N337 = 1.86635 + N370*N461*0.178787 - N461*0.339194

N328 = 1.92819 + N370*N412*0.150056 - N412*0.343041

N370 = 57.5671 - "x1, cubert"*62.6097 + "x1, cubert"*"x6, cubert"*70.5482 - "x6, cubert"*62.8511

N465 = 0.480848 + N477*N494*0.262076

N494 = 1.34928 + "x5, cubert"*N500*0.754855

N477 = 0.205933 + "x4, cubert"*N500*1.22035

N11 = 0.0386873 - N322*0.0460058 + N17*1.034

N17 = -0.45554 + N456*0.157711 + N32*0.983705

N32 = -1.62904 + N478*0.511716 + N124*0.993993

N124 = 0.320162 + N407*0.110294 + N407*N178*0.0381621 + N178*0.606026

N178 = 1.1165 + N284*0.17753 + N284*N340*0.0167736 + N340*0.320193

N340 = 3.01832 - N414*0.998618 + N414*N435*0.591256 - N435*0.834076

N435 = 0.961143 - "x6, cubert"*N443*0.934851 + N443*1.28706

N443 = 0.430114 + "x4, cubert"*13.0173 - "x4, cubert"*"x6, cubert"*12.5971

N414 = -0.138401 + N424*N498*0.326067

N498 = 1.78612 + "x2, cubert"*"x5, cubert"*2.492

N284 = 2.25844 - N365*0.29977 + N365*N366*0.069583

N366 = -8.44548 + "x6, cubert"*11.5603 - "x6, cubert"*N424*6.72422 + N424*5.96334

N365 = 4.30972 - N424*1.45588 + N424*N461*0.749523 - N461*1.28859

N424 = 11.1728 - "x1, cubert"*11.6685 + "x1, cubert"*"x3, cubert"*1.30673

N407 = -2.25843 + N412*0.99435 + N500*0.706746

N412 = 0.90769 - "x1, cubert"*"x5, cubert"*14.9215 + "x5, cubert"*14.2023

N478 = -0.1265 + N491*N500*0.322998

N491 = 0.392842 + "x4, cubert"*3.6826

N456 = -3.16483 + N461*0.999203 + N500*0.983269

N500 = 2.06765 + "x2, cubert"*"x3, cubert"*2.05317

N322 = 3.8446 - N396*1.4466 + N396*N457*0.731271 - N457*1.08228

N457 = 1.33159 - "x6, cubert"*0.284432 - "x6, cubert"*N461*0.774968 + N461*1.1403

N461 = 9.8986 - "x6, cubert"*8.97134

N396 = -1.80904e-12 + N400*0.511926 + N400*N502*0.151515

N502 = 3.22129

N400 = -3.02752 + "x1, cubert"*4.29884 - "x1, cubert"*"x4, cubert"*19.1814 + "x4, cubert"*18.2679

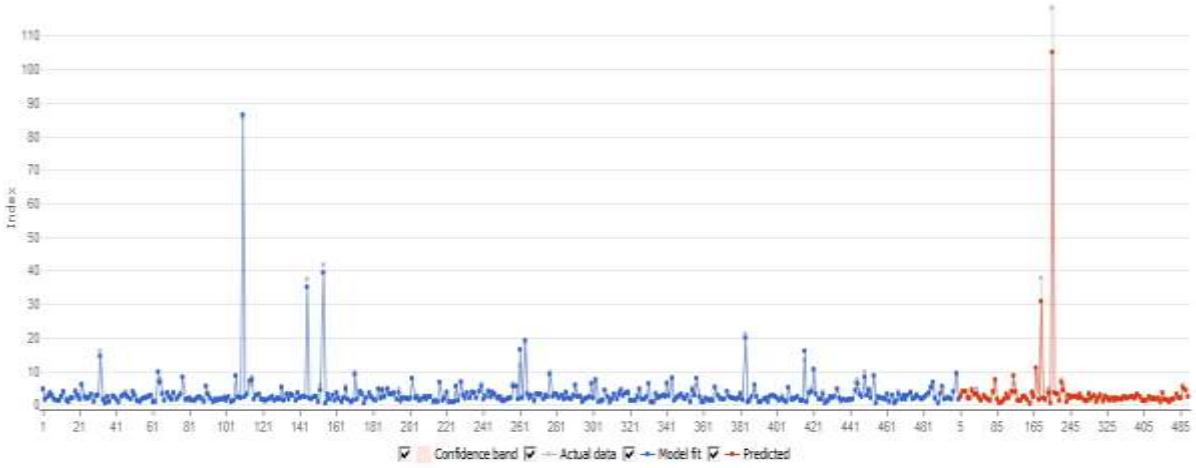

Figure 3: Actual and predicted S-value (using equation 10) from the PNN model during the testing and training phase.

6.3. Real time monitoring of sustainability of an industry:

The Central Control Unit (CCU) here acting as aggregator of the signals using equation 10, and the same data is sent back to the supercomputer to analyse and monitor the sustainability of the industry continuously. The result obtained is sent to CCU, and it passes the signal back to the indicators to adjust their performance indices. The CCU is connected to the supercomputer through the blue line, and the connections between the indicators and CCU are shown by the respective lines. Single arrow line denotes the direct relationship between the indicators and aggregation function and dotted lines denote the inverse relationship between them and a double arrow line indicates the relationship between the indicators. Figure 4 shows the schematic diagram of the real-time monitoring system developed to monitor sustainability of any industry.

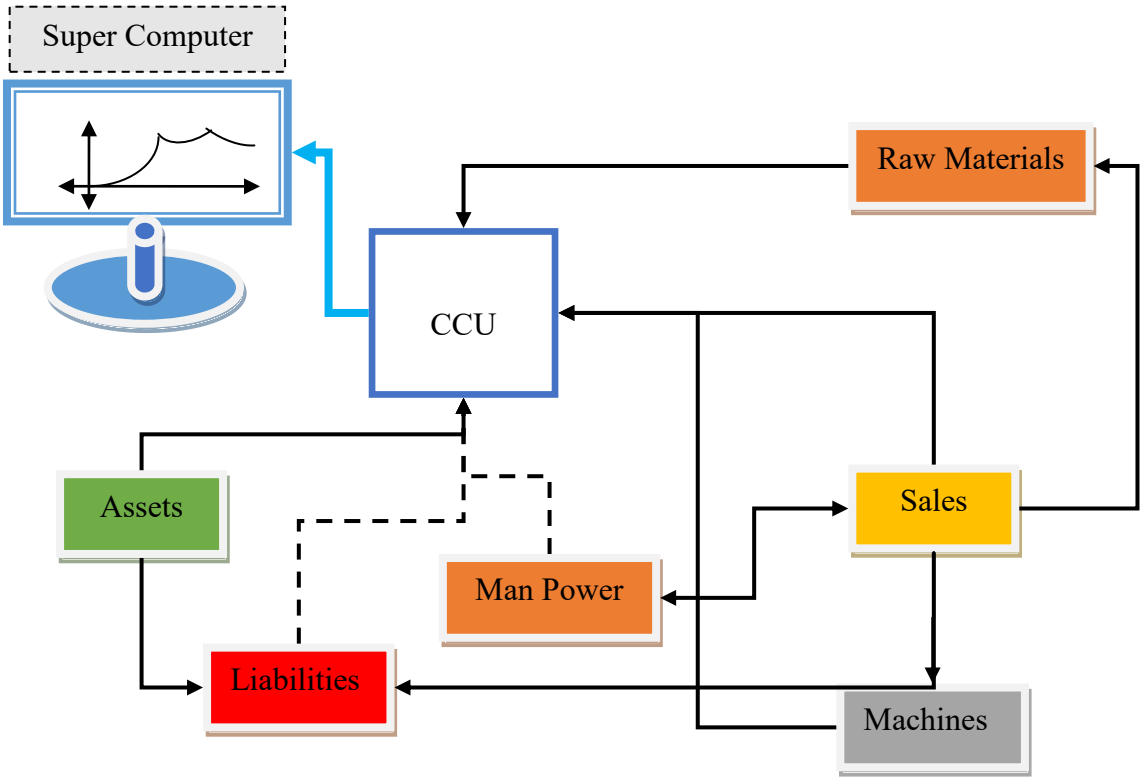

Figure 4: The schematic diagram of the real time monitoring system for monitoring of sustainability of any industry.

7. Conclusion

The COVID-19 pandemic has brought to mankind many difficulties that include the need for new improved medical treatments, social and economical approaches. The present work uses SF-AHP and PNN to analyze the sustainability of any industry to cope up with the impacts of COVID-19. The flow of work starts with the selection of indicators that are responsible for the downfall of the industry and then it is found that 'sales' is the most significant indicator responsible, using cosine similarity measure under SVPNS and PNN. This method is applicable to various problems related decision-making like biotechnology, biological control system, agriculture, etc.

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Modeling Influenced Criteria in Classifiers' Imbalanced Challenges Based on TrSS Bolstered by The Vague Nature of Neutrosophic Theory

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Abstract: Because of the advancements in technology, classification learning has become an essential activity in today's environment. Unfortunately, through the classification process, we noticed that the classifiers are unable to deal with the imbalanced data, which indicates there are many more instances (majority instances) in one class than in another. Identifying an appropriate classifier among the various candidates is a time-consuming and complex effort. Improper selection can hinder the classification model's ability to provide the right outcomes. Also, this operation requires preference among a set of alternatives by a set of criteria. Hence, multi-criteria decision-making (MCDM) methodology is the appropriate methodology can deploy in this problem. Accordingly, we applied MCDM and supported it through harnessing neurotrophic theory as motivators in uncertainty circumstances. Single value Neutrosophic sets (SVNSs) are applied as branch of Neutrosophic theory for evaluating and ranks classifiers and allows experts to select the best classifier So, to select the best classifier (alternative), we use MCDM method called Multi-Attributive Ideal-Real Comparative Analysis (MAIRAC) and the criteria weight calculation method called Stepwise Weight Assessment Ratio Analysis (SWARA) where these methods consider single-value neutrosophic sets (SVNSs) to improve and boost these techniques in uncertain scenarios. All these methods are applied after modeling criteria and its sub-criteria through a novel technique is Tree Soft Sets (TrSS). Ultimately, the findings of leveraging these techniques indicated that the hybrid multi-criteria meta-learner (HML)-based classifier is the best classifier compared to the other compared models.

Keywords: Neutrosophic theory; Multi-Criteria Decision Making; Class Imbalance; Meta-Learner; Ranking Classifiers; Single Values Neutrosophic Sets, Tree Soft Set (Trss).

1. Introduction

Currently, Artificial Intelligence (AI) techniques have been applied in several spheres. As in [1] where AI techniques are leveraged in healthcare, Fiscal fraud [2], and agriculture[3]. As well, machine learning (ML) techniques subset of AI are gleaning valuable knowledge from massive, complicated, diverse, and hierarchical data[4]. Also, ML techniques can be used as a classifier. Just like [5] described classification as ML techniques wherein a computer program learns from historical data

and then applies that knowledge to forecast the class label for data that hasn't yet been observed. These techniques are represented in k Nearest Neighbors (KNN), Support Vector Machines (SVM), and random forest (RF) [5]. Moreover, Johnson et al.[6] affirmed that the majority or minority class is forecasted using binary classification. Besides that [7] categorized data in binary classification into balanced or imbalanced. And [8] demonstrated that the majority of classifiers travail optimally when the response variable's distribution in the dataset is balanced. In spite of that, the techniques mentioned in [5] encounter significant difficulties due to imbalanced data. Due to [9] one class has more instances than the other classes in imbalanced data , and the distribution of classes is skewed toward that class. From perspective of [10] positive instances are typically referred to as the minority class, whereas negative instances are typically called the majority class. Also, [11] indicated that in handling an imbalanced hurdle, the conventional algorithms exhibit a bias in favor of the dominant class.

Hence, [12] exhibited that class imbalance is still a perplexing problem that needs further study to be properly understood and skillfully managed. All that motivated[13] for developing algorithms or techniques that are very dependable and efficient is essential to properly addressing the problems brought on by the imbalanced datasets.

As per the prior literature [13, 14], several techniques have been suggested to tackle the problem of class disparity, and these may be roughly categorized into (1) algorithm-level methods [15]included cost-sensitive learning which employs the expenses of incorrect classifying samples and make an effort to improve the classifiers' favorability for the minority class by adding various cost variables into the algorithms. (2) sampling methods [15] which encompasses over sampling, random under sampling, synthetic minority oversampling technique (SMOTE), and edited nearest neighbour.(3) ensemble learning, this technique applied boosted based methods, pre-processing ensemble, and boosted imbalanced data solving toward endeavor to improve the unbalanced data classification's accuracy by fusing many classifiers to produce a novel, more potent classifier. Additionally, Chamlal et al. [13] proposed Hybrid Multi-criteria Meta-learner (HML) which includes an ensemble-based meta-learner component and a multi-objective optimization component as its two primary parts.

General speaking, selection of optimal and suitable classifier for treating with imbalanced data amongst these techniques is crucial process.

1.1 Motivation of Study

According to the surveys conducted for prior studies. From perspective of [13] it’s difficult to foresee the unpredictable, notably when tackling the problem of class imbalance, which occurs when the training data's class distribution is biased in favor of one particular class. In the same vein [16] demonstrated that the majority and minority samples are included in the imbalance datasets. Comparatively speaking, there are much less sample instances in the minority class than in the majority class. Accordingly, severe skews in the distribution of classes and inadequate rendition of specific data are persistent challenges in many domains as medicine [17], predicting defects for software[18],and in financial services [19].Hence, [20] stated that the performance of conventional classifiers may suffer when there is an imbalanced distribution of classes in a dataset.

Another aspect discussed by [21] in classification issues, depending on only a single criteria is widely used for evaluation. However, the evaluation of only one aspect may mis select the best performance classifier. To select the best performance classifier, several evaluation criteria, including proficiency, time consuming, uniformity, and others, need to be utilized. The multicriteria evaluation aims to achieve a balance between these criteria instead of depending on only one criterion [22] . So, we need an efficient multi-criteria decision-making method that assesses and ranks classifiers and allows experts to select the best classifier for their applications by using the previously mentioned criteria.

1.2 Contribution

Herein, we are evaluating the optimal classifier based on a set of criteria. For conducting this process, we are leveraging MCDM techniques which have ability to treat with such problems. Especially, Stepwise Weight Assessment Ratio Analysis (SWARA) for obtaining criteria's weights. Also, Multi-Attributive Ideal-Real Comparative Analysis (MAIRAC) is applied for ranking the alternatives of classifiers and select optimal classifier.

Neutrosophic theory is deploying in this study and contribute to MCDM techniques for bolstering and supporting expert in ambiguity situations as incomplete data and uncertainty [23]. Due to, ability of this theory to measure membership function as truth (T), also non-membership function false (F) whilst take into consideration indeterminacy (I). Thereby, single value neutrosophic sets (SVNSs) as type of Neutrosophic is implementing in evaluation process.

Also, Tree soft set (TrSS) is leveraging in this problem to model the identified criteria and its sub-criteria in set of nodes which resident into set of levels. TrSS is introduced Smarandache [24] who is founder of this approach as well as introduced Neutrosophic theory.

1.3 Study Outline

This study is organized into a set of sections; each section exhibits the benefits of our study and the followed steps toward achieving study's objectives.

Section one: illustrated the main idea of our study, motivations and the main contributions which are provided through our study. For completing our objectives' study, we conducted survey for prior techniques and studies in section 2. Through the conducted surveys, we determined the effective techniques to treat our problem through conducting SDMM. To validate the accuracy of this model, it forced us to apply the constructed model on real case study in section four. Finally, we recorded the results and conclusions which we reached in this study research through section five.

2. Literature review

In this section we exhibited the earlier studies which related to our study's objectives. Therefore, this section divides into set of sub-sections. Each sub-section introduces previous studies and techniques have been harnessed.

2.1 Around classification of imbalanced data

Several strategies [22, 25] have been mentioned by researchers to deal with the imbalanced data problem. These strategies can be categorized into data-level approaches, algorithm-level approaches, cost-sensitive strategies, and boosting strategies. The first strategy rebalances the data, utilizing the resampling technique to improve accuracy. In the second strategy, the standard classifiers are biased towards the minor class by adjusting their methodology. The third strategy gathers data-level and

algorithm-level strategies by giving higher costs to positive samples and decreasing these costs. The fourth strategy combines multiple learners and then aggregates their predictions.

The Random Over-Sampling (ROS)[26] technique is the simplest over-sampling approach that randomly generates minor instances from the imbalanced data set until the class distribution more balanced. The Random Under-Sampling Strategy (RUS) is the simplest under-sampling approach that picks negative examples at random and discards them from the dataset until the class distribution is balanced. Wang et al. [27] used the SMOTE method to synthesize data with the Tomek Links technique to eliminate some of the majority of cases. Wang et al. [28] introduced Focal-XGBoost and Weighted-XGBoost, which blend the XGBoost algorithm with focal and weighted strategies to cope with imbalanced classification issues by minimizing the significance of well-classified cases. Ref [29] applied integrated optimization and sampling presumptions to address class imbalances. The researchers employed simulated annealing to choose the optimal subset of negative class records based on F-score. The under-sampled training set was then trained using several core classifiers, including SVM, KNN, DA, and DT. Boosted Random Forest [30] is constructed from two components: the boosting technique and the random forest classifier, in which each decision tree in the forest is created based on misclassification penalties. HICD [31] depends on data density; it is a hybrid, unbalanced classification model. It creates subsets for various instance classes, builds ensemble models, splits the data space using a density-based resampling technique, and chooses suitable models according to the instance distribution. Liu et al. [32] developed the fuzzy SVM algorithm and began dealing with borderline noise by employing a new strategy of measuring distance and gaussian fuzzy to decrease the influence of this noise. Zhang et al. [33]trained several classifiers on balanced subsets obtained by POS (perturbation-based oversampling) and used majority voting for ensemble learning. Barua et al. [34]presented the MWMOTE technique, which addresses imbalanced learning through determining key minority class instances, providing penalties based on proximity to the majority class, and creating synthetic instances from the minority class. Choudhary et al. [35] provided a method that employs a fuzzy clustering technique to segment the complex imbalance challenge into smaller issues before allocating ratings to each sub-classifier for a majority vote.

2.2 Influenced Neutrosophic Theory in Evaluation Process

In the recent studies, Neutrosophic with its various types are emerged in various vital domains toward supporting the stakeholders with valuable decisions in anxiety ambience. As Elhenawy et al. [36] employed neutrosophic especially, Triangular Neutrosophic Sets (TriNSs) for weighting criteria which contributed to evaluate alternatives of Metaverse in healthcare domain.Also, SVNSs are merged with TrSS in [37] for modeling criteria and its sub-criteria based blockchain technology (BCT).Portfolio selection model is established in [38] through adopting neutrosophic theory and entropy objective function and this model is applied on real time case study to validate the model accuracy. The optimal warehouse management software is selected through utilizing various MCDM with SVNSs for evaluating warehouse management software programs and select optimal toward achieving sustainable logistics systems [39]

As mentioned previously, we need efficient MCDM techniques to be implemented under authority of SVNSs and modeling the identified criteria and sub-criteria using TrSS which applied to model the criteria and sub-criteria in various applications as indicate the best location for solar hydrogen production [40]. Also, TrSS is leveraged in [41] for recommending the secure enterprise based on modeling blockchain criteria and its sub-criteria using TrSS.

Overall, These techniques are leveraged for constructing a robust decision-making model for assessing and ranking classifiers and allow experts to select the best classifier. So, we will use HML and MESA (boost ensemble imbalanced learning using a meta-sampler) with other familiar solutions based on sampling and cost-sensitive techniques for selecting the best classifier.

3. Soft Decision-Making Model (SDMM)

The objective of this section is to cover the following points:

- How is the evaluation process conducting?
- What techniques are used for serving the objective of model and study? And what is the role of each technique?
- What are the influenced criteria which impact on the quality and performance of classifiers?

The previous questions will be answered through the following sub-sections.

3.1 Preliminaries

The utilized techniques and its basic concept are exhibited in this sub-section.

3.1.1 Tree Soft Sets [37]

A novel technique of TrSS is proposed by Smarandache who is founder for Neutrosophic theory Smarandache [24]. This technique has several concepts which described as:

- Assuming that $\aleph$ be a universe of discourse which includes ϱ a non-empty as subset of $\aleph$, thus the powerset of ϱ expressed as p (ϱ).
- Let TrSS encompasses set of levels, each one has a multitude of nodes as:
  - Level 1: consists of a multitude of nodes where each node represents main criteria, then expressed as: C={C1, C2,..Cn} for integer n ≥ 1.
  - Level 2: includes several sub-nodes of {C1, C2,..Cn} and stated as { C1-1, …C1-n } branched of C1 , and { C2-1, …C2-n } branched of C2 , finally { Cn-m, …Cn-m } branched of Cn .
- We call the leaves of the graph-tree, all terminal nodes (nodes that have no descendants). Then, Tree Soft Set: F: P (Tree (C)) → p (ϱ).
- Tree (C) is the set of all nodes and leaves (from level 1 to level n) of the graph-tree, and P (Tree(δ)) is the powerset of the Tree (Ind). All node sets of TrSS of level n as: Tree (C) = {C nm | nm= 1, 2, ...}.

3.1.2 Single-Valued Neutrosophic Sets (SVNSs)[42]

SVNSs are a branch of Neutrosophic theory that originated from Smarandache's work. Whilst SVNSs consider three measurement and probabilities as Truth (ϑ), Falsity (ν), and Indeterminacy (δ). Hence, three measurements are deployed and represented as:

- Assume that χ is universal set and κ is element in χ and this element is formed as: $\vartheta_{\kappa}(\omega)$, $\nu_{\kappa}(\omega)$, $\delta_{\kappa}(\omega)$.

- $0 \leq \sup \vartheta_{\kappa}(\omega)+\sup \nu_{\kappa}(\omega)+ \sup\delta_{\kappa}(\omega) \leq 3.$
- The operations in SVNSs formed as:
  - Addition of two sets : $\widetilde{\mathrm{Ne}_1}+ \widetilde{\mathrm{Ne}_2}=$
  $\langle(\tau_1 + \tau_2 - \tau_1 \tau_2 , \gamma_1 , \gamma_2, \varphi_1\varphi_2)\rangle$
  - Multiplication of two sets : $\widetilde{\mathrm{Ne}_1}+ \widetilde{\mathrm{Ne}_2}=$
  $\langle(\tau_1 \tau_2 , \gamma_1 + \gamma_2 - \gamma_1 \gamma_2 , \varphi_1 + \varphi_2 - \varphi_1 \varphi_2)\rangle$

3.2 Development of SDMM: Evaluation classifiers and selecting the best classifier in the imbalanced data problem

Developing SDMM for evaluating determined alternatives of classifiers required follow set of steps to implement the mentioned techniques.

Step 1: Structuring criteria and sub-criteria into set of levels.

1.1 Determining set of alternatives of classifiers which involve into evaluation process.

1.2 Determining the influenced criteria and sub-criteria which contribute to evaluating process.

1.3 Modeling and structuring these criteria and its sub-criteria as nodes into several levels.

1.4 DMs panel is formed for rating enterprises based on modelled criteria and sub-criteria.

Step 2: SVNSs based SWARA for generating weighting [43]**.**

2.1 SWARA is deployed for obtaining criteria weights as:

- Expert panel is rating criteria through using SVN scale. Each decision maker (DM) rates the criteria in his/her decision matrix.
- Deneutrosophic rating of each DM according to Eq.(1).

$$\mathrm{De_j} = \frac{2+\vartheta-\nu-\delta}{3} \qquad (1)$$

- Where: ϑ, ν , δ indicated to truth, false, and indeterminacy respectively.
- The constructed deneutrosophic matrices are aggregated into an aggregated matrix through employing Eq.(2).

$$\wp_{\mathrm{j}} = \frac{\sum_{\mathrm{j}}^{\mathrm{n}} \mathrm{De_j}}{\mathbf{U}} \qquad (2)$$

- Where: **U** refers to number of DMs
- Comparative importance of Average value (Sj) is obtaining according to Eq.(3) [44]. The values of Sj facilitate obtaining the values of coefficient (Kj) according to Eq.(4).

$$\mathbf{S_j} = \begin{cases} \mathbf{0} & \mathbf{j = 1} \\ \wp_{\mathbf{j-1}} - \wp_{\mathbf{j}} & \mathbf{j > 1} \end{cases} \quad (\mathbf{3})$$

$$\mathbf{K_j} = \begin{cases} \mathbf{1} & \mathbf{j = 1} \\ \mathbf{s_j} & \mathbf{j > 1} \end{cases} \quad (\mathbf{4})$$

- Generating recalculated weights (qj) through implementing Eq.(5).

$$\mathbf{q_j} = \begin{cases} \mathbf{1} & \mathbf{j = 1} \\ \dfrac{\mathbf{q_j - 1}}{\mathbf{k_j}} & \mathbf{j > 1} \end{cases} \quad (\mathbf{5})$$

- Accordingly, qj contributed to obtain final weights ($\boldsymbol{w_j}$)based on Eq.(6).

$$\boldsymbol{w_j} = \frac{\boldsymbol{q_j}}{\sum_{k=1}^{n} \boldsymbol{q_k}} \quad (\mathbf{6})$$

2.2 **SWARA is deployed for obtaining sub- criteria weights.** We follow the steps of 2.1 to generate sub- criteria weights.

Step 3: Recommending Optimal classifier based on MAIRAC and SVNSs.

3.1 Constructing Neutrosophic decision matrix for DM based on SVN scale.

3.2 Denutrosophic decision matrix for each DM according to Eq.(2). Also, Eq.(3) has vital role for aggregating these matrices into an aggregated matrix.

3.3 Eq.(6) responsible for calculating theoretical evaluation matrix(TP) toward estimating preferences of alternatives.

$$P_{Aj} = \frac{1}{m} \quad (6)$$

Where: m indicates the number of alternatives

3.4 Calculating real evaluation matrix (TR) according to Eq.s(7),(8).

$$\mathbf{tr_{ij}} = \mathbf{tp_{ij}} \left(\frac{\mathbf{x_{ij} - \bar{x}_i}}{\mathbf{x_i^+ - x_i^-}} \right), \text{ for maximum} \quad (7)$$

$$\mathbf{tr_{ij}} = \mathbf{tp_{ij}} \left(\frac{\mathbf{x_{ij} - x_i^+}}{\mathbf{x_i^- - x_i^+}} \right), \text{ for minimum} \quad (8)$$

3.5 Calculating criteria function (Q) based on Eq.(9).

$$Q_i = \sum_{i=1}^{m} g_{ij} \quad (9)$$

Where,

$$g_{ij} = t_{pij} - t_{rij} \quad (10)$$

4 Empirical Case Study.

We are implementing our SDMM for evaluating and ranking classifiers. Herein,we are leveraging five classifiers (alternatives) are Smote-Tomek Link (STL), Focal-XGBoost (FGB), Boosted Random Forest (BRF), MESA, and HML. The evalaution for five alternatives are conducting based on four criteria used for evaluation are described in Table 1.

The evalaution is conducting through implementing the steps are aforementioned.

4.1 Assigning each criterion to certain node and also, sub-criteria through leveraging TrSS technique as in Figure 1.

4.2 Three DMs are rating modeled criteria and sub-criteria through utilizing the SVN scale in Table 1.

4.2.1 For main criteria, DMs are rating the four criteria through SVN scale in Table 1.

- DMs'preferences and transform these preferences into deneutrosophic values according to Eq.(1). Eq.(2) used to aggregate these preferences as in Table 2.

- Sorting the criteria in descending order according toaggreget values of criteria.According to aggregated values in Table 2 , C4 is the more important than C2.Also, C2 is more important than C3.Accordingly, C3 is more important than C1.

- Employing Eq.(3) for genertaing Sj values .also, q_j and criteria weights are obtaining through Eq.s(5),(6).Table 3 involves the findings of applied Eq.s.Figure 2 showcases final weights for criteria.

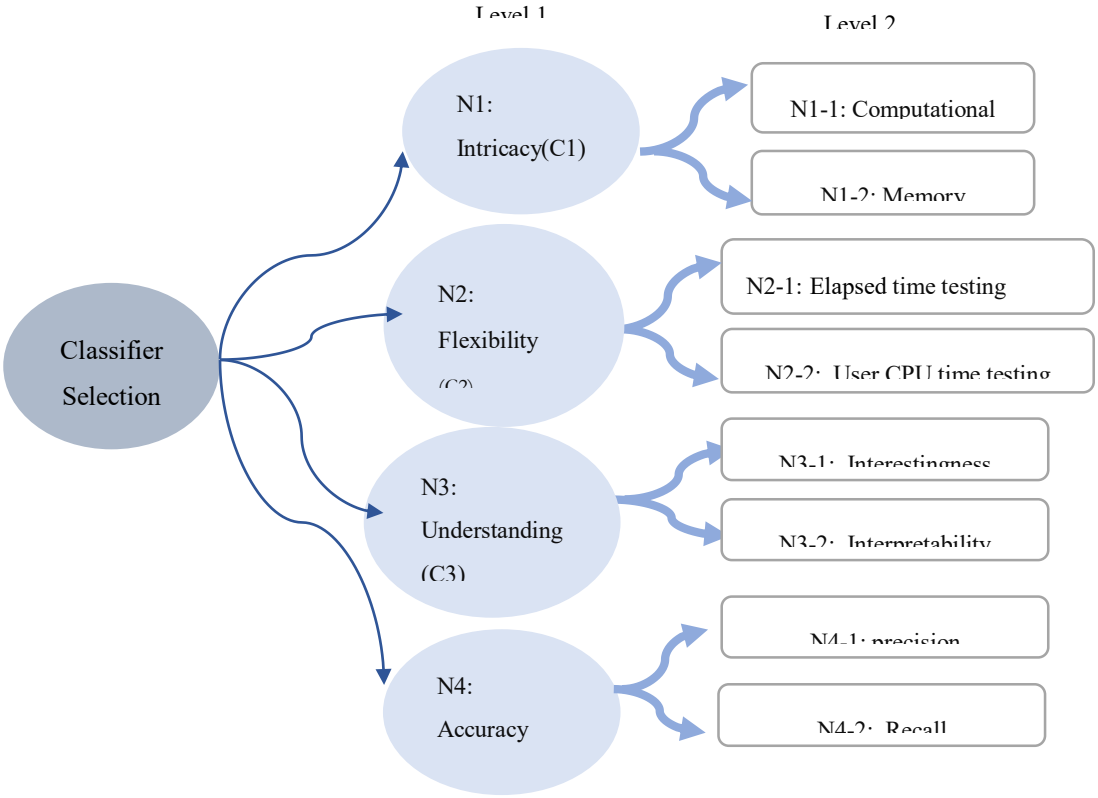

Figure 1.Tree soft for determined criteria and sub-criteria

4.2.2 For Sub- criteria, DMs are rating for each criterion is branched from main criterion according to structure of TrSS in Figure 1. SVN scale in Table 1 is utilized to rate the sub-criteria.

- The steps in 4.2.1 are followed for obtaining weights for these sub-criteria. Final findings formed in Figure 3,4,5,6 for sub-criteria of main criteria.

4.3 Three DMs are utilized SVN scale in Table 1 for second time for rating five alternatives of classifiers .

4.4 Aggregated decision matrix is generated through employing Eq.(2) after convreting three matrices form neutrosophic to deneutrosophic. Table 4 aggregets the three matrices into single matrix.

4.5 Calculating calculating theoretical evaluation matrix(TP) through Eq.(6) and represented in Table 5.

4.6 Calculating real evaluation matrix (TR)through employing Eq.(7) for maximum whlist Eq.(8) for minimum. The findings recorded in Table 6.

4.7 Calculating criteria function (Q) based on Eq.s (9),(10) and final ranking for alternatives is shown in Figure 7. This figure indicated that alternitive 5 (A5) HML is the optimal classifier. In contrast, alternative 2(A2) is the worst.

Table 1. Scale of SVN

Synonmy	Acronym	Scale		
		T	I	F
Extremly Weak	EW	0.00	1.00	1.00
Absolutely Weak	AW	0.10	0.90	0.90
Very Weak	VW	0.20	0.85	0.80
Weak	W	0.30	0.75	0.70
Fairly Weak	FW	0.40	0.65	0.60
Fairly	F	0.50	0.50	0.50
Fairly Well	FW	0.60	0.35	0.40
Well	W	0.70	0.25	0.30
Very Well	VW	0.80	0.15	0.20
Absolutely Well	AW	0.90	0.10	0.10
Extremly Well	EW	1.00	0.00	0.00

Table 2. Denutrosophic Matrix

Criteria	Expert Panel			Aggregeted values
	DM_1	**DM_2**	**DM_3**	
Intricacy(C1)	0.5	0.62	0.9	0.672222222
Flexibility (C2)	0.62	0.82	1	0.811111111

Understanding (C3)	0.82	0.9	0.5	0.738888889
Accuracy (C4)	0.9	1	0.82	0.905555556

Table 3. Final criteria weights

Criteria	Sj	Kj	qj	wj
Accuracy (C4)	0	1	1	0.280523
Flexibility (C2)	0.094444	1.094444	0.913706	0.256316
Understanding (C3)	0.072222	1.072222	0.852161	0.239051
Intricacy(C1)	0.066667	1.066667	0.798901	0.22411
		Sum	3.564767	1

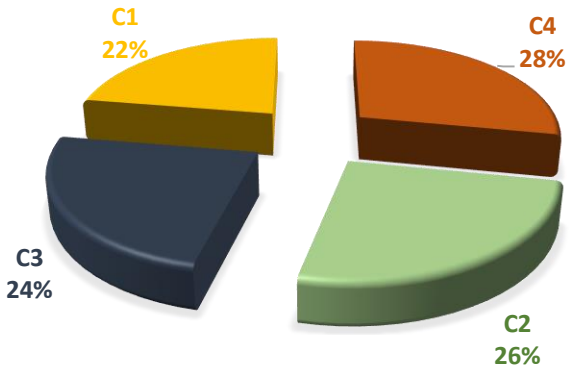

Figure 2. Final criteria weights

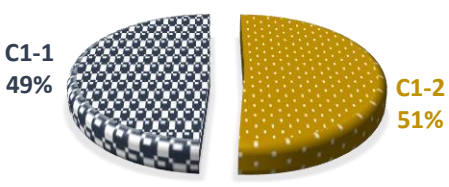

Figure 3.Sub-ceiteria of main criteria 1 weights

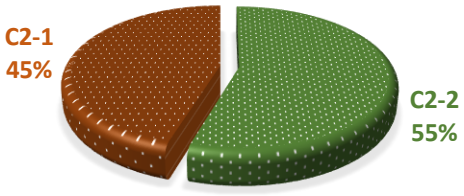

Figure 4. Sub-ceiteria of main criteria 2 weights

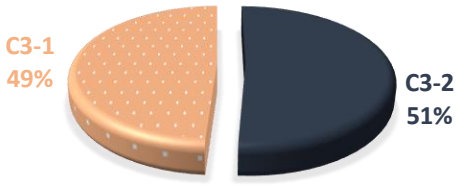

Figure 5.Sub-ceiteria of main criteria 3 weights

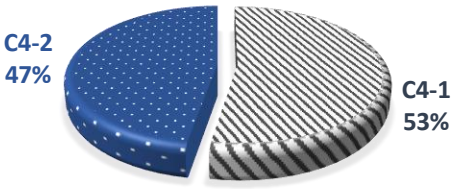

Figure 6.Sub-ceiteria of main criteria 4 weights

Table 4. Aggregeted matrix

Alternatives / criteria	Intricacy(C1)	Flexibility (C2)	Understanding (C3)	Accuracy (C4)
SMOTE Tomek Link	0.5	0.572222222	0.5	0.461111111
FGB	0.65	0.283333333	0.538888889	0.5
BRF	0.816666667	0.5	0.75	0.5
MESA	0.816666667	0.716666667	0.75	0.75
HML	0.9	0.75	0.844444444	0.844444444

Table 5. Theoretical Evaluation Matrix(TP)

Alternatives / criteria	Intricacy(C1)	Flexibility (C2)	Understanding (C3)	Accuracy (C4)
SMOTE Tomek Link	0.04482204	0.051263133	0.047810176	0.056104651
FGB	0.04482204	0.051263133	0.047810176	0.056104651
BRF	0.04482204	0.051263133	0.047810176	0.056104651
MESA	0.04482204	0.051263133	0.047810176	0.056104651
HML	0.04482204	0.051263133	0.047810176	0.056104651

Table 6. Real Evaluation Matrix (TR)

Alternatives / criteria	Intricacy(C1) MIN	Flexibility (C2) MAX	Understanding (C3) MAX	Accuracy (C4) MAX
SMOTE Tomek Link	0.04482204	0.03173432	0	0
FGB	0.028013775	0	0.005397923	0.005691776
BRF	0.009337925	0.02380074	0.034700934	0.005691776
MESA	0.009337925	0.047601481	0.034700934	0.005691776
HML	0	0.051263133	0.047810176	0.056104651

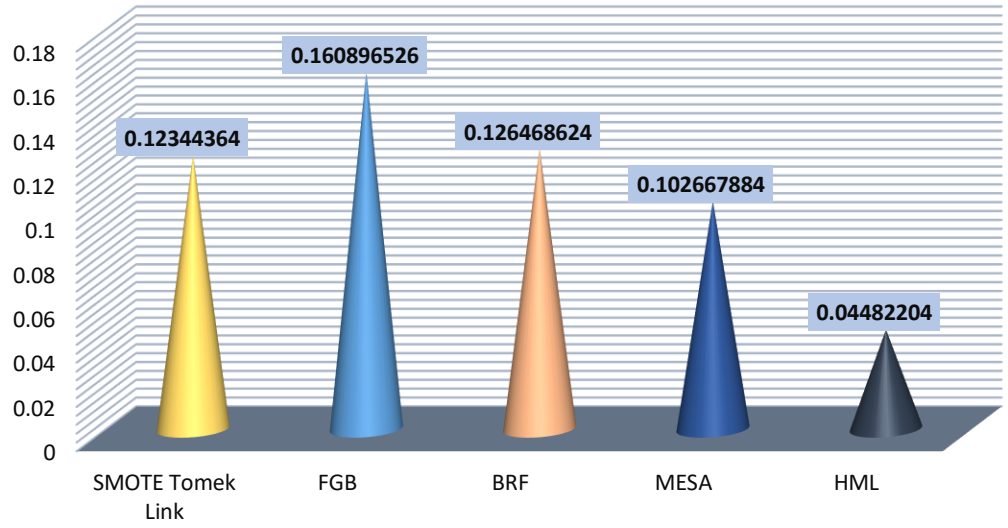

Figure 7. Final Ranking for Alternatives

5. Conclusions

Classification learning is a vital process. However, most classifiers can't deal with the imbalanced data problem, which indicates there are many more instances (majority instances) in one class than in another. Unfortunately, algorithms usually concentrate on only one criterion when facing this problem. Moreover, this became the catalyst for conducting this study and constructing SDMM based

on MCDM techniques to compare different classifiers according to multiple dimensions in the case of an imbalanced data problem based on set of influenced criteria and sub-criteria. Hence, that motivates us for modeling and structuring these criteria and its sub-criteria to illustrate the relation between each other. Accordingly, SWARA as technique of MCDM, is applied to obtain criteria and sub-criteria weights, with the assistance of SVNSs. The findings of SVNSs based SWARA showcased in Figures 2 for main criteria whereas sub-criteria's weights illustrated in Figures 3,4,5,6. After that role of MAIRAC based on SVNSs initializes for rating five alternatives of classifiers through leveraging generated criteria's weights from SWARA -SVNSs. The findings recommended that A5 HML is the optimal classifier otherwise, A2 is the worst as in Figure 7.

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Strategy Selection for Natural Resource Conservation CapturingVagueness through Integrated Neutrosophic CRITIC-MAIRCA Approach

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Abstract

Conservation of natural resources like water, soil, and biodiversity is necessary for ecological balanceand human survival. This article aims to identify potential ways to conserve natural resources while maintain-ing a balance between ecology and human survival. An integrated neutrosophic CRiteria Importance Through Intercriteria (CRITIC)-Multi-Atributive Ideal-Real Comparative Analysis (MAIRCA) approach is developed to handle uncertain and inconsistent information. The linguistic assessments of the criteria are quantified through single-valued neutrosophic sets (SVNSs). The CRITIC method evaluates the criteria weights to avoid subjec-tive and biased assessments. The MAIRCA approach is improved in a neutrosophic context to identify the ranking of the alternatives. The combined neutrosophic CRITIC-MAIRCA approach determines the best way to preserve natural resources. The proposed method reveals that the most favourable ways to conserve natural resources are through research and monitoring, while the last option is sustainable practices. Comparison and sensitivity discussions of the proposed approach are conducted to check its consistency and robustness.

Keywords: Single-valued neutrosophic set; MAIRCA; CRITIC; MCDM; Objective weights; Natural resource conservation

1. Introduction

Managing and preserving water, soil, forests, and wildlife to guarantee their continued availability for future generations is known as natural resource conservation [1]. Protecting areas, restoring damaged ecosystems, and implementing sustainable practices are all part of it. Protecting natural resources aims to satisfy present-day human needs while preserving ecological balance by adopting laws and increasing public awareness [1]. Over-exploitation can result in species extinction, environmental damage, and resource depletion without conservation. Pollination, clean air, and water filtration are examples of ecosystem services maintained when these resources are protected [2]. Rich nations consume six times more resources and cause ten times more climate damage than lower-income countries. According to the survey, worldwide use of natural resources is expected to increase by 60 percent by 2060 compared to 2020 levels [3]. Therefore, it is necessary to address this issue with the utmost priority of conserving the elements of natural resources.

A multifaceted strategy is needed to address these issues, including increasing public awareness, creating sustainable practices, enforcing laws, and encouraging international cooperation. In this situation, decisions are influenced by various factors that cannot be easily quantified or compared directly. Various Multi-criteria decision-making (MCDM) methods, including outranking approaches, reference point approaches, and utility and value theory, are implemented to promote the conservation and management of natural resources [4, 5]. However, these crisp MCDM approaches need to deal with a qualitative assessment of the associated factors in natural resource conservation. In this regard, it becomes necessary to formulate an improved MCDM methodology to deal with vague and indeterminate descriptions of the criteria [6, 7]. Therefore, this article aims to introduce a completely flexible MCDM method for analyzing the available strategies for natural resource conservation and, if necessary, generating a new strategy. In conclusion, the objectives of this article include:

- ☐ To formulate an integrated MCDM method that can deal with indeterminate and incomplete information.
- ☐ To determine the objective criteria weight for avoiding subjective and biased assessment.
- ☐ To formulate an MCDM framework on natural resource conservation by identifying strategies and their related goals.
- ☐ To identify the executive strategy through the implementation of proposed MCDM methods

To reach the mentioned objectives, this article contributes as follows:

- Proposed an integrated neutrosophic Multi-Atributive Ideal-Real Comparative Analysis (MAIRCA) approach to address uncertain and indeterminate MCDM issues.
- Implemented the CRiteria Importance Through Intercriteria (CRITIC) approach to determine the objective criteria weights.
- Formulate an MCDM framework with six strategies and eleven criteria on natural resource conservation.
- Illustrate the proposed method numerically to select the best strategy for natural resource conservation.
- Established the proposed neutrosophic CRITIC-MAIRCA approach through comparison and sensitivity analysis.

To track the remaining article, the following section provides a brief literature study on the fuzzy MAIRCA approach and MCDM application on natural resource conservation. The proposed methodology with preliminary discussion on single valued neutrosophic sets (SVNSs) is demonstrated in Section 3. A framework on strategy selection to conserve natural resources is formulated in Section 4. Section 5 contains a comparison and sensitivity analysis-based numerical demonstration of the proposed approach. Section 6 accomplishes the article with some insights and findings.

2. **Literature review**

MAIRCA [8] simultaneously analyzes several competing criteria to provide an organized and methodical approach to decision-making. MAIRCA offers a fair evaluation of different options and their associated trade-offs by quantifying the criteria and lowering subjective biases through objective assessment. Though MAIRCA is helpful in handling multi-criteria, it cannot quantify qualitative assessment. Therefore, introducing fuzzy sets into the MCDM problem is a significant development to counter the uncertain decision-making process. It improves the capacity to handle ambiguity, imprecision, and qualitative aspects of decision criteria, resulting in decision-making processes that are more adaptable, practical, and focused on people. Chatterjee et al. [9] study the MAIRCA approach to assess the efficiency of environmentally friendly suppliers in the electronics sector. Trung et al. [10] and Nguyen et al. [11] compare the effectiveness of the MAIRCA approach with other MCDM methods. Hadian et al. [12] conducted a study that specifically examined flood susceptibility assessment using the MAIRCA approach. Maruf and Özdemir [13] work on tourism websites ranking by the MAIRCA method.

2.1. *Fuzzy MAIRCA approach*

The introduction of two membership degrees of intuitionistic fuzzy sets (IFSs) [14], Pythagorean fuzzy sets (PFSs) [15], and fermatean fuzzy sets (FFSs) [16] are the primary extensions of fuzzy sets that represent the acceptance and rejection of ambiguous information. However, these extensions fail to represent the indeterminacy of DM in quantifying inconsistent and hesitant data. The neutrosophic sets [17] are defined by membership, non-membership, and indeterminacy, which independently lies in (0, 1) and can represent the indeterminacy in decision-making. The significance of neutrosophic fuzzy logic in decision-making has led to considerable advancements in SVNSs [18, 19]. Researchers developed their idea of neutrosophic fuzzy to analyze more conveniently to handle fuzziness [20, 21]. The fuzzy MAIRCA approach is utilized in several applications like defense system strategy selection [22], analysis of sustainable methods for treating wastewater [23], choosing a COVID-19 vaccination during the coronavirus pandemic [24], occupational health and environmental risk assessment [25], assessment of occupational risks [26]. Haq et al. [27] use the MAIRCA technique throughout an interval neutrosophic framework to determine the most suitable sustainable material for Human-Powered Aircraft.

The CRITIC [28] method establishes the objective weights of the criteria through the inter-criteria relationship established by statistical measures. The objectivity and certainty of selecting the best alternatives can be organized by combining MAIRCA and CRITIC approaches.

2.2. *MCDM in natural resource conservation*

The effective management and protection of natural resources have emerged as significant problems in biodiversity and ecological systems. Academic researchers are demonstrating commendable achievement in the domain of natural resource conservation. The literature has a large number of works about the same. The research by Regan et al. [29] examines the complete criteria for assessing biodiversity in forest conservation planning. Mendoza and Martins [30] examine the application of MCDM for natural resource management, with a specific focus on forest ecosystems. Hassangavyar et al. [31] investigate the methods used to mitigate soil erosion using a comparison of Vise Kriterijumska Optimizajica (VIKOR) and Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) models. Yang et al. [32] examine the connection between environmental preservation and managing natural assets through the VIKOR approach. Fuzzy MCDM techniques make a significant contribution to the preservation of natural resources. Researchers used several different applications of fuzzy MCDM for

various reasons relating to the conservation of resources. Chen et al. [33] use the fuzzy Analytic Hierarchy Process (AHP) methodology to determine the optimal environment-watershed design. Narayanamoorthy et al. [34] conduct a thorough analysis of the selection of appropriate biomass conservation strategies using the fuzzy Decision-Making Trial and Evaluation Laboratory (DEMATEL) method. Table 1 summarizes the case studies conducted on natural resource conservation.

Table 1. Natural resource conservation: Fuzzy decision-making.

Benchmark	Fuzziness	Application	Contributor
AHP, WASPAS		Energy, deforestation, biodiversity	[35]
WASPAS	TFN	Climate change mitigation	[36]
AHP	TFN	Watersheds for conservation measures	[37]
AHP	TFN	Soil erosion conservation	[38]
VIKOR	TFN	Coastal areas conservation	[39]
TOPSIS	TFN	Forest conservation	[40]
TOPSIS	Trapezoidal fuzzy	Water resources	[41]
CoCoSo	Fermatean Fuzzy	Water save	[42]
MARCOS		Solar cite location	[43]
TOPSIS	Hesitant fuzzy set	Energy policy	[44]

It is evident from Table 1 that the researchers primarily used TFN for uncertainty representation while AHP, WASPAS, and TOPSIS dominate in the MCDM approaches. The applications of these fuzzy approaches mainly cover specific conservation, such as energy, deforestation, watersheds, soil, and forests.

2.3. *Research gap*

Through the pertinent literature review on the MAIRCA approach in fuzzy environments and MCDM application for natural resource conservation, we have identified the following as the progressive research area:

- ◇ To conserve natural resources, it is more meaningful to consider all aspects rather than a specific goal. Specific conservation may not be affected due to the interlinked properties of the elements related to natural resources.
- ◇ There needs to be more fuzzy decision-making approaches in natural resource conservation. The existing approaches only consider basic fuzzy sets for representing uncertainty, which may miss the hesitancy component of a decision expert.

- ◇ The researchers often need to pay more attention to the criteria weight determination procedure and deploy subjective assessment in this case. Hence, employing an objective assessment of the criteria is necessary to avoid subjective assessments and biases.
- ◇ The neutrosophic extension of the MAIRCA approach is available in the literature, but the neutrosophic MAIRCA was not combined with the CRITIC approach to produce a combined MCDM method.

3. Material and method

This section provides preliminary information on SVNSs and the proposed methodology.

3.1. *Preliminaries*

Definition 3.1. [17] An SVNS is represented by $A = (x,\ \varphi_A(x),\ \varpi_A(x),\ \varrho_A(x)|x \in X)$ on a fixed set X, where $\varphi_A(x),\ \varpi_A(x),\ \varrho_A(x) \in [0,\ 1]$ are degrees of membership, non-membership, and indeterminacy of the element $x \in X$ to the set A, respectively. For simplicity, $A = (\varphi_A,\ \varpi_A,\ \varrho_A)$ is used as single-valued neutrosophic number (SVNE).

Definition 3.2. [45] Let $A_1 = (x,\ \varphi_1,\ \varpi_1,\ \varrho_1)$ and $A_2 = (x,\ \varphi_2,\ \varpi_2,\ \varrho_2)$ be two SVNEs and $\lambda \geq 0$, the arithmetic operations are defined as follows:

$$A_1 \bigoplus A_2 = (\varphi_1 + \varphi_2 - \varphi_1.\varphi_2, \varpi_1\varpi_2, \varrho_1\varrho_1)$$
$$A_1 \bigotimes A_2 = (\varphi_1.\varphi_2,\ \varpi_1 + \varpi_2 - \varpi_1\varpi_2,\ \varpi_1 + \varpi_2 - \varpi_1\varpi_2,\ \varrho_1\varrho_1)$$
$$\lambda A_1 = (1 - (1 - \varphi_1)^{\lambda},\ \varpi_1^{\lambda},\ \varrho_1^{\lambda})$$

Definition 3.3. [46] **Score function of SVNS:** Let $A = (x,\ \varphi_A(x),\ \varpi_A(x),\ \varrho_A(x)|x \in X)$ be a SVNS, then the score function of A is

$$\mathcal{SC}(A) = \frac{3 + \varphi_A(x) - 2.\varpi_A(x) - \varrho_A(x)}{4} \tag{1}$$

Let $\mathcal{SC}(A_1)$ and $\mathcal{SC}(A_2)$ be the score functions of two SVNEs A_1 and A_2 respectively, then

(i) If $\mathcal{SC}(A_1) \geq \mathcal{SC}(A_2)$ then $A_1 \geq A_2$
(ii) If $\mathcal{SC}(A_1) \leq \mathcal{SC}(A_2)$ then $A_1 \leq A_2$

3.2. *Proposed methodology*

In a neutrosophic context, this section develops a decision-making technique that employs the CRITIC and MAIRCA approaches to address imprecise MCDM problems. The suggested technique includes the following steps:

Step 1:: Formulate an MCDM issue with 'm' options $\mathcal{N} = \{\mathcal{N}_1,\ \mathcal{N}_2,\ ...,\ \mathcal{N}_m\}$ and 'n' criteria $\mathcal{R} = \{\mathcal{R}_1,\ \mathcal{R}_2,\ ...,\ \mathcal{R}_n\}$, where beneficiary and non-beneficiary criteria are denoted by $\mathcal{R}_{\mathcal{B}}$ and $\mathcal{R}_{\mathcal{NB}}$, respectively such that $\mathcal{R}_{\mathcal{B}} \cup \mathcal{R}_{\mathcal{NB}} = \mathcal{R}$ and $\mathcal{R}_{\mathcal{B}} \cap \mathcal{R}_{\mathcal{NB}} = \emptyset$.

Step 2:: Gather opinions of the DM using the seven-point linguistic scale provided in Table 2 to formulate a neutrosophic decision matrix $\mathrm{D} = (\S_{ij})_{m\times n}$, where $\S_{ij} = (\Upsilon_{ij}, \aleph_{ij}, \beth_{ij})$ such that Υ_{ij} is membership degree, $\aleph_{ij}$ is non-membership degree, and $\beth_{ij}$ is indeterminacy degree.

TABLE 2. Scale for criteria assessment.

Linguistic terms	SVNS (Υ, $\aleph$, $\beth$)
Highly Oppose (HO)	(0.01, 0.75, 0.35)
Oppose (O)	(0.25, 0.55, 0.30)
Slightly Oppose (SO)	(0.30, 0.45, 0.25)
Neutral (N)	(0.50, 0.35, 0.20)
Slightly Favour (SF)	(0.75, 0.25, 0.15)
Favour (F)	(0.90, 0.15, 0.10)
Highly Favour (HF)	(0.99, 0.01, 0.01)

Step 3:: Determine the preference degree ($\mathcal{P}_{A_i}$) of the alternatives according to DM. Since the primary presumption of DM is unbiased about the alternatives, hence

$$\mathcal{P}_{A_i} = \frac{1}{m},\ \sum_{i=1}^{m} \mathcal{P}_{A_i} = 1$$

Step 4:: Criteria weight determination using the CRITIC method.

Step 4.1:: Evaluate crisp decision matrix from neutrosophic decision matrix D using equation (1).

Step 4.2:: Compute the criteria' respective standard deviations as follows: $\sigma_j = \sqrt{\frac{1}{m}\sum_{i=1}^{m} \left(\S_{ij} - \overline{\S_i}\right)^2},\ j = 1(1)n.$

Step 4.3:: Calculate the linear correlation coefficient of every element of crisp decision matrix as $\rho_{ij} = \frac{\sum_{j=1}^{n} (\S_{ij} - \overline{\S_i})((\S_{kj} - \overline{\S_k})}{\sqrt{\sum_{j=1}^{n} (\S_{ij} - \overline{\S_i})^2 ((\S_{kj} - \overline{\S_k})^2}}$

Step 4.4:: Determine key indicators of each criterion as:

$$\mathrm{II}_j = \sigma_j . \sum_{i=1}^{m} (1 - \rho_{ik}) \tag{2}$$

and weights as

$$\omega_j = \frac{\amalg_j}{\sum_{j=1}^{n} \amalg_j}. \tag{3}$$

Step 5:: Compute the theoretical evaluation matrix $\mathcal{T}_P = (\mathrm{t}_{p_{ij}})_{m\times n}$ such that $\mathrm{t}_{p_{ij}} = \omega_j \mathcal{P}_{A_i}$.

Step 6:: Determination of real evaluation matrix $\mathcal{T}_r = (\mathrm{t}_{r_{ij}})_{m\times n}$ from equations (4) and (5).

(i) For beneficiary criteria

$$\mathrm{t}_{r_{ij}} = \mathrm{t}_{p_{ij}} \times \left(\frac{\S_{ij} - \S_i^-}{\S_i^+ - \S_i^-}\right) \tag{4}$$

(ii) For non-beneficiary criteria

$$\mathrm{t}_{r_{ij}} = \mathrm{t}_{p_{ij}} \times \left(\frac{\S_{ij} - \S_i^+}{\S_i^- - \S_i^+}\right) \tag{5}$$

where $\S_i^+ = \max_i(\S_{ij})$ and $\S_i^- = \min_i(\S_{ij}),\ i = 1(1)m$.

Step 7:: Compute the overall gap matrix $\mathcal{G} = (\mathrm{g}_{ij})_{m\times n}$, where $\mathrm{g}_{ij} = \mathrm{t}_{p_{ij}} - \mathrm{t}_{r_{ij}}$.

Step 8:: Determine the total of the criterion function ($mathcalQ_i$) for every option derived from equation (6).

$$\mathcal{Q}_i = \sum_{j=1}^{n} \mathrm{g}_{ij},\ i = 1(1)n. \tag{6}$$

Step 9:: Determine alternate ranking based on the decreasing order of the criterion function sum ($\mathcal{Q}_i$).

4. Natural resource conservation via MCDM problem

The criteria of natural resource conservation and the strategies to conserve are identified in this section.

4.1. *Identification of strategies alternatives*

In order to maintain sustainable use for present and future generations, it is necessary to put strategies in place to protect ecosystems, biodiversity, and natural processes. The following are some crucial tactics that are frequently used to protect natural resources:

Protected areas and conservation reserves [47, 48] ($\mathcal{N}_1$):: Establishing designated areas where human activities are limited to protect biodiversity, habitats, and ecosystem functions includes biosphere reserves, marine protected areas, national parks, and wildlife sanctuaries.

Sustainable resource management [49] ($\mathcal{N}_2$)**::** Preserving natural resources while satisfying present demands without jeopardizing the capability of upcoming generations to satisfy their own. Sustainable practices in agriculture, fisheries management, and forestry, such as organic farming and agroforestry, are exemplified by measures like protected breeding areas, quotas, and selective logging and reforestation.

Biodiversity conservation [50] ($\mathcal{N}_3$)**::** Safeguarding an ecosystem's species diversity and genetic diversity, among other aspects of its diversity and variability. A few instances include conservation breeding initiatives for endangered species, species reintroduction initiatives, and habitat restoration and protection initiatives.

Ecosystem restoration [51] ($\mathcal{N}_4$)**::** Revitalization is the process of restoring the biodiversity and ecological functionality of harmed or destroyed ecosystems. Reforestation of degraded land, rehabilitation of coral reefs, and restoration of rivers, as well as wetland habitats, are a few examples.

Climate change adaptation and mitigation [52] ($\mathcal{N}_5$)**::** Employing adaptation of climate change mitigation techniques on ecosystems and natural resources while lowering greenhouse gas emissions. A few examples are implementing carbon sequestration projects (e.g., afforestation and mangrove restoration) and creating climate-resilient ecosystems.

Research and monitoring [53] ($\mathcal{N}_6$)**::** Assessing the condition of natural resources, identifying threats, and assessing the efficacy of conservation strategies through scientific research and monitoring programs. Studies on the ecological impacts caused by people, biodiversity surveys, and ecological monitoring programs are a few examples.

4.2. *Defining criteria*

Several variables influence the conservation and preservation of resources. These features above are considered to be essential factors for the conservation of natural resources. However, further study focuses on the specific criteria used to assess the rehabilitation and preservation of natural resources. The present study has effectively identified an array of eleven significant elements that are described in Table 3.

TABLE 3. Criteria description.

Criteria	Description	Symbol
Ecological Impact [54]	Consider biodiversity, ecosystem services, habitat quality, and climate change resilience when evaluating each strategy	$\mathcal{R}_1$
Economic Costs [55, 56]	Examine the financial effects of every approach, taking into account startup costs, ongoing expenses, and possible sources of income (such as eco-tourism or the sale of timber)	$\mathcal{R}_2$
Social Acceptance [57]	Various stakeholder groups, such as local communities, indigenous peoples, and future generations, should have their benefits and burdens distributed accordingly	$\mathcal{R}_3$
Regulatory Compliance	Ensure adherence to local, national, and international laws and regulations governing natural resource management	$\mathcal{R}_4$
Regulatory Effectiveness [58]	Think about the method's effectiveness in utilizing energy, land, and water as natural resources	$\mathcal{R}_5$
Technological Feasibility [59, 60]	Examine whether the technologies needed to implement the strategy are dependable and readily available	$\mathcal{R}_6$
Public Health and Safety	Determine the possible effects on people's health and safety, taking into account any exposure to risks or pollutants	$\mathcal{R}_7$
Long-Term Sustainability [61]	Evaluate the strategy's capacity to sustain social justice, economic feasibility, and ecological balance over time	$\mathcal{R}_8$
Risk and Uncertainty [62]	Analyze the strategy's degree of risk while taking the economy, society, and environment into consideration	$\mathcal{R}_9$
Scalability and Replicability	Examine the possibility of scaling up or replicating the strategy in different settings or areas.	$\mathcal{R}_{10}$
Stakeholder Acceptance [63]	Assess the degree of acceptance and support from important parties such as businesses, governmental organizations, environmental non-governmental organizations, and communities	$\mathcal{R}_{11}$

Among these criteria $\mathcal{R}_2$ and $\mathcal{R}_9$ are cost-base whereas remaining are benefit-base criteria.

5. Numerical analysis of proposed method on natural resource conservation

Step 1::

Create an MCDM problem with six alternatives $\mathcal{N} = \{\mathcal{N}_1,\ \mathcal{N}_2,\ \mathcal{N}_3,\ \mathcal{N}_4,\ \mathcal{N}_5,\ \mathcal{N}_6\}$ and eleven criteria $\mathcal{R} = \{\mathcal{R}_1,\ \mathcal{R}_2,\ \mathcal{R}_3,\ \mathcal{R}_4,\ \mathcal{R}_5, \mathcal{R}_6,\ \mathcal{R}_7,\ \mathcal{R}_8,\ \mathcal{R}_9,\ \mathcal{R}_{10},\ \mathcal{R}_{11}\}$.

Step 2:: The neutrosophic decision matrix D is formulated in Table 4, using criteria rating from Table 2.

TABLE 4. Linguistic neutrosophic decision matrix.

Alt./Cr.	$\mathcal{R}_1$	$\mathcal{R}_2$	$\mathcal{R}_3$	$\mathcal{R}_4$	$\mathcal{R}_5$	$\mathcal{R}_6$	$\mathcal{R}_7$	$\mathcal{R}_8$	$\mathcal{R}_9$	$\mathcal{R}_{10}$	$\mathcal{R}_{11}$
$\mathcal{N}_1$	SF	F	O	HF	HF	HF	SF	HF	HF	F	HF
$\mathcal{N}_2$	F	F	O	HF	HF	HF	F	HF	HF	HF	F
$\mathcal{N}_3$	F	HF	SO	HF	HF	HF	SF	HF	HF	F	F
$\mathcal{N}_4$	HF	HF	SO	HF	HF	HF	SO	F	F	F	F
$\mathcal{N}_5$	HF	N	HO	N	SF	O	SO	F	HF	F	F
$\mathcal{N}_6$	HF	N	HO	F	HF	HF	F	F	HF	HF	HF

Step 3:: Since the problem regarding natural resource conservation contains six alternatives, the preference degree ($\mathcal{P}_{A_i}$) of each alternative is $\mathcal{P}_{A_i} = \frac{1}{6};\ i = 1,\ 2,\ 3,\ 4,\ 5,\ 6.$

Step 4:: Application of the CRITIC approach:

Step 4.1:: Crisp decision-matrix is determined from the linguistic neutrosophic decision matrix of Table 4 using equation (1).

Step 4.2: The standard deviations $\sigma_j,\ j = 1(1)11$ are computed for each criteria. The outcomes of these two steps are demonstrated in Table 5.

TABLE 5. Crisp decision matrix and standard deviations.

Alt./Cr.	$\mathcal{R}_1$	$\mathcal{R}_2$	$\mathcal{R}_3$	$\mathcal{R}_4$	$\mathcal{R}_5$	$\mathcal{R}_6$	$\mathcal{R}_7$	$\mathcal{R}_8$	$\mathcal{R}_9$	$\mathcal{R}_{10}$	$\mathcal{R}_{11}$
$\mathcal{N}_1$	0.85	0.925	0.612	0.995	0.995	0.995	0.85	0.995	0.995	0.9	0.995
$\mathcal{N}_2$	0.925	0.925	0.612	0.995	0.995	0.995	0.925	0.995	0.995	0.992	0.925
$\mathcal{N}_3$	0.925	0.995	0.662	0.995	0.995	0.995	0.85	0.995	0.995	0.9	0.925
$\mathcal{N}_4$	0.995	0.995	0.662	0.995	0.995	0.995	0.6625	0.925	0.925	0.9	0.925
$\mathcal{N}_5$	0.995	0.75	0.465	0.75	0.85	0.662	0.662	0.925	0.995	0.9	0.925
$\mathcal{N}_6$	0.995	0.75	0.465	0.925	0.995	0.995	0.925	0.925	0.995	0.992	0.995
σ_j	0.059	0.113	0.092	0.098	0.059	0.136	0.121	0.038	0.029	0.048	0.036

Step 4.3:: Derive the linear correlation relationship of every element of the crisp decision matrix

TABLE 6. Correlation coefficient of each criterion.

Criteria	$\mathcal{R}_1$	$\mathcal{R}_2$	$\mathcal{R}_3$	$\mathcal{R}_4$	$\mathcal{R}_5$	$\mathcal{R}_6$	$\mathcal{R}_7$	$\mathcal{R}_8$	$\mathcal{R}_9$	$\mathcal{R}_{10}$	$\mathcal{R}_{11}$
$\mathcal{R}_1$	1.000	-0.448	-0.473	-0.517	-0.396	-0.396	-0.448	-0.885	-0.396	0.165	-0.329
$\mathcal{R}_2$	-0.448	1.000	0.999	0.794	0.608	0.608	-0.019	0.566	-0.456	-0.360	-0.360
$\mathcal{R}_3$	-0.473	0.999	1.000	0.561	0.613	0.613	-0.002	0.586	-0.440	-0.348	-0.348
$\mathcal{R}_4$	-0.517	0.794	0.794	1.000	0.959	0.959	0.485	0.585	-0.261	0.138	0.138
$\mathcal{R}_5$	-0.396	0.608	0.613	0.959	1.000	1.000	0.608	0.447	0.447	-0.200	0.316
$\mathcal{R}_6$	-0.396	0.608	-0.002	0.959	1.000	1.000	0.608	0.447	-0.200	0.316	0.316
$\mathcal{R}_7$	-0.448	-0.019	-0.002	0.485	0.608	0.608	1.000	0.566	0.608	0.721	0.480
$\mathcal{R}_8$	-0.885	0.566	0.586	0.585	0.447	0.447	0.566	1.000	0.447	0.000	0.000
$\mathcal{R}_9$	-0.396	-0.456	-0.440	-0.261	0.447	-0.200	0.608	0.447	1.000	0.316	0.316
$\mathcal{R}_{10}$	0.165	-0.360	-0.348	0.138	-0.200	0.316	0.721	0.000	0.316	1.000	0.250
$\mathcal{R}_{11}$	-0.329	-0.360	-0.348	0.138	0.316	0.316	0.480	0.000	0.316	0.250	1.000

Step 4.4:: Determine key indicators of the criteria using equation 2 and corresponding weights using equation 3. Table 7 demonstrates the computational outcomes of criteria weight determination.

TABLE 7. Key indicator and criteria weight by CRITIC approach.

Criteria	$\mathcal{R}_1$	$\mathcal{R}_2$	$\mathcal{R}_3$	$\mathcal{R}_4$	$\mathcal{R}_5$	$\mathcal{R}_6$	$\mathcal{R}_7$	$\mathcal{R}_8$	$\mathcal{R}_9$	$\mathcal{R}_{10}$	$\mathcal{R}_{11}$
$\mathcal{R}_1$	0.000	1.448	1.473	1.517	1.396	1.396	1.448	1.885	1.396	0.835	1.329
$\mathcal{R}_2$	1.448	0.000	0.001	0.206	0.392	0.392	1.019	0.434	1.456	1.360	1.360
$\mathcal{R}_3$	1.473	0.001	0.000	0.439	0.387	0.387	1.002	0.414	1.440	1.348	1.348
$\mathcal{R}_4$	1.517	0.206	0.206	0.000	0.041	0.041	0.515	0.415	1.261	0.862	0.862
$\mathcal{R}_5$	1.396	0.392	0.387	0.041	0.000	0.000	0.392	0.553	0.553	1.200	0.684
$\mathcal{R}_6$	1.396	0.392	1.002	0.041	0.000	0.000	0.392	0.553	1.200	0.684	0.684
$\mathcal{R}_7$	1.448	1.019	1.002	0.515	0.392	0.392	0.000	0.434	0.392	0.279	0.520
$\mathcal{R}_8$	1.885	0.434	0.414	0.415	0.553	0.553	0.434	0.000	0.553	1.000	1.000
$\mathcal{R}_9$	1.396	1.456	1.440	1.261	0.553	1.200	0.392	0.553	0.000	0.684	0.684
$\mathcal{R}_{10}$	0.835	1.360	1.348	0.862	1.200	0.684	0.279	1.000	0.684	0.000	0.750
$\mathcal{R}_{11}$	1.329	1.360	1.348	0.862	0.684	0.684	0.520	1.000	0.684	0.750	0.000
Sum	11.834	8.047	8.066	8.204	8.216	8.216	10.376	9.701	12.106	11.606	11.584
σ_j	0.059	0.113	0.092	0.098	0.059	0.136	0.121	0.038	0.029	0.048	0.036
II_j	0.696	0.908	0.741	0.807	0.486	1.115	1.255	0.372	0.346	0.554	0.419
ω_j	0.090	0.118	0.096	0.105	0.063	0.145	0.163	0.048	0.045	0.072	0.054

Step 5:: Calculate the theoretical evaluation matrix $\mathcal{T}_P$ and we get

TABLE 8. Theoretical evaluation matrix.

Criteria	$\mathcal{R}_1$	$\mathcal{R}_2$	$\mathcal{R}_3$	$\mathcal{R}_4$	$\mathcal{R}_5$	$\mathcal{R}_6$	$\mathcal{R}_7$	$\mathcal{R}_8$	$\mathcal{R}_9$	$\mathcal{R}_{10}$	$\mathcal{R}_{11}$
$\mathcal{N}_1$	0.020	0.009	0.009	0.010	0.010	0.010	0.017	0.015	0.023	0.021	0.126
$\mathcal{N}_2$	0.020	0.009	0.009	0.010	0.010	0.010	0.017	0.015	0.023	0.021	0.126
$\mathcal{N}_3$	0.020	0.009	0.009	0.010	0.010	0.010	0.017	0.015	0.023	0.021	0.126
$\mathcal{N}_4$	0.020	0.009	0.009	0.010	0.010	0.010	0.017	0.015	0.023	0.021	0.126
$\mathcal{N}_5$	0.020	0.009	0.009	0.010	0.010	0.010	0.017	0.015	0.023	0.021	0.126
$\mathcal{N}_6$	0.020	0.009	0.009	0.010	0.010	0.010	0.017	0.015	0.023	0.021	0.126

Step 6:: Determination of real evaluation Equation $\mathcal{T}_r$ and we get

TABLE 9. Real evaluation matrix.

Cr./Alt.	$\mathcal{N}_1$	$\mathcal{N}_2$	$\mathcal{N}_3$	$\mathcal{N}_4$	$\mathcal{N}_5$	$\mathcal{N}_6$
$\mathcal{R}_1$	0.000	0.011	0.011	0.020	0.020	0.000
$\mathcal{R}_2$	0.004	0.004	0.000	0.000	0.016	0.016
$\mathcal{R}_3$	-0.015	-0.015	-0.012	-0.012	-0.025	-0.025
$\mathcal{R}_4$	0.010	0.010	0.010	0.010	-0.007	-0.007
$\mathcal{R}_5$	0.010	0.010	0.010	0.010	0.000	0.000
$\mathcal{R}_6$	0.010	0.010	0.010	0.010	-0.017	-0.017
$\mathcal{R}_7$	0.000	0.009	0.000	-0.022	-0.022	-0.022
$\mathcal{R}_8$	0.015	0.015	0.015	0.008	0.008	0.008
$\mathcal{R}_9$	0.000	0.000	0.000	0.011	0.000	0.011
$\mathcal{R}_{10}$	0.011	0.021	0.011	0.011	0.011	0.011
$\mathcal{R}_{11}$	0.000	0.061	0.061	0.061	0.061	0.061

Step 7:: Obtain the gap matrix $\mathcal{G}$ shown in table 10

Step 8:: Calculate values of criteria-function $\mathcal{Q}_i$ shown in table 10

Step 9:: The ranking order of the alternatives are evaluated and listed Table 10.

TABLE 10. Gap matrix and ranking order of the alternatives.

Cr./Alt.	$\mathcal{N}_1$	$\mathcal{N}_2$	$\mathcal{N}_3$	$\mathcal{N}_4$	$\mathcal{N}_5$	$\mathcal{N}_6$
$\mathcal{R}_1$	0.015	0.007	0.007	0.000	0.000	0.015
$\mathcal{R}_2$	0.010	0.010	0.020	0.020	-0.014	-0.014
$\mathcal{R}_3$	0.042	0.042	0.037	0.037	0.059	0.059
$\mathcal{R}_4$	0.000	0.000	0.000	0.000	0.030	0.030
$\mathcal{R}_5$	0.000	0.000	0.000	0.000	0.011	0.011
$\mathcal{R}_6$	0.000	0.000	0.000	0.000	0.064	0.064
$\mathcal{R}_7$	0.027	0.013	0.027	0.062	0.062	0.062
$\mathcal{R}_8$	0.000	0.000	0.000	0.004	0.004	0.004
$\mathcal{R}_9$	0.007	0.007	0.007	0.004	0.007	0.004
$\mathcal{R}_{10}$	0.006	0.000	0.006	0.006	0.006	0.006
$\mathcal{R}_{11}$	0.126	0.065	0.065	0.065	0.065	0.065
$\sum_{i=1}^{m} \mathcal{Q}_i$	0.234	0.146	0.169	0.197	0.293	0.305
Rank	3	6	5	4	2	1

5.1. *Sensitivity analysis*

A critical factor in determining the classification order of the alternatives is the influence of criteria weight variations. It is imperative to evaluate the effect on the robustness of the proposed neutrosophic MAIRCA approach. We employ the MEREC [64], Rank-Sum [65], Entropy [66], and FUCOM [67] methods to determine the weight, as illustrated in Table 11. To ensure that the proposed approach remains logically equivalent, the remaining procedures are maintained identically.

TABLE 11. Criteria preferences determined by various methods.

Methods	$\mathcal{R}_1$	$\mathcal{R}_2$	$\mathcal{R}_3$	$\mathcal{R}_4$	$\mathcal{R}_5$	$\mathcal{R}_6$	$\mathcal{R}_7$	$\mathcal{R}_8$	$\mathcal{R}_9$	$\mathcal{R}_{10}$	$\mathcal{R}_{11}$
CRITIC	0.090	0.118	0.096	0.105	0.063	0.145	0.163	0.048	0.045	0.072	0.054
MEREC	0.166	0.025	0.061	0.027	0.198	0.026	0.095	0.026	0.200	0.024	0.151
RANK-SUM	0.167	0.076	0.061	0.030	0.045	0.152	0.106	0.136	0.121	0.091	0.015
ENTROPY	0.091	0.091	0.091	0.091	0.091	0.091	0.091	0.091	0.091	0.091	0.091
FUCOM	0.0001	0.0002	0.0007	0.0001	0.003	0.007	0.018	0.034	0.088	0.245	0.596

The CRITIC approach assigns higher preference to $\mathcal{R}_2$, $\mathcal{R}_6$, and $\mathcal{R}_7$ while sets lower preferences to $\mathcal{R}_8$ and $\mathcal{R}_9$. The MEREC method allocates comparatively higher weights to the criteria $\mathcal{R}_1$, $\mathcal{R}_5$, and $\mathcal{R}_9$ while sets lower wights to the criteria $\mathcal{R}_2$, $\mathcal{R}_4$, $\mathcal{R}_8$, and $\mathcal{R}_{10}$. Surprisingly, the entropy approach assigns identical preferences to the criteria, which results in equal treatment. The FUCOM approach assigns significantly higher preferences to the criteria $\mathcal{R}_{10}$ and $\mathcal{R}_{11}$ compared to the remaining criteria. In conclusion, the cost-based criteria $\mathcal{R}_2$ and

$\mathcal{R}_9$ received higher preference than the remaining criteria except in the FUCOM approach. Figure 1 is drawn to understand the allocation of criteria weights in these approaches.

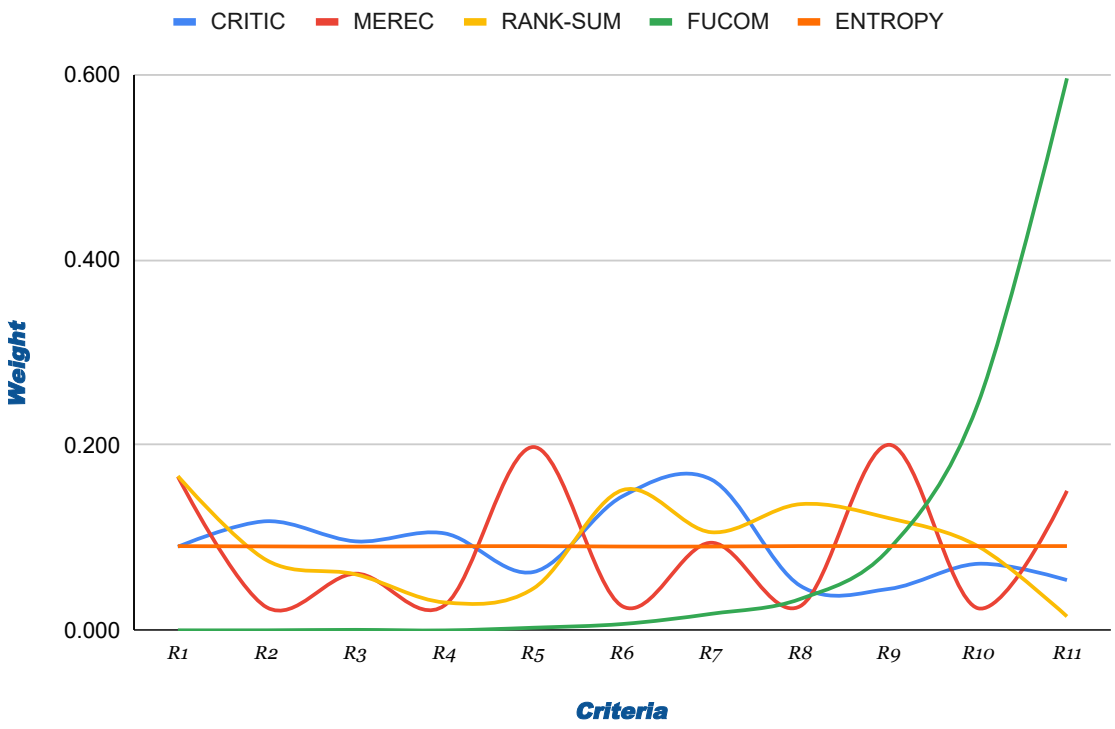

FIGURE 1. Criteria weight allocation in different approaches.

Figure 1 shows that the criteria weights of $\mathcal{R}_1$, $\mathcal{R}_2$, $\mathcal{R}_3$, and $\mathcal{R}_4$ have low variation, $\mathcal{R}_5$, $\mathcal{R}_6$, $\mathcal{R}_7$, $\mathcal{R}_8$ and $\mathcal{R}_9$ have moderate variation, and $\mathcal{R}_{10}$ and $\mathcal{R}_{11}$ have high variation. The FUCOM approach has the highest degree of variation in assigning criteria weights as it gradually increases from the $\mathcal{R}_1$ criterion weight to $\mathcal{R}_8$ criterion weight. However, the curve instantly grows high for the criteria weights from $\mathcal{R}_9$ to $\mathcal{R}_{11}$. The impact of the criteria weights on the ranking sequence of the options is depicted in Table 12.

TABLE 12. Ranking of alternatives based on various methods.

Methods	Performance score						Rank
——	$\mathcal{N}_1$	$\mathcal{N}_2$	$\mathcal{N}_3$	$\mathcal{N}_4$	$\mathcal{N}_5$	$\mathcal{N}_6$	
CRITIC	0.234	0.146	0.169	0.197	0.293	0.305	$\mathcal{N}_6 \succ \mathcal{N}_5 \succ \mathcal{N}_1 \succ \mathcal{N}_4 \succ \mathcal{N}_3 \succ \mathcal{N}_2$
MEREC	0.234	0.149	0.157	0.150	0.226	0.237	$\mathcal{N}_6 \succ \mathcal{N}_1 \succ \mathcal{N}_5 \succ \mathcal{N}_3 \succ \mathcal{N}_4 \succ \mathcal{N}_2$
RANK-SUM	0.232	0.141	0.160	0.170	0.255	0.273	$\mathcal{N}_6 \succ \mathcal{N}_5 \succ \mathcal{N}_1 \succ \mathcal{N}_4 \succ \mathcal{N}_3 \succ \mathcal{N}_2$
ENTROPY	0.226	0.143	0.160	0.172	0.255	0.263	$\mathcal{N}_6 \succ \mathcal{N}_5 \succ \mathcal{N}_1 \succ \mathcal{N}_4 \succ \mathcal{N}_3 \succ \mathcal{N}_2$
FUCOM	0.164	0.082	0.103	0.102	0.113	0.106	$\mathcal{N}_1 \succ \mathcal{N}_5 \succ \mathcal{N}_6 \succ \mathcal{N}_3 \succ \mathcal{N}_4 \succ \mathcal{N}_2$

The alternative $\mathcal{N}_6$ ranks first in all weight determination techniques except FUCOM. In contrast, each technique allocates the alternative $\mathcal{N}_2$ to the last position. The alternative $\mathcal{N}_5$ always occupies the second position, except for the MEREC approach. In all methods, the fourth and fifth places are consistently occupied by either $\mathcal{N}_3$ or $\mathcal{N}_4$. Hence, the alternative $\mathcal{N}_6$ suppresses the remaining alternatives in terms of their performance through almost all weight determination procedures while $\mathcal{N}_2$'s performance is the weakest. This phenomenon can be conveniently viewed from figure 2.

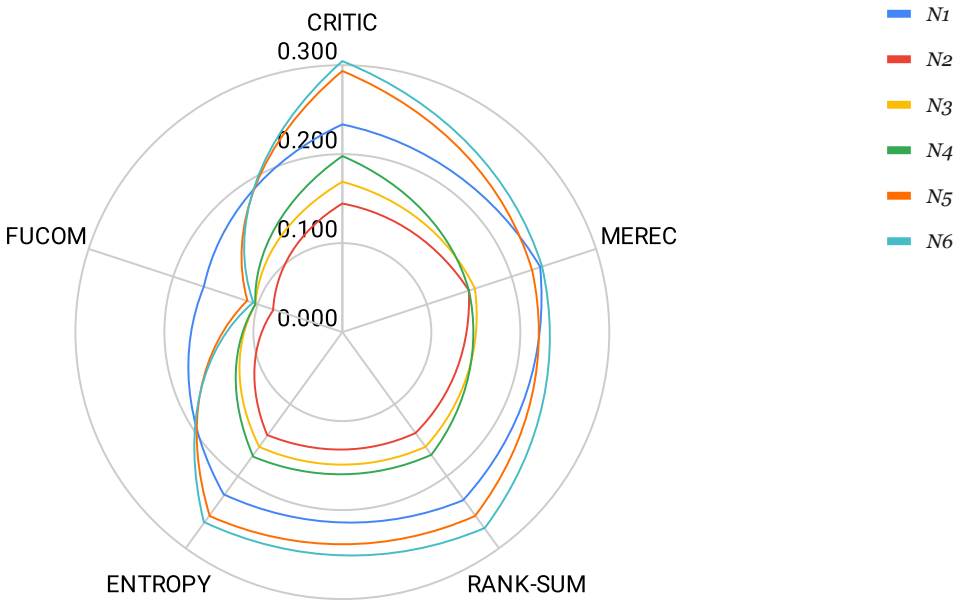

FIGURE 2. Performance score of each alternative by using various methods of weight calculation.

Figure 2 shows the performance score variation of alternatives for several weight methods. A marginal influence on the performance scores of the alternatives for the FUCOM approach is seen in figure 2. Based on the CRITIC approach, a significant disparity in performance scores is seen by which one can easily rank the options, with $\mathcal{N}_6$ occupying the top position and $\mathcal{N}_2$ ranking last. In the context of performance scores for the CRITIC, RANK-SUM, and ENTROPY methods, the figure illustrates how to rank the options easily. The MEREC technique offers an alternative approach that assigns greater performance priority to $\mathcal{N}_6$, $\mathcal{N}_5$, and $\mathcal{N}_1$ while assigning lesser preference to $\mathcal{N}_2$, $\mathcal{N}_3$, and $\mathcal{N}_4$.

5.2. *Comparison analysis*

To establish it, it is imperative to compare the outcomes of the suggested SVN-MAIRCA with those of the current MCDM techniques. We have considered popular MCDM approaches like TOPSIS, MARCOS, and MABAC to compare the outcome of the proposed SVN-MAIRCA approach. Evaluating alternative rankings using the CRITIC criterion weight maintains logical similarity in computation. The crisp MCDM approaches are applied to the crisp decision matrix from Table 7. Table 13 compares the proposed and existing techniques' ranking orders.

TABLE 13. Alternatives' ranking in suggested and existing methods.

Approach	SVN-MAIRCA		TOPSIS		MARCOS		MABAC	
Alternatives	Score	Rank	Score	Rank	Score	Rank	Score	Rank
$\mathcal{N}_1$	0.226	3	0.632	3	0.011	3	1.637	3
$\mathcal{N}_2$	0.136	6	0.634	1	0.011	1	1.748	2
$\mathcal{N}_3$	0.156	5	0.633	2	0.011	2	1.620	4
$\mathcal{N}_4$	0.164	4	0.630	5	0.011	4	1.544	5
$\mathcal{N}_5$	0.227	2	0.617	6	0.009	6	1.208	6
$\mathcal{N}_6$	0.236	1	0.631	4	0.010	5	1.780	1

Table 13 demonstrates that the ranking order derived from the SVN-MAIRCA method exhibits a substantial disparity compared to the rankings generated from other MCDM techniques. The first ranked alternative $\mathcal{N}_6$ in the SVN-MAIRCA model is only equivalent to MABAC. The alternative $\mathcal{N}_5$ obtained the lowest score in all crisp approaches, except for SVN-MAIRCA, which achieved the second highest ranking. The alternative $\mathcal{N}_2$ exhibits significant rank fluctuations when comparing the neutrosophic and crisp MCDM techniques. The remaining options exhibit a moderate degree of difference in ranking between the proposed and current techniques. Figure 3 compares the alternatives' ranking among SVN-MAIRCA and existing approaches.

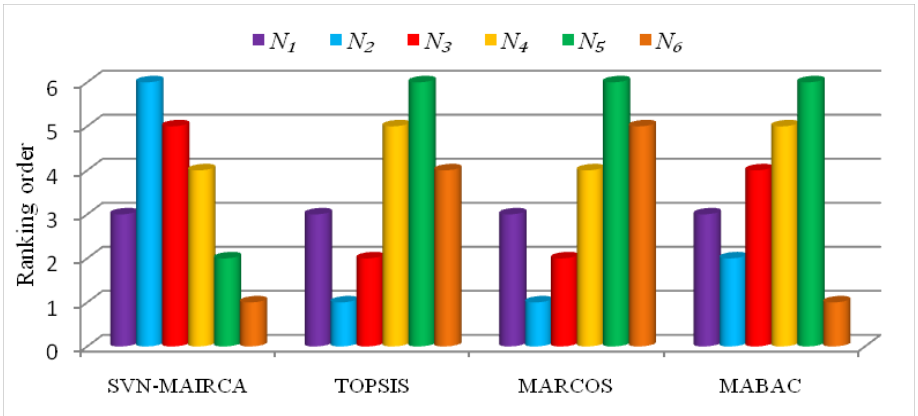

FIGURE 3. Ranking comparison of natural resource conservation strategy

The diagram 3 makes clear that the option $\mathcal{N}_1$ is consistently assigned third place by both the suggested SVN-MAIRCA technique and other approaches. This phenomenon indicates that the suggested technique is consistent. Every approach assigns $\mathcal{N}_4$ to either the third or fourth position. Figure 3 also shows that, in contrast to the suggested SVN-MAIRCA technique, which allocates $\mathcal{N}_5$ at the second place, all considered methods allocate it at the last position.

Conclusion

This paper presented an integrated neutrosophic MCDM methodology, that successfully determined the criteria weight and the ranking of the alternatives. The suggested method incorporates hesitation in decision-making through SVNS-rating of the criteria. The criteria weight variation shows that the proposed method is robust in decision-making. The comparison analysis reveals that the proposed neutrosophic methodology is inconsistent with the outcome of crisp MCDM approaches. The reason behind this inconsistency is the incorporation of uncertainty and hesitation in the computation procedures of the recommended approach. Based on the presented methodology, the alternative "research and monitoring" is the most optimal strategy, while "sustainable resource management" is deemed the least desirable. The critical criteria for ranking preference are ecological impact, economic costs, and stakeholder acceptance. Since the preference of the criterion is high, the financial cost is much higher than the remaining critical criteria; hence, it significantly impacts strategy selection for natural resource conservation.

Although the suggested method has several benefits, it has some constraints, as follows: (i) single DM may be subjective to some particular criterion, so producing a skewed assessment; (ii) the inclusion of uncertainty and hesitation in computation procedures is unavailable in criteria weight determination; and (iii) the availability of hesitant information about the criteria descriptions.

Recently developed fuzzy sets such as the Z-number, D-number, type-2 fuzzy set, and hesitant bi-fuzzy set may be used to update or enhance the criterion rating. The CRITIC approach may be expanded to include uncertainty in the criterion weight calculation in a Pythagorean, fermatean fuzzy environment. Introducing a panel of available DMs, a group decision-making methodology can be developed using the proposed N-MAIRCA approach for a more compact assessment of the criteria.

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Pythagorean Neutrosophic Triplet Groups

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Abstract:
It is a well-known fact that groups are the only algebraic structures having a single binary operation that is mathematically so perfect that it is impossible to introduce a richer structure within it. The main purpose of this study is to introduce the notion of the Pythagorean neutrosophic triplet (PNT) which is the generalization of neutrosophic triplet (NT). The PNT is an algebraic structure of three ordered pairs that satisfy several properties under the binary operation (B-Operation) "*". Furthermore, we used the PNTs to introduce the novel concept of a Pythagorean neutrosophic triplet group (PNTG). The algebraic structure (AS) of PNTG is different from the neutrosophic triplet group (NTG). We discussed some properties, related results, and particular examples of these novel concepts. We further studied Pythagorean neutro-homomorphism, Pythagorean neutro-isomorphism, etc., for PNTGs. Moreover, we discussed the main distinctions between the neutrosophic triplet group (NTG) and the PNTG.

Keywords: Neutrosophic triplet, Pythagorean neutrosophic triplet, Neutrosophic triplet group, Pythagorean neutrosophic triplet group.

1. Introduction

Neutrosophy is a novel tributary of philosophy associated with the origin, nature, and scope of neutralities. Smarandache [21] defined the notion of neutrosophic set (NS) and neutrosophic logic (NL) in 1995. Each proposition in NL is almost to have the percentage of truth in a subset T, the percentage of indeterminacy in a subset I, and the percentage of falsity in a subset F. The NS is the extension of fuzzy sets (FSs) [25], classical sets [12], intuitionistic fuzzy sets (IFSs) [6], Pythagorean fuzzy sets (PFSs) [16], and interval-valued fuzzy sets, etc., while as NL is the generalization of fuzzy logic, intuitionistic fuzzy logic, ete. The theory of neutrosophic set [22] is utilized to discuss problems involving imprecision, uncertainty, indeterminacy, incompleteness, inconsistency, and falsity. Smarandache and Kandasamy used neutrosophic theory to investigate

neutrosophic ASs in [11], [12], [13] by embedding an indeterminacy I'' into the AS. They combine indeterminacy with the elements of the AS under the binary operation $*$ and say it neutrosophic element (NE), and the novel AS is known as neutrosophic AS. Moreover, they further developed several NASs such as neutrosophic fields, neutrosophic groups, neutrosophic N-groups, neutrosophic bigroups, neutrosophic vector spaces, neutrosophic bisemigroups, neutrosophic semigroups, neutrosophic N-semigroup, neutrosophic groupoids, neutrosophic bigroupoids, neutrosophic biloops, neutrosophic loops, and neutrosophic N-loop, and so on.

In algebraic structures, the groups [8], [9], [24] are so significant that they contribute the role of backbone in practically all the theory of ASs. In the study of algebra, Groups are the most foundational and well-off AS under certain binary operations $\cdot$. Groups give concrete foundations for many ASs, such as rings, fields, vector spaces, and so on. Many other fields, including physics, chemistry, combinatorics, biology, and others, use groups to explore symmetries and other behavior among their elements. Group action is the most crucial feature of a group. In daily life problems, matrix groups, permutation groups, lie groups, transformation groups, and other forms of groups are widely employed as a mathematical tools. In this regard, generalized groups [8] are very significant. For the first time, Smarandache and Ali [20] gave the notion of neutrosophic triplet (NT). The newly defined notion of NTs is highly dependable on the binary operation $*$. Furthermore, they utilized the concept of NTs to introduce an NTG. Regarding to the structural and foundational properties, the NTG is distinct from the classical group. The NT has a strong structure than Molaei's Generalized Group [15]. Zhang et al. [26] defined a new congruence relation based on commutative NTG. They induced quotient structure by neutrosophic triplet subgroup and proved neutron-homomorphism basic theorem. Zhang et al. [27] gave the notions of neutrosophic triplet subgroups, strong neutrosophic triplet subgroups and weak commutative neutrosophic triplet groups. Further, they established quotient structures on strong neutrosophic triplet subgroups. Hu and Zhang studied the relationships among various neutrosophic extended triplet cyclic associative semihypergroups [10]. The main properties of strong pure neutrosophic extended triplet cyclic associative semihypergroups are obtained. A lot of researchers have been dealing with neutrosophic triplet metric space, neutrosophic triplet vector space, neutrosophic triplet inner product, and neutrosophic triplet normed space in ([17], [18], [19]). The concept of neutrosophic extended triplet was given by by Smarandache in [23]. The neutrosophic extended triplet is the generalization of neutrosophic triplet. Li et al. [14] studied neutrosophic extended triplet group based on neutrosophic quadruple numbers. They proved some significant results with respect to neutrosophic quadruple numbers. Bal et al. [7] defined the neutrosophic image, neutrosophic inverse-image, neutrosophic kernel, and the NET subgroup. The notion of the neutrosophic triplet coset and its relation with the classical coset were discussed. Furthermore, the neutrosophic triplet normal subgroups, and neutrosophic triplet quotient groups were studied. Zhou and Xin proposed the notion of ideals on neutrosophic extended triplet groups and discussed their different properties [29]. The concept of singular neutrosophic extended triplet group was given by Zhang et al. in [28]. They discussed different properties and proved some significant results.

The concept of neutrosophic ring was defined in [11] which is the generalization of classical rings. Agboola et al. [1,2] introduced the notion of refined neutrosophic ring which is the extension of neutrosophic ring. Ahmad et al. [3] solved the imperfect duplets problem in refined neutrosophic rings. Ali et al. [4] proposed the concepts of zero divisor, neutrosophic triplet subring, neutrosophic triplet ideal, nilpotent integral neutrosophic domain, and neutrosophic triplet ring homomorphism. Moreover, they gave the idea of a neutrosophic triplet field.

In this paper, we define the notion of the PNT which is the generalization of NT. We utilize the PNTs to introduce the novel concept of a PNTG. The algebraic structure of PNTG is different from

the neutrosophic triplet group (NTG). We discuss some properties, related results, and particular examples of these novel concepts. We further study Pythagorean neutro-homomorphism, Pythagorean neutro-isomorphism, etc., for PNTGs. Moreover, we study the main distinctions between the NTG and the PNTG.

3. Neutrosophic Triplet Groups

In this section, we will recall the notions of NTGs.

Definition [20] Let $\aleph$ be a set together with a B-Operation $*$. Let ℓ be an element of $\aleph$. If there exist a neutral of ℓ characterized by $\overset{+}{\aleph}(\ell)$, and an opposite of ℓ characterized by $\overset{-}{\aleph}(\ell)$, with $\overset{+}{\aleph}(\ell)$ and $\overset{-}{\aleph}(\ell)$ are the elements of $\aleph$, such that:

$$\ell * \overset{+}{\aleph}(\ell) = \overset{+}{\aleph}(\ell) * \ell = \ell,$$

and

$$\ell * \overset{-}{\aleph}(\ell) = \overset{-}{\aleph}(\ell) * \ell = \overset{+}{\aleph}(\ell).$$

Then, $\aleph$ is said to be an NT set.

The elements ℓ, $\overset{+}{\aleph}(\ell)$, and $\overset{-}{\aleph}(\ell)$ are collectively called as NT. The NT set is denoted by $(\ell, \overset{+}{\aleph}(\ell), \overset{-}{\aleph}(\ell))$, where ℓ is the first coordinate of NT, and by $\overset{+}{\aleph}(\ell)$, we mean neutral of ℓ.

Note that $\overset{+}{\aleph}(\ell)$ is not same as the classical algebraic unitary element. For the same element ℓ in N, the neutral "$\overset{+}{\aleph}(\ell)$" and opposites "$\overset{-}{\aleph}(\ell)$" of ℓ are not unique.

Definition [20] Let $(\aleph,*)$ be a NTS. Then, $\aleph$ is said to be a NTG, if the following properties are hold.

(i). If $(\aleph,*)$ is well-defined, i.e. for any ℓ, $\wp \in \aleph$, one has $\ell * \wp \in \aleph$.

(ii). If $(\aleph,*)$ is associative, i.e. $(\ell * \wp) * \hbar = \ell * (\wp * \hbar)$ for all ℓ, $\wp$, $\hbar \in \aleph$.

Definition [20] Let $(\aleph_1,*)$ and $(\aleph_2,\#)$ be two NTGs. Let $F : \aleph_1 \to \aleph_2$ be a mapping. Then, F is called neutro-homomorphism if for all ℓ, $\wp \in \aleph_1$, we have

(i). $F(\ell * \wp) = F(\ell) \# F(\wp)$,

(ii). $F(\overset{+}{\aleph}(\ell)) = \overset{+}{\aleph}(F(\ell))$,

and

(iii). $F(\overset{-}{\aleph}(\ell)) = \overset{-}{\aleph}(F(\ell))$.

4. Pythagorean neutrosophic triplet (PNT)

Definition Let $P_{\aleph}$ be a set together with a B-Operation $*$. Let ℓ_1 and ℓ_2 be any two elements of $P_{\aleph}$. If there exists neutrals of "ℓ_1" and "ℓ_2" known $\overset{+}{\aleph}(\ell_1)$ and $\overset{+}{\aleph}(\ell_2)$, are not same as the classical algebraic unitary elements, and an opposite of "ℓ_1" and "ℓ_2" called $\overset{-}{\aleph}(\ell_1)$ and $\overset{-}{\aleph}(\ell_2)$, with $\overset{+}{\aleph}(\ell_1)$, $\overset{+}{\aleph}(\ell_2)$, $\overset{-}{\aleph}(\ell_1)$, and $\overset{-}{\aleph}(\ell_2)$ are elements of $P_{\aleph}$, such that:

$$\ell_1 * \overset{+}{\aleph}(\ell_1) = \overset{+}{\aleph}(\ell_1) * \ell_1 = \ell_1,\ \ell_1 * \overset{-}{\aleph}(\ell_1) = \overset{-}{\aleph}(\ell_1) * \ell_1 = \overset{+}{\aleph}(\ell_1),$$
$$\ell_2 * \overset{+}{\aleph}(\ell_2) = \overset{+}{\aleph}(\ell_2) * \ell_2 = \ell_2,\ \ell_2 * \overset{-}{\aleph}(\ell_2) = \overset{-}{\aleph}(\ell_2) * \ell_2 = \overset{+}{\aleph}(\ell_2),$$

and

$$\ell_1 * \ell_2 = \overset{+}{\aleph}(\ell_1).$$

Then, $P_{\aleph}$ is said to be a PNT set.

Definition The second component of the PNT is $(\wp_1, \wp_2)$, represented as $\overset{+}{\aleph}(\cdot)$, and $\overset{+}{\aleph}(\bullet)$, if there exist other elements (ℓ_1, ℓ_2), $(\hbar_1, \hbar_2)$ in PNT such that $\ell_1 * \wp_1 = \wp_1 * \ell_1 = \ell_1$, $\ell_1 * \hbar_1 = \hbar_1 * \ell_1 = \wp_1$, and $\ell_2 * \wp_2 = \wp_2 * \ell_2 = \ell_2$, $\ell_2 * \hbar_2 = \hbar_2 * \ell_2 = \wp_2$, and $\ell_1 * \ell_2 = \overset{+}{\aleph}(\ell_1)$. The formed PNT is $[(\ell_1, \ell_2), (\wp_1, \wp_2), (\hbar_1, \hbar_2)]$.

Definition The element $(\hbar_1, \hbar_2)$ is the third component, represented as $\overset{-}{\aleph}(\cdot)$, and $\overset{-}{\aleph}(\bullet)$, of a PNT, if there exist other elements (ℓ_1, ℓ_2), $(\wp_1, \wp_2)$ in PNT such that $\ell_1 * \wp_1 = \wp_1 * \ell_1 = \ell_1$, $\ell_1 * \hbar_1 = \hbar_1 * \ell_1 = \wp_1$, and $\ell_2 * \wp_2 = \wp_2 * \ell_2 = \ell_2$, $\ell_2 * \hbar_2 = \hbar_2 * \ell_2 = \wp_2$, and $\ell_1 * \ell_2 = \overset{+}{\aleph}(\ell_1)$. The formed PNT is $[(\ell_1, \ell_2), (\wp_1, \wp_2), (\hbar_1, \hbar_2)]$.

Example Consider $(Z_{10}, *)$, where "*" is defined as $\ell * \wp = 3\ell\wp (\mathrm{mod}\, 10)$ and $Z_{10} = \{0, 1, 2, 3, ..., 9\}$ then,

*	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	3	6	9	2	5	8	1	4	7
2	0	6	2	8	4	0	6	2	8	4
3	0	9	8	7	6	5	4	3	2	1
4	0	2	4	6	8	0	2	4	6	8
5	0	5	0	5	0	5	0	5	0	5
6	0	8	6	4	2	0	8	6	4	2
7	0	1	2	3	4	5	6	7	8	9
8	0	4	8	2	6	0	4	8	2	6
9	0	7	4	1	8	5	2	9	6	3

1 and 9 produce an PNT because $\overset{+}{\aleph}(1) = 7,$ as $1 * 7 = 21 \equiv 1(\text{mod}10).$ Also $\overset{-}{\aleph}(1) = 9,$ as $1 * 9 = 27 \equiv 7(\text{mod}10).$ Now $\overset{+}{\aleph}(9) = 7,$ as $9 * 7 = 189 \equiv 9(\text{mod}10).$ Also $\overset{-}{\aleph}(9) = 1,$ as $9 * 1 = 27 \equiv 7(\text{mod}10).$ Thus, $[(1,9),(7,7),(9,1)]$ is a PNT because $1 * 9 = 27 \equiv 7(\text{mod}10) = \overset{+}{\aleph}(1).$

Similarly, $[(4,6),(2,2),(6,4)]$ is a PNT. But 2 and 4 does not give rise to a PNT. Since $[(2,4),(2,2),(2,6)]$ does not implies that $2 * 4 = \overset{+}{\aleph}(2),$ as $2 * 4 = 24 \equiv 4(\text{mod}10) \neq \overset{+}{\aleph}(2).$ The trivial PNT is denoted by $[(0,0),(0,0),(0,0)],$ as $\overset{+}{\aleph}(0) = 0$, $\overset{-}{\aleph}(0) = 0$ and $0 * 0 = 0 = \overset{-}{\aleph}(0).$

Theorem If $[(\ell, \wp),(\overset{+}{\aleph}(\ell), \overset{+}{\aleph}(\wp)),(\overset{-}{\aleph}(\ell), \overset{-}{\aleph}(\wp))]$ form a PNT, then $[(\overset{-}{\aleph}(\ell), \overset{-}{\aleph}(\wp)),(\overset{+}{\aleph}(\ell), \overset{+}{\aleph}(\wp)),(\ell, \wp)]$ also form a Pythagorean neutrosophic.

Proof Of course $\overset{-}{\aleph}(\ell) * \ell = \overset{+}{\aleph}(\ell)$ and $\overset{-}{\aleph}(\wp) * \wp = \overset{+}{\aleph}(\wp).$ We have to prove that $\overset{-}{\aleph}(\ell) * \overset{+}{\aleph}(\ell) = \overset{-}{\aleph}(\ell)$ and $\overset{-}{\aleph}(\wp) * \overset{+}{\aleph}(\wp) = \overset{-}{\aleph}(\wp).$ Multiply by ℓ and $\wp$ to the L.H.S, we have

$$\overset{-}{\aleph}(\ell) * \overset{+}{\aleph}(\ell) = \overset{-}{\aleph}(\ell) \text{ and } \overset{-}{\aleph}(\wp) * \overset{+}{\aleph}(\wp) = \overset{-}{\aleph}(\wp)$$

$$\ell * \overset{-}{\aleph}(\ell) * \overset{+}{\aleph}(\ell) = \ell * \overset{-}{\aleph}(\ell) \text{ and } \wp * \overset{-}{\aleph}(\wp) * \overset{+}{\aleph}(\wp) = \wp * \overset{-}{\aleph}(\wp)$$

or

$$[\ell * \overset{-}{\aleph}(\ell)] * \overset{+}{\aleph}(\ell) = \overset{+}{\aleph}(\ell) \text{ and } [\wp * \overset{-}{\aleph}(\wp)] * \overset{+}{\aleph}(\wp) = \overset{+}{\aleph}(\wp)$$

or

$$\overset{+}{\aleph}(\ell) * \overset{+}{\aleph}(\ell) = \overset{+}{\aleph}(\ell) \text{ and } \overset{+}{\aleph}(\wp) * \overset{+}{\aleph}(\wp) = \overset{+}{\aleph}(\wp)$$

Again multiply by ℓ and $\wp$ to the L.H.S, we have:

$$\ell * \overset{+}{\aleph}(\ell) * \overset{+}{\aleph}(\ell) = \ell * \overset{+}{\aleph}(\ell) \text{ and } \wp * \overset{+}{\aleph}(\wp) * \overset{+}{\aleph}(\wp) = \wp * \overset{+}{\aleph}(\wp)$$

or

$$[\ell * \overset{+}{\aleph}(\ell)] * \overset{+}{\aleph}(\ell) = \ell \text{ and } [\wp * \overset{+}{\aleph}(\wp)] * \overset{+}{\aleph}(\wp) = \wp$$

or

$$\ell * \overset{+}{\aleph}(\ell) = \ell \text{ and } \wp * \overset{+}{\aleph}(\wp) = \wp$$

Pythagorean neutrosophic triplet group (PNTG)

Definition If $(P_{\aleph},*)$ be a PNT set. Then, $P_{\aleph}$ is said be a PNTG with the properties.

(i). If $(P_{\aleph},*)$ is well-defined, i.e. for any $\ell, \wp \in P_{\aleph}$, one has $\ell * \wp \in \aleph$.

(ii). If $(P_{\aleph},*)$ is associative i.e. $(\ell_1 * \wp_1) * \hbar_1 = \ell_1 * (\wp_1 * \hbar_1)$ and $(\ell_2 * \wp_2) * \hbar_2 = \ell_2 * (\wp_2 * \hbar_2)$ for all ℓ_1, ℓ_2, $\wp_1$, $\wp_2$, $\hbar_1$, $\hbar_2 \in P_{\aleph}$,.

In general, the PNTG is different from the classical algebraic group.

The Pythagorean neutrosophic neutrals is considered as the classical unitary element and the Pythagorean neutrosophic opposites is considered as the classical inverse elements.

Example Consider $(Z_5,*)$, where $*$ is defined as $\ell * \wp = \ell (\text{mod}\, 5)$. Then, $(Z_5,*)$ is a PNTG with the given table.

$*$	0	1	2	3	4
0	0	0	0	0	0
1	1	1	1	1	1
2	2	2	2	2	2
3	3	3	3	3	3
4	4	4	4	4	4

It is associative, i.e. $(\ell_1 * \wp_1) * \hbar_1 = \ell_1 * (\wp_1 * \hbar_1)$ and $(\ell_2 * \wp_2) * \hbar_2 = \ell_2 * (\wp_2 * \hbar_2)$. Now take L. H. S to prove the R. H. S, so

$$\begin{aligned}(\ell_1 * \wp_1) * \hbar_1 &= \ell_1 * \hbar_1 \\ &= \ell_1\end{aligned}$$

also,

$$\begin{aligned}\ell_1 * (\wp_1 * \hbar_1) &= \ell_1 * \wp_1 \\ &= \ell_1.\end{aligned}$$

From (1) and (2), we have

$$(\ell_1 * \wp_1) * \hbar_1 = \ell_1 * (\wp_1 * \hbar_1).$$

Similarly,

$$\begin{aligned}(\ell_2 * \wp_2) * \hbar_2 &= \ell_2 * \hbar_2 \\ &= \ell_2\end{aligned}$$

also,

$$\begin{aligned}\ell_2 * (\wp_2 * \hbar_2) &= \ell_2 * \wp_2 \\ &= \ell_2.\end{aligned}$$

From (1) and (2), we have

$$(\ell_2 * \wp_2) * \hbar_2 = \ell_2 * (\wp_2 * \hbar_2).$$

The Pythagorean triplets are:

$$\left\{\begin{array}{l}[(0,1),(0,1),(0,1)]: \text{ such that } 0*1 = \overset{+}{\aleph}(0) \\ [(1,2),(1,2),(1,2)]: \text{ such that } 1*2 = \overset{+}{\aleph}(1) \\ [(4,3),(4,3),(4,3)]: \text{ such that } 4*3 = \overset{+}{\aleph}(4)\end{array}\right\}.$$

Thus, all the elements of Z_5 give rise to a PNT.

Example Consider $(Z_3,*)$, where $*$ is defined as $\ell * \wp = \ell + \wp + 3 (\mathrm{mod}\, 5)$. Then, $(Z_3,*)$ is a PNTG under the B-Operation $*$ with the below table.

$*$	0	1	2
0	0	1	2
1	1	2	0
2	2	0	1

It is associative, i.e. $(\ell_1 * \wp_1) * \hbar_1 = \ell_1 * (\wp_1 * \hbar_1)$ and $(\ell_2 * \wp_2) * \hbar_2 = \ell_2 * (\wp_2 * \hbar_2)$.

Also, the Pythagorean triplets are:

$$\left\{\begin{array}{l}[(0,1),(0,0),(0,2)]: \text{ such that } 0*1 = \overset{+}{\aleph}(0) \\ [(1,2),(0,1),(2,2)]: \text{ such that } 1*2 = \overset{+}{\aleph}(1)\end{array}\right\}$$

Thus, all the elements of Z_3 give rise to a PNT.

Remark Let $(P_{\aleph},*)$ be a PNTG under $*$ and let $[(\ell,\wp),(\overset{+}{\aleph}(\ell),\overset{+}{\aleph}(\wp)),(\overset{-}{\aleph}(\ell),\overset{-}{\aleph}(\wp))]$ such that $\ell * \wp = \overset{+}{\aleph}(\ell)$, be a PNT. Then, $\overset{+}{\aleph}(\ell)$ and $\overset{+}{\aleph}(\wp)$ is not unique in $P_{\aleph}$, and also ne $\overset{+}{\aleph}(\ell)$, $\overset{+}{\aleph}(\wp)$ depends on the elements ℓ and $\wp$, and the B-Operation $*$.

We consider the below Example to prove the above remark.

In Example 2, consider the PNT, $[(1,2),(1,2),(1,2)]$: such that $1*2=\overset{+}{\aleph}(1)$, so the $\overset{+}{\aleph}(1)=0,1,2,3,4.$ Similarly, $\overset{+}{\aleph}(\wp)=0,1,2,3,4.$ Thus, $\overset{+}{\aleph}(\ell)$ and $\overset{+}{\aleph}(\wp)$ is not unique in $P_{\aleph}$.

Remark Let $(P_{\aleph},*)$ be a PNTG under $*$ and let $[(\ell,\wp),(\overset{+}{\aleph}(\ell),\overset{+}{\aleph}(\wp)),(\overset{-}{\aleph}(\ell),\overset{-}{\aleph}(\wp))]$ such that $\ell * \wp = \overset{+}{\aleph}(\ell),$ be a PNT. Then, $\overset{-}{\aleph}(\ell)$ and $\overset{-}{\aleph}(\wp)$ is not unique in $P_{\aleph}$, and also $\overset{-}{\aleph}(\ell)$, $\overset{-}{\aleph}(\wp)$ depends on the elements ℓ and $\wp$, and the B-Operation $*$.

We consider the below Example to prove the above remark.

In Example 2, consider the PNT, $[(2,3),(2,3),(2,3)]$: such that $2*3=\overset{-}{\aleph}(2),$ so $\overset{-}{\aleph}(2)=0,1,2,3,4.$ Similarly, $\overset{+}{\aleph}(\wp)=0,1,2,3,4.$ Thus, $\overset{+}{\aleph}(\ell)$ and $\overset{+}{\aleph}(\wp)$ is not unique in $P_{\aleph}$.

Theorem All PNTG is a NTG.

Proof Let $P_{\aleph}$ be a PNTG. Then it satisfies the given conditions.

(i). If $(P_{\aleph},*)$ is well-defined, i.e. for any $\ell,\wp\in P_{\aleph}$, one has $\ell*\wp\in\aleph$.

(ii). If $(P_{\aleph},*)$ is associative i.e. $(\ell_1*\wp_1)*\hbar_1=\ell_1*(\wp_1*\hbar_1)$ and $(\ell_2*\wp_2)*\hbar_2=\ell_2*(\wp_2*\hbar_2)$ for all $\ell_1,\ \ell_2,\ \wp_1,\ \wp_2,\ \hbar_1,\ \hbar_2\in P_{\aleph},$.

From conditions (i) and (ii), we have that $P_{\aleph}$ is a NT.

Example In example 1, $(Z_{10},*)$ is a NT but not a PNTG because the PNT $[(2,4),(2,2),(2,6)]$ does not implies that $2*4=\overset{+}{\aleph}(2),$ as $2*4=24\equiv 4(\mathrm{mod}10)\neq\overset{+}{\aleph}(2).$ Hence, not all the elements of Z_{10} give rise to a PNT.

Definition Let $(P_{\aleph},*)$ be a PNTG. Then, $P_{\aleph}$ is called a commutative PNTG if for all $\ell,\wp\in P_{\aleph},$ we have $\ell*\wp=\wp*\ell.$

Example In example 2, $(Z_5,*)$ is a PNTG but not a commutative PNTG because $0*1\neq 1*0.$ Similarly, $1*2\neq 2*1.$

Theorem The idempotent elements ℓ and $\wp$ give rise to PNT under the operation $*$ such that $\ell * \wp = \ell$.

Solution Let ℓ and $\wp$ be idempotent elements. Then, by definition, $\ell^2 = \ell$ and $\wp^2 = \wp$, which clearly implies that $\overset{+}{\aleph}(\ell) = \ell$, $\overset{-}{\aleph}(\ell) = \ell$ and $\overset{+}{\aleph}(\wp) = \wp$, $\overset{-}{\aleph}(\wp) = \wp$. Also, $\ell * \wp = \ell = \overset{+}{\aleph}(\ell)$ or $\wp * \ell = \wp = \overset{+}{\aleph}(\wp)$. Hence ℓ and $\wp$ give rise to a PNT, that is,

$[(\ell, \wp), (\ell, \wp), (\ell, \wp)] : \ell * \wp = \ell.$

Theorem Let $(P_{\aleph}, *)$ be a PNTG with respect to $*$ and let $[(\ell_1, \wp_1), (\ell_2, \wp_2), (\ell_3, \wp_3)] : \ell_1 * \wp_1 = \overset{+}{\aleph}(\ell_1)$, be a PNT then

(i). $\ell_1 * \ell_2 = \ell_1 * \ell_3$ and $\wp_1 * \wp_2 = \wp_1 * \wp_3$ if and only if $\overset{+}{\aleph}(\ell_1) * \ell_2 = \overset{+}{\aleph}(\ell_1) * \ell_3$ and $\overset{+}{\aleph}(\wp_1) * \wp_2 = \overset{+}{\aleph}(\wp_1) * \wp_3$.

(ii). $\ell_2 * \ell_1 = \ell_3 * \ell_1$ and $\wp_2 * \wp_1 = \wp_3 * \wp_1$ if and only if $\overset{+}{\aleph}(\ell_2) * \ell_1 = \overset{+}{\aleph}(\ell_3) * \ell_1$ and $\overset{+}{\aleph}(\wp_2) * \wp_1 = \overset{+}{\aleph}(\wp_3) * \wp_1$.

Proof (i). Suppose that $\ell_1 * \ell_2 = \ell_1 * \ell_3$ and $\wp_1 * \wp_2 = \wp_1 * \wp_3$. Since $P_{\aleph}$ is a PNTG, so $\overset{-}{\aleph}(\ell_1), \overset{-}{\aleph}(\wp_1) \in P_{\aleph}$. Multiply $\overset{-}{\aleph}(\ell_1)$ to the L.H.S with $\ell_1 * \ell_2 = \ell_1 * \ell_3$, we get:

$$\begin{aligned} \overset{-}{\aleph}(\ell_1) * \ell_1 * \ell_2 &= \overset{-}{\aleph}(\ell_1) * \ell_1 * \ell_3 \\ [\overset{-}{\aleph}(\ell_1) * \ell_1] * \ell_2 &= [\overset{-}{\aleph}(\ell_1) * \ell_1] * \ell_3 \\ \overset{+}{\aleph}(\ell_1) * \ell_2 &= \overset{+}{\aleph}(\ell_1) * \ell_3. \end{aligned}$$

Similarly, multiply $\overset{-}{\aleph}(\wp_1)$ to the left side with $\wp_1 * \wp_2 = \wp_1 * \wp_3$, we get:

$$\begin{aligned} \overset{-}{\aleph}(\wp_1) * \wp_1 * \wp_2 &= \overset{-}{\aleph}(\wp_1) * \wp_1 * \wp_3 \\ [\overset{-}{\aleph}(\wp_1) * \wp_1] * \wp_2 &= [\overset{-}{\aleph}(\wp_1) * \wp_1] * \wp_3 \\ \overset{+}{\aleph}(\wp_1) * \wp_2 &= \overset{+}{\aleph}(\wp_1) * \wp_3. \end{aligned}$$

(ii). The proof is similar to (i).

Theorem Let $(P_{\aleph}, *)$ be a PNTG with respect to $*$ and let $[(\ell_1, \wp_1), (\ell_2, \wp_2), (\ell_3, \wp_3)] : \ell_1 * \wp_1 = \overset{+}{\aleph}(\ell_1)$, be a PNT then

(i). $\bar{\aleph}(\ell_1)*\ell_2=\bar{\aleph}(\ell_1)*\ell_3$ and $\bar{\aleph}(\wp_1)*\wp_2=\bar{\aleph}(\wp_1)*\wp_3$ then $\overset{+}{\aleph}(\ell_1)*\ell_2=\overset{+}{\aleph}(\ell_1)*\ell_3$ and $\overset{+}{\aleph}(\wp_1)*\wp_2=\overset{+}{\aleph}(\wp_1)*\wp_3$.

(ii). $\ell_2*\bar{\aleph}(\ell_1)=\ell_3*\bar{\aleph}(\ell_1)$ and $\wp_2*\bar{\aleph}(\wp_1)=\wp_3*\bar{\aleph}(\wp_1)$ then $\ell_2*\overset{+}{\aleph}(\ell_1)=\ell_3*\overset{+}{\aleph}(\ell_1)$ and $\wp_2*\overset{+}{\aleph}(\wp_1)=\wp_3*\overset{+}{\aleph}(\wp_1)$.

Proof (i). Suppose that $\bar{\aleph}(\ell_1)*\ell_2=\bar{\aleph}(\ell_1)*\ell_3$ and $\bar{\aleph}(\wp_1)*\wp_2=\bar{\aleph}(\wp_1)*\wp_3$. Since $P_{\aleph}$ is a PNTG with respect to $*$, so ℓ_1, $\wp_1\in P_{\aleph}$. Multiply ℓ_1 to the left side with $\bar{\aleph}(\ell_1)*\ell_2=\bar{\aleph}(\ell_1)*\ell_3$, we get:

$$\begin{aligned}\ell_1*\bar{\aleph}(\ell_1)*\ell_2&=\ell_1*\bar{\aleph}(\ell_1)*\ell_3\\ [\ell_1*\bar{\aleph}(\ell_1)]*\ell_2&=[\ell_1*\bar{\aleph}(\ell_1)]*\ell_3\\ \overset{+}{\aleph}(\ell_1)*\ell_2&=\overset{+}{\aleph}(\ell_1)*\ell_3.\end{aligned}$$

Similarly, multiply $\wp_1$ to the left side with $\bar{\aleph}(\wp_1)*\wp_2=\bar{\aleph}(\wp_1)*\wp_3$, we get:

$$\begin{aligned}\wp_1*\bar{\aleph}(\wp_1)*\wp_2&=\wp_1*\bar{\aleph}(\wp_1)*\wp_3\\ [\wp_1*\bar{\aleph}(\wp_1)]*\wp_2&=[\wp_1*\bar{\aleph}(\wp_1)]*\wp_3\\ \overset{+}{\aleph}(\wp_1)*\wp_2&=\overset{+}{\aleph}(\wp_1)*\wp_3.\end{aligned}$$

(ii). The proof is same as (i).

Theorem Let $(P_{\aleph},*)$ be a PNTG with respect to $*$ and let $[(\ell_1,\wp_1),(\ell_2,\wp_2),(\ell_3,\wp_3)]:\ell_1*\wp_1=\overset{+}{\aleph}(\ell_1)$, be a PNT then $\overset{+}{\aleph}(\ell_1)*\overset{+}{\aleph}(\wp_1)=\overset{+}{\aleph}(\ell_1*\wp_1)$.

Proof Consider L.H.S, $\overset{+}{\aleph}(\ell_1)*\overset{+}{\aleph}(\wp_1)$. Now, multiply to the L.H.S with ℓ_1 and to the R.H.S with $\wp_1$, we get:

$$\begin{aligned}\ell_1*\overset{+}{\aleph}(\ell_1)*\overset{+}{\aleph}(\wp_1)*\wp_1&=[\ell_1*\overset{+}{\aleph}(\ell_1)]*[\overset{+}{\aleph}(\wp_1)*\wp_1]\\ &=\ell_1*\wp_1.\end{aligned}$$

Now consider R.H.S, we have $\overset{+}{\aleph}(\ell_1*\wp_1)$. Again multiply to the L.H.S with ℓ_1 and to the R.H.S with $\wp_1$, we get:

$$\ell_1*\overset{+}{\aleph}(\ell_1*\wp_1)*\wp_1=[\ell_1*\wp_1]*\overset{+}{\aleph}(\ell_1*\wp_1),$$

as $*$ is associative, so we have

$$\ell_1 * \overset{+}{\aleph}(\ell_1 * \wp_1) * \wp_1 = \ell_1 * \wp_1.$$

Hence proved.

Theorem Let $(P_{\aleph}, *)$ be a PNTG with respect to $*$ and let $[(\ell_1, \wp_1), (\ell_2, \wp_2), (\ell_3, \wp_3)] : \ell_1 * \wp_1 = \overset{+}{\aleph}(\ell_1),$ be a PNT then $\overset{+}{\aleph}(\ell_1) * \overset{+}{\aleph}(\wp_1) = \overset{+}{\aleph}(\ell_1 * \wp_1).$

Proof Consider L.H.S, $\overset{-}{\aleph}(\ell_1) * \overset{-}{\aleph}(\wp_1)$. Now multiply to the L.H.S with ℓ_1 and to the R.H.S with $\wp_1$, we get:

$$\begin{aligned} \ell_1 * \overset{-}{\aleph}(\ell_1) * \overset{-}{\aleph}(\wp_1) * \wp_1 &= [\ell_1 * \overset{-}{\aleph}(\ell_1)] * [\overset{-}{\aleph}(\wp_1) * \wp_1] \\ &= \overset{+}{\aleph}(\ell_1) * \overset{+}{\aleph}(\wp_1) \\ &= \overset{+}{\aleph}(\ell_1 * \wp_1), \text{ By above theorem.} \end{aligned}$$

Now consider R.H.S, we have $\overset{-}{\aleph}(\ell_1 * \wp_1)$. Again multiply to the L.S with ℓ_1 and to the R.S with $\wp_1$, we get:

$$\ell_1 * \overset{-}{\aleph}(\ell_1 * \wp_1) * \wp_1 = [\ell_1 * \wp_1] * \overset{-}{\aleph}(\ell_1 * \wp_1),$$

as $*$ is associative, so we have

$$\ell_1 * \overset{-}{\aleph}(\ell_1 * \wp_1) * \wp_1 = \overset{+}{\aleph}(\ell_1 * \wp_1).$$

Hence proved.

Theorem Let $(P_{\aleph}, *)$ be a commutative PNTG with respect to $*$ and let $[(\ell_1, \wp_1), (\ell_2, \wp_2), (\ell_3, \wp_3)] : \ell_1 * \wp_1 = \overset{+}{\aleph}(\ell_1),$ be a PNT then

$(i).$ $\overset{+}{\aleph}(\ell_1) * \overset{+}{\aleph}(\wp_1) = \overset{+}{\aleph}(\wp_1) * \overset{+}{\aleph}(\ell_1).$

$(ii).$ $\overset{-}{\aleph}(\ell_1) * \overset{-}{\aleph}(\wp_1) = \overset{-}{\aleph}(\wp_1) * \overset{-}{\aleph}(\ell_1).$

Proof Consider the L.H.S $\overset{+}{\aleph}(\ell_1) * \overset{+}{\aleph}(\wp_1)$. According to Theorem 7, we have

$$\begin{aligned} \overset{+}{\aleph}(\ell_1) * \overset{+}{\aleph}(\wp_1) &= \overset{+}{\aleph}(\ell_1 * \wp_1) \\ &= \overset{+}{\aleph}(\wp_1 * \ell_1), \end{aligned}$$

as $P_{\aleph}$ is commutative, so

$$\overset{+}{\aleph}(\ell_1) * \overset{+}{\aleph}(\wp_1) = \overset{+}{\aleph}(\wp_1) * \overset{+}{\aleph}(\ell_1),$$

again by Theorem 7.

Thus $\overset{+}{\aleph}(\ell_1) * \overset{+}{\aleph}(\wp_1) = \overset{+}{\aleph}(\wp_1) * \overset{+}{\aleph}(\ell_1)$.

(ii). The proof is same as (i).

Definition Let $(P_{\aleph}, *)$ be a PNTG with respect to $*$, and let $P'_{\aleph} \subseteq P_{\aleph}$. Then, $P'_{\aleph}$ is said to be a PNT subgroup of $P_{\aleph}$ if $P'_{\aleph}$ itself a PNTG with respect to $*$.

Example Consider $(Z_5, *)$, where $*$ is defined as $\ell * \wp = \ell (\mathrm{mod}\, 5)$. Then, $(Z_5, *)$ is a PNTG under the binary operation $*$, and $P'_{\aleph} = \{0, 1, 2, 3\}$ be a subset of Z_5. Then, clearly $P'_{\aleph}$ is a PNT subgroup of Z_5.

Proposition Let $(P_{\aleph}, *)$ be a PNTG and $P'_{\aleph} \subseteq P_{\aleph}$. Then $P'_{\aleph}$ is a PNT subgroup of N $\Leftrightarrow$ the following properties are satisfied.

(i). $\ell_1 * \wp_1 \in P'_{\aleph}$ for all ℓ_1, $\wp_1 \in P'_{\aleph}$.

(ii). $\overset{+}{\aleph}(\ell_1)$, $\overset{+}{\aleph}(\wp_1) \in P'_{\aleph}$ for all ℓ_1, $\wp_1 \in P'_{\aleph}$.

(iii). $\overset{-}{\aleph}(\ell_1)$, $\overset{-}{\aleph}(\wp_1) \in P'_{\aleph}$ for all ℓ_1, $\wp_1 \in P'_{\aleph}$.

Proof It is easy to prove.

Definition Let $P_{\aleph}$ be a PNTG and let $\ell \in P_{\aleph}$. The smallest positive integer $n \geq 1$ such that $\ell^n = \overset{+}{\aleph}(\ell)$ is called PNT order. It is denoted by $pnto(\ell)$.

Example Consider $(Z_5, *)$, where $*$ is defined as $\ell * \wp = \ell (\mathrm{mod}\, 5)$ then

$$pnto(1) = 1,\ pnto(2) = 1,\ pnto(3) = 1,\ pnto(4) = 1.$$

Theorem Let $(P_{\aleph}, *)$ be a PNTG with respect to $*$ and let $\alpha \in P_{\aleph}$. Then $\overset{+}{\aleph}(\alpha) * \overset{+}{\aleph}(\alpha) = \overset{+}{\aleph}(\alpha)$. In general $(\overset{+}{\aleph}(\alpha))^n = \overset{+}{\aleph}(\alpha)$, where $n \geq 1$.

Proof Consider $\overset{+}{\aleph}(\alpha) * \overset{+}{\aleph}(\alpha) = \overset{+}{\aleph}(\alpha)$. Multiply α to the left side, we get;

$$\alpha * \overset{+}{\aleph}(\alpha) * \overset{+}{\aleph}(\alpha) = \alpha * \overset{+}{\aleph}(\alpha)$$
$$[\alpha * \overset{+}{\aleph}(\alpha)] * \overset{+}{\aleph}(\alpha) = [\alpha * \overset{+}{\aleph}(\alpha)]$$
$$\alpha * \overset{+}{\aleph}(\alpha) = \alpha$$
$$\alpha = \alpha$$

Similarly, we can easily see that $(\overset{+}{\aleph}(\alpha))^n = \overset{+}{\aleph}(\alpha)$ for a nonzero positive integer n.

Definition Let $P_{\aleph}$ be a PNTG and $\alpha \in P_{\aleph}$. Then, $P_{\aleph}$ is called Pythagorean neutro-cyclic triplet group if $P_{\aleph} = \langle \alpha \rangle$. We say that α is a generator part of the PNT.

5. Pythagorean neutro-homomorphism (PN-h)

In this section, we introduce PN-h and Pythagorean neutro-isomorphisms (PN-i) for the PNTGs.

Definition Let $(P_{\aleph_1}, *)$ and $(P_{\aleph_2}, \#)$ be two PNTGs. Let $F : P_{\aleph_1} \to P_{\aleph_2}$ be mapping. Then, F is called PN-h if for all ℓ, $\wp \in P_{\aleph_1}$, we have

(i). $F(\ell * \wp) = F(\ell) \ \# \ F(\wp)$.

(ii). $F(\overset{+}{\aleph}(\ell)) = \overset{+}{\aleph}(F(\ell))$.

(iii). $F(\overset{-}{\aleph}(\ell)) = \overset{-}{\aleph}(F(\ell))$.

Example Let $P_{\aleph_1}$ be a PNTGs with respect to $*$ modulo 10 in $(Z_{10}, *)$, where $*$ is defined as: $\ell * \wp = \ell (\mathrm{mod}\, 10)$, and $Z_{10} = \{0,1,2,3,...,9\}$. Let $F : P_{\aleph_1} \to P_{\aleph_1}$ be a mapping defined as:

$$F(0) = 0,\ F(1) = 1,\ F(2) = 2,\ F(3) = 3,\ F(4) = 4,\ F(5) = 5,\ F(6) = 6,$$
$$F(7) = 7,\ F(8) = 8,\ F(9) = 9.$$

Then, clearly F is a PN-h because conditions (i), (ii) and (iii) are satisfied easily.

Definition A PN-h is called PN-i if it is one--one and onto.

Example Let $P_{\aleph_1}$ be a PNTGs with respect to $*$ modulo 5 in $(Z_5, *)$, where $*$ is defined as: $\ell * \wp = \ell (\mathrm{mod}\, 5)$, and $Z_5 = \{0,1,2,3,4\}$. Let $F : P_{\aleph_1} \to P_{\aleph_1}$ be a mapping defined as:

$$F(0) = 0,\ F(1) = 1,\ F(2) = 2,\ F(3) = 3,\ F(4) = 4.$$

Then, clearly F is a PN-i.

Note that F further defines that,

(i). F is called Pythagorean neutro-endomorphism of $P_{\aleph_1}$, if $P_{\aleph_1} = P_{\aleph_2}$.

(ii). F is called Pythagorean neutro-epimorphism if F is onto.

(iii). F is called Pythagorean neutro-monomorphism if F is $(1-1)$.

(iv). A Pythagorean neutro-endomorphism F of a Pythagorean triplet group $P_{\aleph}$ is called a Pythagorean neutro-automorphism of G if F is $(1-1)$ and onto.

Definition Two given PNTGs $(P_{\aleph_1},*)$ and $(P_{\aleph_2},\#)$ are said to be Pythagorean neutro-isomorphic to each other if there exists a PN-i between $P_{\oplus 1}$ and $P_{\oplus 2}$.

They are written as $P_{\aleph_1} \cong P_{\aleph_2}$, and read as "the PNTG $P_{\aleph_1}$ is Pythagorean neutro-isomorphic to the PNTG $P_{\aleph_2}$".

Theorem The relation "$\cong$" of Pythagorean neutro-isomorphism over the set of all the Pythagorean neutrosophic triplet group is an equivalence relation.

Proof Let $P_{\aleph_1}$, $P_{\aleph_2}$ and $P_{\aleph_3}$ be three Pythagorean neutrosophic triplet groups. Then $P_{\aleph_1} \cong P_{\aleph_1}$ by the identity Pythagorean neutro-isomorphism.

For symmetry, suppose $P_{\aleph_1} \cong P_{\aleph_2}$. Then there exists a Pythagorean neutro-isomorphism $F : P_{\aleph_1} \to P_{\aleph_2}$. Since F is $(1-1)$ and onto Pythagorean neutro-homomorphism from $P_{\aleph_1}$ onto $P_{\aleph_2}$, therefore $F^{-1} : P_{\aleph_2} \to P_{\aleph_1}$ is a Pythagorean neutro-isomorphism. Thus $P_{\aleph_2} \cong P_{\aleph_1}$.

For transitivity, suppose, $P_{\aleph_1} \cong P_{\aleph_2}$ and $P_{\aleph_2} \cong P_{\aleph_3}$. Since $P_{\aleph_1}$ is Pythagorean neutro-isomorphism to $P_{\aleph_2}$ and $P_{\aleph_2}$ is Pythagorean neutro-isomorphism to $P_{\aleph_3}$, therefore, there exists, Pythagorean neutro-isomorphism F and g such that $F : P_{\aleph_1} \to P_{\aleph_2}$ and $g : P_{\aleph_2} \to P_{\aleph_3}$.

Since $g \circ F : P_{\aleph_1} \to P_{\aleph_3}$ is a Pythagorean neutro-isomorphism from $P_{\aleph_1}$ into $P_{\aleph_3}$. It proves that $P_{\aleph_1} \cong P_{\aleph_3}$.

Hence the relation "$\cong$" is an equivalence relation over the set of all Pythagorean neutrosophic triplet groups.

6. Distinctions and comparison

The distinctions between NT [20] and PNTG are:

(i). In the NT, any single element $\ell \in \aleph$, gives rise to a neutrosophic triplet, while in the PNTG, two elements ℓ_1, $\wp_1 \in P_{\aleph}$ give rise to a PNT with the extra condition, that is, $\ell_1 * \wp_1 = \overset{+}{\aleph}(\ell_1)$.

(ii). The structure of neutrosophic triplet is $(\ell, \wp, \hbar)$, where $\wp = \overset{+}{\aleph}(\ell)$, and $\hbar = \overset{-}{\aleph}(\ell)$, while the structure of PNT is

$$[(\ell_1, \wp_1), (\ell_2, \wp_2), (\hbar_1, \hbar_2)]: \text{ such that } \ell_1 * \wp_1 = \overset{+}{\aleph}(\ell_1),$$

where $\ell_2 = \overset{+}{\aleph}(\ell_1)$, $\ell_3 = \overset{-}{\aleph}(\ell_1)$, $\wp_2 = \overset{+}{\aleph}(\wp_1)$, and $\wp_3 = \overset{-}{\aleph}(\wp_1)$.

Clearly, the NTG has a weaker structure than the PNTG.

7. Conclusions

Inspiring from the neutrosophic triplet, we developed the notion of PNT which is a group of three ordered pairs that satisfy certain properties with some binary operation. The purpose of this paper is first to introduce the PNT and then used these PNTs to introduce the novel concept of a PNTG, which differs from a classical group in terms of structural features. The PNTG is completely different from the neutrosophic group. We discussed some properties, basic results, and particular examples of these novel concepts. We further studied PN-h, PN-i, etc., for NTs. Moreover, we discussed the main distinctions between the NTG and the PNTG.

Conflict of Interest

The authors declare that they have no conflict of interests.

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q-Rung Neutrosophic Sets and Topological Spaces

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Abstract: The concept of q-rung orthopair neutrosophic set is introduced in this paper and fundamental properties of it are studied. Also the ordinary notion of topological space is extended to q-rung orthopair neutrosophic environment, as well as the fundamental concepts of convergence, continuity, compactness and Hausdorff topological space. All these generalizations are illustrated by suitable examples.

Keywords: Fuzzy Set; Intuitionistic Fuzzy Set; Neutrosophic Set; q-Rung Orthopair Fuzzy Set; q-Rung Orthopair Neutrosophic Set; q-Rung Orthopair Neutrosophic Topological Space.

1. Introduction

Zadeh, in 1965, extended the concept of a crisp set to that of a *fuzzy set* [1], on the purpose of tackling mathematically the existing in everyday life partial truths, as well as the definitions having no clear boundaries, like "high mountains", "clever people", "good players", etc. Zadeh's idea was to replace the objective function of crisp sets with the membership function in fuzzy sets taking values in the interval [0, 1]. In this way a membership degree between 0 and 1 is assigned to each element of the universal set within the corresponding fuzzy set.

Before the introduction of fuzzy sets, probability used to be the unique mathematical tool in hands of the experts for managing the existing in real world uncertainty, caused by the shortage of knowledge about an observed phenomenon. Several types of uncertainty appear in everyday life, including *randomness, imprecision, vagueness, ambiguity, inconsistency*, etc. [2]**.** The uncertainty due to randomness is related to well-defined events whose outcomes cannot be predicted in advance, like the turn of a coin, the throwing of a die, etc. Imprecision occurs when the corresponding events are well defined, but the possible outcomes cannot be expressed with an exact numerical value; e.g. "The temperature tomorrow will be over 30^{o} C". Vagueness is created when one is unable to clearly differentiate between two properties, like a good and a mediocre student. In case of ambiguity the existing information leads to several interpretations by different observers. For example, the phrase "Boy no girl" written as "Boy, no girl" means boy, but written as "Boy no, girl" means girl. Inconsistency appears when two or more pieces of information cannot be true at the same time. As a result the obtainable in this case information is conflicted or undetermined. For example, "The chance of raining tomorrow is 80%, but this does not mean that the chance of not raining is 20%, because they might be hidden weather factors".

Probability, however, was proved to be effective only for tackling the uncertainty due to randomness, in contrast to fuzzy sets which were proved to be suitable for tackling other forms of uncertainty as well, and in particular the uncertainty due to vagueness [3]. Following the introduction of fuzzy sets, several generalizations of them and theories related to them have been proposed on the purpose of tackling more effectively all the forms of the existing uncertainty, e.g. see [4].None of these generalizations or theories, however, was proved to be suitable for tackling all the

forms of the uncertainty alone, but the synthesis of all of them forms an effective framework for this purpose.

In 1986, Atanassov expanded the definition of fuzzy set to *intuitionistic fuzzy set* by adding the degree of non-membership to Zadeh's degree of membership [5]. Intuitionistic fuzzy sets can be used everywhere the ordinary fuzzy sets can be applied, but this is not always necessary. An example where the use of intuitionistic fuzzy sets is necessary is the case of an election, where a candidate can have voted for (membership), or voted against (non-membership) by the electoral vote. The intuitionistic fuzzy sets are suitable for tackling the uncertainty due to imprecision, which appears frequently in human reasoning [6].

Yager introduced later the concept of the *q-rung orthopair fuzzy set* with q a positive integer, in which the sum of the q-th powers of the membership and non-membership degrees of the elements of the universal set is bounded by 1 [7]. When q=1 we have an intuitionistic fuzzy set, when q=2 a *Pythagorean fuzzy set* [8] and when q=3 a *Fermatean fuzzy set* [9]. It has been established that Pythagorean and Fermatean fuzzy sets have stronger ability than intuitionistic fuzzy sets for tackling the uncertainty in decision making problems [10].

In 1995, Smarandache proposed the concept of the degree of indeterminacy or neutrality and further expanded the idea of intuitionistic fuzzy set to that of *neutrosophic set* [11], drawing inspiration from the neutralities that frequently surface in everyday life, like <friend, neutral, enemy>, <win, draw, defeat>, <high, medium, short>, etc. Neutrosophic sets are effective for tackling the uncertainty due to inconsistency and ambiguity.

In 2019 the concept of Pythagorean fuzzy set was extended to neutrosophic environments [12] and in 2021 the same happened with the Fermatean fuzzy set [13].

In this work we introduce the concept of the *q-rung orthopair neutrosophic set* focusing on fundamental properties of this kind of set and on *q-rung orthopair neutrosophic topological spaces.* The rest of the paper is formulated as follows: Section 2 contains the necessary mathematical background for the understanding of the paper. The concept of the q-rung orthopair neutrosophic set is presented in Section 3 together with basic properties of these sets. In Section 4 the classical notion of topological space is extended to *q-rung orthopair neutrosophic topological spaces* together with fundamental properties and concepts like convergence, continuity, compactness and Hausdorff topological spaces. The paper closes with the final conclusions and some hints for further research included in its last Section 5.

2. Mathematical Background

The exact definition of a fuzzy set [1] is the following:

Definition 1: A fuzzy set A in the universal set of the discourse U is a set of ordered pairs of the form:

$$A = \{(x, m(x)): x \in U, m(x) \in [0, 1]\} \quad (1)$$

In Eq. (1) m: U → [0, 1] is the membership function of A, and the real value y = m(x) is the membership degree of x in A, for all x in U. The greater m(x), the better x satisfies the characteristic property of A.

The definition of the membership function of A is not unique depending on the personal goals of each observer. For example, if A is the fuzzy set of "tall men", one may consider all the men with heights greater than 1.90 m tall and another one all those with heights greater than 2 m. As a result, the first observer will assign membership degree 1 to all men with heights between 1.90 m and 2 m, whereas the second one will assign membership degrees <1 to them. The only restriction for the definition of the membership function is to be compatible with common sense; otherwise it does not give a creditable description of the real situation represented by the corresponding fuzzy set. This could happen for instance in the previous example, if men with heights less than 1.60 m had membership degrees > 0.5. Most of the properties and operations of crisp sets can be extended in a natural way to fuzzy sets; e.g. see [2].

Atanassov extended the concept of fuzzy set to that of intuitionistic fuzzy set [5] as follows:

Definition 2: An intuitionistic fuzzy set A in the universal set U is of the form

$A = \{(x, m(x), n(x)): x \in U, m(x), n(x) \in [0, 1], \quad 0 \leq m(x) + n(x) \leq 1\}$ (2)

In Eq. (2) m: U → [0, 1] is the membership function and n: U → [0, 1] is the non-membership function of A and m(x), n(x) are the degrees of membership and non-membership respectively for each x in A. For simplicity we write A = <m, n>. Further, h(x) = 1 - m(x) - n(x), is said to be the degree of hesitation of x in A. If h(x) = 0, then A is a fuzzy set.

The term intuitionistic fuzzy set is due to Atanassov's collaborator Gargov [14] in analogy to the idea of intuitionism introduced by Brouwer at the beginning of the last century [15]. It is recalled that intuitionism rejects Aristotle's law of the excluded middle by stating that a proposition is either true, or not true, or we do not know if it is true or not true. The first part of this statement corresponds to Zadeh's membership degree, the second to Atanassov's non-membership and the third to the degree of hesitation.

Example 1: Let U be the set of the students of a class, let A be the intuitionistic fuzzy set of the good students of the class. Then each student x of U is characterized by an *intuitionistic fuzzy pair* (m, n) with respect to A, with m, n in [0, 1].For example, if (x, 0.5, 0.3) is in A, then, there is a 50% belief that x is a good student, but also a 30% belief that he is not a good student and a 20% hesitation to characterize him as a good student or not.

The properties and operations of fuzzy sets can be extended to intuitionistic fuzzy sets; e.g. see [6].

When defining the membership and non-membership degrees of the elements of the universal set U, it could happen that m(x) + n(x) >1. In such cases the corresponding structure cannot be treated as an intuitionistic fuzzy set. This motivated Yager to define the wider class of q-rung orthopair fuzzy sets [7] as follows:

Definition 3: A q-rung orthopair fuzzy set A in the universal set U, where q is a positive integer, is of the fo

$A = \{(x, m(x), n(x)): x \in U, m(x), n(x) \in [0, 1], \quad 0 \leq [m(x)]^q + [n(x)]^q \leq 1\}$ (3)

In Eq. (3) m(x) is the membership and n(x) is the non-membership degree of x in A respectively.

For q = 1, a 1-rung orthopair fuzzy set is an ordinary intuitionistic fuzzy set. Further, a 2-rung orthopair fuzzy set is referred to as a Pythagorean fuzzy set [8] and a 3-rung orthopair fuzzy set is referred to as a Fermatean fuzzy set [9].

The following Proposition helps to clarify the Yager's motivation for introducing the notion of the q-rung orthopair fuzzy set:

Proposition 1: Let q_1, q_2 be positive integers, with $q_2 > q_1$. Then the set $\{(m, n): m, n \in [0, 1], 0 \leq m^{q_2} + n^{q_2} \leq 1\}$ is larger than the set $\{(m, n): m, n \in [0, 1], 0 \leq m^{q_1} + n^{q_1} \leq 1\}$.

Proof: Since m, n ∈ [0, 1], is $m^{q_2} + n^{q_2} < m^{q_1} + n^{q_1}$. Consequently, if $m^{q_1} + n^{q_1} \leq 1$, it is also $m^{q_2} + n^{q_2} \leq 1$ and the result follows.

Example 2: Let (x, 0.8, 9.7) be an element of the q-rung orthopair fuzzy set A. Then, since 0.8 + 0.7 >1, A is not an intuitionistic fuzzy set, Also, since $(0.8)^2 + (0.7)^2 = 0.64 + 0.49 > 1$, A is not a Pythagorean fuzzy set too. But $(0.8)^3 + (0.7)^3 = 0.512 + 0.343 < 1$. Thus A could be a Fermatean fuzzy set, this depending on the form of its other elements.

Proposition 2: Let A be q_1-rung and B be q_2-rung orthopair fuzzy sets respectively, with $q_2 > q_1$. Then A is also a q_2-rung orthopair fuzzy set.

Proof: Let x(m, n) be an element of A. Then $0 \leq m^{q_1} + n^{q_1} \leq 1$, with m, n ∈ [0, 1]. But $q_2 > q_1$, therefore, $0 \leq m^{q_2} + + n^{q_2} \leq m^{q_1} + n^{q_1} \leq 1$ and the result follows.

In particular, an intuitionistic fuzzy set is a Pythagorean fuzzy set, which is a Fermatean fuzzy set.

The simplest form of a neutrosophic set is the *single valued neutrosophic set*, which is defined as follows [16]:

Definition 4: A single valued neutrosophic set A in the universe U is of the form

$A = \{(x, m(x),i(x),n(x)): x \in U, m(x), i(x), n(x) \in [0,1], 0 \leq m(x)+i(x)+n(x) \leq 3\}$ (4)

In Eq. (4) m(x), i(x), n(x) are the degrees of membership (or truth), indeterminacy (or neutrality) and non-membership (or falsity) with respect to A, for all x in U, referred to as the neutrosophic components of x. For simplicity, we write A = <m, i, n>.

The etymology of the term "neutrosophy" comes from the adjective "neutral" and the Greek word "sophia" (wisdom) and, according to Smarandanche, who introduced it, means the "knowledge of the neutral thought".

Example 3: Let U be the set of the players of a football team and let A be the single valued neutrosophic set of the good players of U. Then each player x of U is characterized by a *neutrosophic triplet* (m, i, n) with respect to A, with m, i, n in [0, 1]. For example, x(0.6, 0.2, 0.4) ∈ A, means that there is a 60% belief that x is a good player, but simultaneously a 20% doubt about and a 40% belief that x may not be a good player. In particular, x(0,1,0) ∈ A means that we do not know absolutely nothing about x's affiliation with A.

The concepts and operations defined on intuitionistic fuzzy sets can be extended in a natural way to neutrosophic sets [11]**.**

Remark 1:

(i). Indeterminacy is understood to be everything which is between the opposites of truth and falsity [17]. In an intuitionistic fuzzy set the indeterminacy is equal by default with the hesitancy, i.e. we have i(x)=1- m(x) – n(x). Also, in a fuzzy set is i(x) = 0 and n(x) = 1 – m(x), whereas in a crisp set it is m(x) =1 (or 0) and n(x)= 0 (or 1). In other words, crisp sets, fuzzy sets and intuitionistic fuzzy sets are special cases of single valued neutrosophic sets.

(ii). When the sum m(x) + i(x) + n(x) of the neutrosophic components of x ∈ U in a neutrosophic set in U is < 1, then it leaves room for incomplete information about x, when it is equal to 1 for complete information and when is greater than 1 for inconsistent (i.e. contradiction tolerant) information about x. A single valued neutrosophic set may contain simultaneously elements leaving room for all the previous types of information.

(iii). When m(x) + i(x) + n(x) < 1, ∀ x ∈ U, then the corresponding single valued neutrosophic set is usually referred to as *picture fuzzy set* [18]. In this case 1- m(x) -i(x) -n(x) is called the degree of refusal membership of x in A. The picture fuzzy sets based models are suitable for describing situations where we face human opinions involving answers of types yes, abstain, no and refusal to express an opinion. Voting is a representative example of such a situation.

(iv). The difference between the general definition of a neutrosophic set and of the previously given definition of a single valued neutrosophic set is that in the general definition m(x), i(x) and n(x) may take values in the non-standard unit interval]−0, 1+[including values < 0 or > 1. This is something that can happen in everyday life situations; e.g. see an example, in [11].

3. Extending the Concept of Orthopair Fuzzy Set to Neutrosophic Environment

The concept of Pythagorean fuzzy set was extended to neutrosophic environment [12] as follows:

Definition 5: A Pythagorean neutrosophic set A in the universe U is of the form:

$$A = \{(x, m(x), i(x), n(x)): x \in U, m(x), i(x), n(x) \in [0,1], 0 \leq m^2(x) + n^2(x) \leq 1\} \quad (5)$$

In Eq. (5) m(x), i(x), n(x) are the degrees of membership, indeterminacy and non-membership with respect to A, for all x in U. Since $0 \leq m^2(x)+n^2(x) \leq 1$, the neutrosophic components m(x) and n(x) are dependent and the component i(x) is independent.

Proposition 3: Let A be a Pythagorean neutrosophic set, then $0 \leq m^2(x) + i^2(x) + n^2(x) \leq 2$, for all x in U.

Proof: Since $i(x) \in [0, 1]$, is $0 \leq i^2(x) \leq 1$ and the result follows by $0 \leq m^2(x) + n^2(x) \leq 1$.

Remark 2: By Corollary 1 an intuitionistic fuzzy set is a Pythagorean fuzzy set too. A neutrosophic set, however, need not be a Pythagorean neutrosophic set. For example, let A be a neutrosophic set and let x(0.8, 0.3. 0.7) be an element of A. Then $(0.8)^2 + (0.7)^2 = 0.64 + 0.49 > 1$, therefore A is not a Pythagorean neutrosophic set.

The concept of Fermatean neutrosophic set has also been defined [13] as follows:

Definition 6: A Fermatean neutrosophic set A in the universe U is of the form

$$A = \{(x, m(x), i(x), n(x)): x \in U, m(x), i(x), n(x) \in [0,1], 0 \leq m^3(x) + n^3(x) \leq 1\} \quad (6)$$

In Eq. (6) m(x), i(x), n(x) are the degrees of membership, indeterminacy and non-membership with respect to A, for all x in U. The components m(x) and n(x) are dependent and the component i(x) is independent. Similarly with Proposition 3 it can be shown that $0 \leq m^3(x) + i^3(x) + n^3(x) \leq 2$, for all x in U.

Pythagorean and Fermatean neutrosophic sets have found some interesting applications; e.g. see [12, 19, 20-23], etc.

Here we extend the concept of a Pythagorean (Fermatean) neutrosophic set as follows:

Definition 7: A *q-rung orthopair neutrosophic set* A, with q a positive integer, is of the form

$$A = \{(x, m(x), i(x), n(x)): x \in U, m(x), i(x), n(x) \in [0,1], 0 \leq m^q(x) + n^q(x) \leq 1\} \quad (7)$$

In Eq. (7) m(x), i(x), n(x) are the degrees of membership, indeterminacy and non-membership with respect to A, for all x in U. The components m(x) and n(x) are dependent and the component i(x) is independent. Similarly with Proposition 3 it can be shown that $0 \leq m^q(x) + i^q(x) + n^q(x) \leq 2$, for all x in U. For simplicity we write A = <m, i, n>.

A 1-rung orthopair neutrosophic set is referred to as *intuitionistic neutrosophic set*; e.g. see Table 1of [19]. Also, for q = 2 we have a Pythagorean neutrosophic set and for q = 3 a Fermatean neutrosophic set. Following the proof of Proposition 2 one can show:

Proposition 4: Let A be q_1-rung and B be q_2-rung orthopair neutrosophic sets respectively, with $q_2 > q_1$, then A is also a q_2-rung orthopair neutrosophic set.

Proof: The same with the proof of Proposition 2.

In particular, an intuitionistic neutrosophic set is a Pythagorean neutrosophic set, which is a Fermatean neutrosophic set.

The classical operations on crisp sets can be generalized for q-rung orthopair neutrosophic sets. Here we define the subset and the complement of a q-rung orthopair neutrosophic set, as well as the union and intersection of two such sets.

Definition 8: Let $A = \langle m_A, i_A, n_A \rangle$ and $B = \langle m_B, i_B, n_B \rangle$ be two q-rung orthopair neutrosophic sets in the universe U. Then:

i. A is called a *subset* of B ($A \subseteq B$), if, and only if, $m_A(x) \leq m_B(x)$, $i_A(x) \leq i_B(x)$ and $n_A(x) \geq n_B(x)$, $\forall x \in U$. If we have simultaneously $A \subseteq B$ and $B \subseteq A$, then A and B are called *equal* q-rung orthopair neutrosophic sets (A=B).
ii. The *complement* of $A = \langle m_A, i_A, n_A \rangle$ is the q-rung orthopair neutrosophic set $A^C = \langle n_A, 1-i_A, m_A \rangle$ in U.
iii. The *union* $A \cup B$ is the q-rung orthopair neutrosophic set $C = \langle m_C, i_C, n_C \rangle$ in U with $m_C = \max\{m_A, m_B\}$, $i_C = \max\{i_A, i_B\}$ and $n_C = \min\{n_A, n_B\}$.
iv. The *intersection* $A \cap B$ is the q-rung orthopair neutrosophic set $D = \langle m_D, i_D, n_D \rangle$ in U with $m_D = \min\{m_A, m_B\}$, $i_C = \min\{i_A, i_B\}$ and $n_C = \max\{n_A, n_B\}$.

Remark 3:

(i). It is easy to check that all the above relations are well defined. For the union, for example, set $m = \max\{m_A, m_B\}$ and $n = \min\{n_A, n_B\}$. If $m = m_A$ and $n = n_B$, then $0 \leq m^q + n^q = (m_A)^q + (n_B)^q \leq (m_A)^q + (n_A)^q \leq 1$. In an analogous way one can show that we always have $m^q + n^q \leq 1$ for all the other possible combinations, which means that $A \cup B$ is a q-rung orthopair neutrosophic set.

(ii). When A and B are crisp sets, it is straightforward to check that the previous definitions are reduced to the corresponding ordinary definitions for crisp sets.

(iii). With the help of the previous definitions it is straightforward to check that most of the laws and properties of crisp sets are also true for q-rung orthopair neutrosophic sets, like the commutative and associative laws for the union and intersection, the distributive law of the

union with respect to intersection and vice versa, the double complement property $(A^C)^C = A$, etc.

Example 4: Let U= {x_1, x_2, x_3} be the universal set and let A={(0.3,0.3,0.6,x_1),(0.5,0.3,0.4,x_2), (0.7,0.2,0.5,x_3)}and B={(0.6,0.1,0.2,x_1), (0.3,0.2,0.5, x_2), (0.3,0.1,0.6,x_3)}be two q-rung orthopair neutrosophic set in U, q>1. Then:

(i). Neither $A \subseteq B$, nor $B \subseteq A$

(ii). c(A)={(0.6,0.7,0.3,x_1),(0.4,0.7,0.5,x_2), (0.5,0.8, 0.7,x_3)} and c(B)={(0.2,0.9,0.6,x_1),0.5,0.8,0.3,x_2) (0.6,0.9,0.3..,x_3)}.

(iii). $A \cup B$={(0.6,0.3,0.2,x_1),(0.5,0.3,0.4,x_2), (0.7,0.2,0.5,x_3)}

(iv). $A \cap B$={(0.3,0.1,0.6,x_1),(0.3,0.2,0.5,x_2), (0.3,0.1,0.6,x_3)}

Definition 9:

(i). The *empty* q-rung orthopair neutrosophic set in the universe U is defined to be $\emptyset_U$ = {(x, 0, 0, 1): x ∈ U}.

(ii). The *universal* q-rung orthopair neutrosophic set in U is defined to be I_U = {(x, 1, 1, 0): x ∈ U}. It is straightforward to check that for each q-rung orthopair neutrosophic set A in U is $A \cup I_U = I_U$, $A \cap I_U = A$, $A \cup \emptyset_U = A$ and $A \cap \emptyset_U = \emptyset_U$.

4. q-Rung Orthopair Neutrosophic Topological Spaces

Topological spaces are the most general category of mathematical spaces, on which fundamental properties like convergence, continuity, compactness, etc. are defined [24]. The ordinary notion of topological space has been extended to fuzzy [25], to intuitionistic fuzzy [26], to soft [27], to neutrosophic topological space [26], etc. Here we generalize the notion of topological space to the notion to *q-rung orthopair neutrosophic topological space* and we study the previously mentioned properties on such kind of spaces.

Definition 10: A *q-rung orthopair neutrosophic topology* T on a non-empty set U is defined as a collection of q-rung orthopair neutrosophic sets in U such that:

1 I_U and $\emptyset_U$ belong to T.

2 The intersection of any two elements of T belongs to T.

3 The union of any number (finite or infinite) of elements of T belongs also to T.

Trivial examples are the *discrete q-rung neutrosophic topology* of all q-rung orthopair neutrosophic sets in U and the *non-discrete q-rung neutrosophic topology* T= {I_U, $\emptyset_U$}.

The elements of a q-rung neutrosophic topology Ton U are called *open* q-rung orthopair neutrosophic sets of U and their complements are called *closed* q-rung orthopair neutrosophic sets of U. The pair (U, T) is referred to as a *q-rung neutrosophic topological space* on U.

Example 5: Let U = {x} and let A = {(x ,0.5,0.5,0.4)}, B = {(x ,0.4,0.6,0.8)}, C = {(x ,0.5,0.6,0.4)}, D ={(x ,0.4,0.5,0.8)} be q-rung orthopair neutrosophic sets in U, q > 1. Then it is straightforward to check that the collection T = {$\emptyset_U$, I_U, A, B, C, D} is a q-rung orthopair neutrosophic topology on U.

We close this work by extending the concepts of *convergence*, *continuity*, *compactness* and Hausdorff topological space to q-rung orthopair neutrosophic topological spaces.

Definition 11: Given two q-rung orthopair neutrosophic sets A and B of the q-rung neutrosophic topological space (U, T), B is said to be a *neighborhood* of A, if there exists an open q-rung orthopair neutrosophic set Q such that $A \subseteq Q \subset B$. Further, we say that a sequence {A_n} of q-rung orthopair neutrosophic sets of (U, T) *converges* to the q-rung orthopair neutrosophic set A of (U, T), if there exists a positive integer m such that for each integer $n \geq m$ and each neighborhood B of A we have that $A_n \subset B$.

The following Proposition generalizes Zadeh's extension principle for fuzzy sets (see [2], pp. 20-21) to q-rung orthopair neutrosophic sets.

Proposition 5: Let U and V be two non-empty crisp sets and let g: U → V be a function. Then g can be extended to a function G mapping q-rung orthopair neutrosophic sets in U to q-rung orthopair neutrosophic sets in V.

Proof: Let A<m_A, i_A, n_A> be a q-rung orthopair neutrosophic set in U. Then its image G(A) is a q-rung orthopair neutrosophic set B in V, whose neutrosophic components are defined as follows: Given y in V, consider the set $g^{-1}(y)=\{x \in U: g(x)=y\}$. If $g^{-1}(y) = \emptyset$, then $m_B(y)=0$, and if $g^{-1}(v)\neq\emptyset$, then $m_B(y)$ is equal to the maximal value of all $m_A(x)$ such that $x \in g^{-1}(y)$. Conversely, the inverse image $G^{-1}(B)$ is the q-rung orthopair neutrosophic set A in U with membership function $m_A(x)=m_B(g(ux)$, for each x ∈ U. In an analogous way one can determine the neutrosophic components i_B and n_B of B.

Definition 12: Let (U, T) and (V, S) be two q-rung neutrosophic topological spaces on the non-empty crisp sets U and V respectively and let g be a function g: U → V. Then, according to Proposition 5, g can be extended to a function G which maps q-rung orthopair neutrosophic sets of U to q-rung orthopair neutrosophic sets of V. We say then that g is a *q-rung orthopair neutrosophical continuous* function, if, and only if, the inverse image of each open q-rung orthopair neutrosophic set of V through G is an open q-rung orthopair neutrosophic set of U.

Definition 13: A family A = $\{A_i, i\in I\}$ of q-rung orthopair neutrosophic sets of the q-rung orthopair neutrosophic topological space (U, T) is called a *cover* of U, if $U = \bigcup_{i\in I} A_i$. If the elements of A are open q-rung orthopair neutrosophic sets, then A is called an *open cover* of U. Also, each q-rung orthopair neutrosophic subset of A which is also a cover of U is called a sub-cover of A. The q-rung orthopair neutrosophic topological space (U, T) is said to be *compact*, if every open cover of U contains a sub-cover with finitely many elements.

Definition 14: A q-rung orthopair neutrosophic topological space (U, T) is called a *T_1- q-rung orthopair neutrosophic topological space* if, and only if, for each pair of elements x_1, x_2 of U with $x_1 \neq x_2$, there exist at least two open q-rung orthopair neutrosophic sets Q_1 and Q_2 such that $x_1 \in Q_1$, $x_2 \notin O_1$ and $x_2 \in Q_2$, $x_1 \notin O_2$.

Definition 15: A q-rung orthopair neutrosophic topological space (U, T) is called a *T_2- q-rung orthopair neutrosophic topological space* if, and only if, for each pair of elements x_1, x_2 of U with $x_1 \neq x_2$, there exist at least two open q-rung orthopair neutrosophic sets Q_1 and Q_2 such that $x_1 \in O_1$, $x_2 \in O_2$ and $O_1 \cap O_2 = \emptyset_U$.

A T_2-q-rung orthopair neutrosophic topological space is also called a *Hausdorff* or a *separable* q-rung orthopair neutrosophic topological space. Obviously a T_2-rung orthopair neutrosophic topological space is always a T_1- q-rung orthopair neutrosophic topological space.

5. Conclusions

In this work we introduced the concept of q-rung orthopair neutrosophic set and we extended the classical notion of topological space and the fundamental properties of convergence, continuity, compactness and Hausdorff space defined in it to q-rung orthopair neutrosophic topological spaces. Examples were also given to illustrate our results.

It looks that proper combinations of the theories developed for tackling the existing in real life uncertainty is a promising tool for obtaining better results in a variety of human activities characterized by uncertainty (see also [29, 30]). This is, therefore, a fruitful area for future research

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Data availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

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The authors declare that there is no conflict of interest in the research.

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Ambiguous Set is a subclass of the Double Refined Indeterminacy Neutrosophic Set, and of the Refined Neutrosophic Set in general

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Abstract: In this short note we show that the so-called Ambiguous Set (2019) is a subclass of the Double Refined Indeterminacy Neutrosophic Set (2017) and is a particular case of the Refined Neutrosophic Set (2013). Also, the Ambiguous Set is similar to the Quadripartitioned Neutrosophic Set (2016), and Belnap's Four-Valued Logic (1975).

Keywords: Double Refined Indeterminacy Neutrosophic Set (DRINS); Refined Neutrosophic Set (RNS); Ambiguous Set (AS); Quadripartitioned Neutrosophic Set (QNS); Belnap's Four-Valued Logic (BFVL).

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1. Introduction

We provide the definitions of the previous five types of sets, and we prove that the Ambiguous Set is a particular case of the Refined Neutrosophic Set (RNS), Quadripartitioned Neutrosophic Set (QNS), and Belnap's Four-Valued Logic (BFVL), and mostly that the Ambiguous Set coincides with the Double Refined Indeterminacy Neutrosophic Set with the distinction that the sum of quadruple components is ≤ 2 for the AS, which makes it a subclass of the DRINS where the sum is any number between 0 and 4.

2. Ambiguous Set

The definition of the Ambiguous Set (AS) according to [1, 2] is given as follows:

Let U = {g} be the universe for any event g, which is fixed. An AS $\acute{S}$ for $g \in U$ is defined by:

$$\acute{S} = \{g, \Pi t(g), \Pi f(g), \Pi ta(g), \Pi fa(g) \mid g \in U\}$$

where, $\Pi t(g): U \to [0,1]$, $\Pi f(g): U \to [0,1]$, $\Pi ta(g): U \to [0,1]$, and $\Pi fa(g): U \to [0,1]$ are

called the true membership degree (TMD), false membership degree (FMD), true-ambiguous membership degree (TAMD), and false-ambiguous membership degree (FAMD), respectively.

Where $\Pi t(g)$, $\Pi f(g)$, $\Pi ta(g)$ and $\Pi fa(g)$ must satisfy the following condition as:

$$0 \leq \Pi t(g) + \Pi f(g) + \Pi ta(g) + \Pi fa(g) \leq 2$$

3. Double Refined Indeterminacy Neutrosophic Set (DRINS)

The definition of Double Refined Indeterminacy Neutrosophic Set is given in [3] as follows:

Let X be a space of points (objects) with generic elements in X denoted by x.

A Double Refined Indeterminacy Neutrosophic Set (DRINS) A in X is characterized by four components:

truth membership function $T_A(x)$, indeterminacy leaning towards truth membership function $I_{TA}(x)$,

indeterminacy leaning towards falsity membership function $I_{FA}(x)$, and falsity membership function $F_A(x)$.

For each generic element $x \in X$, there are $T_A(x)$, $I_{TA}(x)$, $I_{FA}(x)$, $F_A(x) \in [0, 1]$,

and $0 \leq T_A(x)+I_{TA}(x)+I_{FA}(x)+ F_A(x) \leq 4$.

Therefore, a DRINS A can be represented by

$A = \{\langle x, T_A(x), I_{TA}(x), I_{FA}(x), F_A(x)\rangle \mid x \in X\}$.

4. Ambiguous Set vs. Double Refined Indeterminacy Neutrosophic Set

Let's compare the two definitions.

The definition of Ambiguous Set, as presented by Singh, Huang, & Lee [1, 2] in 2019 and in 2023, coincides with that of Double Refined Indeterminacy Neutrosophic Set introduced by Ilanthenral & Smarandache [3] in 2017, ahead of them.

They only renamed:

the indeterminacy leaning towards truth membership function $I_{TA}(x)$, as true-ambiguous membership degree (TAMD),

and the indeterminacy leaning towards falsehood membership function $I_{TA}(x)$, as false-ambiguous membership degree (FAMD).

The only distinction between AS and DRINS is that:

the sum of AS quadruple components is restricted to be ≤ 2,

while the sum of DRINS quadruple components is ≤ 4 (no restriction), which means that one can take any number between 0 and 4, in the particular case they took the number 2, whence AS is a subclass of the DRINS.

5. Refined Neutrosophic Set

The Definition of Refined Neutrosophic Set is the following.

Let X be a space of points (objects) with generic elements in X denoted by x.

A Refined Neutrosophic Set (RNS) A in X is characterized by *n* sub-components:

sub-truth membership functions $T_{1A}(x), T_{2A}(x), \dots, T_{pA}(x)$;

sub-indeterminacy membership functions $I_{1A}(x), I_{2A}(x), \dots, I_{rA}(x)$;

and sub-falsehood membership functions $F_{1A}(x), F_{2A}(x), \dots, F_{sA}(x)$;

where p, r, s ≥ 0 are integers, and p + r + s = n ≥ 2, such that at least one of p, r, s is ≥ 2 for assuring the refinement of at least one neutrosophic component amongst T, I, or F.

For each generic element x ∈ X, the functions

$T_{1A}(x), T_{2A}(x), \dots, T_{pA}(x), I_{1A}(x), I_{2A}(x), \dots, I_{rA}(x), F_{1A}(x), F_{2A}(x), \dots, F_{sA}(x) \in [0, 1]$,

with their sum

$$0 \le T_{1A}(x) + T_{2A}(x) + \dots + T_{pA}(x) + I_{1A}(x) + I_{2A}(x) + \dots + I_{rA}(x) + F_{1A}(x) + F_{2A}(x) + \dots + F_{sA}(x) \le n$$

Therefore, a RNS A can be represented by

$$A_{RNS} = \{\langle x, T_{1A}(x), T_{2A}(x), \dots, T_{pA}(x), I_{1A}(x), I_{2A}(x), \dots, I_{rA}(x), F_{1A}(x), F_{2A}(x), \dots, F_{sA}(x)\rangle, \mid x \in X\}.$$

The Ambiguous Set is a particular case of the Refined Neutrosophic Set, since one takes

p = 1 (only one true membership);

r = 2 (two types of indeterminacy memberships,

I_1 = true-ambiguous membership degree (TAMD),

and

I_2 = false-ambiguous membership degree (FAMD);

s = 1 (only one false membership).

Therefore, the Ambiguous Set is a particular case of the Refined Neutrosophic Set.

In the same way it is proven that the Double Refined Indeterminacy Neutrosophic Set is a particular of the Refined Neutrosophic Set.

6. Ambigous Set vs. Refined Neutrosophic Set

Both, the so-called Ambiguous Set and the Double Refined Indeterminacy Neutrosophic Set are particular cases of the Refined Neutrosophic Set [4] introduced by Smarandache in 2013.

7. Quadripartitioned Neutrosophic Set

The Definition of single-valued Quadripartitioned Neutrosophic Set [5]

Let X be a non-empty set. The Quadripartitioned single-valued Neutrosophic Set (QNS) A over X characterizes each element x in X by a truth-membership function T_A, a contradiction membership function C_A, an ignorance–membership function U_A and a falsity membership function F_A such that:

for each $x \in X$ one has $T_A(x), C_A(x), U_A(x), F_A(x) \in [0,1]$ and

$0 \le T_A(x) + C_A(x) + U_A(x) + F_A(x) \in [0,1] \le 4$.

When X is discrete, A is represented as

$$A = \sum_{i=1}^{n} < T_A(x_i), C_A(x_i), U_A(x_i), F_A(x_i) > / x_i, x_i \in X.$$

However, when X is continuous, A is represented as:

$$\int_X < T_A(x), C_A(x), U_A(x), F_A(x) > / x, x \in X.$$

It is clear the Quadripartitioned Neutrosophic Set (no matter if it is single-valued, interval-valued, or set-valued in general) is a particular case of the Refined Neutrosophic Set, of the form T, F, and indeterminacy I is split into two parts: I_1 = C (contradiction-membership) and I_2 = U (ignorance-membership).

While the Ambiguous Set is similar with the Quadripartitioned Neutrosophic Set, where the two types of sub-indeterminacies I_1 and I_2 are named differently: true-ambiguous membership and respectively false-ambiguous membership.

Surely, one can rename the sub-indeterminacies I_1 and I_2 in many ways, since there are many types of indeterminacies / uncertainties / vagueness / conflicting informations etc.

8. Belnap's Four-Valued Logic

In 1975 Belnap has considered a logic of four values: true, false, both (true and false), and neither (neither true, nor false). We can denote them by T (true), F (false), C (true and false = contradiction), U (neither true not false = ignorance) respectively and we see that the Ambiguous Set and Quadripartitioned Neutrosophic Set are similar to Belnap's Logic

Further on, the *Belnap's 4-valued Logic* is a particular case of the *Refined Neutrosophic n-valued Logic* that has types of truths $T_1, T_2, \ldots, T_p$, types of indeterminacies $I_1, I_2, \ldots, I_r$, and types of falsehoods: $F_1, F_2, \ldots, F_s$.

9. Conclusion

We proved that the so-called Ambiguous Set coincides with the Double Refined Indeterminacy Neutrosophic Set with respect their quadruple structures, while, with respect to the sum of components, AS is a subclass of the DRINS.

Also, AS is similar with the Quadripartitioned Neutrosophic Set and Belnap Four-Valued Logic as well.

Further on, we proved that the AS, DRINS, QNS and BFVL are particular cases of the Refined Neutrosophic Set / Logic respectively.

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NEUTROSOPHIC STATISTICS: HANDLING IMPRECISE DATA

Short Introduction to Standard and NonStandard Neutrosophic Set and Logic (review paper)

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Smarandache, Florentin. "Short Introduction to Standard and NonStandard Neutrosophic Set and Logic (review paper)." *Neutrosophic Sets and Systems*, Vol. 77, 2025, pp. 395-404.

Abstract: The definitions of Single-Valued Neutrosophic Set (SVNS), Interval-Valued Neutrosophic Set (IVNS), Subset-Valued Neutrosophic Set (SVNS), and respectively NonStandard Neutrosophic Set [Single-Valued (NoS SVNS), Interval-Valued (NoS IVNS), and Subset-Valued (NoS SVNS)], are recalled together with their operators. Similarly for Standard and NonStandard Neutrosophic Logic.

1. Introduction

The neutrosophic set was introduced in 1995 by F. Smarandache as extension of Intuitionistic Fuzzy Set. The first publications were in 1998-2005 [25, 26].

2. Standard Neutrosophic Set and Logic

Let $\mathcal{U}$ be a universe of discourse, and $\mathcal{S}$ a non-empty subset of $\mathcal{U}$. Let $t(x), i(x), f(x)$ be the degrees of truth (membership), indeterminacy, and falsehood (nonmembership) respectively of the generic element x with respect to the set $\mathcal{S}$.

Let $\mathcal{S} = \left\{x_{(t(x),i(x),f(x))}, \text{for } x \in \mathcal{U}\right\}$.

2.1 Definition of Single-Valued Neutrosophic Set (SVNS)

$\mathcal{S}$ is a Single-Valued Neutrosophic Set if:

$$t(x), i(x), f(x): \mathcal{S} \to [0,1]$$

(or $t(x), i(x), f(x)$ are single numbers in [0, 1]) such that:

$$0 \leq t(x) + i(x) + f(x) \leq 3$$

2.2 Definition of Interval-Valued Neutrosophic Set (IVNS)

$\mathcal{S}$ is an Interval-Valued Neutrosophic Set if:

$$t(x), i(x), f(x): \mathcal{S} \to \mathcal{P}([0,1])$$

where $\mathcal{P}([0,1])$ is the powerset of [0, 1], and $t(x), i(x), f(x)$ are intervals included in [0, 1] such that:

$$0 \leq inf\big(t(x)\big) + inf\big(i(x)\big) + inf\big(f(x)\big) \leq sup\big(t(x)\big) + sup\big(i(x)\big) + sup\big(f(x)\big) \leq 3$$

2.3 Definition of Subset-Valued Neutrosophic Set (SVNS)

$\mathcal{S}$ is a Subset-Valued Neutrosophic Set if:

$$t(x), i(x), f(x): \mathcal{S} \to \mathcal{P}([0,1])$$

and $t(x), i(x), f(x)$ are subsets included in [0, 1] such that:

$$0 \leq inf\big(t(x)\big) + inf\big(i(x)\big) + inf\big(f(x)\big) \leq sup\big(t(x)\big) + sup\big(i(x)\big) + sup\big(f(x)\big) \leq 3$$

3. Necessity to Introduce the NonStandard Neutrosophic Set and Logic

In order to make distinction between Relative Truth (truth in at least one world, according to Leibniz) and Absolute Truth (truth in all possible words, again according to Leibniz), we considered the NonStandard Analysis:

$t(x) = 1$, meaning relative truth (membership),

and $t(x) = 1^+$, meaning absolute truth (membership).

Similarly, for Relative Indeterminacy and Absolute Indeterminacy respectively:

$i(x) = 1$, for relative indeterminacy,

and $i(x) = 1^+$, for absolute indeterminacy.

And for Relative Falsehood (NonMembership), and Absolute Falsehood (NonMembership):

$f(x) = 1$, for relative falsehood (nonmembership),

and $f(x) = 1^+$, for absolute falsehood (nonmembership).

4. Introduction to NonStandard Analysis

Abraham Robinson in 1960s has developed the non-standard analysis [15, 16, 27, 28], by rigurously defining the infinitesimals and infinite numbers.

An *infinitesimal number* (ε) is a number ε such that its absolute value $|\varepsilon| < \frac{1}{n}$, for any non-null positive integer *n*. An infinitesimal is close to zero, and so small that it cannot be measured.

The infinitesimal is a number smaller, in absolute value, than anything positive nonzero. Infinitesimals are used in calculus, but interpreted as tiny real numbers.

An *infinite number* (ω) is a number greater than anything:

$$1 + 1 + 1 + \cdots + 1 \text{ (for any finite number terms).}$$

The infinites are reciprocals of infinitesimals.

The set of *hyperreals* (*non-standard reals*), denoted as R^*, is the extension of set of the real numbers, denoted as R, and it comprises the infinitesimals and the infinites, that may be represented on the *hyperreal number line*

$$\frac{1}{\varepsilon} = \frac{\omega}{1}$$

The set of hyperreals satisfies the *transfer principle*, which states that the statements of first order in *R* are valid in R^* as well [according to the classical NonStandard Analysis].

A *monad* (*halo*) of an element $a \in R^*$, denoted by $\mu(a)$, is a subset of numbers infinitesimally close to *a*.

Let's denote by R_+^* the set of positive nonzero hyperreal numbers.

Left Monad and Right Monad were defined as follows:

Left Monad {that we denote, for simplicity, by $({}^-a)$}, is defined as:

$$\mu({}^-a) = ({}^-a) = \{a - x, x \in R_+^* | x \text{ is infinitesimal}\}$$

Right Monad {that we denote, for simplicity, by (a^+)}, is defined as:

$$\mu(a^+) = (a^+) = \{a + x, x \in R_+^* | x \text{ is infinitesimal}\}$$

5. Extensions of NonStandard Analysis

In 1998, Smarandache [25] introduced the Pierced Binad.

5.1. **Pierced Binad** {that we denote, for simplicity, by $(^{-}a^{+})$} is defined as:

$$\mu(^{-}a^{+}) = (^{-}a^{+}) =$$

$$= \{a - x, x \in R_{+}^{*} | x \text{ is a positive infinitesimal}\} \cup \{a + x, x \in R_{+}^{*} | x \text{ is a positive infinitesimal}\}$$

$$= \{a - x, x \in R^{*} | x \text{ is a positive or negative infinitesimal}\}.$$

Later on, in 2019, Smarandache [23] also introduced the Left Monad Closed to the Right, The Right Monad closed to the Left, and the Unpierced Binad (all defined as below) in order to have the Nonstandard Real Monad/Binad Set closed under arithmetic operations.

5.2. Left Monad Closed to the Right

$$\mu\left(\begin{matrix}-0\\ a\end{matrix}\right) = \left(\begin{matrix}-0\\ a\end{matrix}\right) = \left\{a - x | x = 0, \text{or } x \in R_{+}^{*}, \text{and } x \text{ is infinitesimal}\right\}$$

$$= \mu(^{-}a) \cup \{a\}$$

By notation, $\mu\left(\begin{matrix}0\\ a\end{matrix}\right) = \left(\begin{matrix}0\\ a\end{matrix}\right) = {}^{0}a = a$.

And by $x = {}^{-0}a$ we understand the hyperreal $x = a - \varepsilon$, or $x = a$, where ε is a positive infinitesimal. So, x is not clearly known, $x \in \{a - \varepsilon, a\}$.

5.3. Right Monad Closed to the Left

$$\mu\left(\begin{matrix}0+\\ a\end{matrix}\right) = \left(\begin{matrix}0+\\ a\end{matrix}\right) = \left\{a + x | x = 0, \text{or } x \in R_{+}^{*}, \text{and } x \text{ is infinitesimal}\right\}$$

$$= \mu(a^{+}) \cup \{a\}$$

And by $x = {}^{0+}a$ we understand the *hyperreal* $x = a + \varepsilon$, or $x = a$, where ε is a positive infinitesimal. So, x is not clearly known, $x \in \{a, a + \varepsilon\}$.

5.4. Unpierced Binad

$$\mu\left(\overset{-0+}{a}\right)=\left(\overset{-0+}{a}\right)=\{a+x|x=0, \text{or } x\in R^*, \text{and } x \text{ is positive or negative infinitesimal}\}$$
$$=\mu(^-a)\cup(a^+)\cup\{a\}=(^-a)\cup(a^+)\cup\{a\}$$

And by $x=\overset{-0+}{a}$ we understand the *hyperreal* $x=a-\varepsilon$, or $x=a$, or $x=a+\varepsilon$, where ε is a positive infinitesimal. So, x is not clearly known, $x\in\{a-\varepsilon, a, a+\varepsilon\}$.

The left monad, left monad closed to the right, right monad, right monad closed to the left, the pierced binad, and the unpierced binad are subsets of R^*, while the above hyperreals are numbers from R^*.

6. NonStandard Neutrosophic Set and Logic

Let $\mathcal{U}$ be a universe of discourse, and $\mathcal{S}$ a non-empty subset of $\mathcal{U}$.

Let $\mathcal{S}=\left\{x_{(t(x),i(x),f(x))}, \text{for } x\in\mathcal{U}\right\}$.

6.1 Definition of Single-Valued NonStandard Neutrosophic Set and Logic

$\mathcal{S}$ is an Single-Valued NonStandard Neutrosophic Set and Logic, if:

$$t(x), i(x), f(x): \mathcal{S}\to\,]^-0, 1^+[$$

where $]^-0, 1^+[$ is the nonstandard unit interval, which is a set that contains, besides the ordinary real numbers between 0 and 1, also:

Left Monads, Right Monads;

As well as the newly introduced by Smarandache [23, 25] in 1998-2019:

Left Monads closed to the Right,

Right Monads closed to the Left,

Pierced Binads,

and Unpierced Binads.

Of course, $[0, 1]\subset\,]^-0, 1^+[$.

Also,

$$^{-}0 \leq inf\big(t(x)\big) + inf\big(i(x)\big) + inf\big(f(x)\big) \leq sup\big(t(x)\big) + sup\big(i(x)\big) + sup\big(f(x)\big) \leq 3^{+}$$

6.2 Definition of Interval-Valued NonStandard Neutrosophic Set and Logic

$\mathcal{S}$ is an Interval-Valued NonStandard Neutrosophic Set and Logic, if:

$$t(x), i(x), f(x): \mathcal{S} \to \mathcal{P}(\,]^{-}0, 1^{+}[\,)$$

where $\mathcal{P}(\,]^{-}0, 1^{+}[\,)$ is the powerset of all standard and nonstandard subset of $]^{-}0, 1^{+}[$

with $t(x), i(x), f(x)$ being nonstandard intervals of the form:

$$\left]\overset{*}{a}, \overset{*}{b}\right[$$

where $0 \leq a \leq b \leq 1$, and $\overset{*}{a} < \overset{\#}{b}$,

where $\overset{*}{a} \in \{a, \overset{-}{a}, \overset{0}{a}, \overset{+}{a}, \overset{-0}{a}, \overset{-+}{a}, \overset{0+}{a}, \overset{-0+}{a}\}$

similarly $\overset{\#}{b} \in \{b, \overset{-}{b}, \overset{0}{b}, \overset{+}{b}, \overset{-0}{b}, \overset{-+}{b}, \overset{0+}{b}, \overset{-0+}{b}\}$.

6.3 Definition of Subset-Valued NonStandard Neutrosophic Set and Logic

$\mathcal{S}$ is a Subset-Valued NonStandard Neutrosophic Set and Logic, if:

$$t(x), i(x), f(x): \mathcal{S} \to \mathcal{P}(\,]^{-}0, 1^{+}[\,)$$

with $t(x), i(x), f(x)$ being nonstandard subsets of the nonstandard interval $]^{-}0, 1^{+}[$.

7. Standard and NonStandard Neutroosphic Operators

Firstly we need to recall the t-norm and t-conorm from the fuzzy set and logic:

7.1. A *t-norm* [29] is a function t: [0, 1] × [0, 1] → [0, 1] that satisfies the following properties:

- Commutativity: t(a, b) = t(b, a)
- Monotonicity: t(a, b) ≤ t(c, d) if $a \leq c$ and $b \leq d$
- Associativity: t(a, t(b, c)) = t(t(a, b), c)
- The number 1 acts as identity element: t(a, 1) = a

A common notation in fuzzy set and logic is t(a, b) = a /\F b, meaning intersection respectively conjunction.

A *t-conorm* [29] is a function $\perp$: [0, 1] × [0, 1] → [0, 1] that satisfies the following properties:

- Commutativity: $\perp(a, b) = \perp(b, a)$
- Monotonicity: $\perp(a, b) \leq \perp(c, d)$ if $a \leq c$ and $b \leq d$
- Associativity: $\perp(a, \perp(b, c)) = \perp(\perp(a, b), c)$
- Identity element: $\perp(a, 0) = a$

A common notation in fuzzy set and logic is t(a, b) = a $\vee_F$ b, meaning union respectively disjunction.

All the above types of Neutroosphic Standard and NonStandard Operators are based on the fuzzy t-norm ($\wedge_F$) and fuzzy t-conorm ($\vee_F$).

Let x(T, I, F)

7.2. Neutroosphic Standard and NonStandard Intersection/Conjunction ($\wedge_N$)

$$x(T_1, I_1, F_1) \wedge_N x(T_2, I_2, F_2) = x(T_1 \wedge_F T_2, I_1 \vee_F I_2, F_1 \vee_F F_2)$$

7.3. Neutroosphic Standard and NonStandard Union/Disjunction ($\vee_N$)

$$x(T_1, I_1, F_1) \vee_N x(T_2, I_2, F_2) = x(T_1 \vee_F T_2, I_1 \wedge_F I_2, F_1 \wedge_F F_2)$$

7.4. Neutroosphic Standard and NonStandard Complement/Negation ($\neg_N$)

$$\neg_N x(T, I, F) = \bar{x}(F, 1 - I, T)$$

7.5. Neutrosophic Standard and NonStandard Implication ($\rightarrow_N$)

$$A_1(T_1, I_1, F_1) \rightarrow_N A_2(T_2, I_2, F_2)$$

is neutrosophically equivalent with

$$\neg_N A_1(T_1, I_1, F_1) \vee_N A_2(T_2, I_2, F_2)$$

7.6. Neutrosophic Standard and NonStandard Equivalence ($\leftrightarrow_N$)

$$A_1(T_1, I_1, F_1) \leftrightarrow_N A_2(T_2, I_2, F_2)$$

is neutrosophically equivalent with

$A_1(T_1, I_1, F_1) \to_N A_2(T_2, I_2, F_2)$ and $A_2(T_2, I_2, F_2) \to_N A_1(T_1, I_1, F_1)$.

Conclusion

In this paper we reviewed the various types of standard and nonstandard neutrosophic sets and logics, used for about three decades of scientific research and applications.

With the help of the fuzzy t-norm and fuzzy t-conorm we recalled the definitions of the neutrosophic standard and nonstandard operators.

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Foundation of Appurtenance and Inclusion Equations for Constructing the Operations of Neutrosophic Numbers Needed in Neutrosophic Statistics

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Abstract: We introduce for the first time the appurtenance equation and inclusion equation, which help in understanding the operations with neutrosophic numbers within the frame of neutrosophic statistics. The way of solving them resembles the equations whose coefficients are sets (not single numbers).

Keywords: Real Neutrosophic Numbers; Neutrosophic Statistics; True Value; True Set-Value; Appurtenance Relationship; Appurtenance Equation; Inclusion Relationship; Inclusion Equation; Equality Equation with Set-Coefficients; NonAppurtenance Equation; NonInclusion Equation; NonEquality Equation; Operations with Neutrosophic Numbers.

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1. Introduction

In neutrosophic statistics, from the fact that the single true value v is in I, it does not result that v is in $a + bI = N$ as well, but: $a + bv \in a + bI$. That's why the appurtenance relationship and equation must be introduced and studied.

Even more, if one has a set of true values, from the fact that the set of true values V is included in I, it does not mean that V is included in $a + bI$ too, but $a + bV \subset a + bI$ (or $a + bV \subseteq a + bI$). That's why the inclusion relationship and equation must be introduced [1, 2].

In the same way as the "=" symbol is used for an equality relationship or an equality equation, we use the symbol "$\in$" {belong(s) to} for an *appurtenance relationship* or *appurtenance equation* of a number to a set, respectively the symbol $\subset$ (or $\subseteq$) {included in, or included in or equal to} for an *inclusion relationship* or *inclusion equation*.

We use in this paper the tautological denomination Equality Equation with Set-Coefficients (=), in order to distinguish it from the Appurtenance Equation ($\in$) and Inclusion Equation {$\subset$ or $\subseteq$}.

Whatever operation we do on the left-hand side of an *appurtenance relationship* or *appurtenance equation* (respectively, *inclusion relationship* or *inclusion equation*), we must do the same on the right-hand side as well of the *appurtenance relationship* or *appurtenance equation* (respectively, *inclusion relationship* or *inclusion equation*).

In addition, we also present their complementary *NonAppurtenance Equation*, *NonInclusion Equation*, and the elementary *NonEquality Equation* respectively.

2. Definition of the Real Neutrosophic Number

A *Real Neutrosophic Number* (N) has the form:
$N = a + bI$, where a, b are real numbers, "a" is called the determinate part of N, while "bI" is the indeterminate part of N, while I is a real subset, $I \subset \mathbb{R}$.

They are mostly used in *Neutrosophic Statistics*.

The neutrosophic numbers frequently occur in our real world, where one often has imprecise, unclear data to deal with.

For example, let's have a right triangular shape land whose legs are 5 and 6 kilometers respectively, then the length of its hypotenuse is $\sqrt{5^2+6^2}=\sqrt{61}=7.81024967\ldots$ that we need to approximate with some required accuracy.

Another example, let's consider the real circular land of radius 10 km, compute its area $A=\pi\cdot 10^2=100\pi=314.159265\ldots \text{km}^2$, or to compute the volume or surface of a sphere, but π is a transcendental number (a number that is not the root of a non-zero polynomial with rational coefficients and of finite degree), having infinitely many decimals with no repeated pattern.

Similarly, the Euler's constant e = 2.71828182…is a transcendental number and occurs in many formulas.

In the same way when, from real world applications, one arises inexact results (radicals, exponential or logarithmic or trigonometric equations, differential equations, transcendental functions, etc.).

Examples of Real Neutrosophic Numbers.

(i). $N_1=2+3I$, where $I_1=[0,1]$ is an interval.

Or

$N_1=2+3\cdot[0,1]=2+[3.0,3.1]=2+[0,3]=[2+0,2+3]=[2,5]$

(ii). Let $I_2=\{0.6,0.8,0.9\}$, which is a finite discrete set of three elements.

Then:

$$N_2=2+3I_2=2+3\cdot\{0.6,0.8,0.9\}=2+\{3\cdot(0.6),3\cdot(0.8),3\cdot(0.9)\}=2+\{1.8,2.4,2.7\}$$
$$=\{2+1.8,2+2.4,2+2.7\}=\{3.8,4.4,4.7\}\subset[2,5].$$

(iii). Let $I_3=\left\{\frac{1}{n},1\le n\le\infty,n \text{ is integer}\right\}$, which is an infinite discrete set.

Then:

$$N_3=2+3I_3=\left\{2+3\cdot\frac{1}{n},1\le n\le\infty,n \text{ is integer}\right\}$$
$$=\left\{2+3\cdot\frac{1}{1},2+3\cdot\frac{1}{2},2+3\cdot\frac{1}{3},\ldots,2+3\cdot\frac{1}{n},\ldots\right\}=\left\{5,3.5,3,\ldots,2+3\cdot\frac{1}{n},\ldots\right\}$$
$$\subset[2,5].$$

3. Foundation of Appurtenance Relationship and Appurtenance Equation

The below Theorems 1 and 2 allow us to do operations on both sides of an appurtenance relationship and appurtenance equation respectively.

3.1 Theorem 1

Let A and B be real sets.

If $a\in A$ and $b\in B$, then:

- *Addition* $a+b\in A+B$
- *Subtraction* $a-b\in A-B$
- *Multiplication* $a\times b\in A\times B$
- *Division* $\frac{a}{b}\in\frac{A}{B}$
- *Power* $a^b\in A^B$

Proof

Let $\star$ be any of the above operation, then:
$A \star B = \{x \star y;$ where $x \in A, y \in B\}$, and the operation $\star$ is well-defined.

If we let $x = a \in A$, and $y = b \in B$ into the above definition, then:
$a \star b = A \star B$.

3.2 Theorem 1

Let A be a real set, $a \in A$, β be a real scalar, and m, n be (positive) integers. Then:

- Scalar Multiplication of a Set $\beta \cdot a \in \beta \cdot A$ (1)
- Raising to a Power of a Set $a^n \in A^n$
- Root Index of a Set $\sqrt[n]{a} \in \sqrt[n]{A}$
- Negative Exponent of a Set $a^{-n} \in A^{-n}$
- Rational Exponent of a Set $a^{\frac{m}{n}} \in A^{\frac{m}{n}}$

Proof

Similarly.

$$\beta \cdot A = \left\{\beta \cdot x, x \in A\right\} \quad (2)$$

Letting $x = a \in A$ into Definition (2), it results that

$$\beta \cdot a \in \beta \cdot A.$$

For the next four appurtenance relationships, let p be any of the exponents, n, $\frac{1}{n}$, $-n$, or $\frac{m}{n}$, then:

$$A^p = \{x^p, x \in A\}. \quad (3)$$

Letting $x = a$ into the Definition (3), it results that: $a^p \in A^p$, for p being any of n, $\frac{1}{n}$, $-n$, or $\frac{m}{n}$.

3.3 Examples of Operations with Appurtenance Relationships

- Addition in an Appurtenance Relationship
  $(3 + 0.67) \in \big(3 + (0,1)\big)$ or $3.67 \in (3 + 0, 3 + 1)$ or $3.67 \in (3,4)$, which is true.
- Scalar Multiplication of an Appurtenance Relationship
  $2 \cdot (0.67) \in 2 \cdot (0,1)$ or $1.34 \in (2.0, 2.1)$ or $1.34 \in (0,2)$, which is true.
- Power
  $0.67^2 \in (0,1)^2$ or $0.4489 \in (0^2, 1^2)$ or $00.4489 \in (0,1)$, which is true.
- Division by a Scalar of an Appurtenance Relationship
  $\frac{0.67}{-2} \in \frac{(0,1)}{-2}$ or $-0.335 \in (-0.5, 0)$, which is true.

3.4 Appurtenance Equations

Its general form is defined as follows.

Let *R* be the set of real numbers, and *f* and *g* be real HyperFunctions {"hyper" stands for the fact that their domain and/or codomain are powersets *P(R)*},

$$f, g : P(R) \to P(R)$$

$$f(x) \in g(x)$$

All procedures done to solve a classical equation (but whose coefficients are sets, not single numbers) are similarly allowed to do for solving an *appurtenance equation*.

Because sometimes it is not clear what number, either *a* or *b* is bigger, we consider that:
$(a, b) \equiv (b, a)$ and $[a, b] \equiv [b, a]$.

3.5 Solution of an Appurtenance Equation

The solution of an appurtenance equation means a real set *S*, or $S \in P(R)$, to whom the unknown *x* belongs to, or $x \in S$.

3.6 Example 1 of Appurtenance Equation

Solve for *x*.

$4 - 5x \in 1 + 2 \cdot (0.5, 0.8)$.

Subtract 4 from both sides:

$(-5x) \in \big(-3 + 2(0.5, 0.8)\big)$.

We use parentheses ( ) in order to clearly distinguish between the left-hand side and the right-hand side of the appurtenance equation.

$(-5x) \in \big(-3 + (1, 1.6)\big)$

$(-5x) \in (-3 + 1, -3 + 1.6)$

Divide both sides by -5:

$\frac{(-5x)}{-5} \in \frac{(-3+1,-3+1.6)}{-5}$

$x \in \left(\frac{1.4}{5}, \frac{2}{5}\right)$

$x \in (0.28, 0.40)$.

There are infinitely many particular solutions of this appurtenance equation, i.e. all the numbers inside the open interval $(0.28, 0.40)$. We do not take the subsets of *(0.28, 0.40)* as particular solutions, since they are included (or) in, not appurtenant ($\in$) to *(0.28, 0.40).*

Check the maximal solution of the appurtenance equation.

$4 - 5 \cdot x \in 1 + 2 \cdot (0.5, 0.8) \subseteq$

$4 - 5 \cdot (0.28, 0.40) \overset{?}{\in} 1 + 2 \cdot (0.5, 0.8)$

where $\overset{?}{\in}$ means that we must check the appurtenance.

$4 - (1.4, 2.0) \overset{?}{\in} 1 + (1.0, 1.6)$

$(4 - 2.0, 4 - 1.4) \overset{?}{\in} (1 + 1.0, 1 + 1.6)$

or $(2, 2.6) \in (2, 2.6)$

Actually we have an equality here above, which means that any number x, and any subset inside of the left-hand side interval, are solutions.

Therefore, this appurtenance equation has infinitely solutions, $x \in (0.28, 0.40)$.

Let's check some of them, a particular solution as a single number:

$x = 0.35 \in (0.28, 0.40)$

The appurtenance equation:

$4 - 5x \in 1 + 2 \cdot (0.5, 0.8)$ becomes, after substituting x,

$4 - 5 \cdot (0.35) \in 1 + 2(0.5, 0.8)$

$4 - (1.75) \in 1 + (1.0, 1.6)$

$2.25 \in (2.00, 2.60)$, which is true.

Let's check a particular solution-subset of (0.28, 0.40):

$x = (0.30, 0.34) \subset (0.28, 0.40)$.

The appurtenance equation: $4 - 5x \in 1 + 2 \cdot (0.5, 0.8)$ becomes:

$4 - 5 \cdot (0.30, 0.34) \in (2.00, 2.60)$

$4 - (5 \cdot 0.30, 5 \cdot 0.34) \in (2.00, 2.60)$

$4 - (1.50, 1.70) \in (2.00, 2.60)$

$(4 - 1.70, 4 - 1.50) \in (2.00, 2.60)$

$(2.30, 2.50) \in (2.00, 2.60)$,

Actually, the left-hand side is included into the right-hand side.

3.7 Example of Equality Equation with Set-Coefficients

The maximal solution-set $x = (0.28, 0.40)$ of this appurtenance equation becomes the set-solution of the following *equality equation with set-coefficients*:

$4 - 5x = 1 + 2 \cdot (0.5, 0.8)$.

This equation, whose one of the coefficients is a set, (0.5, 0.8), is solved in the same way:

- Subtract *4* from both sides: $-5x = -3 + 2 \cdot (0.5, 0.8)$,
- Then multiply and add the sets:
  $-5x = -3 + (1.0, 1.6)$
  $-5x = (-3 + 1.0, -3 + 1.6)$
  $-5x = (-2, -1.4)$,
- And divide by -5 to get:

  $x = \left(\frac{-1.4}{-5}, \frac{-2}{-5}\right)$

  $x = (0.28, 0.40)$, which is a set-solution.

4. Foundation of Inclusion Relationship and Inclusion Equation

Similarly for the *Inclusion* ($\subset$ or $\subseteq$) *Relationships* and *Inclusion Equation* as we did for Appurtenance Relationships and Appurtenance Equation respectively.

For the case where one has $\subseteq$, the below theorems will be the same, just using $\subseteq$ instead of $\subset$.

4.1 Inclusion Equations

Its general form is defined as follows.

Let *R* be the set of real numbers, and *f* and *g* be real HyperFunctions {"hyper" stands for the fact that their domain and/or codomain are powersets *P(R)*},

$f, g : P(R) \to P(R)$

Then, $f(x) \subset g(x)$ or $f(x) \subseteq g(x)$ are called inclusion equations.

4.2 Theorem 3

Let A and B be real sets, and A_1, B_1 also real sets, but: $A_1 \subset \mathrm{A}$, and $B_1 \subset B$.

Then:

- Addition of Sets $A_1 + B_1 \subset \mathrm{A} + B$
- Subtraction of Sets $A_1 - B_1 \subset \mathrm{A} - B$
- Multiplication of Sets $A_1 \times B_1 \subset \mathrm{A} \times B$
- Division of Sets $\frac{A_1}{B_1} \subset \frac{\mathrm{A}}{B}$
- Power of Sets $\mathrm{A}_1^{B_1} \subset A^B$

Proof:
In the same way, let $\star$ be any of above operations $+, -, \times, \div$, ^ (power), then:

$$A \star B = \{x \star y; \text{ where } x \in A, y \in B\} \quad (3)$$

and the operation $\star$ is well-defined.
We let $x = a_1 \in A_1 \subset A$, and $y = b_1 \in B_1 \subset B$ into the Definition (3), then:

$a_1 \star b_1 \in A \star B$, for all $a_1 \in A_1$ and $b_1 \in B_1$, which means that:
$A_1 \star B_1 \subset A \star B$.

4.3 Theorem 4

Let A and A_1 be real sets, with $A_1 \subset$ A, $\beta \neq 0$ a real number, and m, n positive integers.
Then:

- Scalar Multiplication of a Set $\beta \cdot A_1 \subset \beta \cdot$ A
- Raising to the Power n of a Set $A_1^n \subset A^n$
- Root Index n of a Set $\sqrt[n]{A_1} \subset \sqrt[n]{A}$
- Negative Exponent of a Set $A_1^{-n} \subset A^{-n}$
- Rational Exponent of a Set $A_1^{\frac{m}{n}} \subset A^{\frac{m}{n}}$

Proof
Similarly to the previous theorem.

$\beta \cdot A = \{\beta \cdot x, x \in A\}$ and $\beta \cdot A_1 = \{\beta \cdot x, x \in A_1 \subset A\} \subset \{\beta \cdot x, x \in A\} = \beta \cdot A$.
For the other inclusion relationships, we let again p be any of the exponents n, $\frac{1}{n}$, $-n$, $\frac{m}{n}$, then:
$A_1^p = \{x^p, x \in A_1\} \subset \{x^p, x \in A\} = A$, since $A_1 \subset A$, where for any $x \in A$, and any p, all x^p operations are well-defined.
Analogously, these theorems 3 and 4 allow us to do many operations on both sides of an inclusion relationship or inclusion equation.

4.4 Examples of Inclusion Relationships

$(2, 3] \subset [0, 4]$

- *Let's add 1 in both sides:*
  $1 + (2, 3] \subset 1 + [0, 4]$ or $(1 + 2, 1 + 3] \subset [1 + 0, 1 + 4]$
  $(3, 4] \subset [1, 5]$, which is true.
- *Let's add an interval* $(-1, 5)$ *on both sides:*
  $(2, 3] + (-1, 5) \subset \{0, 4] + (-1, 5)$
  $(2 - 1, 3 + 5) \subset (0 - 1, 4 + 5)$
  $(1, 8) \subset (-1, 90)$, which is true.
- *Let's subtract 2 from both sides:*
  $(2, 3] - 2 \subset [0, 4] - 2$
  $(2 - 2, 3 - 2] \subset [0 - 2, 4 - 2]$
  $(0, 1] \subset [-2, 2]$, which is true.
- *Let's subtract a set [0.5, 0.6] from both sides.*
  $(2,3] - [0.5, 0.6] \subset [0, 4] - [0.5, 0.6]$
  $(2 - 0.6, 3 - 0.5] \subset [0 - 0.6, 4 - 0.5]$
  $(1.4, 2.5] \subset [-0.6, 3.5]$, which is true.
- Let's multiply both sides by a positive (non-zero) number 7:

$7 \cdot (2,3] \subset 7 \cdot [0,4]$

$(7.2, 7.3] \subset [7.0, 7.4]$

$(14, 21] \subset [0, 28]$, which is true.

- Let's multiply both sides by a negative (non-zero) number -5:

$-5 \cdot (2,3] \subset -5 \cdot [0,4]$

$(-5.3, -5.2] \subset [-5.4, -5.0]$

$(-15, -10] \subset [-20, 0]$, which is true.

- Let's multiply both sides with a set $(-1,1)$.

$(-1,1) \cdot (2,3] \subset (-1,1) \cdot (0,4]$

$(-3,3) \subset (-4,4)$, which is true.

- Let's raise to the second power both sides:

$(2,3]^2 \subset [0,4]^2$

$(2^2, 3^2] \subset [0^2, 4^2]$

$(4,9] \subset [0,16]$, which is true.

- Let's divide by -5 both sides:

$\frac{(2,3]}{-5} \subset \frac{[0,4]}{-5}$

$\left[-\frac{3}{5}, -\frac{2}{5}\right) \subset \left[-\frac{4}{5}, -\frac{0}{5}\right]$

$[-0.6, -0.4) \subset [-0.8, 0]$, which is true.

- Let's divide each side by a real set [4, 5].

$\frac{(2,3]}{[4,5]} \subset \frac{[0,4]}{[4,5]}$

$\left(\frac{2}{5}, \frac{3}{4}\right] \subset \left[\frac{0}{5}, \frac{4}{4}\right]$

$(0.40, 0.75] \subset [0,1]$, which is true.

4.5 Example 2 of Inclusion Equation

Solve for x.

$1 + x \cdot (1,2) \subset (0,5)$

$1 + (1, 2x) \subset (0,5)$

$(x + 1, 2x + 1) \subset (0,5)$

whence $0 < x + 1 < 5$ or $-1 < x < 4$ and $0 < 2x + 1 < 5$ or $-1 < 2x < 4$ or $-0.5 < x < 2$

whence $(-1,4) \cap (-0.5, 2) = (-0.5, 2)$

So $x = (-0.5, 2)$ is the maximal solution. All subsets of $(-0.5, 2)$ are particular solutions – therefore one has infinitely many particular solutions.

Check it:

$1 + x(1,2) \subset (0,5)$

$1 + (-0.5, 2) \cdot (1,2) \subset (0,5)$

$1 + (-1,2) \subset (0,5)$

$(1 - 1, 1 + 2) \subset (0,5)$

$(0,3) \subset (0,5)$, which is true.

4.6 Another Example of Inclusion Equation

Solve for x.

$(4, 5) + x \cdot [1, 2] \subseteq [6, 10]$

$(4, 5) + [1 \cdot x, 2 \cdot x] \subseteq [6, 10]$

$(4, 5) + [x, 2x] \subseteq [6, 10]$, for $x \geq 0$, one gets: $(4 + x, 5 + 2x) \subseteq [6, 10]$

Hence:

$6 \leq 4 + x \leq 10$,

whence $2 \leq x \leq 6$

and $6 \leq 5 + 2x \leq 10$

or $1 \leq 2x \leq 5$

or $1.5 \leq x \leq 2.5$

Thus, solution for $x \geq 0$ is $[2, 6] \cap [1.5, 2.5] = [2, 2.5]$.

For $x < 0$

$(4, 5) + x \cdot [1, 2] \subseteq [6, 10]$

$(4, 5) + [2x, x] \subseteq [6, 10]$

$(4 + 2x, 5 + x) \subseteq [6, 10]$

Whence

$6 \leq 4 + 2x \leq 10$, or $2 \leq 2x \leq 6$ or $1 \leq x \leq 3$

and $6 \leq 5 + x \leq 10$, or $1 \leq x \leq 5$.

But x must be negative, therefore this situation doesn't produce any solution.

The maximum solution is $x = [2, 2.5]$.

Let's check the maximal solution.

$(4, 5) + x \cdot [1, 2] \subseteq [6, 10]$

Then:

$(4, 5) + [2, 2.5] \subseteq [6, 10]$

$(4 + 2, 5 + 2.5) \subseteq [6, 10]$

$(6, 7.5) \subseteq [6, 10]$, which is true.

As particular solutions are all subsets of the maximal solution [2, 2.5], therefore infinitely many.

Verifications:

Let $x = 2 \in [2, 2.5]$. We may also write *x* as a set, *x = [2, 2]* $\subseteq [2, 2.5]$.

$(4, 5) + x \cdot [1, 2] \subseteq [6, 10]$

$(4, 5) + 2 \cdot [1, 2] \subseteq [6, 10]$

$(4, 5) + [2, 4] \subseteq [6, 10]$

$(6, 9) \subseteq [6, 10]$, which is true.

Let $x = 2.3 \in [2, 2.5]$.

Then: $(4, 5) + 2.3[1.2] \subseteq [6, 10]$

$(4, 5) + [2.3, 4.6] \subseteq [6, 10]$

$(6.3, 9.6) \subseteq [6, 10]$, which is true.

Let $x = [2.1, 2.4) \subseteq \{2, 2.5]$.

Then $(4,5) + x \cdot [1,2] \subseteq [6,10]$

$(4,5) + [2.1, 2.4) \cdot [1,2] \subseteq [6,10]$

$(4,5) + [2.1, 4.8] \subseteq [6,10]$

$(6.1, 9.8) \subseteq [6,10]$, which is true.

In conclusion, this inclusion equation has one maximal solution and infinitely many particular solutions which are actually included into the maximal solution.

In order to deal with inclusion only, in this problem, since it is an *inclusion* equation, we take as solutions only the subsets of the maximal solution, since: *subset* $\subseteq$ *maximal_solution*, not the single numbers, since: *number* $\in$ *maximal_solution* (not$\subseteq$); this should better be adjusted as [*number, number*] $\subseteq$ *maximal_solution*, for example$[2.1, 2.1] \subseteq [2, 2.5]$.

4.7 Example of Inclusion Equation which has No Solution

Solve for x.

$(1,2) - 2x \subseteq (0, 0.5)$.

Hence:

$(1 - 2x, 2 - 2x) \subseteq (0, 0.5)$,

whence $0 \leq 1 - 2x \leq 0.5$ and $0 \leq 2 - 2x \leq 0.5$,

or $-1 \leq -2x \leq -0.5$ and $-2 \leq -2x \leq -1.5$,

or $0.25 \leq x \leq 0.50$ and $0.75 \leq x \leq 1$.

But $[0.25, 0.50] \cap [0.75, 1] = \emptyset$, therefore there is no solution x.

4.8 Inclusion Equation which has only One Solution

Solve for x.

$(1,2) - 2x \subseteq (0,1)$.

Hence: $(1 - 2x, 2 - 2x) \subseteq (0,1)$,

whence $0 \leq 1 - 2x \leq 1$, and $0 \leq 2 - 2x \leq 1$,

or $-1 \leq -2x \leq 0$, and $-2 \leq -2x \leq -1$,

or $0 \leq x \leq 0.5$, and $0.5 \leq x \leq 1$.

From $[0, 0.5] \cap [0.5, 1] = 0.5$, one gets that the only solution is $x = 0.5$ = [0.5, 0.5].

Let's check the inclusion solution:

$(1,2) - 2x \subseteq (0,1)$

$(1,2) - 2 \cdot 0.5 \subseteq (0,1)$

$(1,2) - 1 \subseteq (0,1)$

$(1 - 1, 2 - 1) \subseteq (0,1)$

$(0,1) \subseteq (0,1)$, which is true.

5. First Method of Operating with Real Neutrosophic Numbers used in Neutrosophic Statistics

5.1 The True Value v is a Single Number

A Neutrosophic Real Number has the form $N = a + bI$, where a and b are real numbers, while "I" is a real set. The determinate part of N is "a" and indeterminate (unclear) part of N is "bI".

Let's consider the real true value being the single value number v, that we are looking for in statistical problems where indeterminate, unclear, partially unknown data occur, where this single number v belongs to the real set I, or $v \in I$.

From the fact that the single true value v is in I, it does not result that v is in $a + bI = N$ as well, but: $a + bv \in a + bI$.

Let $N_1 = a_1 + b_1 I$ and $N_2 = a_2 + b_2 I$ be two real neutrosophic numbers, where $a_1, b_1, a_2, b_2 \in R$ and I is a subset (not necessarily interval) of real numbers.

Let the true value, we are looking for in statistics, under indeterminate (unclear, vague) data, be $v \in I$. Then:

$$a_1 + b_1 v \in a_1 + b_1 I = N_1$$
$$a_2 + b_2 v \in a_2 + b_2 I = N_2$$

The previous Theorems 1 and 2 allow us to do straightforward operations with real neutrosophic numbers.

- *Addition of Real Neutrosophic Numbers*

$$N_1 + N_2 = (a_1 + a_2) + (b_1 + b_2) I$$

Proof:

According to Theorem 1, since

$$a_1 + b_1 v \in a_1 + b_1 I$$

$$a_2 + b_2 v \in a_2 + b_2 I$$

We add, the left-hand sides, then the right-hand sides, referred to the appurtenance symbol ($\in$), and we get:

$$(a_1 + b_1 v) + (a_2 + b_2 v) \in (a_1 + b_1 I) + (a_2 + b_2 I) = N_1 + N_2$$

Therefore:

$$(a_1 + a_2) + (b_1 + b_2) v \in N_1 + N_2.$$

By similar proofs we can do the next operations with real neutrosophic numbers.

- *Subtraction of Real Neutrosophic Numbers*

$$N_1 - N_2 = (a_1 - a_2) + (b_1 - b_2) I$$

- *Scalar Multiplication of Real Neutrosophic Numbers*

Let $\beta \neq 0$ be a real scalar. Then:

$$\beta \cdot N_1 = \beta \cdot (a_1 + b_1 I) = \beta \cdot a_1 + \beta \cdot b_1 I$$

- *Multiplication of Real Neutrosophic Numbers*

$$N_1 \cdot N_2 = (a_1 + b_1 I) \cdot (a_2 + b_2 I) = a_1 a_2 + (a_1 b_2 + a_2 b_1) I + b_1 b_2 I^2$$

- *Square of Real Neutrosophic Numbers*

$$N^2 = (a + bI)^2 = a^2 + 2abI + b^2 I^2$$

5.2 The True Value V is a Set

Let's consider a set of true values V, that we are looking for in statistical problems where indeterminate, unclear, partially unknown data occur, where V is included in I, or $V \subset I$ (or $V \subseteq I$).

From the fact that the set of true values V is in I, it does not result that V is included in $a + bI = N$ as well, but: $a + bV \subset a + bI$.

Let $N_1 = a_1 + b_1 I$ and $N_2 = a_2 + b_2 I$ be two real neutrosophic numbers, where $a_1, b_1, a_2, b_2 \in R$ and I is a subset (not necessarily interval) of real numbers.

Let the set of true values, we are looking for in statistics, under indeterminate (unclear, vague) data, be $V \subset I$. Then:

$$a_1 + b_1 V \subset a_1 + b_1 I = N_1$$

$$a_2 + b_2 V \subset a_2 + b_2 I = N_2$$

Whence:

$$(a_1 + b_1 V) + (a_2 + b_2 V) \subset (a_1 + b_1 I) + (a_2 + b_2 I) = N_1 + N_2 .$$

Similarly, for subtraction, scalar multiplication, multiplication, etc.

$$(a_1 + b_1 V) - (a_2 + b_2 V) \subset (a_1 + b_1 I) - (a_2 + b_2 I) = N_1 - N_2 .$$

$$\beta \cdot (a_1 + b_1 V) \subset \beta \cdot (a_1 + b_1 I) = \beta \cdot N_1 .$$

$$(a_1 + b_1 V) \cdot (a_2 + b_2 V) \subset (a_1 + b_1 I) \cdot (a_2 + b_2 I) = a_1 a_2 + (a_1 b_2 + a_2 b_1) I + b_1 b_2 I^2$$

$$= N_1 \cdot N_2 .$$

The previous Theorems 1, 2, 3, and 4 allow us to do straightforward operations with real neutrosophic numbers (for both cases: a single true value v, or a set of true values V).

6. Second Method of Operating with Neutrosophic Numbers

This method is to transform each real neutrosophic number into a real set:

$N = a + bI = \{a + b \cdot x, x \in I\}$ and do operations using sets as below.

In this case it is not necessarily to have the same indeterminate real set "I".

Firstly, we need to recall the operations with real sets.

6.1 Operations with Sets

Let $\mathbb{R}$ be the set of real numbers, $\mathbb{C}$ the set of complex numbers, and $\mathcal{M}$ the set of other types of numbers.

Let A and B be two real or complex, or other type of number sets.

One or both may also be a scalar, because a scalar $\alpha \in \mathbb{R}$ may be written as a set, $[\alpha, \alpha]$.

Then, $A \star B = \{a \star b$, where $a \star b$ is well defined; $a \in A, b \in B\}$, where $\star$ means any operations: addition, subtraction, scalar multiplication, multiplication, division, power, radical (root).

Afterwards, one computes *min/inf* and *max/sup* of $A \star B$.

In the next sections we are referring only to the sets of real numbers, since they are needed in *Neutrosophic Statistics*, but for the other types of sets the research is similar.

- Addition of Sets

  $A + B = \{a + b; a \in A, b \in B\}$

  Examples:

  $A = (2, 3),\ B = (0, 1)$

  $A + B = (2, 3) + (0, 1) = (2 + 0, 3 + 1)$

  $A + A = (2, 3) + (2, 3) = (2 + 2, 3 + 3) = (4, 6) = 2 \cdot (2, 3) = 2A$

  The last one is similar to the addition of a number to itself, for example:

$5 + 5 = 2 \cdot 5$.

- *Subtraction of Sets*

  $A - B = \{a - b; a \in A, b \in B\}$

  Examples:

  $A = (2,3),\ B = (0,1)$

  $A - B = (2,3) - (0,1) = (2 - 1, 3 - 0) = (1,3)$

  $A - A = (2,3) - (2,3) = (2 - 3, 3 - 2) = (-1,1)$

  Therefore: $A - A \neq 0$ (zero) and $A - A \neq \emptyset$ (empty set), contrarily to the subtraction of a number from itself (for example, 5 - 5 = 0).

- *Scalar Multiplication of Sets*

  Let the scalar $\beta \in R$, then:

  $\beta \cdot A = \{\beta \cdot a; a \in A\}$.

  Examples:

  (i). $\beta = 6$, $A = (2, 3)$, then: $\beta \cdot A = 6 \cdot (2,3) = (6 \cdot 2, 6 \cdot 3) = (12,18)$.

  (ii). $\beta = 0$, $A = (2, 3)$, then: $\beta \cdot A = 0 \cdot (2,3) = (0 \cdot 2, 0 \cdot 3) = (0,0) = \emptyset$ (empty set).

  (iii). $\beta = 0$, $B = [2, 3]$, then: $\beta \cdot B = 0 \cdot [2,3] = [0 \cdot 2, 0 \cdot 3] = [0,0] =$

  $\{0\}$(a set that has only one element, 0).

- *Multiplication of Sets*

  $A \cdot B = \{a \cdot b; a \in A, b \in B\}$

  Examples:

  $A = (2,3),\ B = (0,1)$

  $A \cdot B = (2,3) \cdot (0,1) = (2 \cdot 0, 3 \cdot 1) = (0,3)$

  $A \cdot A = (2,3) \cdot (2,3) = (2 \cdot 2, 3 \cdot 3) = (4,9) = A^2$.

- *Division of Sets*

  $A \div B = \frac{A}{B} = \{a \div b; a \div b$ is well defined, $a \in A, b \in B\}$

  Examples:

  $A = (2,3),\ B = (0,1)$

  (i). For (A, B) intervals, one has $A \div B = \left(\frac{minA}{maxB}, \frac{maxA}{minB}\right) = \left(\frac{2}{1}, \frac{3}{0}\right) \to (2, +\infty)$ since the undefined $\frac{3}{0} \to +\infty$ (not $-\infty$, because B has only positive elements).

  (ii). Let $A = (2,3)$, and $C = (-1,0)$ be two intervals. Then:

  $A \div C = \left(\frac{minA}{maxC}, \frac{maxA}{minC}\right) = \left(\frac{2}{0}, \frac{3}{-1}\right) = \left(\frac{2}{0}, -3\right) \to (-\infty, -3)$,

We take $\frac{2}{0}$ as $-\infty$, because the set C contains only negative elements.

(iii). $A \div A = \left(\frac{minA}{maxA}, \frac{maxA}{minA}\right) = \left(\frac{2}{3}, \frac{3}{2}\right)$.

Therefore, $A \div A \neq 1$, contrarily to the division of real numbers, where a non-zero number divided by itself is equal to 1, for example: $\frac{5}{5} = 1$.

(iv). $B \div B = \left(\frac{minB}{maxB}, \frac{maxB}{minB}\right) = \left(\frac{0}{1}, \frac{1}{0}\right) \rightarrow \mathbf{(O, +\infty) \neq 1}$

- *Power and Root of Sets*

Let r be a rational number, i.e. $r = \frac{m}{n}$, where m, n are integers, $n \neq 0$.

$A^r = \{a^r;$ where a^r is well defined, $a \in A\}$.

(i). Positive integer power

$A = (2, 3),\ r = 4$

$A^4 = (2, 3)^4 = (2^4, 3^4) = (16, 81)$.

Let $E = (-2, 3)$.

$E^2 = (-2, 3) \cdot (-2, 3) = (-2 \cdot 3, 3 \cdot 3) = (-6, 9) \neq ((-2)^2, 3^2) = (4, 9)$.

(ii). Power zero

$A = (2, 3),\ r = 0$

$A^0 = (2, 3)^0 = (2^0, 3^0) = (1, 1) = \emptyset$ (Empty set).

Therefore, $A^0 \neq 1$, contrarily to the real numbers, where for example $7^0 = 1$.

Let $D = [2, 3]$, then $D^0 = [2^0, 3^0] = [1, 1] \equiv \{1\}$, as for real numbers, where for example $7^0 = 1$.

(iii). Square Root

$\sqrt{A} = \sqrt{(2, 3)} = \left(\sqrt{2}, \sqrt{3}\right)$.

(iv). *Partial Square Root*

Let the set $D = (-2, 3)$, then:

$\sqrt{D} = \left[\sqrt{0}, \sqrt{3}\right) = [0, \sqrt{3})$, since in the set of real numbers one cannot compute square root of the negative numbers from the interval $(-2, 0)$. We have only computed a partial square root of D.

(v). Negative Power

$A = (2, 3), r = -2$.

$A^{-2} = (2, 3)^{-2} = (2^{-2}, 3^{-2}) = \left(\frac{1}{4}, \frac{1}{9}\right) \equiv \left(\frac{1}{9}, \frac{1}{4}\right)$.

Now, the operations with the Real Neutrosophic Numbers follow the rules of operations with real sets presented above, because a Real Neutrosophic Number is equivalent to a real subset:

Let I be a real subset, $I \subset \mathbb{R}$.

$N = a + bI = \{a + b \cdot x,$ where $x \in I\}$, which is a real subset of the form as of I.

Actually N is the enlarged subset I.

If I is an interval of the form $I = (c, d)$, or $[c, d)$, or $(c, d]$, or $[c, d]$, then N will also be an interval of the same corresponding open/closed form.

If $I = \{c_1, c_2, \dots, c_n\}$ is a real discrete subset, of cardinal n, $1 \leq n \leq \infty$, then N will also be a real discrete subset of cardinal n.

It is a union of several subsets, $I = I_1 \cup I_2 \cup \dots \cup I_m$, then N will also be a union of corresponding subsets:

$N = N_1 \cup N_2 \cup \dots \cup N_m = \bigcup_{k=1}^{m} N_k$,

Where

$N_k = a + b \cdot I_k = \{a + bx, \text{where } x \in I_k\}$.

6.2 Second Method of Operations with Real Neutrosophic Numbers is the following.

Transform each real neutrosophic number into an equivalent real subset, especially when the indeterminacy (I) are not the same.

Examples:

$N_1 = 1 + 2I_1$, where $I_1 = \{02., 0.5, 0.8\}$

$N_2 = 3 - I_2$, where $I_2 = [0, 1)$

Then:

$N_1 = 1 + 2 \cdot \{0.2, 0.5, 0.8\} = 1 + \{0.4, 1.0, 1.6\} = \{1.4, 2.0, 2.6\}$

and

$N_2 = 3 - [0, 1) = (3 - 1, 3 - 0] = (2, 3]$

- *Addition of Real Neutrosophic Numbers*

$$N_1 + N_2 = \{1.4, 2.0, 2.6\} + (2, 3] = \{1.4 + (2, 3]\} \cup \{2.0 + (2, 3]\} \cup \{2.6 + (2, 3]\}$$
$$= (1.4 + 2, 1.4 + 3] \cup (2.0 + 2, 2.0 + 3] \cup (2.6 + 2, 2.6 + 3]$$
$$= (3.4, 4.4] \cup (4, 5] \cup (4.6, 5.6] = (3.4, 5.6].$$

- *Addition of a Scalar with a Neutrosophic Set*

$0.9 + N_1 = \{1.4, 2.0, 2.6\} + 0.9 = \{1.4 + 0.9,\ 2.0 + 0.9,\ 2.6 + 0.9\} = \{2.3, 2.9, 3.5\}$.

$0.9 + N_2 = 0.9 + (2, 3] = (0.9 + 2, 0.9 + 3] = (2.9, 3.9]$.

- *Subtraction of Real Neutrosophic Numbers*

$$N_1 - N_2 = \{1.4, 2.0, 2.6\} - (2, 3] = \{1.4 - (2, 3]\} \cup \{2.0 - (2, 3]\} \cup \{2.6 - (2, 3]\}$$
$$= [1.4 - 3, 1.4 - 2) \cup [2.0 - 3, 2.0 - 2) \cup [2.6 - 3, 2.6 - 2)$$
$$= [-1.6, -0.6) \cup [-1, 0) \cup [-0.4, 0.6) = [-1.6, 0.6).$$

- *Multiplication of Real Neutrosophic Numbers*

$$N_1 \cdot N_2 = \{1.4, 2.0, 2.6\} \cdot (2, 3] = \{1.4 \cdot (2, 3]\} \cup \{2.0 \cdot (2, 3]\} \cup \{2.6 \cdot (2, 3]\}$$
$$= (1.4 \cdot 2, 1.4 \cdot 3] \cup (2.0 \cdot 2, 2.0 \cdot 3] \cup (2.6 \cdot 2, 2.6 \cdot 3]$$
$$= (2.8, 4.2] \cup (4, 6] \cup (5.2, 7.8] = (2.8, 7.8].$$

- *Multiplication of a Scalar with a Neutrosophic Number*

$4 \cdot N_1 = 4 \cdot \{1.4, 2.0, 2.6\} = \{4 \cdot (1.4), 4 \cdot (2.0), 4 \cdot (2.6)\} = \{5.6, 8.0, 10.4\}$.

- Division of Real Neutrosophic Numbers

$$\frac{N_1}{N_2} = \frac{\{1.4, 2.0, 2.6\}}{(2, 3]} = \frac{1.4}{(2, 3]} \cup \frac{2.0}{(2, 3]} \cup \frac{2.6}{(2, 3]} = \left(\frac{1.4}{3}, \frac{1.4}{2}\right] \cup \left(\frac{2.0}{3}, \frac{2.0}{2}\right] \cup \left(\frac{2.6}{3}, \frac{2.6}{2}\right]$$
$$= (0.4\overline{6}, 0.7] \cup (0.\overline{6}, 1] \cup (0.8\overline{6}, 1.3] = (0.4\overline{6}, 1.3].$$

- *Another Example of Division of Neutrosophic Numbers*

Let $N_1 = 2 - 3I_1$ where $I_1 = [4, 5]$, and $N_2 = 1 + 4I_2$ where $I_2 = \{-1, 3, 5\}$.

Then:

$$N_1 = 2 - 3I_1 = 2 - 3 \cdot [4,5] = 2 - [3 \cdot 4, 3 \cdot 5] = 2 - [12,15] =$$
$$= [2-15, 2-12] = [-13,-10]$$

$$N_2 = 1 + 4I_2 = 1 + 4 \cdot \{-1,3,5\} = 1 + \{4 \cdot (-1), 4 \cdot 3, 4 \cdot 5\} =$$
$$= 1 + \{-4,12,20\} = \{1 + (-4), 1 + 12, 1 + 20\} = \{-3,13,21\}$$

$$\frac{N_1}{N_2} = \frac{[-13,-10]}{\{-3,13,21\}} = [\frac{-13}{-3}, \frac{-10}{-3}] \cup [\frac{-13}{13}, \frac{-10}{13}] \cup [\frac{-13}{21}, \frac{-10}{21}] =$$
$$= [\frac{10}{3}, \frac{13}{3}] \cup [\frac{-13}{13}, \frac{-10}{13}] \cup [\frac{-13}{21}, \frac{-10}{21}] =$$
$$= [\frac{-13}{13}, \frac{-10}{13}] \cup [\frac{-13}{21}, \frac{-10}{21}] \cup [\frac{10}{3}, \frac{13}{3}]$$

- *Division between a Real Neutrosophic Number and a Scalar*

$\frac{N_1}{4} = \frac{\{1.4,2.0,2.6\}}{4} = \left\{\frac{1.4}{4}, \frac{2.0}{4}, \frac{2.6}{4}\right\} = \{0.35, 0.50, 0.65\}.$

$\frac{4}{N_1} = \frac{4}{\{1.4,2.0,2.6\}} = \left\{\frac{4}{1.4}, \frac{4}{2.0}, \frac{4}{2.6}\right\} = \{2.857, 2.000, 1.538\}.$

$\frac{4}{N_2} = \frac{4}{(2,3]} = \left[\frac{4}{3}, \frac{4}{2}\right) \approx [1.333, 2.000)$

$\frac{N_2}{4} = \frac{(2,3]}{4} = \left[\frac{2}{4}, \frac{3}{4}\right) = [0.50, 0.75).$

- *Power of Real Neutrosophic Numbers*

$$N_1{}^{N_2} = \{1.4, 2.0, 2.6\}^{(2,3]} = 1.4^{(2,3]} \cup 2.0^{(2,3]} \cup 2.6^{(2,3]} = (1.4^2, 1.4^3] \cup (2.0^2, 2.0^3] \cup (2.6^2, 2.6^3]$$
$$= (1.960, 2.744] \cup (4, 8] \cup (6.760, 17.576] = (1.960, 2.744] \cup (4.000, 17.576].$$

$$*$$

$$N_2{}^{N_1} = (2,3]^{\{1.4,2.0,2.6\}} = \{(2^{1.4}, 3^{1.4}], (2^{2.0}, 3^{2.0}], (2^{2.6}, 3^{2.6}]\}$$
$$\approx \{(2.639, 4.656], (4.0, 9.0], (6.063, 17.399]\}$$
$$\equiv \{2.639, 4.656] \cup (4.0, 9.0] \cup (6.063, 17.399]$$
$$= (2.639, 17.399].$$

- *Power of a Neutrosophic Real Numbers to a Scalar*

$(N_1)^4 = \{1.4, 2.0, 2.6\}^4 = \{1.4^4, 2.0^4, 2.6^4\} = \{3.8416, 16.000, 45.6976\}$; and $4^{N_1} = 4^{\{1.4,2.0,2.6\}} = \{4^{1.4}, 4^{2.0}, 4^{2.6}\} \approx \{6.9644, 16.000, 36.7583\}.$

- *Real Root of a Neutrosophic Real Number*

$\sqrt{N_1} = \sqrt{\{1.4, 2.0, 2.6\}} = \{\sqrt{1.4}, \sqrt{2.0}, \sqrt{2.6}\} \approx \{1.183, 1.414, 1.612\}$
$\sqrt[3]{N_2} = \sqrt[3]{(2,3]} = (\sqrt[3]{2}, \sqrt[3]{3}) \simeq (1.260, 1.442].$

- *Real Neutrosophic Root of a Neutrosophic Real Number*

$$\sqrt[N_1]{N_2} = N_2^{\frac{1}{N_1}} = (2,3]^{\frac{1}{\{1.4,2.0,2.6\}}} = \left\{(2,3]^{\frac{1}{1.4}}, (2,3]^{\frac{1}{2.0}}, (2,3]^{\frac{1}{2.6}}\right\} = \left\{\left(2^{\frac{1}{1.4}}, 3^{\frac{1}{1.4}}\right], \left(2^{\frac{1}{2.0}}, 3^{\frac{1}{2.0}}\right], \left(2^{\frac{1}{2.6}}, 3^{\frac{1}{2.6}}\right]\right\}$$

$$\approx \{(1.641, 2.192], (1.414, 1.732], (1.306, 1.526]\} \equiv (1.306, 2.192]$$

*

$$\sqrt[N_2]{N_1} = N_1^{\frac{1}{N_2}} = \{1.4, 2.0, 2.6\}^{\frac{1}{(2,3]}} = \{1.4, 2.0, 2.6\}^{[\frac{1}{3},\frac{1}{2})} = \left\{1.4^{[\frac{1}{3},\frac{1}{2})}, 2.0^{[\frac{1}{3},\frac{1}{2})}, 2.6^{[\frac{1}{3},\frac{1}{2})}\right\}$$

$$= \left\{\left[1.4^{\frac{1}{3}}, 1.4^{\frac{1}{2}}\right), \left[2.0^{\frac{1}{3}}, 2.0^{\frac{1}{2}}\right), \left[2.6^{\frac{1}{3}}, 2.6^{\frac{1}{2}}\right)\right\}$$

$$\approx \{[1.119, 1.183), [1.260, 1.414), [1.375, 1.612)\} \equiv [1.119, 1.183) \cup [1.260, 1.612).$$

7. Literal Neutrosophic Numbers

The Literal Neutrosophic Numbers (*LNN*) have the form: $LNN = a + bI$, where a, b are real or complex numbers, and $I =$ literal indeterminacy, where $I^2 = I$, and $I/I =$ undefined.

Their Addition, Subtraction, Scalar Multiplication, Multiplication, Division, Power, Radical are straightforward. The Literal Neutrosophic Numbers are not used in Neutrosophic Statistics, but in Neutrosophic Algebraic Structures, that's why we do not present their operations herein.

8. NonAppurtenance Equation, NonInclusion Equation, and NonEquality Equation

They are complementarians of the Appurtenance Equation, Inclusion Equation, and Equality Equation respectively.

We present them as a curiosity, or as recreational mathematics.

(i). The Appurtenance Equation from previous Example 1 was:
$4 - 5x \in 1 + 2 \cdot (0.5, 0.8)$ whose solutions are all real numbers $x \in (0.28, 0.40)$.
Its corresponding NonAppurtenance Equation is:
$4 - 5x \notin 1 + 2 \cdot (0.5, 0.8)$ whose solutions are all real numbers $x \notin (0.28, 0.40)$, or all real numbers $x \in R - (0.28, 0.40)$.

(ii). The Inclusion Equation from previous Example 2 was:

$1 + x \cdot (1, 2) \subset (0, 5)$, whose maximal solution is $x = (-0.5, 2)$.

Its corresponding NonInclusion Equation is:

$1 + x \cdot (1, 2) \not\subset (0, 5)$, whose maximum solution is R - (- 0.5, 2).

(iii). An elementary Equality Equation

3x + 4 = 7, has the unique solution x = 1.

Its corresponding NonEquality Equation is:

$3x + 4 \neq 7$ has, of course, infinitely many solutions $x \in R - \{1\}$.

9. Conclusions

In neutrosophic statistics, from the fact that the single true value v is in I, it does not result that v is in $a + bI = N$ as well, but: $a + bv \in a + bI$. That's why the appurtenance relationship and equation must be introduced and studied.

Even more, if one has a set of true values, from the fact that the set of true values V is included in I, it does not mean that V is included in $a + bI$ too, but $a + bV \not\subseteq a + bI$ (or $a + bV \not\subseteq a + bI$). That's why the inclusion relationship and equation must be introduced.

In the same way as the "=" symbol is used for an equality relationship or an equality equation, we use the symbol "$\in$" {belong(s) to} for an *appurtenance relationship* or *appurtenance equation* of a number to a set, respectively the symbol $\subseteq$ (or $\subseteq$) {included in, or included in or equal to} for an *inclusion relationship* or *inclusion equation*.

We just introduced for the first time the Appurtenance Equation and Inclusion Equation, which help in understanding the operations with neutrosophic numbers within the frame of neutrosophic statistics. The way of solving them resembles the equations whose coefficients are sets (no single numbers).

In addition, we also presented their complementary NonAppurtenance Equation, NonInclusion Equation, and the elementary NonEquality Equation respectively.

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Foundation of Appurtenance and Inclusion Equations for Constructing the Operations of Neutrosophic Numbers Needed in Neutrosophic Statistics (revised)

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Abstract

We introduce for the first time the appurtenance equation and inclusion equation, which help in understanding the operations with neutrosophic numbers within the frame of neutrosophic statistics. The way of solving them resembles the equations whose coefficients are sets (not single numbers).

Keywords: Real Neutrosophic Numbers; Neutrosophic Statistics;True Value; True Set-Value; Appurtenance Relationship; Appurtenance Equation; Inclusion Relationship; Inclusion Equation; Equality Equation with Set-Coefficients; NonAppurtenance Equation; NonInclusion Equation; NonEquality Equation; Operations with Neutrosophic Numbers

1. Introduction

In neutrosophic statistics, from the fact that the single true value v is into the indeterminate set I, it does not result that v is in the neutrosophic number $N = a + bI$ as well, but: $a + bv \in a + bI = N$. That's why the appurtenance relationship and equation must be introduced and studied.
Even more, if one has a set of true values V, from the fact that the set of true values V is included in I, it does not mean that V is included in $N = a + bI$ too,
but $a + bV \subset a + bI = N$ (or $a + bV \subseteq a + bI = N$). That's why the inclusion relationship and equation must be introduced.

In the same way as the "=" symbol is used for an equality relationship or an equality equation, we use the symbol "$\in$" {belong(s) to} for an *appurtenance (belongness) relationship* or *appurtenance (belongness) equation* of a number to a set, respectively the symbol $\subset$ (or $\subseteq$) {included in, or included in or equal to} for an *inclusion relationship* or *inclusion equation*.

We use in this paper the tautological denomination Equality Equation with Set-Coefficients (=), in order to distinguish it from the Appurtenance Equation ($\in$) and Inclusion Equation $\{\subset$ or $\subseteq\}$.

Whatever operation we do on the left-hand side of an *appurtenance relationship* or *appurtenance equation* (respectively, *inclusion relationship* or *inclusion equation*), we must do the same on the right-hand side as well of the *appurtenance relationship* or *appurtenance equation* (respectively, *inclusion relationship* or *inclusion equation*).

In addition, we also present their complementary *NonAppurtenance Equation*, *NonInclusion Equation*, and the elementary *NonEquality Equation* respectively.

2. Definition of the Real Neutrosophic Number

A *Real Neutrosophic Number* (N) has the form:

$N = a + bI$, where a, b are real numbers, "a" is called the determinate part of N, while "bI" is the indeterminate part of N, while I is a real subset, $I \subset \mathbb{R}$.

They are mostly used in *Neutrosophic Statistics*.

The neutrosophic numbers frequently occur in our real world, where one often has imprecise, unclear data to deal with.

For example, let's have a right triangular shape land whose legs are 5 and 6 kilometers respectively, then the length of its hypotenuse is $\sqrt{5^2 + 6^2} = \sqrt{61} = 7.81024967\ldots$ that we need to approximate with some required accuracy.

Another example, let's consider the real circular land of radius 10 km, compute its area $A = \pi \cdot 10^2 = 100\pi = 314.159265\ldots \text{km}^2$, or to compute the volume or surface of a sphere, but π is a transcendental number (a number that is not the root of a non-zero polynomial with rational coefficients and of finite degree), having infinitely many decimals with no repeated pattern.

Similarly, the Euler's constant e = 2.71828182…is a transcendental number and occurs in many formulas.

In the same way when, from real world applications, one arises inexact results (radicals, exponential or logarithmic or trigonometric equations, differential equations, transcendental functions, etc.).

Examples of Real Neutrosophic Numbers

(i) $N_1 = 2 + 3I$, where $I_1 = [0, 1]$ is an interval.

Or

$$N_1 = 2 + 3 \cdot [0, 1] = 2 + [3.0, 3.1] = 2 + [0, 3] = [2 + 0, 2 + 3] = [2, 5]$$

(ii) Let $I_2 = \{0.6, 0.8, 0.9\}$, which is a finite discrete set of three elements.

Then:

$$N_2 = 2 + 3I_2 = 2 + 3 \cdot \{0.6, 0.8, 0.9\} = 2 + \{3 \cdot (0.6), 3 \cdot (0.8), 3 \cdot (0.9)\} = 2 + \{1.8, 2.4, 2.7\}$$
$$= \{2 + 1.8, 2 + 2.4, 2 + 2.7\} = \{3.8, 4.4, 4.7\} \subset [2, 5].$$

(iii) Let $I_3 = \left\{\frac{1}{n}, 1 \leq n \leq \infty, n \text{ is integer}\right\}$, which is an infinite discrete set.

Then:

$$N_3 = 2 + 3I_3 = \left\{2 + 3 \cdot \frac{1}{n}, 1 \leq n \leq \infty, n \text{ is integer}\right\} = \left\{2 + 3 \cdot \frac{1}{1}, 2 + 3 \cdot \frac{1}{2}, 2 + 3 \cdot \frac{1}{3}, \ldots, 2 + 3 \cdot \frac{1}{n}, \ldots\right\}$$
$$= \left\{5, 3.5, 3, \ldots, 2 + 3 \cdot \frac{1}{n}, \ldots\right\} \subset [2, 5].$$

3. Foundation of Appurtenance Relationship and Appurtenance Equation

The below theorems 1 and 2 allow us to do operations on both sides of an appurtenance relationship and appurtenance equation respectively.

3.1. Theorem 1

Let A and B be real sets.

If $a \in A$ and $b \in B$, then:

Addition

$a + b \in A + B$

Subtraction

$a - b \in A - B$

Multiplication
$a \times b \in A \times B$

Division
$\frac{a}{b} \in \frac{A}{B}$

Power
$a^b \in A^B$

Proof
Let $\star$ be any of the above operation, then:

$A \star B = \{x \star y; \text{where } x \in A, y \in B\}$,

and the operation $\star$ is well-defined.

If we let $x = a \in A$, and $y = b \in B$ into the above definition, then:

$a \star b = A \star B$.

3.2. Theorem 2
Let A be a real set, $a \in A$, β be a real scalar, and m, n be (positive) integers. Then:

Scalar Multiplication of a Set

$\beta \cdot a \in \beta \cdot A$

Raising to a Power of a Set
$a^n \in A^n$

Root Index of a Set

$\sqrt[n]{a} \in \sqrt[n]{A}$

Negative Exponent of a Set
$a^{-n} \in A^{-n}$

Rational Exponent of a Set
$a^{\frac{m}{n}} \in A^{\frac{m}{n}}$

Proof
Similarly.

$\beta \cdot A = \{\beta \cdot x, x \in A\}$ (2)

Letting $x = a \in A$ into definition (2), it results that

$\beta \cdot a \in \beta \cdot A$.

For the next four appurtenance relationships, let p be any of the exponents, n, $\frac{1}{n}$, $-n$, or $\frac{m}{n}$, then:

$A^p = \{x^p, x \in A\}$. (3)

Letting $x = a$ into the definition (3), it results that:

$a^p \in A^p$,

for p being any of n, $\frac{1}{n}$, $-n$, or $\frac{m}{n}$.

3.3. Examples of Operations with Appurtenance Relationships
Let's take the true appurtenance:

$0.67 \in (0, 1)$.

Scalar Addition of an Appurtenance Relationship
$(3 + 0.67) \in \big(3 + (0, 1)\big)$

or $3.67 \in (3 + 0, 3 + 1)$

or $3.67 \in (3,4)$, which is true.

Scalar Multiplication of an Appurtenance Relationship
$2 \cdot (0.67) \in 2 \cdot (0, 1)$

or $1.34 \in (2 \cdot 0, 2 \cdot 1)$

or $1.34 \in (0, 2)$, which is true.

Power
$0.67^2 \in (0, 1)^2$

or $0.4489 \in (0^2, 1^2)$

or $00.4489 \in (0, 1)$, which is true.

Division by a Scalar of an Appurtenance Relationship
$\frac{0.67}{-2} \in \frac{(0,1)}{-2}$

or $-0.335 \in (-0.5, 0)$, which is true.

3.4. Appurtenance Equations

Its general form is defined as follows.

Let *R* be the set of real numbers, and *f* and *g* be real HyperFunctions { "hyper" stands for the fact that their domain and/or codomain are powersets *P(R)* },

$$f, g : P(R) \to P(R)$$

$$f(x) \in g(x)$$

All procedures done to solve a classical equation (but whose coefficients are sets, not single numbers) are similarly allowed to do for solving an *appurtenance equation*.

Because sometimes it is not clear what number, either *a* or *b* is bigger, we consider that:

$(a, b) \equiv (b, a)$ and $[a, b] \equiv [b, a]$.

3.5. Solution of an Appurtenance Equation

The solution of an appurtenance equation means a real set *S*, or $S \in P(R)$ the powerset of R, to whom the unknown *x* belongs to, or $x \in S$.

3.6. Example 1 of Appurtenance Equation
Solve for *x*.

$4 - 5x \in 1 + 2 \cdot (0.5, 0.8)$.

Subtract 4 from both sides:

$(-5x) \in \big(-3 + 2(0.5, 0.8)\big)$.

We use parentheses ( ) in order to clearly distinguish between the left-hand side and the right-hand side of the appurtenance equation.

$(-5x) \in \big(-3 + (1, 1.6)\big)$

$(-5x) \in (-3 + 1, -3 + 1.6)$

Divide both sides by -5:

$\frac{(-5x)}{-5} \in \frac{(-3+1,-3+1.6)}{-5}$

$x \in \left(\frac{1.4}{5}, \frac{2}{5}\right)$

$x \in (0.28, 0.40)$.

There are infinitely many particular solutions of this appurtenance equation, i.e. all the numbers inside the open interval $(0.28, 0.40)$. We do not take the subsets of *(0.28, 0.40)* as particular solutions, since they are included ($\subset$ or $\subseteq$) in, not appurtenant ($\in$) to *(0.28, 0.40)*.

Check the maximal solution of the appurtenance equation.

$4 - 5 \cdot x \in 1 + 2 \cdot (0.5, 0.8)$

$4 - 5 \cdot (0.28, 0.40) \overset{?}{\in} 1 + 2 \cdot (0.5, 0.8)$

where $\overset{?}{\in}$ means that we must check the appurtenance.

$4 - (1.4, 2.0) \overset{?}{\in} 1 + (1.0, 1.6)$

$(4 - 2.0, 4 - 1.4) \overset{?}{\in} (1 + 1.0, 1 + 1.6)$

or $(2, 2.6) \in (2, 2.6)$

actually we have an equality here above, which means that any number x, and any subset inside of the left-hand side interval, are solutions.

Therefore, this appurtenance equation has infinitely solutions, $x \in (0.28, 0.40)$.

Let's check some of them, a particular solution as a single number:

$x = 0.35 \in (0.28, 0.40)$

The appurtenance equation:

$4 - 5x \in 1 + 2 \cdot (0.5, 0.8)$ becomes, after substituting x,

$4 - 5 \cdot (0.35) \in 1 + 2(0.5, 0.8)$

$4 - (1.75) \in 1 + (1.0, 1.6)$

$2.25 \in (2.00, 2.60)$, which is true.

Let's check a particular solution-subset of (0.28, 0.40):

$x = (0.30, 0.34) \subset (0.28, 0.40)$.

The appurtenance equation: $4 - 5x \in 1 + 2 \cdot (0.5, 0.8)$ becomes:

$4 - 5 \cdot (0.30, 0.34) \in (2.00, 2.60)$

$4 - (5 \cdot 0.30, 5 \cdot 0.34) \in (2.00, 2.60)$

$4 - (1.50, 1.70) \in (2.00, 2.60)$

$(4 - 1.70, 4 - 1.50) \in (2.00, 2.60)$

$(2.30, 2.50) \in (2.00, 2.60)$,

actually, the left-hand side is included into the right-hand side.

3.7. Example of Equality Equation with Set-Coefficients

The maximal solution-set $x = (0.28, 0.40)$ of this appurtenance equation becomes the set-solution of the following *equality equation with set-coefficients*:

$4 - 5x = 1 + 2 \cdot (0.5, 0.8)$.

This equation, whose one of the coefficients is a set, (0.5, 0.8),

is solved in the same way:

subtract *4* from both sides:

$-5x = -3 + 2 \cdot (0.5, 0.8)$,

then multiply and add the sets:

$-5x = -3 + (1.0, 1.6)$

$-5x = (-3 + 1.0, -3 + 1.6)$

$-5x = (-2, -1.4)$,

and divide by -5 to get:

$x = \left(\frac{-1.4}{-5}, \frac{-2}{-5}\right)$

$x = (0.28, 0.40)$, which is a set-solution.

4. Foundation of Inclusion Relationship and Inclusion Equation

Similarly for the *Inclusion* (⊂ or ⊆) *Relationships* and *Inclusion Equation* as we did for Appurtenance Relationships and Appurtenance Equation respectively.

For the case where one has ⊆, the below theorems will be the same, just using ⊆ instead of ⊂.

4.1. Inclusion Equations

Its general form is defined as follows.

Let *R* be the set of real numbers, and *f* and *g* be real HyperFunctions { "hyper" stands for the fact that their domain and/or codomain are powersets *P(R)* },

$$f, g : P(R) \to P(R)$$

Then, $f(x) \subset g(x)$ or $f(x) \subseteq g(x)$ are called inclusion equations.

4.2. Theorem 3

Let A and B be real sets, and A_1, B_1 also real sets, but:

$A_1 \subset$ A, and $B_1 \subset B$.

Then:

Addition of Sets
$A_1 + B_1 \subset \mathrm{A} + B$

Subtraction of Sets
$A_1 - B_1 \subset \mathrm{A} - B$

Multiplication of Sets
$A_1 \times B_1 \subset \mathrm{A} \times B$

Division of Sets
$\frac{A_1}{B_1} \subset \frac{\mathrm{A}}{B}$

Power of Sets
$\mathrm{A}_1^{\,B_1} \subset A^B$

Proof
In the same way, let $\star$ be any of above operations $+, -, \times, \div, \wedge$ (power), then:

$A \star B = \{x \star y; \text{where } x \in A, y \in B\}$ (3)

and the operation $\star$ is well-defined.

We let $x = a_1 \in A_1 \subset A$, and $y = b_1 \in B_1 \subset B$ into the definition (3), then:

$a_1 \star b_1 \in A \star B$, for all $a_1 \in A_1$ and $b_1 \in B_1$, which means that:

$A_1 \star B_1 \subset A \star B$.

4.3. Theorem 4
Let A and A_1 be real sets, with $A_1 \subset \mathrm{A}$, $\beta \neq 0$ a real number, and m, n positive integers.

Then:

Scalar Multiplication of a Set
$\beta \cdot A_1 \subset \beta \cdot \mathrm{A}$

Raising to the Power n of a Set
$A_1^n \subset A^n$

Root Index n of a Set
$\sqrt[n]{A_1} \subset \sqrt[n]{A}$

Negative Exponent of a Set
$A_1^{-n} \subset A^{-n}$

Rational Exponent of a Set
$A_1^{\frac{m}{n}} \subset A^{\frac{m}{n}}$

Proof
Similarly to the previous theorem.

$\beta \cdot A = \{\beta \cdot x, x \in A\}$

and $\beta \cdot A_1 = \{\beta \cdot x, x \in A_1 \subset A\} \subset \{\beta \cdot x, x \in A\} = \beta \cdot A$.

For the other inclusion relationships, we let again p be any of the exponents n, $\frac{1}{n}$, $-n$, $\frac{m}{n}$, then:

$A_1^p = \{x^p, x \in A_1\} \subset \{x^p, x \in A\} = A$, since $A_1 \subset A$,

where for any $x \in A$, and any p, all x^p operations are well-defined.

Analogously, these theorems 3 and 4 allow us to do many operations on both sides of an inclusion relationship or inclusion equation.

4.4. Examples of Inclusion Relationships

Let's consider the true inclusion relationship:

$(2, 3] \subset [0, 4]$

Let's add 1 in both sides:

$1 + (2, 3] \subset 1 + [0, 4]$

or $(1 + 2, 1 + 3] \subset [1 + 0, 1 + 4]$

$(3, 4] \subset [1, 5]$, which is true.

Let's add an interval $(-1, 5)$ on both sides:

$(2, 3] + (-1, 5) \subset [0, 4] + (-1, 5)$

$(2 - 1, 3 + 5) \subset (0 - 1, 4 + 5)$

$(1, 8) \subset (-1, 9)$, which is true.

Let's subtract 2 from both sides:

$(2, 3] - 2 \subset [0, 4] - 2$

$(2 - 2, 3 - 2] \subset [0 - 2, 4 - 2]$

$(0, 1] \subset [-2, 2]$, which is true.

Let's subtract a set [0.5, 0.6] from both sides.

$(2,3] - [0.5, 0.6] \subset [0, 4] - [0.5, 0.6]$

$(2 - 0.6, 3 - 0.5] \subset [0 - 0.6, 4 - 0.5]$

$(1.4, 2.5] \subset [-0.6, 3.5]$, which is true.

Let's multiply both sides by a positive (non-zero) number 7:

$7 \cdot (2, 3] \subset 7 \cdot [0, 4]$

$(7 \cdot 2, 7 \cdot 3] \subset [7 \cdot 0, 7 \cdot 4]$

$(14, 21] \subset [0, 28]$, which is true.

Let's multiply both sides by a negative (non-zero) number -5:

$-5 \cdot (2, 3] \subset -5 \cdot [0, 4]$

$(-5 \cdot 3, -5 \cdot 2] \subset [-5 \cdot 4, -5 \cdot 0]$

$(-15, -10] \subset [-20, 0]$, which is true.

Let's multiply both sides with a set $(-1, 1)$.

$(-1, 1) \cdot (2, 3] \subset (-1, 1) \cdot (0, 4]$

$(-3, 3) \subset (-4, 4)$, which is true.

Let's raise to the second power both sides:

$(2,3]^2 \subset [0,4]^2$

$(2^2, 3^2] \subset [0^2, 4^2]$

$(4,9] \subset [0,16]$, which is true.

Let's divide by -5 both sides:

$\frac{(2,3]}{-5} \subset \frac{[0,4]}{-5}$

$\left[-\frac{3}{5}, -\frac{2}{5}\right) \subset \left[-\frac{4}{5}, -\frac{0}{5}\right]$

$[-0.6, -0.4) \subset [-0.8, 0]$, which is true.

Let's divide each side by a real set [4, 5].

$\frac{(2,3]}{[4,5]} \subset \frac{[0,4]}{[4,5]}$

$\left(\frac{2}{5}, \frac{3}{4}\right] \subset \left[\frac{0}{5}, \frac{4}{4}\right]$

$(0.40, 0.75] \subset [0,1]$, which is true.

4.5. Example 2 of Inclusion Equation
Solve for x.

$1 + x \cdot (1,2) \subset (0,5)$

$1 + (x, 2x) \subset (0,5)$

$(x + 1, 2x + 1) \subset (0,5)$

whence $0 < x + 1 < 5$ or $-1 < x < 4$ or $x \subset$ (-1, 4),

and $0 < 2x + 1 < 5$ or $-1 < 2x < 4$ or $-0.5 < x < 2$ or $x \subset$ (-0.5, 2),

whence $x \subset (-1,4) \cap (-0.5,2) = (-0.5,2)$.

So $x = (-0.5, 2)$ is the maximal solution. All subsets of $(-0.5, 2)$ are particular solutions – therefore one has infinitely many particular solutions.

Check it:

$1 + x(1,2) \subset (0,5)$

$1 + (-0.5, 2) \cdot (1,2) \subset (0,5)$

$1 + (-1,2) \subset (0,5)$

$(1 - 1, 1 + 2) \subset (0,5)$

$(0,3) \subset (0,5)$, which is true.

4.6. Another Example of Inclusion Equation
Solve for x.

$(4,5) + x \cdot [1,2] \subseteq [6,10]$

$(4,5) + [1 \cdot x, 2 \cdot x] \subseteq [6,10]$

$(4,5) + [x, 2x] \subseteq [6,10]$

For $x \geq 0$, one gets:

$(4 + x, 5 + 2x) \subseteq [6, 10]$

Hence:

$6 \leq 4 + x \leq 10,$

whence $2 \leq x \leq 6$

and $6 \leq 5 + 2x \leq 10$

or $1 \leq 2x \leq 5$

or $1.5 \leq x \leq 2.5$

thus, solution for $x \geq 0$ is $[2, 6] \cap [1.5, 2.5] = [2, 2.5]$.

For $x < 0$

$(4, 5) + x \cdot [1, 2] \subseteq [6, 10]$

$(4, 5) + [2x, x] \subseteq [6, 10]$

$(4 + 2x, 5 + x) \subseteq [6, 10]$

Whence

$6 \leq 4 + 2x \leq 10$, or $2 \leq 2x \leq 6$ or $1 \leq x \leq 3$

and $6 \leq 5 + x \leq 10$, or $1 \leq x \leq 5$.

But x must be negative in this case, therefore this situation doesn't produce any solution.

The maximum solution is $x = [2, 2.5]$.

Let's check the maximal solution.

$(4, 5) + x \cdot [1, 2] \subseteq [6, 10]$

Then:

$(4, 5) + [2, 2.5] \subseteq [6, 10]$

$(4 + 2, 5 + 2.5) \subseteq [6, 10]$

$(6, 7.5) \subseteq [6, 10]$, which is true.

As particular solutions are all subsets of the maximal solution [2, 2.5], therefore infinitely many.

Verifications.

Let $x = 2 \in [2, 2.5]$. We may also write *x* as a set, *x = [2, 2]* $\subseteq [2, 2.5]$.

$(4, 5) + x \cdot [1, 2] \subseteq [6, 10]$

$(4, 5) + 2 \cdot [1, 2] \subseteq [6, 10]$

$(4, 5) + [2, 4] \subseteq [6, 10]$

$(6, 9) \subseteq [6, 10]$, which is true.

Let $x = 2.3 \in [2, 2.5]$.

Then: $(4, 5) + 2.3[1.2] \subseteq [6, 10]$

$(4, 5) + [2.3, 4.6] \subseteq [6, 10]$

$(6.3, 9.6) \subseteq [6, 10]$, which is true.

Let $x = [2.1, 2.4) \subseteq [2, 2.5]$.

Then $(4,5)+x\cdot[1,2]\subseteq[6,10]$

$(4,5)+[2.1,2.4)\cdot[1,2]\subseteq[6,10]$

$(4,5)+[2.1,4.8]\subseteq[6,10]$

$(6.1,9.8)\subseteq[6,10]$, which is true.

In conclusion, this inclusion equation has one maximal solution and infinitely many particular solutions which are actually included into the maximal solution.

In order to deal with inclusion only, in this problem, since it is an *inclusion* equation, we take as solutions only the subsets of the maximal solution, since:

subset $\subseteq$ *maximal_solution*,

not the single numbers, since:

number $\in$ *maximal_solution* (not $\subseteq$); this should better be adjusted as

[*number*, *number*] $\subseteq$ *maximal_solution*, for example $[2.1,2.1]\subseteq[2,2.5]$.

4.7. Example of Inclusion Equation which has No Solution
Solve for x:

$(1,2)-2x\subseteq(0,0.5)$.

Hence:

$(1-2x,2-2x)\subseteq(0,0.5)$,

whence $0\leq 1-2x\leq 0.5$ and $0\leq 2-2x\leq 0.5$,

or $-1\leq -2x\leq -0.5$ and $-2\leq -2x\leq -1.5$,

or $0.25\leq x\leq 0.50$ and $0.75\leq x\leq 1$.

But $[0.25,0.50]\cap[0.75,1]=\emptyset$, therefore there is no solution x.

4.8. Inclusion Equation which has only One Solution
Solve for x:

$(1,2)-2x\subseteq(0,1)$.

Hence: $(1-2x,2-2x)\subseteq(0,1)$,

whence $0\leq 1-2x\leq 1$, and $0\leq 2-2x\leq 1$,

or $-1\leq -2x\leq 0$, and $-2\leq -2x\leq -1$,

or $0\leq x\leq 0.5$, and $0.5\leq x\leq 1$.

From $[0,0.5]\cap[0.5,1]=0.5$, one gets that the only solution is $x=0.5$ = [0.5, 0.5].

Let's check the inclusion solution:

$(1,2)-2x\subseteq(0,1)$

$(1,2)-2\cdot 0.5\subseteq(0,1)$

$(1,2)-1\subseteq(0,1)$

$(1-1,2-1)\subseteq(0,1)$

$(0,1)\subseteq(0,1)$, which is true.

5. First Method of Operating with Real Neutrosophic Numbers used in Neutrosophic Statistics

5.1. The true value v is a single number

A Neutrosophic Real Number has the form $N = a + bI$, where a and b are real numbers, while "I" is a real set. The determinate part of N is "a" and indeterminate (unclear) part of N is "bI".

Let's consider the real true value being the single value number v, that we are looking for in statistical problems where indeterminate, unclear, partially unknown data occur, where this single number v belongs to the real set I, or $v \in I$.

From the fact that the single true value v is in I, it does not result that v is in $a + bI = N$ as well,

but: $a + bv \in a + bI$.

Let $N_1 = a_1 + b_1 I$ and $N_2 = a_2 + b_2 I$ be two real neutrosophic numbers, where $a_1, b_1, a_2, b_2 \in R$ and I is a subset (not necessarily interval) of real numbers.

Let the true value, we are looking for in statistics, under indeterminate (unclear, vague) data, be $v \in I$. Then:

$$a_1 + b_1 v \in a_1 + b_1 I = N_1$$
$$a_2 + b_2 v \in a_2 + b_2 I = N_2$$

The previous Theorems 1 and 2 allow us to do straightforward operations with real neutrosophic numbers.

Addition of Real Neutrosophic Numbers

$$N_1 + N_2 = (a_1 + a_2) + (b_1 + b_2) I$$

Proof:

According to Theorem 1, since

$$a_1 + b_1 v \in a_1 + b_1 I$$

$$a_2 + b_2 v \in a_2 + b_2 I$$

we add, the left-hand sides, then the right-hand sides, referred to the appurtenance symbol ($\in$), and we get:

$$(a_1 + b_1 v) + (a_2 + b_2 v) \in (a_1 + b_1 I) + (a_2 + b_2 I) = N_1 + N_2$$

Therefore:

$$(a_1 + a_2) + (b_1 + b_2) v \in N_1 + N_2 .$$

By similar proofs we can do the next operations with real neutrosophic numbers.

Subtraction of Real Neutrosophic Numbers

$$N_1 - N_2 = (a_1 - a_2) + (b_1 - b_2) I$$

Scalar Multiplication of Real Neutrosophic Numbers

Let $\beta \neq 0$ be a real scalar. Then:

$$\beta \cdot N_1 = \beta \cdot (a_1 + b_1 I) = \beta \cdot a_1 + \beta \cdot b_1 I$$

Multiplication of Real Neutrosophic Numbers

$$N_1 \cdot N_2 = (a_1 + b_1 I) \cdot (a_2 + b_2 I) = a_1 a_2 + (a_1 b_2 + a_2 b_1) I + b_1 b_2 I^2$$

Square of Real Neutrosophic Numbers

$$N^2 = (a + bI)^2 = a^2 + 2abI + b^2 I^2$$

n-Power of Real Neutrosophic Numbers

$N^n = (a + bI)^n = \sum_{k=0}^{n} C_n^k (a^{n-k} b^k I^k)$, for integer $n \geq 1$, where by definition $I^0 \overset{def}{=} [1,1] \equiv 1$.

n-Root of Real Neutrosophic Numbers

$$\sqrt[n]{N} = \sqrt[n]{a + bI}$$

Division

$$\frac{N_1}{N_2} = \frac{a_1 + b_1 I}{a_2 + b_2 I}$$

Remark

The above operations become easier when the indeterminacy *I = (0, 1), or (0, 1], [0, 1), [0, 1]* that are mostly used in applications, because $I^n = I$ for any real number *n > 0*.

5.2. The true value V is a set

Let's consider a set of true values *V*, that we are looking for in statistical problems where indeterminate, unclear, partially unknown data occur, where *V* is included in *I*, or $V \subset I$ (or $V \subseteq I$).
From the fact that the set of true values *V* is in *I*, it does not result that *V* is included in $a + bI = N$ as well, but: $a + bV \subset a + bI$.
Let $N_1 = a_1 + b_1 I$ and $N_2 = a_2 + b_2 I$ be two real neutrosophic numbers, where $a_1, b_1, a_2, b_2 \in R$ and *I* is a subset (not necessarily interval) of real numbers.
Let the set of true values, we are looking for in statistics, under indeterminate (unclear, vague) data, be $V \subset I$. Then:

$$a_1 + b_1 V \subset a_1 + b_1 I = N_1$$
$$a_2 + b_2 V \subset a_2 + b_2 I = N_2$$

Whence the **addition**:

$$(a_1 + b_1 V) + (a_2 + b_2 V) \subset (a_1 + b_1 I) + (a_2 + b_2 I) = N_1 + N_2 .$$

Similarly, for **subtraction**, **scalar multiplication**, **multiplication**, **division**, **n^th^-power**, **n^th^-root**, etc.

$$(a_1 + b_1 V) - (a_2 + b_2 V) \subset (a_1 + b_1 I) - (a_2 + b_2 I) = N_1 - N_2 .$$

$$\beta \cdot (a_1 + b_1 V) \subset \beta \cdot (a_1 + b_1 I) = \beta \cdot N_1 .$$

$$(a_1 + b_1 V) \cdot (a_2 + b_2 V) \subset (a_1 + b_1 I) \cdot (a_2 + b_2 I) = a_1 a_2 + (a_1 b_2 + a_2 b_1) I + b_1 b_2 I^2 =$$

$$= N_1 \cdot N_2 .$$

$$\frac{a_1+b_1V}{a_2+b_2V} \subset \frac{N_1}{N_2}.$$

$$(a+bV)^n \subset N^n.$$

$$\sqrt[n]{a+bV} \subset \sqrt[n]{N}, \text{ etc.}$$

The previous Theorems 1, 2, 3, and 4 allow us to do straightforward operations with real neutrosophic numbers (for both cases: a single true value *v,* or a set of true values *V*).

6. Second Method of Operating with Neutrosophic Numbers

This method is to transform each real neutrosophic number into a real set:

$$N = a + bI = \{a + b \cdot x, x \in I\}$$

And do operations using sets as below.

In this case it is not necessarily to have the same indeterminate real set "I".

Firstly, we need to recall the operations with real sets.

6.1. Operations with Sets

Let $\mathbb{R}$ be the set of real numbers, $\mathbb{C}$ the set of complex numbers, and $\mathcal{M}$ the set of other types of numbers.

Le A and B be two real or complex, or other type of number sets.

One or both may also be a scalar, because a scalar $\alpha \in \mathbb{R}$ may be written as a set, $[\alpha, \alpha]$.

Then $A \star B = \{a \star b, \text{where } a \star b \text{ is well defined}; a \in A, b \in B\}$, where $\star$ means any operations: addition, subtraction, scalar multiplication, multiplication, division, power, radical (root).

Afterwards, one computes *min/inf* and *max/sup* of $A \star B$.

In the next sections we are referring only to the sets of real numbers, since they are needed in *Neutrosophic Statistics*, but for the other types of sets the research is similar.

Addition of Sets

$A + B = \{a + b; a \in A, b \in B\}$

Examples:

$A = (2,3), B = (0,1)$

$A + B = (2,3) + (0,1) = (2 + 0, 3 + 1)$

$A + A = (2,3) + (2,3) = (2 + 2, 3 + 3) = (4,6) = 2 \cdot (2,3) = 2A$

The last one is similar to the addition of a number to itself, for example:

$5 + 5 = 2 \cdot 5.$

Subtraction of Sets

$A - B = \{a - b; a \in A, b \in B\}$

Examples:

$A = (2,3), B = (0,1)$

$A - B = (2,3) - (0,1) = (2 - 1, 3 - 0) = (1,3)$

$A - A = (2,3) - (2,3) = (2-3, 3-2) = (-1,1)$

Therefore:

$A - A \neq 0$ (zero) and $A - A \neq \emptyset$ (empty set),

contrarily to the subtraction of a number from itself (for example, 5 - 5 = 0).

Scalar Multiplication of Sets
Let the scalar $\beta \in R$, then:

$$\beta \cdot A = \{\beta \cdot a; a \in A\}.$$

Examples:

i). $\beta = 6$, $A = (2, 3)$, then: $\beta \cdot A = 6 \cdot (2,3) = (6 \cdot 2, 6 \cdot 3) = (12, 18)$.

ii). $\beta = 0$, $A = (2, 3)$, then: $\beta \cdot A = 0 \cdot (2,3) = (0 \cdot 2, 0 \cdot 3) = (0,0) = \emptyset$ (empty set).

iii) $\beta = 0$, $B = [2, 3]$, then: $\beta \cdot B = 0 \cdot [2,3] = [0 \cdot 2, 0 \cdot 3] = [0,0] =$
$\{0\}$ (a set that has only one element, 0).

Multiplication of Sets
$A \cdot B = \{a \cdot b; a \in A, b \in B\}$

Examples:

$A = (2,3), B = (0,1)$

$A \cdot B = (2,3) \cdot (0,1) = (2 \cdot 0, 3 \cdot 1) = (0,3)$

$A \cdot A = (2,3) \cdot (2,3) = (2 \cdot 2, 3 \cdot 3) = (4,9) = A^2$.

Division of Sets
$A \div B = \frac{A}{B} = \{a \div b; a \div b \text{ is well defined}, a \in A, b \in B\}$

Examples:

$A = (2,3), B = (0,1)$

i). For (*A*, *B*) intervals, one has $A \div B = \left(\frac{minA}{maxB}, \frac{maxA}{minB}\right) = \left(\frac{2}{1}, \frac{3}{0}\right) \to (2, +\infty)$ since the undefined $\frac{3}{0} \to +\infty$ (not $-\infty$, because B has only positive elements).

ii). Let $A = (2,3)$, and $C = (-1,0)$ be two intervals. Then:

$A \div C = \left(\frac{minA}{maxC}, \frac{maxA}{minC}\right) = \left(\frac{2}{0}, \frac{3}{-1}\right) = \left(\frac{2}{0}, -3\right) \to (-\infty, -3)$,

We take $\frac{2}{0}$ as $-\infty$, because the set C contains only negative elements.

iii). $A \div A = \left(\frac{minA}{maxA}, \frac{maxA}{minA}\right) = \left(\frac{2}{3}, \frac{3}{2}\right)$.

Therefore $A \div A \neq 1$, contrarily to the division of real numbers, where a non-zero number divided by itself is equal to 1, for example: $\frac{5}{5} = 1$.

iv). $B \div B = \left(\frac{minB}{maxB}, \frac{maxB}{minB}\right) = \left(\frac{0}{1}, \frac{1}{0}\right) \to \mathbf{(O, +\infty) \neq 1}$

Power and Root of Sets

Let r be a rational number, i.e. $r = \frac{m}{n}$, where m, n are integers, $n \neq 0$.

$A^r = \{a^r; \text{where } a^r \text{ is well defined}, a \in A\}$.

i). Positive integer power

$A = (2, 3),\ r = 4$

$A^4 = (2, 3)^4 = (2^4, 3^4) = (16, 81)$.

Let $E = (-2, 3)$.

$E^2 = (-2, 3) \cdot (-2, 3) = (-2 \cdot 3, 3 \cdot 3) = (-6, 9) \neq ((-2)^2, 3^2) = (4, 9)$.

ii). Power zero

$A = (2, 3),\ r = 0$

$A^0 = (2, 3)^0 = (2^0, 3^0) = (1, 1) = \emptyset$ (empty set).

Therefore, $A^0 \neq 1$, contrarily to the real numbers, where for example $7^0 = 1$.

Let $D = [2, 3]$, then $D^0 = [2^0, 3^0] = [1, 1] \equiv \{1\}$, as for real numbers, where for example $7^0 = 1$.

iii). Square Root

$\sqrt{A} = \sqrt{(2, 3)} = (\sqrt{2}, \sqrt{3})$.

iv). Partial Square Root

Let the set $D = (-2, 3)$, then:

$\sqrt{D} = [\sqrt{0}, \sqrt{3}) = [0, \sqrt{3})$,

since in the set of real numbers one cannot compute square root of the negative numbers from the interval $(-2, 0)$.

We have only computed a partial square root of D.

v). Negative Power

$A = (2, 3), r = -2$.

$A^{-2} = (2, 3)^{-2} = (2^{-2}, 3^{-2}) = \left(\frac{1}{4}, \frac{1}{9}\right) \equiv \left(\frac{1}{9}, \frac{1}{4}\right)$.

Now, the operations with the Real Neutrosophic Numbers follow the rules of operations with real sets presented above, because a Real Neutrosophic Number is equivalent to a real subset:

Let I be a real subset, $I \subset \mathbb{R}$.

$N = a + bI = \{a + b \cdot x, \text{where } x \in I\}$,

which is a real subset of the form as of I.

Actually N is the enlarged subset I.

If I is an interval of the form

$I = (c, d)$, or $[c, d)$, or $(c, d]$, or $[c, d]$,

then N will also be an interval of the same corresponding open/closed form.

If $I = \{c_1, c_2, \dots, c_n\}$ is a real discrete subset, of cardinal n, $1 \leq n \leq \infty$, then N will also be a real discrete subset of cardinal n.

It is a union of several subsets, $I = I_1 \cup I_2 \cup ... \cup I_m$, then N will also be a union of corresponding subsets:

$N = N_1 \cup N_2 \cup ... \cup N_m = \bigcup_{k=1}^{m} N_k$,

where

$N_k = a + b \cdot I_k = \{a + bx, \text{where } x \in I_k\}$.

Second Method of Operations with Real Neutrosophic Numbers is the following.
Transform each real neutrosophic number into an equivalent real subset, especially when the indeterminacy (*I*) are not the same.

Examples:

$N_1 = 1 + 2I_1$, where $I_1 = \{02., 0.5, 0.8\}$

$N_2 = 3 - I_2$, where $I_2 = [0, 1)$

Then:

$N_1 = 1 + 2 \cdot \{0.2, 0.5, 0.8\} = 1 + \{0.4, 1.0, 1.6\} = \{1.4, 2.0, 2.6\}$

and

$N_2 = 3 - [0, 1) = (3 - 1, 3 - 0] = (2, 3]$

Addition of Real Neutrosophic Numbers

$$N_1 + N_2 = \{1.4, 2.0, 2.6\} + (2, 3] = \{1.4 + (2, 3]\} \cup \{2.0 + (2, 3]\} \cup \{2.6 + (2, 3]\}$$
$$= (1.4 + 2, 1.4 + 3] \cup (2.0 + 2, 2.0 + 3] \cup (2.6 + 2, 2.6 + 3]$$
$$= (3.4, 4.4] \cup (4, 5] \cup (4.6, 5.6] = (3.4, 5.6].$$

Addition of a Scalar with a Neutrosophic Set

$0.9 + N_1 = \{1.4, 2.0, 2.6\} + 0.9 = \{1.4 + 0.9,\ 2.0 + 0.9,\ 2.6 + 0.9\}$

$= \{2.3, 2.9, 3.5\}$.

$0.9 + N_2 = 0.9 + (2, 3] = (0.9 + 2, 0.9 + 3] = (2.9, 3.9]$.

Subtraction of Real Neutrosophic Numbers

$$N_1 - N_2 = \{1.4, 2.0, 2.6\} - (2, 3] = \{1.4 - (2, 3]\} \cup \{2.0 - (2, 3]\} \cup \{2.6 - (2, 3]\}$$
$$= [1.4 - 3, 1.4 - 2) \cup [2.0 - 3, 2.0 - 2) \cup [2.6 - 3, 2.6 - 2)$$
$$= [-1.6, -0.6) \cup [-1, 0) \cup [-0.4, 0.6) = [-1.6, 0.6).$$

Multiplication of Real Neutrosophic Numbers

$$N_1 \cdot N_2 = \{1.4, 2.0, 2.6\} \cdot (2, 3] = \{1.4 \cdot (2, 3]\} \cup \{2.0 \cdot (2, 3]\} \cup \{2.6 \cdot (2, 3]\}$$
$$= (1.4 \cdot 2, 1.4 \cdot 3] \cup (2.0 \cdot 2, 2.0 \cdot 3] \cup (2.6 \cdot 2, 2.6 \cdot 3] = (2.8, 4.2] \cup (4, 6] \cup (5.2, 7.8]$$
$$= (2.8, 7.8].$$

Multiplication of a Scalar with a Neutrosophic Number

$$4 \cdot N_1 = 4 \cdot \{1.4, 2.0, 2.6\} = \{4 \cdot (1.4), 4 \cdot (2.0), 4 \cdot (2.6)\}$$

$$= \{5.6, 8.0, 10.4\}.$$

Division of Real Neutrosophic Numbers

$$\frac{N_1}{N_2} = \frac{\{1.4, 2.0, 2.6\}}{(2, 3]} = \frac{1.4}{(2, 3]} \cup \frac{2.0}{(2, 3]} \cup \frac{2.6}{(2, 3]} = \left(\frac{1.4}{3}, \frac{1.4}{2}\right] \cup \left(\frac{2.0}{3}, \frac{2.0}{2}\right] \cup \left(\frac{2.6}{3}, \frac{2.6}{2}\right]$$
$$= (0.4\bar{6}, 0.7] \cup (0.\bar{6}, 1] \cup (0.8\bar{6}, 1.3] = (0.4\bar{6}, 1.3].$$

Another Example of Division of Neutrosophic Numbers

Let $N_1 = 2 - 3I_1$ where $I_1 = [4, 5]$, and $N_2 = 1 + 4I_2$ where $I_2 = \{-1, 3, 5\}$.

Then:

$$N_1 = 2 - 3I_1 = 2 - 3 \cdot [4,5] = 2 - [3 \cdot 4, 3 \cdot 5] = 2 - [12,15] =$$
$$= [2-15, 2-12] = [-13,-10]$$

$$N_2 = 1 + 4I_2 = 1 + 4 \cdot \{-1,3,5\} = 1 + \{4 \cdot (-1), 4 \cdot 3, 4 \cdot 5\} =$$
$$= 1 + \{-4,12,20\} = \{1 + (-4), 1 + 12, 1 + 20\} = \{-3,13,21\}$$

$$\frac{N_1}{N_2} = \frac{[-13,-10]}{\{-3,13,21\}} = [\frac{-13}{-3}, \frac{-10}{-3}] \cup [\frac{-13}{13}, \frac{-10}{13}] \cup [\frac{-13}{21}, \frac{-10}{21}] =$$
$$= [\frac{10}{3}, \frac{13}{3}] \cup [\frac{-13}{13}, \frac{-10}{13}] \cup [\frac{-13}{21}, \frac{-10}{21}] =$$
$$= [\frac{-13}{13}, \frac{-10}{13}] \cup [\frac{-13}{21}, \frac{-10}{21}] \cup [\frac{10}{3}, \frac{13}{3}]$$

Division between a Real Neutrosophic Number and a Scalar

$\frac{N_1}{4} = \frac{\{1.4,2.0,2.6\}}{4} = \left\{\frac{1.4}{4}, \frac{2.0}{4}, \frac{2.6}{4}\right\} = \{0.35, 0.50, 0.65\}.$

$\frac{4}{N_1} = \frac{4}{\{1.4,2.0,2.6\}} = \left\{\frac{4}{1.4}, \frac{4}{2.0}, \frac{4}{2.6}\right\} = \{2.857, 2.000, 1.538\}.$

$\frac{4}{N_2} = \frac{4}{(2,3]} = \left[\frac{4}{3}, \frac{4}{2}\right) \approx [1.333, 2.000)$

$\frac{N_2}{4} = \frac{(2,3]}{4} = \left[\frac{2}{4}, \frac{3}{4}\right) = [0.50, 0.75).$

Power of Real Neutrosophic Numbers

$$N_1{}^{N_2} = \{1.4, 2.0, 2.6\}^{(2,3]} = 1.4^{(2,3]} \cup 2.0^{(2,3]} \cup 2.6^{(2,3]} = (1.4^2, 1.4^3] \cup (2.0^2, 2.0^3] \cup (2.6^2, 2.6^3]$$
$$= (1.960, 2.744] \cup (4, 8] \cup (6.760, 17.576] = (1.960, 2.744] \cup (4.000, 17.576].$$

*

$$N_2{}^{N_1} = (2,3]^{\{1.4,2.0,2.6\}} = \{(2^{1.4}, 3^{1.4}], (2^{2.0}, 3^{2.0}], (2^{2.6}, 3^{2.6}]\}$$
$$\approx \{(2.639, 4.656], (4.0, 9.0], (6.063, 17.399]\}$$
$$\equiv \{2.639, 4.656] \cup (4.0, 9.0] \cup (6.063, 17.399]$$
$$= (2.639, 17.399].$$

Power of a Neutrosophic Real Numbers to a Scalar

$(N_1)^4 = \{1.4, 2.0, 2.6\}^4 = \{1.4^4, 2.0^4, 2.6^4\} = \{3.8416, 16.000, 45.6976\};$

and $4^{N_1} = 4^{\{1.4,2.0,2.6\}} = \{4^{1.4}, 4^{2.0}, 4^{2.6}\} \approx \{6.9644, 16.000, 36.7583\}.$

Real Root of a Neutrosophic Real Number

$\sqrt{N_1} = \sqrt{\{1.4, 2.0, 2.6\}} = \left\{\sqrt{1.4}, \sqrt{2.0}, \sqrt{2.6}\right\} \approx \{1.183, 1.414, 1.612\}$

$\sqrt[3]{N_2} = \sqrt[3]{(2,3]} = (\sqrt[3]{2}, \sqrt[3]{3}) \simeq (1.260, 1.442].$

Real Neutrosophic Root of a Neutrosophic Real Number

$$\sqrt[N_1]{N_2} = N_2^{\frac{1}{N_1}} = (2,3]^{\frac{1}{\{1.4,2.0,2.6\}}} = \left\{(2,3]^{\frac{1}{1.4}}, (2,3]^{\frac{1}{2.0}}, (2,3]^{\frac{1}{2.6}}\right\} = \left\{\left(2^{\frac{1}{1.4}}, 3^{\frac{1}{1.4}}\right], \left(2^{\frac{1}{2.0}}, 3^{\frac{1}{2.0}}\right], (2^{\frac{1}{2.6}}, 3^{\frac{1}{2.6}}]\right\}$$
$$\approx \{(1.641, 2.192], (1.414, 1.732], (1.306, 1.526]\} \equiv (1.306, 2.192]$$

*

$$\sqrt[N_2]{N_1} = N_1^{\frac{1}{N_2}} = \{1.4, 2.0, 2.6\}^{\frac{1}{(2,3]}} = \{1.4, 2.0, 2.6\}^{[\frac{1}{3},\frac{1}{2})} = \left\{1.4^{[\frac{1}{3},\frac{1}{2})}, 2.0^{[\frac{1}{3},\frac{1}{2})}, 2.6^{[\frac{1}{3},\frac{1}{2})}\right\}$$
$$= \left\{\left[1.4^{\frac{1}{3}}, 1.4^{\frac{1}{2}}\right), \left[2.0^{\frac{1}{3}}, 2.0^{\frac{1}{2}}\right), \left[2.6^{\frac{1}{3}}, 2.6^{\frac{1}{2}}\right)\right\} \approx \{[1.119, 1.183), [1.260, 1.414), [1.375, 1.612)\}$$
$$\equiv [1.119, 1.183) \cup [1.260, 1.612).$$

7. Literal Neutrosophic Numbers

The Literal Neutrosophic Numbers (*LNN*) have the form:

$LNN = a + bI$, where a, b are real or complex numbers, and $I =$ literal indeterminacy, where $I^2 = I$, and $I/I =$ undefined.

Their Addition, Subtraction, Scalar Multiplication, Multiplication, Division, Power, Radical are straightforward. The Literal Neutrosophic Numbers are not used in Neutrosophic Statistics, but in Neutrosophic Algebraic Structures, that's why we do not present their operations herein.

8. NonAppurtenance Equation, NonInclusion Equation, and NonEquality Equation

They are complementarians of the Appurtenance Equation, Inclusion Equation, and Equality Equation respectively.

We present them as a curiosity, or as recreational mathematics.

(i) The Appurtenance Equation from previous Example 1 was:

$4 - 5x \in 1 + 2 \cdot (0.5, 0.8)$ whose solutions are all real numbers $x \in (0.28, 0.40)$.

Its corresponding **NonAppurtenance Equation** is:

$4-5x \notin 1+2\cdot(0.5,0.8)$ whose solutions are all real numbers $x \notin (0.28,0.40)$, or all real numbers $x \in R-(0.28,0.40)$.

(ii) The Inclusion Equation from previous Example 2 was:

$1 + x \cdot (1, 2) \subset (0, 5)$, whose maximal solution is $x = (-0.5, 2)$.

Its corresponding **NonInclusion Equation** is:

$1+x\cdot(1,2) \not\subset (0,5)$, whose maximum solution is R - (- 0.5, 2).

(iii) An elementary Equality Equation

3x + 4 = 7, has the unique solution x = 1.

Its corresponding **NonEquality Equation** is:

$3x+4 \neq 7$ has, of course, infinitely many solutions $x \in R-\{1\}$

9. Conclusion

In neutrosophic statistics, from the fact that the single true value v is in I, it does not result that v is in $a + bI = N$ as well, but: $a + bv \in a + bI$. That's why the appurtenance relationship and equation must be introduced and studied.

Even more, if one has a set of true values, from the fact that the set of true values V is included in I, it does not mean that V is included in $a + bI$ too, but $a + bV \subset a + bI$ (or $a + bV \subseteq a + bI$). That's why the inclusion relationship and equation must be introduced.

In the same way as the "=" symbol is used for an equality relationship or an equality equation, we use the symbol "∈" {belong(s) to} for an *appurtenance relationship* or *appurtenance equation* of a number to a set, respectively the symbol $\subset$ (or $\subseteq$) {included in, or included in or equal to} for an *inclusion relationship* or *inclusion equation*.

We just introduced for the first time the Appurtenance Equation and Inclusion Equation, which help in understanding the operations with neutrosophic numbers within the frame of neutrosophic statistics. The way of solving them resembles the equations whose coefficients are sets (no single numbers).

In addition, we also presented their complementary NonAppurtenance Equation, NonInclusion Equation, and the elementary NonEquality Equation respectively.

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Chi-square test for imprecise data in consistency table

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Abstract. In this paper, we propose the introduction of a neutrosophic chi-square-test for consistency, incorporating neutrosophic statistics. Our aim is to modify the existing chi-square -test for consistency in order to analyze imprecise data. We present a novel test statistic for the neutrosophic chi-square -test for consistency, which accounts for the uncertainties inherent in the data. To evaluate the performance of the proposed test, we compare it with the traditional chi-square -test for consistency based on classical statistics. By conducting a comparative analysis, we assess the efficiency and effectiveness of our proposed neutrosophic chi-square -test for consistency. Furthermore, we illustrate the application of the proposed test through a numerical example, demonstrating how it can be utilized in practical scenarios. Through this implementation, we aim to provide empirical evidence of the improved performance of our proposed test when compared to the traditional chi-square-test for consistency based on classical statistics. We anticipate that the proposed neutrosophic chi-square -test for consistency will outperform its classical counterpart, offering enhanced accuracy and reliability when dealing with imprecise data. This advancement has the potential to contribute significantly to the field of statistical analysis, particularly in situations where data uncertainty and imprecision are prevalent.

KEYWORDS

neutrosophic statistics, chi-square -test for consistency, imprecise data, comparative analysis, data uncertainty

1. Introduction

In statistical analysis, the chi-square -test for consistency, also known as the chi-square test, is a commonly used method to determine if there is a significant association between two categorical variables in a 2×2 contingency table. This test allows researchers to assess whether the observed frequencies in the table deviate significantly from what would be expected under the assumption of independence between the variables. The 2×2 table also referred to as a contingency table or cross-tabulation table presents the frequencies or counts of two categorical variables. The resulting test statistic follows a chi-square distribution with one degree of freedom. If the calculated chi-square statistic exceeds a critical value from the chi-square distribution, it indicates a significant departure from independence. This suggests that there is an association or relationship between the variables under investigation. On the other hand, if the calculated chi-square statistic is smaller than the critical value, it suggests no significant association, implying that the variables are independent. Recent research has expanded upon the application and interpretation of the chi-square-test for consistency, exploring its use in various fields such as healthcare, social sciences, and marketing. In conclusion, the chi-square-test for consistency is a valuable statistical tool for assessing the association between two categorical variables in a 2×2 contingency table. More details on the application of chi-square-test can be seen in Dutton and Dutton [1], McHugh [2], Rana and Singhal [3], Lin et al. [4], Benhamou and Melot [5], and Ahammed and Smith [6].

Imprecise data, also referred to as data with imprecise, interval, and fuzzy observations, encompasses various scenarios. In practical terms, imprecise data may arise when measuring water levels, collecting survey responses, or determining the lifetimes or failure times of electronic components. Neutrosophic statistics is a specialized branch of statistics that deals with uncertainties and imprecise information using the framework of neutrosophy. Neutrosophy is a philosophical concept introduced by Smarandache [7] aiming to analyze and study the indeterminate, uncertain, and ambiguous nature of various phenomena. In traditional statistics, uncertainty is often handled using probabilistic methods, which assume that events can be described by precise probabilities. However, in many real-world scenarios, uncertainties cannot be accurately represented by traditional probability theory. Neutrosophic statistics offers an alternative approach to address these limitations and provides a framework for handling uncertain, imprecise, and incomplete data. The fundamental principle of neutrosophic statistics is the recognition that most real-world problems involve not only true and false values but also indeterminacy, which represents the degree of truth or falsity. Neutrosophic statistics extends the notion of probability by introducing a third parameter called indeterminacy. This additional parameter allows for a more comprehensive representation of uncertainty and ambiguity in statistical analysis. Neutrosophic statistics is particularly useful in situations where information is incomplete, imprecise, or contradictory. It provides a formal framework for representing and manipulating uncertain data, making it applicable to a wide range of fields, including decision making, artificial intelligence, pattern recognition, and data mining. One of the significant advantages of neutrosophic statistics is its ability to handle incomplete and imprecise data. Traditional statistical methods often struggle when faced with missing data or imprecise measurements. Neutrosophic statistics, on the other hand, provides mechanisms to handle such situations, enabling researchers to make meaningful inferences even in the presence of incomplete information. Moreover, neutrosophic statistics offers a flexible framework for modeling uncertainty. It allows for the integration of various types of uncertainties, including random uncertainties, fuzzy uncertainties, and subjective uncertainties. By capturing and analyzing multiple dimensions of uncertainty, neutrosophic statistics provides a more realistic and nuanced representation of complex real-world phenomena. In conclusion, neutrosophic statistics is an innovative and powerful approach to handle uncertainty and imprecise information in statistical analysis. By incorporating the concept of neutrosophy, this field provides a more comprehensive framework for representing and analyzing uncertainties. Neutrosophic statistics has the potential to significantly impact various disciplines, enabling researchers to gain deeper insights and make more informed decisions in the face of uncertainty.

Smarandache [8] demonstrated the superior effectiveness of neutrosophic statistics when compared to classical and interval statistics. Shahzadi [9] introduced neutrosophic statistical analysis for temperature data collected from various cities in Pakistan. Additionally, Al Aita and Talebi [10] in the same year presented a method for analyzing imprecise data using neutrosophic augmented experimental design. Furthermore, Aslam and Saleem [11] put forth a neutrosophic test for evaluating linearity, while Aslam [12] conducted research on neutrosophic statistical testing methods for imprecise sequential contingency data. More applications of neutrosophic statistics can be seen in Chen et al. [13], Alhabib and Salama [14], Polymenis [15], Aslam [16], Raghav [17], Al Aita and Aslam [18], and Chen et al. [19].

In this paper, our main contribution is the introduction of a neutrosophic chi-square test for consistency, which incorporates the principles of neutrosophic statistics. The existing chi-square test for consistency is widely used in statistical analysis, but it assumes precise and deterministic data. Our aim is to modify this test to handle imprecise data by considering uncertainties inherent in the data. To achieve this, we propose a novel test statistic for the neutrosophic chi-square test for consistency. This test statistic takes into account the imprecise nature of the data and provides a more accurate assessment of consistency. We intend to evaluate the performance of our proposed test by comparing it with the traditional chi-square test for consistency based on classical statistics. This comparative analysis will allow us to assess the efficiency and effectiveness of our approach. Additionally, we plan to illustrate the practical application of the proposed test through a numerical example. By demonstrating how it can be utilized in real-world scenarios, we aim to provide empirical evidence of the improved performance of our test compared to the traditional chi-square test for consistency based on classical statistics. This empirical evidence will highlight the enhanced accuracy and reliability of our proposed test when dealing with imprecise data. The anticipated outcome of our research is that the proposed neutrosophic chi-square test for consistency will outperform its classical counterpart. By incorporating neutrosophic statistics and considering the uncertainties in the data, our test has the potential to offer more accurate and reliable results. This advancement in statistical analysis, particularly in situations where data uncertainty and imprecision are prevalent, will contribute significantly to the field.

2. Methods

In order to explore the statistical significance of the disparities between the observed frequencies within two separate dichotomous distributions, a comprehensive investigation will be conducted. This analysis aims to delve into the significance of the variations observed between the frequencies in each distribution, ultimately shedding light on the underlying factors that contribute to these differences. By examining the statistical significance, we can gain a deeper understanding of the implications and potential impact of these disparities within the context of the given distributions. The existing test given in Kanji [20] can be applied when the data is precise. Under complexity and uncertainty, the data may be imprecise and indeterminate therefore the existing test cannot be applied. Now, we present the modification of chi-square test under neutrosophic statistics in this section as follows:

When presented with two distinct samples, each categorized into two classes, it is possible to construct a comprehensive 2 $\times$ 2 table. This table serves as a valuable tool for organizing and analyzing the neutrosophic data obtained from the samples,

facilitating a deeper understanding of the relationships between the variables under investigation. By systematically organizing the data into rows and columns, the 2 × 2 table allows for a clear visualization of the neutrosophic frequency distribution within each class of the two samples. The imprecise data in 2 × 2 table is shown in Table 1. The neutrosophic 2 × 2 table having the measure of indeterminacy (I_N) is shown in Table 2. The first values in Table 2 present the determinate values and the second values are known as the indeterminate values and I_N is the measure of indeterminacy. Note that neutrosophic 2 × 2 table reduces to 2 × 2 table under classical statistics when I_L =0. The neutrosophic test statistic is given as:

$$\chi_N^2 = \frac{(n_L - 1)\left(a_L d_L - b_L c_L\right)^2}{\left(a_L + b_L\right)\left(a_L + c_L\right)\left(c_L + d_L\right)\left(c_L + d_L\right)} + \frac{(n_U - 1)\left(a_U d_U - b_U c_U\right)^2}{\left(a_U + b_U\right)\left(a_U + c_U\right)\left(c_U + d_U\right)\left(c_U + d_U\right)} I_N;\ I_N \epsilon\ [I_L, I_U] \quad (1)$$

The test statistic proposed here conforms to the chi-square distribution with a single degree of freedom. Note that the neutrosophic chi-square test is the generalization of the chi-square test statistic under classical statistics. The first part presents the test statistic under classical statistics and the second part denote the indeterminate part. In accordance with the guidelines outlined in Kanji [20], the suggested test should be utilized when the sample size exceeds 20. When I_L =0, the neutrosophic chi-square test simplifies to the test statistic in classical statistics, and this is expressed as follows:

$$\chi_N^2 = \frac{(n_L - 1)\left(a_L d_L - b_L c_L\right)^2}{\left(a_L + b_L\right)\left(a_L + c_L\right)\left(c_L + d_L\right)\left(c_L + d_L\right)} \quad (2)$$

The methodology for the proposed test is outlined in the following steps:

Step 1: Formulate the null hypothesis H_0 asserting independence between two samples, in contrast to the alternative hypothesis H_1 suggesting a lack of independence between the two samples.

TABLE 1 Neutrosophic 2 × 2 table.

	Class 1	Class 2	Total
Sample 1	$[a_L, a_U]$	$[b_L, b_U]$	$[a_L + b_L, a_U + b_U]$
Sample 2	$[c_L, c_U]$	$[d_L, d_U]$	$[c_L + d_L, c_U + d_U]$
Total	$[a_L + c_L, a_U + c_U]$	$[c_L + d_L, b_U + d_U]$	$[n_L, n_U]$

$n_L = a_L + b_L + c_L + d_L$ and $n_U = a_U + b_U + c_U + d_U$.

Step 2: Specify the significance level (α) and determine the critical value using the chi-square table from Kanji [20].
Step 3: Calculate the following statistic:

$$\chi_N^2 = \frac{(n_L - 1)\left(a_L d_L - b_L c_L\right)^2}{\left(a_L + b_L\right)\left(a_L + c_L\right)\left(c_L + d_L\right)\left(c_L + d_L\right)} + \frac{(n_U - 1)\left(a_U d_U - b_U c_U\right)^2}{\left(a_U + b_U\right)\left(a_U + c_U\right)\left(c_U + d_U\right)\left(c_U + d_U\right)} I_N;\ I_N \epsilon\ [I_L, I_U] \quad (3)$$

Step 4: Reject the null hypothesis (H_0) if the computed χ_N^2 value exceeds the critical value.

3. Application

In this section, we will discuss the application of the proposed test using data collected from the production process. The data represents the number of defective articles produced by two machines and has been obtained from Parthiban and Gajivaradhan [21]. The specific data can be found in Table 3. The data consists of recorded counts of defective articles produced by the two machines within an hour. Upon analyzing the data, it becomes evident that the existing test mentioned in Kanji [20] is not suitable for testing the null hypothesis (H_0), which assumes that both machines produce the same number of defectives. Instead, the alternative hypothesis (H_1) states that the two machines do not produce the same number of defectives. Therefore, to test these hypotheses, the application of the neutrosophic chi-square test is deemed appropriate. This test allows for the examination of both the null and alternative hypotheses. For the actual data, we proceed to implement the proposed test, and the resulting value of the neutrosophic test statistic is calculated as follows:

$$\chi_N^2 = 0.4430 + (-0.2795) I_N;\ I_N \epsilon\ [0, 0.5848] \quad (4)$$

The proposed test will be implemented as follows:

Step-1: H_0: two machines produce the same number of defectives vs. H_1: two machines do not produce the same number of defectives.
Step-2: Specified the level of significance α =0.05 and the tabulated value is 5.02.
Step-3: The calculated value of neutrosophic test statistic is $\chi_N^2 = 0.4430 + (-0.2795) I_N;\ I_N \epsilon\ [0, 0.5848]$.
Step-4: Compare the calculated value of χ_N^2 with the tabulated value of 5.02. If χ_N^2 is ≤5.02, the null hypothesis cannot be rejected. Therefore, it is concluded that both machines produce the same number of defectives within an hour.

TABLE 2 Neutrosophic 2 × 2 table with measure of indeterminacy.

	Class 1	Class 2	Total
Sample 1	$a_N = a_L + a_U I_N;\ I_N \epsilon\ [I_L, I_U]$	$b_N = b_L + b_U I_N;\ I_N \epsilon\ [I_L, I_U]$	$[a_N, b_N]$
Sample 2	$c_N = c_L + c_U I_N;\ I_N \epsilon\ [I_L, I_U]$	$d_N = d_L + d_U I_N;\ I_N \epsilon\ [I_L, I_U]$	$[c_N, d_N]$
Total	$[a_L + c_L, a_U + c_U]$	$[c_L + d_L, b_U + d_U]$	$[n_L, n_U]$

TABLE 3 The numerical data.

Machines	Machine-I	Machines-II	Total
Production time (in hours)	[1,1]	[1,1]	[2,2]
Number of defectives	$10 + 15I_N; I_N\epsilon [0, 0.33]$	$26 + 32I_N; I_N\epsilon [0, 0.19]$	$36 + 47I_N; I_N\epsilon [0, 0.23]$
Total	$11 + 16I_N; I_N\epsilon [0, 0.31]$	$27 + 33I_N; I_N\epsilon [0, 0.18]$	$\mathbf{38} + \mathbf{49}I_N; I_N\epsilon [0, 0.22]$

4. Comparative study

Now, let us compare the performance of the proposed chi-square test with the existing chi-square test in terms of flexibility, informativeness, and adequacy. As previously mentioned, the neutrosophic chi-square test serves as a generalization of the existing chi-square test. When there are no indeterminate observations in the data, the proposed test simplifies to the existing chi-square test. In the numerical example provided, the neutrosophic value of the test statistic is represented as $\chi^2_N = 0.4430 - (0.2795)I_N; I_N\epsilon [0, 0.5848]$, where I_N falls within the range of [0, 0.5848]. The initial value of 0.4430 signifies the values obtained from the existing test statistic under classical statistics. The subsequent part $(0.2795)I_N$, represents the indeterminate component, and the measure of indeterminacy is 0.5848. From the analysis conducted, it becomes evident that the proposed test yields results within an indeterminate interval instead of providing an exact value. Considering the nature of the data, which is presented within an indeterminate interval, the use of the existing test could potentially mislead decision-makers. Hence, the existing test mentioned in Kanji [20] is not suitable for datasets containing indeterminate intervals. On the other hand, the proposed test provides results for the test statistic ranging from 0.4430 to 0.2795. Additionally, the proposed test supplies information regarding the measure of indeterminacy, which is calculated to be 0.5848. This measure indicates a high level of indeterminacy during the test implementation. Consequently, the proposed test demonstrates greater efficiency than the existing test in terms of flexibility and provision of information.

5. Concluding remarks

In this paper, we proposed a neutrosophic chi-square test for consistency, which incorporates neutrosophic statistics to handle imprecise data. Our test modifies the existing chi-square test for consistency by considering the uncertainties inherent in the data. We introduced a novel test statistic that accounts for the imprecise nature of the data, providing a more accurate assessment of consistency. To evaluate the performance of our proposed test, we conducted a comparative analysis with the traditional chi-square test based on classical statistics. Through our comparative analysis, we demonstrated that the proposed neutrosophic chi-square test for consistency outperforms its classical counterpart. The traditional chi-square test assumes precise and deterministic data, which can be inadequate for scenarios involving imprecise data. In contrast, our test incorporates the principles of neutrosophic statistics, allowing for a more comprehensive representation and analysis of uncertainties. The neutrosophic chi-square test provides results within an indeterminate interval, which accurately reflects the imprecise nature of the data. We illustrated the application of our test using data from the production process, showcasing its effectiveness in practical scenarios. The proposed neutrosophic chi-square test for consistency offers enhanced accuracy and reliability when dealing with imprecise data. By considering uncertainties and indeterminacies, our test provides a more realistic and nuanced analysis, contributing significantly to the field of statistical analysis. Neutrosophic statistics, as a specialized branch of statistics, offers a powerful framework for handling uncertainty and imprecise information. By incorporating neutrosophy, our test enables researchers to gain deeper insights and make more informed decisions in the face of uncertainty. In conclusion, the proposed neutrosophic chi-square test for consistency presents a valuable advancement in statistical analysis, particularly in situations where data uncertainty and imprecision are prevalent. Its ability to handle imprecise and incomplete data, along with its flexibility in modeling uncertainty, makes it applicable to a wide range of fields. The integration of neutrosophic statistics provides a more comprehensive framework for representing and analyzing uncertainties, thereby contributing to the improvement of statistical analysis methodologies. There are several limitations and drawbacks associated with the proposed test within the framework of neutrosophic statistics. Given that neutrosophic tests are designed for handling complex or imprecise data, the interpretation of test results becomes notably challenging. Additionally, there is a shortage of specialized computer software for the analysis of imprecise data, representing a promising avenue for future research and development. Further research opportunities also exist in the exploration of various statistical properties of the proposed test.

Data availability statement

The original contributions presented in the study are included in the article/supplementary material, further inquiries can be directed to the corresponding author.

Author contributions

MA: Data curation, Software, Writing—original draft, Writing—review and editing. FS: Funding acquisition, Methodology, Validation, Writing—review and editing.

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Analyzing Imprecise Data from Wireless Temperature Sensor

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Abstract

Various sensors play an important role in the monitoring and development of robotic technology. We present a contemporary statistical analysis method for evaluating datasets generated by robotic systems. Specifically, this data set originates from the physical structure of the robot and is acquired by a wireless temperature sensor. The data collection process spans a temporal period of 1 to 10 hours during the operational period of the robot. The collected data is subjected to a rigorous analysis using neutrosophic methodology. To facilitate this, a modern neutrosophic formula has been devised, drawing on definitions established within the field. To benchmark the effectiveness of the proposed approach, a conventional formula is also used for comparison purposes. Our results clearly indicate that the proposed method achieves higher levels of information richness and adaptability compared to classical methods. This indicates the improved utility of the proposed approach in solving the complexities of robotic data analysis.

1. Introduction

The use of the robotic technology in every field of life got importance. One can easily observe that man has created a good interaction system between human and robots and both work in combination (de Graaf et al. [13] and Etemad-Sajadi et al. [14]). Robotics technology is an effective example of the internet of things (IoT). However, for monitoring the robot, different sensors are used (Hadi Hosseinabadi and Salcudean [17]) as shown in Figure 1.

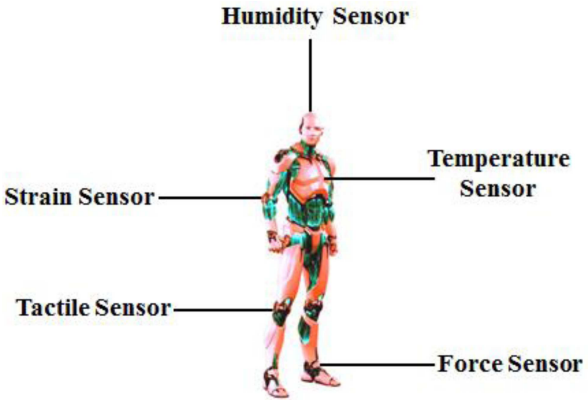

Figure 1. Use of different sensors to monitor.

The above figure is expressing the use of the sensors to monitor and measure different factors. Like temperature sensor is used for the detection of temperature of robotic body (Afzal et al. [1, 5]), humidity is used to measure the humidity effects (Afzal et al. [3]), strain sensor is used to measure strain effect (Liu et al. [20]), force sensor is used to measure applied force on robot body or by robot body (Ko et al. [19]) and tactile sensor is used for the detection of physical interaction (Afzal et al. [23]). If we talk about the communication medium, generally two types of medium are used for this purpose, i.e., wireless or non-wireless. However, nowadays wireless communication medium is widely used due to its high rate data transmission (Gu and Peng [16]). Figure 2 shows the wireless sensing data communication from robot body to the server. The sensors are responsible for the detection and then processors process sensing data. After, this communication unit transfers this data to the server.

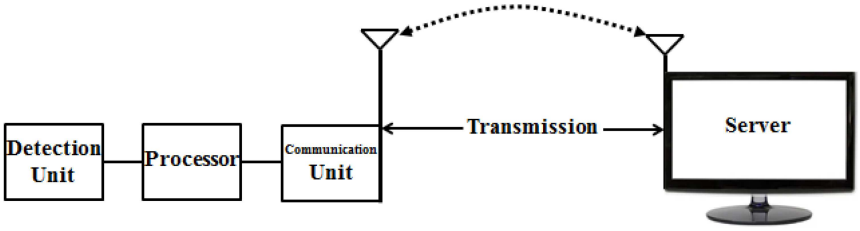

Figure 2. Communication setup.

After, collecting the data through server, it is analyzed through different statistics methods and approaches. Generally, classical statistics is widely used for analysis. It analyzes the single value or fixed value in well manner. But for interval values, it is not enough to analysis and explains the flexibility and complexity of data. That is why for proper analysis, we have to move other statistical methods and approaches like neutrosophic statistics. Neutrosophic is a method which is used to visualize the data and proposed by Smarandache [21]. Nowadays, the use of this technique has been enhanced to analyze the data in various fields, i.e., in medicine for diagnoses data analysis (Ye [24]), in applied sciences to analysis the conductor data (Afzal et al. [4]), in astrophysics for earth speed analysis (Aslam [8]) and others. Usama Afzal and his company have developed many applications of material statistics under the neutrosophic statistics for the analysis of sensors data (Afzal et al. [1-3, 5, 6]). The neutrosophic approach is based on the interval data with its indeterminacy to explain the difference of data (Aslam [9]). But the ancient method follows to deal with single value data and unable to explain the indefiniteness or difference of data (Afzal et al. [3], Chen et al. [11, 12] and Smarandache [22]).

1.1. Aim of the study

We develop an application framework to use neutrosophic statistical techniques in the analysis of interval-based temperature data. The temperature data set under consideration is acquired by wireless sensors integrated into the robotic structure. Notably, our initiative marks an important application of neutrosophic analysis in the realm of temperature data, particularly in describing the temporal variability exhibited by a robotic entity. A comparative aspect is realized through the inclusion of traditional methods, thus facilitating a comprehensive evaluation of the results.

2. Methodology

2.1. Neutrosophic approach

Let 'A_{iN}' be the neutrosophic numbers having A_{iL} lower values and A_{iU} upper values. Hence, the neutrosophic expression for the *i*th interval is as follows:

$$A_{iL} + A_{iU} I_{iN}; \quad I_{iN} \in [I_{iL}, I_{iU}]. \tag{1}$$

Here, $I_{iN} \in [I_{iL}, I_{iU}]$ is an indeterminacy interval and $A_{iN} \in [A_{iL}, A_{iU}]$ is an unsystematically neutrosophic variable having size $n_{iN} \in [n_{iL}, n_{iU}]$. The lower part/value of $A_{iN} \in [A_{iL}, A_{iU}]$ is the classical part/value and the upper part/value is the indeterminate part/value.

Similarly, neutrosophic mean $\overline{A}_N \in [\overline{A}_L, \overline{A}_U]$ is defined as follows:

$$\overline{A}_N = \overline{A}_L + \overline{A}_U I_N; \quad I_N \in [I_L, I_U]. \tag{2}$$

Here

$$\overline{A}_L = \sum_{i=1}^{nL} (A_{iL}/n_L) \quad \text{and} \quad \overline{A}_U = \sum_{i=1}^{nU} (A_{iU}/n_U).$$

The computational algorithm of the neutrosophic statistics is stated as follows:

- Start program.
- Select the interval $[A_L, A_U]$ and let $A_N \in [A_L, A_U]$.
- Run a loop for indeterminacy $I_N = I_L$ to $I_N = I_U$.
- Execute formula $A_{iN} = A_{iL} + A_{iU} I_{iN}$; calculate values.
- Draw graph.
- End program.

2.2. Classical approach

In classical statistical approach, we use the fixed point values. The data is in the interval form. So, we use the mean formula to convert interval point

values into fix point values which is as follows:

$$A = (A_L + A_U)/2. \quad (3)$$

Here, 'A' is the interval with 'A_L' lower value and 'A_U' upper/higher value.

The computational algorithm of the classical statistics is stated as follows:

- Start program.
- Select the interval $[A_L, A_U]$ and let $A_N \in [A_L, A_U]$.
- Execute formula $A = (A_L + A_U)/2;$ calculate values.
- Draw point/graph.
- End program.

3. Data Collection

The data through the wireless temperature sensor of robot body can be seen in Table 1. These values are not measured from experiment but by concerning with different already published works (Gong et al. [15], Husain et al. [18] and Zukowski et al. [25]) and also using some consumptions. Because, in this work, we only want to explain the data analysis through modern statistics approach.

Table 1. Collection of data

Working time (hour)	**Temperature** (^{o}C)
1	[23.5, 24.8]
2	[24.2, 25.1]
3	[25.8, 26.4]
4	[27.2, 29.3]
5	[28.8, 29.9]
6	[32.1, 33.1]
7	[32.9, 34.6]
8	[34.9, 36.2]
9	[35.1, 35.9]
10	[35.7, 37.4]

The above table is expressing the interval data of temperature of robot with respect to working hours. With increase in working time, the temperature of the robot body has been also increased. However, our main concern is with the analysis of the above data.

4. Analysis of Data

As it is mentioned above, two types of statistical approaches have been used for the analysis of the electricity production data, i.e., classical and neutrosophic approaches.

Table 2. Classical and neutrosophic analyses of electricity production

Working time (hour)	**Temperature** (^{o}C)	
	Classical	**Neutrosophic**
1	24.15 ± 0.65	24.8 + 23.5 I_N; $I_N \in$ [0, 0.052]
2	24.65 ± 0.45	25.1 + 25.1 I_N; $I_N \in$ [0, 0.036]
3	26.1 ± 0.3	25.8 + 26.4 I_N; $I_N \in$ [0, 0.023]
4	28.25 ± 1.05	27.2 + 29.3 I_N; $I_N \in$ [0, 0.072]
5	29.35 ± 0.55	28.8 + 29.9 I_N; $I_N \in$ [0, 0.037]
6	32.6 ± 0.5	32.1 + 33.1 I_N; $I_N \in$ [0, 0.030]
7	33.75 ± 0.85	32.9 + 34.6 I_N; $I_N \in$ [0, 0.049]
8	35.55 ± 0.65	34.9 + 36.2 I_N; $I_N \in$ [0, 0.036]
9	35.3 ± 0.4	35.1 + 35.9 I_N; $I_N \in$ [0, 0.022]
10	36.55 ± 0.85	35.7 + 37.4 I_N; $I_N \in$ [0, 0.045]

Table 2 highlights the classical and neutrosophic analyses of electricity production. From the above table, it is seen that for classical approach, all the interval data have lost their indeterminacy as classical formula converts intervals into single value. However, there is some uncertainty is present for each interval due to some error. If we talk about the data variation, uncertainty does not accurately explain it. Because, error only expresses the

specific addition or deduction in the value, for example 24.15 ± 0.65 means one can get either only 23.5 or 24.8. That is why, the classical analysis is not efficient in order to explain the difference of data interval by its maximum and minimum values and one cannot fully depend on it in other to making some decision or concluding a problem. The classical graph of the temperature with respect to working time can be seen in Figure 3. Similarly, if we see the neutrosophic formula, it provides much flexible as it gives an equation for each interval with indeterminacy interval (AlAita and Aslam [7]). It means intervals do not loss their indeterminacy (Aslam and Albassam [10]). Moreover, it is more efficient in explaining the variation of data interval with its maximum and minimum values. The neutrosophic graph of the temperature with respect to working time can be seen in Figure 4.

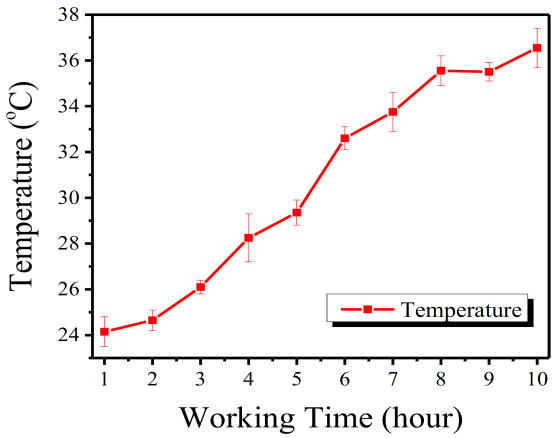

Figure 3. Classical graph of the temperature data.

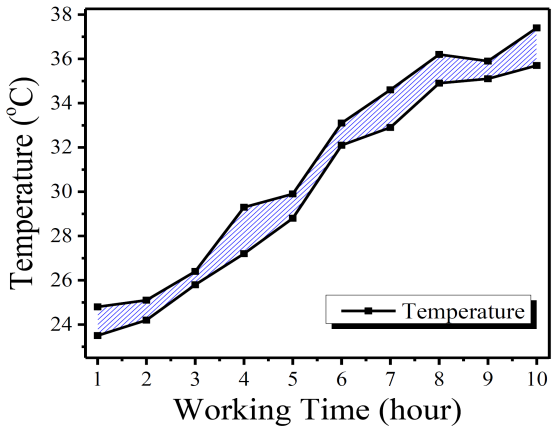

Figure 4. Classical graph of the temperature data.

For example, consider the temperature interval [23.5, 24.8] for the first working hour. The classical approach yields a point value of 24.15, while the neutrosophic approach straddles the range between 23.5 and 24.8. This neutrosophic value can be calculated using the neutrosophic formula using its indefinite interval.

To further evaluate the effectiveness of the proposed approach compared to the established classical method, a combined graph is constructed showing both classical and neutrosophic analyses. This assessment is summarized in Figure 5.

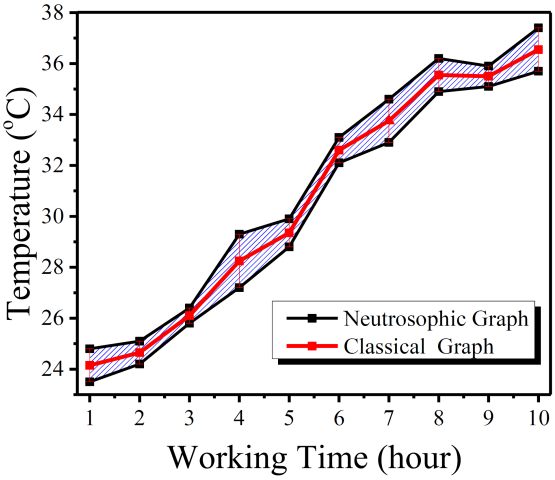

Figure 5. Combined graphs of classical and neutrosophic analyses.

The visual representation presented above provides meaningful insight. Graphs showing the results of classical analysis show that their adaptability is limited because their representation is limited to discrete single points. In contrast, graphs resulting from neutrosophic analysis exhibit the highest degree of adjustability, affording a more comprehensive and nuanced depiction of underlying trends. This observation confirms the notion that neutrosophic statistics offer a more efficient analytical framework.

Table 3. Comparison between classical and neutrosophic methods

Classical method	Neutrosophic method
The classical method relies on deterministic values adjusting for a degree of inherent uncertainty.	The neutrosophic method works based on interval values that are characterized by inherent uncertainty.
The classical method treats intervals as single values.	The neutrosophic method encompasses interval variations.
It exhibits relatively low flexibility and information content.	It exhibits high flexibility and information richness, which is attributed to the generalization status of the classical method.
It lacks a comprehensive treatment of probabilities, being characteristic of the classical method of assigning properties to true or false at any given time.	This addresses potential concerns well. This skill arises from the neutrosophic method's ability to simultaneously accommodate truth and falsehood for a statement within an indefinite interval.

This leads to the final conclusion that neutrosophic statistics inherently embody the properties of informativeness, flexibility and sufficiency that are postulated by classical statistical approaches. This deduction is strengthened by a comparative examination of the two methods, summarized in Table 3.

5. Conclusion

This investigation deals with the comprehensive analysis of data obtained from robotic systems using wireless temperature sensors with specific reference to temporal parameters. In particular, discernible trends emerge in which robotic body temperature shows a direct correlation with the operational duration range increasing from 23.5 to 37.4 degrees Celsius.

This dataset, captured in interval format, undergoes complex evaluation using both classical and neutrosophic statistical methods.

The resulting results clearly illustrate the limitations of the classical approach, which fails to adequately accommodate the multidimensional variability and complexity inherent to robotic temperature datasets. In stark contrast, the neutrosophic statistical approach emerges as distinctly more flexible and informative, better illustrating the relative variability and complexity within the data. Consequently, this investigation advocates the adoption of neutrosophic statistical techniques as the preferred method for interpreting interval-based robotic temperature data.

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Row-column Designs: A Novel Approach for Analyzing Imprecise and Uncertain Observations

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Abstract

Classical row-column designs cannot be applied when the underlying data set contains some imprecise, uncertain, or undetermined observations. In this paper, we discuss row-column design under a neutrosophic statistical framework.

1. Introduction

Experimental design is crucial to the planning, analysis, and interpretation of experiments in order to produce accurate and objective results. The row-column design is one of the most widely used experimental designs in a variety of fields. These designs use two blocking factors, one representing the rows and the other representing the columns, in order to group the experimental units. Row-column designs were first introduced by Fisher [12]. Chang and Notz [10] discussed some optimal nested row-column designs through optimal designs for estimating treatment effects. Williams and John [23] used row-column factorial designs in agricultural field trials. A detailed discussion of row-column designs has been given by Shah and Sinha [20]. Several row-column designs are reviewed and illustrated in Dean and Voss [11]. We can find related articles and books about row-column design in John and Eccleston [13], Majumdar and Tamhane [14], Morgan and Parvu [15], and Morgan and Uddin [16].

The concept of neutrosophic statistics (NS) was introduced by Smarandache [21] for dealing with the problem of data which have some degree of indeterminacy. Neutrosophic statistics is more informative and

flexible than classical statistics (Aslam [4]). The differences between fuzzy statistics, NS, and classical statistics are explained in a table (Aslam [6]). Aslam [5] has highlighted neutrosophic ANOVA. A more recent article of AlAita and Aslam [1] discussed the application of neutrosophic analysis of covariance to a variety of experimental designs. Besides, post hoc multiple comparison tests under NS have been proposed by Aslam and Albassam [8]. Aslam and Al-Marshadi [7] introduced the Watson-Williams test under NS for the analysis of circular and angular data that is uncertain, imprecise, and indeterminate. In another paper, diagnosis test under NS has been discussed by Aslam et al. [9]. Salama et al. [19] discussed neutrosophic correlation and simple linear regression. Nagarajan et al. [17] have proposed in their paper an analysis of neutrosophic multiple regression. Numerous statistical tests have been discussed under NS in Alvaracín Jarrín et al. [3], Smarandache [22], Alpaslan [2], Yazdani et al. [24] and Zolfani et al. [25].

A primary motivation for doing this research is to resolve problems caused by studies involving imprecise and uncertain data that require row-column designs. For example, in 5×3 row-column experiment, three different diets (treatments) were compared on the milk production of dairy cows (five rows) during three periods (three columns) for the purpose of determining whether they affect no protein nitrogen in rat colostrum. In order to analyze this example, under NS, we calculate the NANOVA table, and then derive the F_N-test in the neutrosophic form, which identifies the measure of indeterminacy. The row-column design under classical statistics cannot be used to analyze and interpret neutrosophic data that have some degree of indeterminacy. Accordingly, this type of research requires the proposed design.

According to our literature review, we are not aware of any work on row-column designs under NS. This paper introduces a neutrosophic row-column design. In addition, the F_N-test of row-column design is introduced under NS. Numerical examples are used to illustrate the hypothesis testing procedure. We anticipate that the proposed row-column design performs better than the existing design in the event of uncertainty.

The remaining portions of this paper are arranged as follows: The following section discusses some concepts related to neutrosophic statistics and classical row-column design. In Section 3, we present a statistical analysis of the neutrosophic row-column design. Detailed numerical examples are provided in Section 4. The discussion and comparative study are presented in Section 5. Finally, Section 6 presents the conclusion.

2. Preliminaries

This section discusses the ANOVA approach of row-column design and neutrosophic statistics in brief.

2.1. Model and ANOVA for a row-column design

A row-column design with a rows and b columns can be represented by the following statistical model:

$$y_{hqi} = \mu + \gamma_h + \delta_q + \tau_i + \varepsilon_{hqi}; \quad \begin{cases} h = 1, 2, ..., a, \\ q = 1, 2, ..., b, \\ i = 1, 2, ..., t, \end{cases} \tag{1}$$

where y_{hqi} represents the observation in the hth row and qth column for the ith treatment, μ represents an overall mean, γ_h represents the effect of the hth row, δ_q represents the effect of the qth column, τ_i represents the effect of the ith treatment, and ε_{hqi} represents the random error assumed to have mean of zero and variance σ^2. Let the total number of all plots in the blocks (rows and columns) be n. Then $n = ab$. Table 1 presents the ANOVA for row-column design.

Table 1. ANOVA table for row-column design

Source	*SS*	*Df*	*MS*	*F*
Rows (unadj)	SS_R	$a-1$	$MS_R = \frac{SS_R}{a-1}$	
Columns (unadj)	SS_C	$b-1$	$MS_C = \frac{SS_C}{b-1}$	
Treatments (adj)	$SS_{Tr(adj)}$	$t-1$	$MS_{Tr(adj)} = \frac{SS_{Tr(adj)}}{t-1}$	$\frac{MS_{Tr(adj)}}{MS_E}$
Error	SS_E	$ab-a-b-t+2$	$\frac{SS_E}{ab-a-b-t+2}$	
Total	SS_T	$n-1$		

The computing formulas for SSs can be expressed as follows:

$$SS_T = \sum_{h=1}^{a}\sum_{q=1}^{b}\sum_{i=1}^{t} n_{hqi} y_{hqi}^2 - \frac{y_{...}^2}{n},$$

$$SS_R = \frac{1}{b}\sum_{h=1}^{a} y_{h..}^2 - \frac{y_{...}^2}{n},$$

$$SS_C = \frac{1}{a}\sum_{q=1}^{b} y_{.q.}^2 - \frac{y_{...}^2}{n},$$

$$T_i = \sum_{h=1}^{a}\sum_{q=1}^{b} n_{hqi} y_{hqi},$$

$$Q_i = T_i - \frac{1}{b}\sum_{h=1}^{a} n_{h.i} y_{h.i} - \frac{1}{a}\sum_{q=1}^{b} n_{.qi} y_{.qi} + \frac{r y_{...}^2}{n},$$

$$SS_{Tr(adj)} = \sum_{i=1}^{t} Q_i \hat{\tau}_i,\ SS_E = SS_T - SS_R - SS_C - SS_{Tr(adj)},$$

where $y_{h..}$ represents the total number of observations in the hth row, $y_{.q.}$ represents the total number of observations in the qth column, and $y_{...}$ represents the total number of observations, $n_{h.i}$ stands for the number of times that treatment i appears in row h, and $n_{.qi}$ stands for the number of times that treatment i appears in column q.

2.2. Hypotheses and decision rule

Under classical statistics, a null hypothesis and an alternative hypothesis are presented as follows:

$$H_0 : \tau_1 = \tau_2 = \cdots = \tau_t = 0 \text{ vs } H_1 : \text{at least one } \tau_i \neq 0.$$

The null hypothesis is accepted if p-value $\geq \alpha$, where α is a level of significance. The null hypothesis is rejected if p-value $< \alpha$.

2.3. Neutrosophic statistics (NS)

It has been observed in recent years that many real-world examples have been applied under NS, which are generalizations of classical statistics. Furthermore, it is characterized by its flexibility and efficiency in uncertain environments, as well as its ability to calculate measure of indeterminacy resulting from the state of uncertainty. A brief introduction to some basic concepts related to NS is provided below.

Suppose that a neutrosophic random variable (NRV) $X_N \in [X_L, X_U]$ follows the neutrosophic normal distribution (NND) with a neutrosophic population mean $\mu_N \in [\mu_L, \mu_U]$ and a neutrosophic population variance $\sigma_N^2 \in [\sigma_L^2, \sigma_U^2]$, where X_L and X_U are smaller and larger values of indeterminacy interval. Let $X_N = X_L + X_U I_N$ be the neutrosophic form of NRV having determinate part X_L and indeterminate part $X_U I_N$; $I_N \in [I_L, I_U]$, where $I_N \in [I_L, I_U]$ is an indeterminate interval. (For more information, see (Aslam [5].)

3. Analysis of Neutrosophic Row-column Design

3.1. Model and NANOVA for a neutrosophic row-column design

The neutrosophic statistical model for a row-column design with a_N rows and b_N columns can be expressed as:

$$y_{Nhqi} = \mu_N + \gamma_{Nh} + \delta_{Nq} + \tau_{Ni} + \varepsilon_{Nhqi}; \quad \begin{cases} h = 1, 2, ..., a_N, \\ q = 1, 2, ..., b_N, \\ i = 1, 2, ..., t_N. \end{cases} \tag{2}$$

The neutrosophic form of y_{Nhqi} can be defined as

$$y_{Nhqi} = y_{Lhqi} + y_{Uhqi} I_N; \quad I_N \in [I_L, I_U], \tag{3}$$

where y_{Nhqi} represents the neutrosophic observation in the hth row and qth column for the ith treatment, μ_N represents a neutrosophic overall mean, γ_{Nh} represents the neutrosophic effect of the hth row, δ_{Nq} represents the neutrosophic effect of the qth column, τ_{Ni} represents the neutrosophic effect of the ith treatment, and ε_{Nhqi} represents the neutrosophic random error assumed to have mean of zero and variance $\sigma_N^2 \in [\sigma_L^2, \sigma_U^2]$. Let the total neutrosophic number of all plots in the blocks (rows and columns) be n_{NT}. Then $n_{NT} = a_N b_N$. Table 2 presents ANOVA of neutrosophic row-column design.

Table 2. NANOVA table for neutrosophic row-column design

Source	*NSS*	*Ndf*	*NMS* (neutrosophic mean squares)	F_N
Rows (unadj)	SS_{NR}	$a_N - 1$	$MS_{NR} = \frac{SS_{NR}}{a_N - 1}$	
Columns (unadj)	SS_{NC}	$b_N - 1$	$MS_{NC} = \frac{SS_{NC}}{b_N - 1}$	
Treatments (adj)	$SS_{NTr(adj)}$	$t_N - 1$	$MS_{NTr(adj)} = \frac{SS_{NTr(adj)}}{t_N - 1}$	$\frac{MS_{NTr(adj)}}{MS_{NE}}$
Error	SS_{NE}	$a_N b_N - a_N - b_N - t_N + 2$	$MS_{NE} = \frac{SS_{NE}}{a_N b_N - a_N - b_N - t_N + 2}$	
Total	SS_{NT}	$n_{NT} - 1$		

The computing formulas for NSSs can be expressed as follows:

$$SS_{NT} = \sum_{h=1}^{a_N} \sum_{q=1}^{b_N} \sum_{i=1}^{t_N} n_{Nhqi} y_{Nhqi}^2 - \frac{y_{N...}^2}{n_{NT}};\ SS_{NT} \in [SS_{LT}, SS_{UT}],$$

$$SS_{NR} = \frac{1}{b_N} \sum_{h=1}^{a_N} y_{Nh..}^2 - \frac{y_{N...}^2}{n_{NT}};\ SS_{NR} \in [SS_{LR}, SS_{UR}],$$

$$SS_{NC} = \frac{1}{a_N} \sum_{q=1}^{b_N} y_{N.q.}^2 - \frac{y_{N...}^2}{n_{NT}};\ SS_{NC} \in [SS_{LC}, SS_{UC}],$$

$$T_{Ni} = \sum_{h=1}^{a_N} \sum_{q=1}^{b_N} n_{Nhqi} y_{Nhqi},$$

$$Q_{Ni} = T_{Ni} - \frac{1}{b_N} \sum_{h=1}^{a_N} n_{Nh.i} y_{Nh.i} - \frac{1}{a_N} \sum_{q=1}^{b_N} n_{N.qi} y_{N.qi} + \frac{r_N y_{N...}^2}{n_{NT}},$$

$$SS_{NTr(adj)} = \sum_{i=1}^{t_N} Q_{Ni} \hat{\tau}_{Ni};\ SS_{NTr(adj)} \in [SS_{LTr(adj)}, SS_{UTr(adj)}],$$

$$SS_{NE} = SS_{NT} - SS_{NR} - SS_{NC} - SS_{NTr(adj)};\ SS_{NE} \in [SS_{LE}, SS_{UE}],$$

where $y_{Nh..}$ represents the total number of the neutrosophic observations in the hth neutrosophic row, $y_{N.q.}$ represents the total number of the neutrosophic observations in the qth neutrosophic column, and $y_{N...}$ represents the total number of all the neutrosophic observations, $n_{Nh.i}$ represents the number of times that neutrosophic treatment i appears in neutrosophic row h, and $n_{N.qi}$ represents the number of times that neutrosophic treatment i appears in neutrosophic column q.

Neutrosophic mean squares are defined as:

$$MS_{NR} = \frac{SS_{NR}}{a_N - 1};\ MS_{NR} \in [MS_{LR}, MS_{UR}],$$

$$MS_{NC} = \frac{SS_{NC}}{b_N - 1};\ SS_{NT} \in [MS_{LC}, MS_{UC}],$$

$$MS_{NTr(adj)} = \frac{SS_{NTr(adj)}}{t_N - 1};\ MS_{NTr(adj)} \in [MS_{LTr(adj)},\ MS_{UTr(adj)}],$$

$$MS_{NE} = \frac{SS_{NE}}{a_N b_N - a_N - b_N - t_N + 2};\ MS_{NE} \in [MS_{LE},\ MS_{UE}].$$

The neutrosophic statistic F_N test becomes

$$F_N = \frac{MS_{NTr(adj)}}{MS_{NE}};\quad F_N \in [F_L,\ F_U].$$

F_N can be expressed in the neutrosophic form as follows:

$$F_N = F_L + F_U I_{F_N};\quad I_{F_N} \in [I_{F_L},\ I_{F_U}],$$

where F_L and $F_U I_{F_N}$ are determinate and indeterminate parts of the proposed test. This test reduces to test under classical statistic if $I_{F_N} = 0$.

3.2. Neutrosophic hypotheses and decision rule

Under neutrosophic statistics, a null hypothesis and an alternative hypothesis are presented as follows:

$$H_{N0} : \tau_{N1} = \tau_{N2} = \cdots = \tau_{Ni} = 0 \text{ vs } H_{N1} : \text{at least one } \tau_{Ni} \neq 0.$$

The null hypothesis is accepted if $\min\{p_N\text{-value}\} \geq \alpha$, where α is a level of significance. The null hypothesis is rejected if $\max\{p_N\text{-value}\} < \alpha$.

4. Numerical Examples

Example 4.1. Following is a case study using yields (in kgs.) of paddy with imprecise observations from the article of Parthiban and Gajivaradhan [18]. The last line of data is generated randomly by the R program. The 5×4 neutrosophic row-column design is used. Table 3 illustrates the distribution of data.

Table 3. Data for the neutrosophic row-column design

	Columns			
Rows	1	2	3	4
1	$A_3[22, 27]$	$A_2[20, 25]$	$A_1[15, 20]$	$A_4[17, 22]$
2	$A_1[16, 21]$	$A_4[15, 19]$	$A_3[19, 24]$	$A_2[15, 20]$
3	$A_2[17, 22]$	$A_1[10, 15]$	$A_4[14, 19]$	$A_3[18, 24]$
4	$A_4[13, 17]$	$A_3[15, 20]$	$A_2[19, 24]$	$A_1[10, 16]$
5	$A_2[15, 16]$	$A_3[14, 18]$	$A_1[13, 15]$	$A_4[11, 14]$

Table 4. ANOVA table for the neutrosophic row-column design

Source	*NSS*	*ndf*	*NMS*	F_N	Neutrosophic form F_N	p_N -value
Rows (unadj)	[66.80, 127.30]	[4, 4]	[16.70, 31.82]			
Columns (unadj)	[18.00, 7.40]	[3, 3]	[6.75, 4.12]			
Treatments (adj)	[86.25, 98.35]	[3, 3]	[28.75, 32.78]	[10.05, 9.59]	$10.05 - 9.59I_{F_N}$; $I_{F_N} \in [0, 0.048]$	[0.003, 0.004]
Error	[25.75, 30.75]	[9, 9]	[2.86, 3.42]			
Total	[196.80, 263.80]	[19, 19]				

Example 4.2. Suppose that a researcher conducts an experiment to examine the effects of neutrosophic treatments on a particular experiment. The 6×3 neutrosophic row-column design is used. Table 5 illustrates the distribution of data.

Table 5. Data for the neutrosophic row-column design

	Column		
Row	1	2	3
1	B[10.71, 11]	C[12.11, 12.18]	A[8.79, 9.75]
2	C[11.24, 12.2]	A[11.34, 11.75]	B[6.34, 7.08]
3	C[12.09, 12.86]	B[10.05, 10.2]	A[10.84, 11.17]
4	A[8.1, 8.79]	B[9.65, 9.83]	C[10.07, 10.33]
5	B[8.34, 8.56]	A[10.41, 10.46]	C[10.77, 11.74]
6	A[6.36, 6.94]	C[11.42, 11.73]	B[11.9, 12.08]

Table 6. ANOVA table for the neutrosophic row-column design

Source	*NSS*	*ndf*	*NMS*	F_N	Neutrosophic form F_N	p_N -value
Rows (unadj)	[5.89, 5.77]	[5, 5]	[1.18, 1.15]			
Columns (unadj)	[6.06, 2.94]	[2, 2]	[3.03, 1.47]			
Treatments (adj)	[14.26, 16.63]	[2, 2]	[7.13, 8.32]	[2.03, 2.57]	$2.03 + 2.57I_{F_N}$; $I_{F_N} \in [0, 0.210]$	[0.194, 0.138]
Error	[28.13, 25.93]	[8, 8]	[3.52, 3.24]			
Total	[54.34, 51.27]	[17, 17]				

In order to conduct the proposed F_N-test for neutrosophic row-column design, the following steps will need to be taken:

Step 1. We test the following statistical hypothesis:

$H_{N0}: \tau_{N1} = \tau_{N2} = \cdots = \tau_{Nt} = 0$ vs H_{N1}: at least one $\tau_{Ni} \neq 0$.

Step 2. We organize the NANOVA table for the proposed design.

Step 3. At the level of significance $\alpha = 0.05$, we calculate the p_N-value.

From the NANOVA Table 4 in Example 4.1: p_N-value $= [0.003, 0.004]$ and the NANOVA Table 6 in Example 4.2: p_N-value $= [0.194, 0.138]$.

Step 4. We accept the null hypothesis H_{N0} if p_N-values ≥ 0.05, and reject it otherwise.

In Example 4.1: We reject the null hypothesis H_{N0} because p_N-value $= [0.003, 0.004] < 0.05$, i.e., there is a difference in mean between the three treatments.

In Example 4.2: We accept the null hypothesis H_{N0} because p_N-value $= [0.194, 0.138] \geq 0.05$, i.e., the treatments have the same mean.

5. Discussion and Comparative Study

The aim of this section is to define the advantages of the proposed design in the presence of uncertainty in which, the neutrosophic logic literature has indicated that a method based on data in an indeterminate interval is more effective and suitable for use in uncertainty than determined values under classical statistics. This paper proposes the row-column design under NS as a generalization of the existing row-column design under classical statistics. As long as uncertainty does not exist, the proposed design reduces to the existing row-column design. In order to determine the effectiveness of the proposed F_N-test, we assess its measure of indeterminacy, adequacy, information, and flexibility. For example, in Table 6, the neutrosophic form of the F_N-test is $F_N = 2.03 + 2.57I_{F_N}$; $I_{F_N} \in [0, 0.210]$. There are two parts to this neutrosophic form: an *F*-test of classical statistics and an indeterminate part. The neutrosophic form of the neutrosophic F_N-test reduces to the *F*-test when $I_{F_N} = 0$. In other words, the value 2.03 represents the *F*-test value under classical statistics for the existing row-column design. As for the second part, $2.57I_N$, it contains an indeterminate part that has a measure of indeterminacy of 0.210.

According to Table 6, the proposed F_N-test has a range of values between 2.03 and 2.57. On the other hand, the existing *F*-test only considers a single value that is adequate when dealing with uncertainty. At α significance level, the p_N-value is $[0.194, 0.138] \geq 0.05$. In light of this, the neutrosophic null hypothesis is accepted while the neutrosophic alternative hypothesis is rejected. This indicates that there are no significant differences between the means of the assumed treatments. Additionally, 0.210 is the measure of indeterminacy associated with the proposed F_N-test. The existing *F*-tests cannot provide information regarding measure of indeterminacy. In the light of the results of the study, it can be concluded that the proposed F_N-test is more informative and flexible than the existing *F*-test.

6. Conclusion

In this paper, we presented a row-column design under neutrosophic statistics that is suitable for analyzing indeterminate, uncertain, and imprecise data. The statistical model and NANOVA table for the proposed design have been discussed. Furthermore, we examined the effectiveness of the proposed F_N-test for neutrosophic row-column design compared with the existing row-column design test. A numerical example of the application of the proposed row-column design has been presented. The results demonstrate that the proposed F_N-test provides greater flexibility, applicability, and information than the existing *F*-test.

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Neutrosophic Extension of the Negative Binomial Distribution: Properties and Applications to Interval-Valued Data

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Abstract

In real world while addressing the challenges of ambiguity and vagueness, the problems call for a scientific approach to quantify the uncertainties. The negative binomial distribution, a cornerstone in classical probability theory, traditionally examines the probability of achieving the kth success in n trials, following $n - 1$ failures, with precise observations. However, the available literature does not address the occurrence of observations in the intervals and hence the handling of interval-valued data remained unattended. This study introduces the neutrosophic negative binomial distribution which is extended from the classical version by incorporating both indeterminacy and crisp intervals. We derive various properties of this generalized distribution which includes; characteristic function, probability function and moment generating function. This study also presents the reliability analysis metrics like cumulative hazard rate, Mills ratio, survival hazard rate, odds ratio and reversed hazard rate. This study explores the order statistics for neutrosophic negative binomial distribution, encompassing with joint, maximum, minimum and median order statistics. Special case studies are discussed which demonstrate the application of the proposed distribution, paving the way for addressing complex problems that combine conventional approaches with non-precisely determined information.

1. Introduction

Smarandache (Smarandache [23]) introduced the term neutrosophy which is extended rationally, motivated branch of fuzzy logic which is the intuitionistic generalized fuzzy logic (Atanassov [24]). Neutrosophy logic is established on blurred, unclear, vague and fuzzy realms (ideas, glitches) which are significant in prevailing global multifaceted aspects of economic and social system.

The fuzzy logic is an exclusive representative of precise cognitive accustomed to enumerate the indefinite approaches of reasoning (Zadeh [20]). The theory is useful to comprehend the choices regarding problems faced during decision making in states of uncertain and imprecise circumstances (Akram and Shum [3]; Zadeh [21]). The theory and applications of fuzzy logic are conferred recently in a precise way (Lotfi A Zadeh and Aliev [22]) while further developments and clarifications about fuzzy logic, the literature (Akram and Luqman [2]; Akram and Shum [3]; Aouf et al. [6]; Feng et al. [9]; Koam et al. [11]; Philipo et al. [13]) may be consulted. Generally, the ordinary fuzzy sets incorporate the precise values whereas in real life the parameters move within an interval and it becomes difficult to find exact single values data. Turksen (Turksen [25]) presented the interval fuzzy sets to cater the situation. Smarandache (Smarandache [23]) introduced the neutrosophic statistics and sets by generalizing the fuzzy logic. The neutrosophic statistics incorporates the range valued information. Classical statistical approach becomes helpless when the situations generate indefinite imprecise data in an interval. For methodological and applied description of neutrosophic statistics the available literature may be consulted (Aslam et al. [8]; Jarrín et al. [10]; Sherwani et al. [17]; Sherwani et al. [18]). Sherwani et al. [27] introduced the neutrosophic negative binomial distribution and examined its properties and applications.

The conventional approach overlooks an important aspect of uncertainty and ignores ambiguous values which provided the gap of development. In this regard, Smarandache (Smarandache [19]) explained the neutrosophich

sets in statistical point of view which led the foundations to draw conclusions by incorporating the indeterminate data. Neutrosophic approach is an extended case of classical statistics. In neutrosophic logic, the principal objective is to incorporate the logical argument by considering the 3D neutrosophic space and correspondingly, the dimensions describe the Truth (T), False (F) and Indeterminacy (I). The available literature endorsed the development of extended conventional distributions using neutrosophic logic which comprises of; neutrosophic multinomial distribution (Smarandache [19]), neutrosophic binomial distribution and neutrosophich normal distribution (Patro and Smarandache [12]; Sherwani et al. [17]), neutrosophic gamma distribution (Aslam et al. [7]), Poisson, exponential and uniform distributions (Alhabib et al. [4]), neutrosophic beta distribution (Sherwani et al. [26]) and neutrosophic Weibull distribution and its various extensions (Alhasan and Smarandache [5]). This investigation develops the notion of two-parameter negative binomial distribution with neutrosophic negative binomial distribution by incorporating the neutrosophic logic. Moreover, (Sherwani et al. [15]) can be consulted for further detail.

2. Neutrosophic Negative Binomial Distribution

Current study generalizes the classical negative binomial distribution by establishing the associated indeterminacy to the probabilistic experiments. Supposedly, during the experiments the outcome of every trail is the result which may categorized into Failure (U) or Success (S). Now suppose, while tossing the coin on an uneven surface, the edge of the coin may hit into the crack and resultantly, outcome will neither be tail nor the head but may be indeterminate.

The neutrosophic negative random variable may be illustrated as an adjustable number of trails to acquire the specific number of "*S*" and the distribution may be defined as Neutrosophic Negative Binomial Distribution (NNBD). Firstly, finding the indeterminacy of every experiment clarifies the presence of uncertainty for all trails. Secondly, in case of indeterminacy for

zero trails means absence of indeterminacy for all the experiments. Thirdly, the situation may occur where one may obtain indeterminacy for some of the trails and determinacy for rest of the trials. In this case the study proposed an indeterminate threshold and let the th^* be an indeterminate number of trails where $th^* = (\text{Abbas, } 2023 3, ..., \infty)$. Now if threshold is greater than th^*, then it will classify in the indeterminacy and if the threshold is less than th^*, then it will classify into the determinate part.

For $x = \{s^{\cdot}, s^{\cdot} + 1, s^{\cdot} + 2, ...\}$, Np_r (the occurrence of a fixed number of successes over a variable number of trials) $= (T_x, U_x, I_x)$, probability mass function and cumulative distribution function of NNBD are given as, respectively:

$$T_x = (p_r(S))^{s^{\cdot}} \sum_{t=0}^{th^*} \binom{x - s^{\cdot}}{t} (p_r(I))^t (p_r(U))^{x - s^{\cdot} - t} \tag{1}$$

and,

$$U_x = \sum_{\substack{y=s^{\cdot} \\ y \neq x}}^{\infty} T_y = (p_r(S))^{s^{\cdot}} \sum_{\substack{y=s^{\cdot} \\ y \neq x}}^{\infty} \sum_{t=0}^{th^*} \binom{y - s^{\cdot}}{t} (p_r(I))^t (p_r(U))^{y - s^{\cdot} - t} \tag{2}$$

similarly

$$I_x = (p_r(S))^{s^{\cdot}} \sum_{z^* = th^* + 1}^{x - s^{\cdot}} \binom{x - s^{\cdot}}{z^*} (p_r(I))^{z^*}$$

$$\times \sum_{x=s^{\cdot}}^{\infty} \sum_{t=0}^{x - s^{\cdot} \cdot z^*} \binom{x - s^{\cdot} \cdot z^*}{t} (p_r(U))^{x - s^{\cdot} \cdot z^* - t}. \tag{3}$$

The cumulative distribution function to equation (1) will be

$$F(x) = 1 - (p_r(S))^{\acute{s}} (p_r(U))^{x - \acute{s} + 1} \sum_{m'=1}^{\infty} \binom{x - \acute{s} + m'}{m' - 1} (p_r(I))^{m' - 1}. \tag{4}$$

The cumulative distribution function corresponding to equation (3) will be

$$F(x) = 1 - (p_r(S))^{\acute{s}} \sum_{m'=1}^{\infty} \binom{x - \acute{s} + m'}{m'}$$

$$\times (p_r(I))^{m'} \sum_{m'=0}^{\infty} \binom{x - \acute{s}}{m'} (p_r(U))^{x-\acute{s}-m'}. \quad (5)$$

2.1. A special case of neutrosophic negative binomial distribution (NNBD)

Note that the proposed NNBD reduces to neutrosophic geometric distribution when $s^{\cdot} = 1$.

2.2. Physical conditions

(i) Outcome of each trial will be exhaustive and mutual exclusive as success, failure and indeterminacy.

(ii) The trials are independent.

(iii) The probability of success for each trail is fixed.

(iv) An experiment is repetitive to harvest the fixed number of successes.

2.3. Case studies

Case study 1. Suppose that during an election survey, at polling booth while asking the voters, whether they voted independently, it was noted that about 20% voters cast their votes independently and 60% did not cast their vote independently, 10% remained unclear either they cast the vote independently or not. We are required to find the probability of eight (08) voters, before we find three voters cast the votes independently?

According to the given piece of information let we estimate the probability for three (03) parts of probability mass function (pmf).

$X \rightarrow$ Number of people who voted.

$s' \rightarrow$ Number of people who voted independently will be considered a success.

So, $X = 3, 4, ..., 8$ and $s' = 3$.

Suppose that the threshold value for th^* is 2. Then

p_r (voters independently cast the vote) $= p_r(S) = 0.2$.

p_r (voters not independently cast the vote) $= p_r(U) = 0.6$.

p_r (voters not sure) $= p_r(I) = 0.1$,

$$T_x = T_3 = (0.2)^3 \sum_{t=0}^{2} \binom{8-3}{t} (0.1)^t (0.6)^{8-3-t}$$

$$= (0.2)^3 \left\{ \binom{5}{0} (0.1)^0 (0.6)^5 + \binom{5}{1} (0.1)^1 (0.6)^4 + \binom{5}{2} (0.1)^2 (0.6)^3 \right\}$$

$$= (0.2)^3 \{0.7776 + 0.0648 + 0.0216\}$$

$$= 0.0013132,$$

$$I_3 = (0.2)^3 \sum_{z^*=3}^{8-3=5} \frac{(8-3)!}{z^*!} (0.1)^{z^*} \sum_{t=0}^{5-z^*} \frac{(0.6)^{5-z^*-t}}{t!(5-z^*-t)!}$$

$$= (0.2)^3 \left[\left\{ \frac{5!}{3!} (0.1)^3 \sum_{t=0}^{2} \frac{(0.6)^{2-t}}{t!(2-t)!} \right\} + \left\{ \frac{5!}{4!} (0.1)^4 \sum_{t=0}^{1} \frac{(0.6)^{1-t}}{t!(1-t)!} \right\} \right.$$

$$\left. + \left\{ \frac{5!}{5!} (0.1)^5 \sum_{t=0}^{0} \frac{(0.6)^{0-t}}{t!(0-t)!} \right\} \right]$$

$$= (0.2)^3 \{1.28 + 8 \times 10^{-4} + 0.1^5\}$$

$$= 0.010246.$$

U_3 may be enumerated as

$$T_x + I_x + U_x = (p_r(S) + p_r(I) + p_r(U))^n.$$

Here x represents the number of trials so

$$T_x + I_x + U_x = (p_r(S) + p_r(I) + p_r(U))^x.$$

U_3 may be computed as

$$\begin{aligned} U_3 &= (p_r(S) + p_r(I) + p_r(U))^x - T_3 - I_3 \\ &= (0.2 + 0.1 + 0.6)^8 - 0.0013132 - 0.010246 \\ &= 0.41891. \end{aligned}$$

By normalizing the computed vector,

$$(T_3, I_3, U_3) = (0.001313, 0.41891, 0.010246).$$

By dividing the component of a vector with their total sum

$$0.001313 + 0.41891 + 0.010246 = 0.430469.$$

Hence,

$$(T_3, I_3, U_3) = (0.003051, 0.02380, 0.97315).$$

Case study 2. The capital city has 35% rain on a specific day, it is likely about 70% that the day will be sunny while there are 15% chances that the day will not be sunny and it will not be rainy as well (indeterminacy). Now find the probability of three specific days of a week those will be rainy?

According to the given information, we may calculate the likelihood for all three (03) parts of pmf

$X \rightarrow$ Number of days in a week.

$s' \rightarrow$ Number of rainy days.

So, $X = 3, 4, 5, 6, 7$ and $s^{\cdot} = 3$. Let $th^{*} = 3$.

p_r (the rainy day) $= p_r(S) = 0.35$.

p_r (the sunny day) $= p_r(U) = 0.7$.

p_r (indeterminacy) $= p_r(I) = 0.15$,

$$T_x = T_3 = (0.35)^3 \sum_{t=0}^{3} \binom{7-3}{l} (0.15)^t (0.7)^{7-3-t}$$

$$= (0.35)^3 \left\{ \binom{4}{0} (0.15)^0 (0.7)^4 + \binom{4}{1} (0.15)^1 (0.7)^3 + \binom{4}{2} (0.15)^2 (0.7)^2 + \binom{4}{3} (0.15)^3 (0.7)^1 \right\}$$

$$= (0.35)^3 \{0.2401 + 0.2058 + 0.06615 + 0.00945\}$$

$$= 0.022359,$$

$$I_3 = (0.35)^3 \sum_{z^*=4}^{7-4=3} \frac{(7-4)!}{z^*!} (0.15)^z \sum_{t=0}^{4-z^*} \frac{(0.7)^{4-z^*-t}}{t!(4-z^*-t)!}$$

$$= (0.35)^3 \sum_{z^*=4}^{4} \frac{3!}{4!} (0.15)^4 \sum_{t=0}^{0} \frac{(0.7)^{4-4-t}}{t!(4-4-t)!}$$

$$= (0.35)^3 \left\{ \frac{3!}{4!} (0.15)^4 (1) \right\}$$

$$= 5.42636 \times 10^{-6},$$

$$U_3 = (p_r(S) + p_r(I) + p_r(U))^x - T_3 - I_3$$

$$= (0.35 + 0.15 + 0.7)^7 - 0.022359 - 5.42636 \times 10^{-6}$$

$$= 3.560816.$$

After normalization, the computed vector is given by

$$(T_3,\ I_3,\ U_3) = (0.02236,\ 5.42636 \times 10^{-6},\ 3.56082).$$

By dividing each component of a vector by the total sum of the vectors,

$$0.02236 + 5.42636 + 3.56082 = 3.58318.$$

Hence

$$(T_3,\ I_3,\ U_3) = (0.00624,\ 1.51440 \times 10^{-6},\ 0.99376).$$

Case study 3. Suppose that Samiullah is a hockey team captain with 70% success rate of striking the goal and 40% failure rate while there are 15% chances of uncertainty whether he may hit the goal or not. What is the probability that Samiullah may hit the second goal during his fifth attempt?

On the basis of information given above we may enumerate the probability of all three parts of pmf

$$\acute{s} = 2 \text{ and } X = 2,\ 3,\ 4,\ 5.$$

Assume that the threshold be 1, i.e., $th^* = 1$,

p_r (goal) $= p_r(S) = 0.7.$

p_r (not goal) $= p_r(U) = 0.4.$

p_r (indeterminacy) $= p_r(l) = 0.15,$

$$T_x = T_2 = (0.7)^2 \sum_{t=0}^{1} \binom{5-2}{t} (0.15)^t \cdot (0.4)^{5-2-t}$$

$$= (0.7)^2 \left\{ \binom{3}{0} (0.15)^0 (0.4)^3 + \binom{3}{1} (0.15)^1 (0.4)^2 \right\}$$

$$= (0.7)^2\{0.064 + 0.072\}$$

$$= 0.06664,$$

$$I_2 = (0.7)^2 \sum_{z^*=2}^{5-2=3} \frac{3!}{z^*!}(0.15)^{z^*} \sum_{t=0}^{3-z^*} \frac{(0.4)^{3-z^*-t}}{t!(3-z^*-t)!}$$

$$= (0.7)^2\left[\left\{\frac{3!}{2!}(0.15)^2 \cdot \sum_{t=0}^{1} \frac{(0.4)^{1-t}}{t!(1-t)!}\right\} + \left\{\frac{3!}{3!}(0.15)^3 \cdot \sum_{t=0}^{0} \frac{(0.4)^{0-t}}{t!(0-t)!}\right\}\right]$$

$$= (0.7)^2\{0.0945 + (0.15)^3\},$$

$$I_2 = 0.047959,$$

$$U_2 = (p_r(S) + p_r(I) + p_r(U))^x - T_2 - I_2$$

$$= (0.7 + 0.15 + 0.4)^5 - 0.06664 - 0.047959$$

$$= 2.937159.$$

After normalization, the computed vector is given by

$$(T_2,\ I_2,\ U_2) = (0.06664,\ 0.047959,\ 2.937159).$$

By dividing each component of a vector by the total sum of the vectors

$$0.06664 + 0.047959 + 2.937159 = 3.051758.$$

Hence,

$$(T_2,\ I_2,\ U_2) = (0.02184,\ 0.96245,\ 0.01572).$$

3. Main Properties of NNBD

3.1. Moment generating function

Moment generating function (mgf) of $X \sim NNBD(x;\ s',\ p_r(S))$ for true

part of pmf will be

$$M_0(t) = \left(\frac{p_r(S)}{p_r(U)}\right)^{\acute{s}} \left(1 - \frac{p_r(I)}{p_r(U)}\right)^{-1} \sum_{m'=0}^{\infty} (p_r(U)e^t)^{\acute{s}+m'}. \tag{6}$$

pmf for indeterminate part will be

$$M_0(t) = A^*(p_r(S))^{\acute{s}} \left(\sum_{n=1}^{\infty} - \frac{(p_r(I))^n}{n}\right)\left(\sum_{m'=0}^{\infty} (p_r(U)\cdot e^t)^{\acute{s}+m'}\right), \tag{7}$$

where

$$A^* = -\frac{(p_r(U))^{-(\acute{s}+1)}}{1!} + \frac{(p_r(U))^{-(\acute{s}+3)}}{2!} - \frac{(p_r(U))^{-(\acute{s}+5)}}{2!3!} + \cdots.$$

3.2. Characteristic function

The characteristic function of $X \sim NNBD(x;\ \acute{s},\ p_r(S))$ for true part of pmf is given by

$$\phi_i(t) = \left(\frac{p_r(S)}{p_r(U)}\right)^{\acute{s}} \left(1 - \frac{p_r(I)}{p_r(U)}\right)^{-1} \sum_{m'=0}^{\infty} (p_r(U)e^{it})^{\acute{s}+m'}. \tag{8}$$

Pmf for indeterminate part will be

$$\phi_i(t) = A^*(p_r(S))^{\acute{s}} \cdot \left\{\sum_{n=1}^{\infty} - \frac{(P(I))^n}{n}\right\}\left\{\sum_{m'=0}^{\infty} (p_r(U)e^{it})^{\acute{s}+m'}\right\}. \tag{9}$$

3.3. Probability generating function

Probability generating function of $X \sim NNBD(x;\ \acute{s},\ p_r(S))$ for true part of pmf will be

$$G(\theta) = \left(\frac{p_r(S)}{p_r(U)}\right)^{\acute{s}} \left(1 - \frac{p_r(I)}{p_r(U)}\right)^{-1} \sum_{m'=0}^{\infty} (p_r(U)\cdot\theta)^{\acute{s}+m'}. \tag{10}$$

pmf for indeterminate part will be

$$G(\theta) = A^*(p_r(S))^{\acute{s}} \left\{ \sum_{n=1}^{\infty} - \frac{(p_r(I))^n}{n} \right\} \left\{ \sum_{m'=0}^{\infty} (p_r(U) \cdot \theta)^{\acute{s}+m'} \right\}. \quad (11)$$

4. Reliability Analysis

This section deals with multiple reliability properties such as hazard function, survival function, hazard rate function, reversed hazard rate, and cumulative hazard rate function, along with odds ratio and Mill's ratio are derived for the proposed NNBD.

4.1. Survival function

Survival function of random variable $X \sim NNBD(x;\ \acute{s},\ p_r(S))$ and true part of pmf will be:

$$S(x) = (p_r(S))^{\acute{s}} (p_r(U))^{x-\acute{s}+1} \sum_{m'=1}^{\infty} \binom{x - \acute{s} + m'}{m' - 1} (p_r(I))^{m'-1} \quad (12)$$

and the indeterminate part of pmf will be

$$S(x) = (p_r(S))^{\acute{s}} \sum_{m'=1}^{\infty} \binom{x - \acute{s} + m'}{m'} (p_r(I))^{m'} \sum_{m'=0}^{\infty} \binom{x - \acute{s}}{m'} (p_r(U))^{x-\acute{s}-m'}. \quad (13)$$

4.2. Hazard rate or failure rate function

Hazard rate function of r.v $X \sim NNBD(x;\ \acute{s},\ p_r(S))$ for true part of pmf is given as

$$h(x) = \frac{\sum_{t=0}^{th^*} \binom{x - \acute{s}}{t} (p_r(I))^t (p_r(U))^{x-\acute{s}-t}}{(p_r(U))^{x-\acute{s}+1} \sum_{m'=1}^{\infty} \binom{x - \acute{s} + m'}{m' - 1} (p_r(I))^{m'-1}} \quad (14)$$

and the indeterminate part of pmf will be

$$h(x)=\frac{\sum_{z^*=th^*+1}^{x-\acute{s}}\binom{x-\acute{s}}{z^*}(p_r(I))^{z^*}\times\sum_{x=\acute{s}}^{\infty}\sum_{t=0}^{x-\acute{s}-z^*}\binom{x-\acute{s}-z^*}{t}(p_r(U))^{x-\acute{s}-z^*-t}}{\sum_{m'=1}^{\infty}\binom{x-\acute{s}+m'}{m'}(p_r(I))^{m'}\sum_{m'=0}^{\infty}\binom{x-\acute{s}}{m'}(p_r(U))^{x-\acute{s}-m'}}. \quad (15)$$

4.3. Reversed hazard rate function

Reversed hazard rate function (RHRF) of r.v $X \sim NNBD(x;\ \acute{s},\ p_r(S))$ for true part of pmf is given as

$$\partial(x)=\frac{(p_r(S))^{\acute{s}}\sum_{t=0}^{th^*}\binom{x-\acute{s}}{t}(p_r(I))^{t}(p_r(U))^{x-\acute{s}-t}}{1-(p_r(S))^{\acute{s}}(p_r(U))^{x-\acute{s}+1}\sum_{m'=1}^{\infty}\binom{x-\acute{s}+m'}{m'-1}(p_r(I))^{m'-1}}. \quad (16)$$

For the indeterminate portion of the probability mass function (pmf), RHRF is defined as

$$\partial(x)=\frac{(p_r(S))^{\acute{s}}\sum_{z^*=th^*+1}^{x-\acute{s}}\binom{x-\acute{s}}{z^*}(p_r(I))^{z^*}\times\sum_{x=\acute{s}}^{\infty}\sum_{t=0}^{x-\acute{s}-z^*}\binom{x-\acute{s}-z^*}{t}(p_r(U))^{x-\acute{s}-z^*-t}}{1-(p_r(S))^{\acute{s}}\sum_{m'=1}^{\infty}\binom{x-\acute{s}+m'}{m'}(p_r(I))^{m'}\times\sum_{m'=0}^{\infty}\binom{x-\acute{s}}{m'}(p_r(U))^{x-\acute{s}-m'}}. \quad (17)$$

4.4. Cumulative hazard rate function

Cumulative hazard rate function (CHRF) of r.v $X \sim NNBD(x;\ \acute{s},\ p_r(S))$ for true part of pmf is given as

$$H(x) = -\ln\left\{(p_r(S))^{s'}\,(p_r(U))^{x-s'+1}\sum_{m'=1}^{\infty}\binom{x-s'+m'}{m'-1}(p_r(I))^{m'-1}\right\}. \quad (18)$$

For the indeterminate portion of the pmf and CHRF is defined as

$$H(x) = -\ln\left\{(p_r(S))^{s'}\sum_{m'=1}^{\infty}\binom{x-s'+m'}{m'}(p_r(I))^{m'}\sum_{m'=0}^{\infty}\binom{x-s'}{m'}(p_r(U))^{x-s'-m'}\right\}. \quad (19)$$

4.5. Mills ratio

Mills ratio of r.v $X \sim NNBD(x;\, s',\, p_r(S))$ for true part of pmf is given as

$$m(x) = \frac{(p_r(U))^{x-s'+1}\sum_{m'=1}^{\infty}\binom{x-s'+m'}{m'-1}(p_r(I))^{m'-1}}{\sum_{t=0}^{th^*}\binom{x-s'}{t}(p_r(I))^{t}(p_r(U))^{x-s'-t}}. \quad (20)$$

For the indeterminate portion of the pmf, the Mills ratio is defined as:

$$m(x) = \frac{\begin{array}{c}\sum_{m'=1}^{\infty}\binom{x-s'+m'}{m'}(p_r(I))^{m'} \\ \times\sum_{m'=0}^{\infty}\binom{x-s'}{m'}(p_r(U))^{x-s'-m'}\end{array}}{\begin{array}{c}\sum_{z^*=th^*+1}^{x-s'}\binom{x-s'}{z^*}(p_r(I))^{z^*} \\ \times\sum_{x=s'}^{\infty}\sum_{t=0}^{x-s'-z^*}\binom{x-s'-z^*}{t}(p_r(U))^{x-s'-z^*-t}\end{array}}. \quad (21)$$

4.6. Odds ratio

The odds ratio of r.v $X \sim NNBD(x;\, s',\, p_r(S))$ and the true part of pmf will be

$$\varphi(x) = \frac{1-(p_r(S))^{\acute{s}}\,(p_r(U))^{x-\acute{s}+1}\sum_{m'=1}^{\infty}\binom{x-\acute{s}+m'}{m'-1}(p_r(I))^{m'-1}}{(p_r(S))^{\acute{s}}\,(p_r(U))^{x-\acute{s}+1}\sum_{m'=1}^{\infty}\binom{x-\acute{s}+m'}{m'-1}(p_r(I))^{m'-1}}. \quad (22)$$

For the indeterminate portion of the pmf and the odds ratio is defined as:

$$\varphi(x) = \frac{\begin{array}{l}1-(p_r(S))^{\acute{s}}\sum_{m'=1}^{\infty}\binom{x-\acute{s}+m'}{m'}(p_r(I))^{m'} \\ \times\sum_{m'=0}^{\infty}\binom{x-\acute{s}}{m'}(p_r(U))^{x-\acute{s}-m'}\end{array}}{\begin{array}{l}(p_r(S))^{\acute{s}}\sum_{m'=1}^{\infty}\binom{x-\acute{s}+m'}{m'}(p_r(I))^{m'} \\ \times\sum_{m'=0}^{\infty}\binom{x-\acute{s}}{m'}(p_r(U))^{x-\acute{s}-m'}\end{array}}. \quad (23)$$

5. Order Statistics

This section derives the order statistics for the proposed distribution, i.e., NNBD like *w*th order statistics, joint, median, minimum, and maximum order statistics, smallest and largest order statistics.

5.1. *w*th order statistics

Let $X_1, X_2, X_3, ..., X_w$ represent a random sample from the negative binomial distribution (NNBD), and let $X_{(1)}, X_{(2)}, X_{(3)}, ..., X_{(w)}$ denote the corresponding order statistics. Then the *w*th order statistic for the true part of the NNBD can be expressed as:

$$f_{w:n}(x) = \frac{n!}{(w-1)!(n-w)!}(p_r(S))^{\acute{s}+n-w}\sum_{t=0}^{th^*}\binom{x-\acute{s}}{t}(p_r(I))^{t}(p_r(U))^{x-\acute{s}-t}$$

$$\times\left[1-(p_r(S))^{\acute{s}}\,(p_r(U))^{x-\acute{s}+1}\sum_{m'=1}^{\infty}\binom{x-\acute{s}+m'}{m'-1}(P(I))^{m'-1}\right]^{w-1}$$

$$\times\left[(p_r(U))^{x-\acute{s}+1}\sum_{m'=1}^{\infty}\binom{x-\acute{s}+m'}{m'-1}(p_r(I))^{m'-1}\right]^{n-w}, \tag{24}$$

*w*th order statistics for the indeterminate part of NNBD will be

$$f_{w:n}(x)$$

$$=\frac{n!}{(w-1)!(n-w)!}(p_r(S))^{n+\acute{s}-w}$$

$$\times\sum_{z^*=th^*+1}^{x-\acute{s}}\binom{x-\acute{s}}{z^*}(p_r(I))^{z^*}\sum_{x=k}^{\infty}\sum_{t=0}^{x-\acute{s}-z^*}\binom{x-\acute{s}-z^*}{t}(p_r(U))^{x-\acute{s}-z^*-t}$$

$$\times\left[1-(p_r(S))^{\acute{s}}\sum_{m'=1}^{\infty}\binom{x-\acute{s}+m'}{m'}(p_r(I))^{m'}\sum_{m'=0}^{\infty}\binom{x-\acute{s}}{m'}(p_r(U))^{x-\acute{s}-m'}\right]^{w-1}$$

$$\times\left[\sum_{m'=1}^{\infty}\binom{x-\acute{s}+m'}{m'}(p_r(I))^{m'}\sum_{m'=0}^{\infty}\binom{x-\acute{s}}{m'}(p_r(U))^{x-\acute{s}-m'}\right]^{n-w}. \tag{25}$$

5.2. Joint order statistics

Joint order statistics of $y_{1:b}$ and $y_{u:b}$ for true part of NNBD is explained as

$$f_{a:u:b}(x)$$

$$=\frac{b!}{(a-1)!(u-a-1)!(b-u)!}(p_r(S))^{\acute{s}(b-a+1)}$$

$$\times\left(\sum_{t=0}^{th^*}\binom{y-\acute{s}}{t}(p_r(I))^{t}(p_r(U))^{y-\acute{s}-t}\right)$$

$$\times\left(\sum_{t=0}^{th^*}\binom{z-\acute{s}}{t}(p_r(I))^t(p_r(U))^{z-\acute{s}-t}\right)$$

$$\times\left[1-(p_r(S))^{\acute{s}}(p_r(U))^{y-\acute{s}+1}\sum_{m'=1}^{\infty}\binom{y-\acute{s}+m'}{m'-1}(p_r(I))^{m'-1}\right]^{a-1}$$

$$\times\left[(p_r(U))^{y-\acute{s}+1}\sum_{m'=1}^{\infty}\binom{y-\acute{s}+m'}{m'-1}(p_r(I))^{m'-1}\right.$$

$$\left.-(p_r(U))^{z-\acute{s}+1}\sum_{m'=1}^{\infty}\binom{z-\acute{s}+m'}{m'-1}(p_r(I))^{m'-1}\right]^{u-a-1}$$

$$\times\left[(p_r(U))^{y-\acute{s}+1}\sum_{m'=1}^{\infty}\binom{y-\acute{s}+m'}{m'-1}(p_r(I))^{m'-1}\right]^{b-u}. \tag{26}$$

Joint order statistics of the indeterminate part will be

$$f_{a:u:b}(x)$$

$$=\frac{b!}{(a-1)!(u-a-1)!(b-u)!}(p_r(S))^{\acute{s}(b-a+1)}$$

$$\times\left\{\sum_{z^*=th+1}^{y-\acute{s}}\binom{y-\acute{s}}{z^*}(p_r(I))^{z^*}\sum_{x=\acute{s}}^{\infty}\sum_{t=0}^{y-\acute{s}-z^*}\binom{y-\acute{s}-z^*}{t}(p_r(U))^{y-\acute{s}-z^*-t}\right\}$$

$$\times\left\{\sum_{z^*=th^*+1}^{z-\acute{s}}\binom{z-\acute{s}}{z^*}(p_r(I))^{z^*}\sum_{z=\acute{s}}^{\infty}\sum_{t=0}^{z-\acute{s}-z^*}\binom{z-\acute{s}-z^*}{t}(p_r(U))^{z-\acute{s}-z^*-t}\right\}$$

$$\times\left[\sum_{m'=1}^{\infty}\binom{y-\acute{s}+m'}{m'}(p_r(I))^{m'}\sum_{m'=0}^{\infty}\binom{y-\acute{s}}{m'}(p_r(U))^{y-\acute{s}-m'}\right.$$

$$- \sum_{m'=1}^{\infty} \binom{z - \acute{s} + m'}{m'} (p_r(I))^{m'} \sum_{m'=0}^{\infty} \binom{z - \acute{s}}{m'} (p_r(U))^{x-\acute{s}-m'} \Bigg]^{u-a-1}$$

$$\times \left[\sum_{m'=1}^{\infty} \binom{y - \acute{s} + m'}{m'} (p_r(I))^{m'} \sum_{m'=0}^{\infty} \binom{y - \acute{s}}{m'} (p_r(U))^{y-\acute{s}-m'} \right]^{b-u}. \quad (27)$$

5.3. Largest order statistics

The largest order statistics for the true part of NNBD will be

$$f_{\grave{\eta}:\grave{\eta}}(x) = \grave{\eta} (p_r(S))^{\acute{s}} \sum_{t=0}^{th^*} \binom{x - \acute{s}}{t} (p_r(I))^t (p_r(U))^{x-\acute{s}-t}$$

$$\times \left[1 - (p_r(S))^{\acute{s}} (p_r(U))^{x-\acute{s}+1} \sum_{m'=1}^{\infty} \binom{x - \acute{s} + m'}{m' - 1} (p_r(I))^{m'-1} \right]^{\grave{\eta}-1}. \quad (28)$$

Largest order statistics for an indeterminate part of NNBD will be

$$f_{\grave{\eta}:\grave{\eta}}(x) = \grave{\eta} (p_r(S))^{\acute{s}} \sum_{z^*=th^*+1}^{x-\acute{s}} \binom{x - \acute{s}}{z^*} (p_r(I))^{z^*}$$

$$\times \sum_{x=\acute{s}}^{\infty} \sum_{t=0}^{x-\acute{s}-z^*} \binom{x - \acute{s} - z^*}{t} (p_r(U))^{x-\acute{s}-z^*-t}$$

$$\times \left[1 - (p_r(S))^{\acute{s}} \sum_{m'=1}^{\infty} \binom{x - \acute{s} + m'}{m'} (p_r(I))^{m'} \right.$$

$$\left. \times \sum_{m'=0}^{\infty} \binom{x - \acute{s}}{m'} (p_r(U))^{x-\acute{s}-m'} \right]^{\grave{\eta}-1}. \quad (29)$$

5.4. Smallest order statistics

For $u = 1,$ the smallest order statistics for the true part of NNBD will be

$$f_{1:n}(x) = n(p_r(S))^{n\acute{s}} \sum_{t=0}^{th^*} \binom{x-\acute{s}}{t} (p_r(I))^t (p_r(U))^{x-\acute{s}-t}$$

$$\times \left[(p_r(U))^{x-\acute{s}+1} \sum_{m'=1}^{\infty} \binom{x-\acute{s}+m'}{m'-1} (p_r(I))^{m'-1} \right]^{n-1}. \quad (30)$$

Smallest order statistics of indeterminate part of NNBD will be

$$f_{1:n}(x) = n(p_r(S))^{n\acute{s}} \sum_{z^*=th^*+1}^{x-\acute{s}} \binom{x-\acute{s}}{z^*} (p_r(I))^{z^*}$$

$$\times \sum_{x=\acute{s}}^{\infty} \sum_{t=0}^{x-\acute{s}-z^*} \binom{x-\acute{s}-z^*}{t} (p_r(U))^{x-\acute{s}-z^*-t}$$

$$\times \left[\sum_{m'=1}^{\infty} \binom{x-\acute{s}+m'}{m'} (p_r(I))^{m'} \sum_{m'=0}^{\infty} \binom{x-\acute{s}}{m'} (p_r(U))^{x-\acute{s}-m'} \right]^{n-1}. \quad (31)$$

5.5. Median order statistics

For $u = m + 1,$ the median order statistics for true part of NNBD will be

$$f_{m+1:n}(x) = \frac{(2m+1)!}{m!n!} (p_r(S))^{\acute{s}(m+1)}$$

$$\times \left[1 - (p_r(S))^{\acute{s}} (p_r(U))^{x-\acute{s}+1} \sum_{m'=1}^{\infty} \binom{x-\acute{s}+m'}{m'-1} (p_r(I))^{m'-1} \right]^{m}$$

$$\times \left[(p_r(U))^{x-\acute{s}+1} \sum_{m'=1}^{\infty} \binom{x-\acute{s}+m'}{m'-1} (p_r(I))^{m'-1} \right]^{m}$$

$$\times \sum_{t=0}^{th^*} \binom{x-s'}{t} (p_r(I))^t (p_r(U))^{x-s'-t}. \quad (32)$$

Median order statistics for the indeterminate part of NNBD will be

$$f_{m+1:n}(x)$$

$$= \frac{(2m+1)!}{m!n!} (p_r(S))^{s'(m+1)}$$

$$\times \left[1 - (p_r(S))^{s'} \sum_{m'=1}^{\infty} \binom{x-s'+m'}{m'} (p_r(I))^{m'} \sum_{m'=0}^{\infty} \binom{x-s'}{m'} (p_r(U))^{x-s'-m'} \right]^m$$

$$\times \left[\sum_{m'=1}^{\infty} \binom{x-s'+m'}{m'} (p_r(I))^{m'} \sum_{m'=0}^{\infty} \binom{x-s'}{m'} (p_r(U))^{x-s'-m'} \right]^m$$

$$\times \sum_{z^*=th^*+1}^{x-s'} \binom{x-s'}{z^*} (p_r(I))^{z^*} \sum_{x=s'}^{\infty} \sum_{t=0}^{x-s'-z^*} \binom{x-s'-z^*}{t} (p_r(U))^{x-s'-z^*-t}. \quad (33)$$

5.6. Minimum and maximum joint order statistics

Maximum and minimum joint order statistics for the true part of NNBD will be

$$f_{1:a:a}(y) = n(n-1)(p_r(S))^{as'} \left\{ \sum_{t=0}^{th^*} \binom{y-s'}{t} (p_r(I))^t (p_r(U))^{y-s'-t} \right\}$$

$$\times \left\{ \sum_{t=0}^{th^*} \binom{z-s'}{t} (p_r(I))^t (p_r(U))^{z-s'-t} \right\}$$

$$\times \left[(p_r(U))^{y-s'+1} \sum_{m'=1}^{\infty} \binom{y-s'+m'}{m'-1} (p_r(I))^{m'-1} \right.$$

$$-(p_r(U))^{z-ś+1}\sum_{m'=1}^{\infty}\binom{z-ś+m'}{m'-1}(p_r(I))^{m'-1}\Bigg]^{a-2}. \quad (34)$$

Maximum and minimum joint order statistics for indeterminate NNBD will be

$$\begin{aligned} f_{1:a:a}(y) &= a(a-1) \\ &= n(n-1)(p_r(S))^{aś}\Bigg\{\sum_{z^*=th^*+1}^{y-ś}\binom{y-ś}{z^*}(p_r(I))^{z^*} \\ &\quad\times\sum_{x=ś}^{\infty}\sum_{t=0}^{y-ś-z^*}\binom{y-ś-z^*}{t}(p_r(U))^{y-ś-z^*-t}\Bigg\} \\ &\quad\times\Bigg\{\sum_{z^*=th^*+1}^{z-ś}\binom{y-ś}{z^*}(p_r(I))^{z^*} \\ &\quad\quad\times\sum_{x=ś}^{\infty}\sum_{t=0}^{z-ś-z^*}\binom{z-ś-z^*}{t}(p_r(U))^{z-ś-z^*-t}\Bigg\} \\ &\quad\times\Bigg[(p_r(U))^{y-ś+1}\sum_{m'=1}^{\infty}\binom{y-ś+m'}{m'-1}(p_r(I))^{m'-1} \\ &\quad\quad-(p_r(U))^{z-ś+1}\sum_{m'=1}^{\infty}\binom{z-ś+m'}{m'-1}(p_r(I))^{m'-1}\Bigg]^{a-2}. \end{aligned} \quad (35)$$

6. Conclusion

This study suggests the discrete neutrosophic negative binomial probability distribution by incorporating neutrosophic logic. By using the proposed distribution, this investigation briefly explained the multiple case studies. Many mathematical properties which include; probability generating

function of the discrete neutrosophic negative binomial probability distribution, mgf and characteristics function have been evolved and presented. On reliability review, this study has explained the attributes such as survival function, hazard rate function, cumulative hazard rate function, reversed hazard rate function, Mills's ratio and odd ratio. Moreover, it has evolved the order statistics for the anticipated distribution. The NNBD is significantly suitable while modeling the *k*th success in sequence of *n* independent trails before the occurrence of specific number "*U*".

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A New Mood's Median Test for Imprecise Data

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ABSTRACT. The existing Mood's median test is a non-parametric test used to compare two or more sample data sets whether they are from the same population or not. There can be no application of this test to uncertain and indeterminate data. Therefore, it is necessary to find a generalization of this test that will enable us to apply it in uncertain environments. This study will present a new approach that utilizes neutrosophic statistics to apply Mood's median test. The approach involves defining hypotheses, determining a decision rule, and performing the test in an uncertain environment. An evaluation of the performance of the proposed test will be conducted using numerical examples. The results reveal that the proposed test under neutrosophic statistics is more informative, efficient, and flexible than the existing test under classical statistics in the presence of uncertain data.

1. INTRODUCTION

Nonparametric tests are techniques used in statistics for the analysis of data without specifying a particular distribution to meet the required assumptions to be analysed. For this reason, they are sometimes referred to as distribution-free tests. A wide variety of nonparametric tests have been used in the literature on statistical experimental design. One of the most useful non-parametric tests is Mood's median test, which is used to determine whether

the medians of two or more independent samples are the same. In this context, Mood's median test was first introduced by Mood [1]. Brown and Mood [2] discussed median tests for linear hypotheses. Mood [3] studied the asymptotic efficiency of certain nonparametric two-sample tests. Fligner and Rust [4] described a modification of Mood's median test which allows one to test for differences between two medians without making any assumptions about the shape of the underlying populations. In a more recent article, Chen [5] proposed an extension of Mood's median test for survival data. Various articles and books discussing the Mood's median test can be found in ([6], [7], [8]). As mentioned above, the researchers applied this test to determinant datasets; however, experimenters are sometimes confronted with indeterminate data under uncertain conditions. Therefore, it is therefore necessary to look for an appropriate generalization of this test so that we may use indeterminate data in an uncertain environment.

Neutrosophic logic is an extension of fuzzy logic that provides information about the measure of indeterminacy, whereas fuzzy logic does not. Smarandache [9] claimed that the neutrosophic logic is more efficient than of fuzzy logic. Smarandache [10] demonstrated that neutrosophic statistics (NS) may be regarded as a generalization of classical statistics. A discussion on the differences between fuzzy statistics, NS, and classical statistics has been made by Aslam [11]. Aslam [12] in his publication highlighted the use of neutrosophic ANOVA. In a newer paper, AlAita and Aslam [13] discussed neutrosophic analysis of covariance with respect to completely randomized designs, randomized complete block designs, and split-plot designs. The neutrosophic sign test was applied to COVID-19 data by Sherwani, et al. [14]. The neutrosophic Kruskal Wallis H test was used to analyze COVID-19 data by Sherwani, et al. [15]. Aslam and Aldosari [16] presented a discussion about analyzing alloy melting points data using the Mann-Whitney test. Miari, et al. [17] suggested single valued neutrosophic Kruskal-Wallis and Mann Whitney tests. The median test for interval-valued data was highlighted by Grzegorzewski and Śpiewak [18]. Aslam and Saleem [19] introduced the F-test of testing linearity under neutrosophic statistics. Aslam, et al. [20] examined the monitoring of temperature using a moving average under uncertainty. Afzal and Aslam [21] presented a study of the use of neutrosophic statistics to analyze changes in blood pressure during pregnancy in women. An analysis of the temperature data of five different cities in Pakistan was conducted by Shahzadi [22] using the neutrosophic statistics. In the context of neutrosophic statistics, several statistical tests have been discussed ([23], [24], [25], [26]).

In complex systems, all observations are not determinable, thus the existing Mood's median test cannot be applied. It is the principal objective of this research to solve problems associated with studies that require the application of the neutrosophic mood's median test in instances of imprecise and uncertain data. For example, a study was conducted on women and men in Pakistan to analyze daily ICU occupancy data representing Corona-positive patients under NS. During an experiment, the experimenter often encounters numerous instances in which the results are uncertain. Therefore, it is not possible to apply classical tests to analyze this data. Accordingly, the neutrosophic mood's median test is necessary for this example.

In our literature review, and to the best of our knowledge, there are no studies on Mood's median test under NS. In this paper, we will introduce neutrosophic Mood's median test for the first time. We will determine the neutrosophic hypotheses and decision rule. Moreover, we will compute χ_N^2-test for the proposed test under uncertain conditions. Based on real data, we will evaluate the performance of the proposed test. Considering neutrosophic Mood's median test as an alternative to the existing test in the presence of uncertainty, we can conclude that the proposed test is more informative and efficient than those under CS.

The remainder of this paper is organized as follows: In the following section, we discuss some basic concepts under NS. Section 3 provides Mood's median test under NS. Section 4 presents and explains numerical examples. Section 5 introduces comparison analysis. Section 6 presents conclusions.

2. PRELIMINARIES

Suppose that a neutrosophic random variable (NRV) $X_N \in [X_L, X_U]$ hasn't particular neutrosophic distribution, where X_L and X_U are smaller and larger values of indeterminacy interval. Let $X_N = X_L + X_U I_N$ is the neutrosophic form of NRV having determinate part X_L and indeterminate part $X_U I_N$; $I_N \in [I_L, I_U]$, where $I_N \in [I_L, I_U]$ is measure of uncertainty.

Suppose n is a neutrosophic random sample selected from a population of size N having indeterminate observations. The neutrosophic population median θ_N is expressed as follows;

If n is odd, $\theta_N(x) = x_{N\left(\frac{n+1}{2}\right)}$.

If n is even, $\theta_N(x) = \frac{x_{N\left(\frac{n}{2}\right)} + x_{N\left(\frac{n}{2}+1\right)}}{2}$.

The neutrosophic form of $\theta_N(x)$ is $\theta_N(x) = \theta_L(x) + \theta_U(x) I_N;\ I_N \in [I_L, I_U]$.

3. MOOD'S MEDIAN TEST UNDER NEUTROSOPHIC STATISTICS

In neutrosophic statistics, neutrosophic nonparametric tests refer to methods of statistical analysis that require no particular distribution to be specified (especially when data are not normally distributed). The neutrosophic nonparametric tests can be used as an alternative to neutrosophic parametric tests, such as the neutrosophic T-test or NANOVA, if the underlying neutrosophic data fits certain criteria and assumptions. According to the literature on neutrosophic nonparametric tests, many statistical neutrosophic tests have been studied (e.g., neutrosophic Kruskal Wallis H test, neutrosophic sign test, etc.). Our proposed test is similar to the sign test. The main objective of the proposed neutrosophic Mood's median test is to compare two or more terms of population medians when the data contain neutrosophic numbers.

Under neutrosophic statistics, the neutrosophic Mood's median test is used to test the null hypothesis that all two or more samples have equal medians against the alternative hypothesis that the medians population are not equal. In other words; Suppose θ_{Ni} represents the population median for the ith population, the neutrosophic null and alternative hypotheses are formulated as follows;

$H_{N0}: \theta_{N1} = \theta_{N2} = \cdots = \theta_{Nk}$ vs H_{N1}: At least one median differs.

This test can be performed by following the steps below:

Step 1: Compute the median θ_N of the samples.

Step 2: Establish a $2 \times k$ contingency table consisting of the number of elements that are greater than θ_N in each sample and of the number of elements that are less than or equal to θ_{Ni} in each sample. Both samples are represented by columns. The frequencies of neutrosophic data is shown in Table 1.

TABLE 1. Frequencies of neutrosophic data

	Sample 1	Sample 2	…	Sample k	Total
>Median	O_{N11}	O_{N12}	…	O_{N1k}	$n_{N1.}$
≤Median	O_{N23}	O_{N24}	…	O_{N2k}	$n_{N2.}$
Total	n_{N1}	n_{N2}	…	n_{Nk}	n_N

$E_{Nij} = \frac{n_{Ni.} \times n_{Nj}}{n_N}, i = 1,2; j = 1,2, \ldots, k.$

Step 3: Conduct a chi-square test of independence.

$$\chi_N^2 = \sum \frac{\left(O_{Nij} - E_{Nij}\right)^2}{E_{Nij}} = [\chi_L^2, \chi_U^2], \qquad (1)$$

where O_{Nij} represents the observed neutrosophic frequencies, E_{Nij} represents the expected neutrosophic frequencies, where $i = 1,2,\ j = 1,2,\dots,k$. The degrees of freedom are $k-1$.

Step 4: Compute the $p_N - value$.

Step 5: According to Smarandache [10], the neutrosophic decision rule can be summarized as follows:

If $min\{p_N - value\} > \alpha$, then we accept the null hypothesis H_{N0} at the level α.

If $max\{p_N - value\} \leq \alpha$, then we reject the null hypothesis H_{N0} at the level α.

If $min\{p_N - value\} < \alpha < max\{p_N - value\}$, then there is indeterminacy. Thus $\frac{\alpha - min\{p_N - value\}}{max\{p_N - value\} - min\{p_N - value\}}$ represents the chance to reject H_{N0} at the level α, and $\frac{max\{p_N - value\} - \alpha}{max\{p_N - value\} - min\{p_N - value\}}$ represents the chance to accept H_{N0} at the level α.

4. NUMERICAL EXAMPLES

In this section, we numerically assess the performance of our proposed test by examples using uncertain observations. For assessing the efficiency of the proposed test, the proposed test is calculated and compared with the existing test under classical statistics in terms of a measure of uncertainty.

Example 4.1. A study was conducted on the melting points of two alloys based upon not precise observations. Assume that each observation within and between samples is independent and symmetrical distribution. The purpose of this study is to determine whether the melting point of the first alloy (x) is much higher than that of the second alloy (y). Table 2 summarizes the data. For more information see [18]. Data frequencies of melting points of two alloys are shown in Table 3.

TABLE 2. Measurement of melting point of two alloys

	x	y
1	[545.5, 563.3]	[426.1, 444.1]
2	[511.6, 529.4]	[406.7, 430.5]
3	[503.5, 523.1]	[387.3, 407.2]
4	[449.2, 470.1]	[450.9, 475.8]
5	[489.0, 506.7]	[440.2, 458.5]
6	[479.1, 495.6]	[490.6, 507.7]
7	[467.9, 495.3]	[480.2, 496.8]
8	[495.6, 520.9]	[503.8, 520.9]
9	[472.8, 496.9]	[482.8, 503.6]
10	[519.1, 542.9]	[432.8, 458.2]
11	[484.0, 505.4]	[453.3, 480.5]
12	[525.9, 550.7]	[446.9, 473.8]
13	[500.9, 517.7]	[451.2, 468.4]
14	[483.0, 499.2]	[466.1, 482.2]
15	[485.0, 500.6]	[459.4, 477.0]
16	[499.6, 516.8]	[479.4, 496.3]
17	[515.1, 535.0]	
18	[464.4, 489.3]	
$Median$	[492.30, 511.75]	[452.25, 476.40]

The neutrosophic null and alternative hypotheses are

$H_{N0}: \theta_{N\mathrm{x}} = \theta_{N\mathrm{y}}$ vs $H_{N1}: \theta_{N\mathrm{x}} > \theta_{N\mathrm{y}}$

$Median = [479.80, 496.85]$.

$\chi_N^2 = [7.556, 11.806]$.

TABLE 3. Data frequencies of melting points of two alloys

	x	y	Total
$> Median$	[13, 14]	[4, 3]	[17, 17]
$\leq Median$	[5, 4]	[12, 13]	[17, 17]
Total	[18, 18]	[16, 16]	[34, 34]

The neutrosophic form of the proposed test-χ_N^2 can be expressed as $\chi_N^2 = 7.556 + 11.806I_N;\ I_N \in [0,0.360]$.

$p_N - value = [0.006, 0.001] < 0.05$. Then we reject the null hypothesis H_{N0} and accept H_{N1} at the level $\alpha = 0.05$. That is, the melting point of the first alloy (x) is much higher than that of the second alloy (y).

Example 4.2. To apply the proposed neutrosophic Mood's median test, daily ICU occupancy data representing Corona-positive patients from Pakistan have been analyzed. In this study, the hypothesis being tested is whether there is a statistically significant difference in ICU occupancy of Covid-19 patients according to their age group. The data shown in Table 4 is uncertainty data. This study uses the neutrosophic Mood's median test to test the null hypothesis that there are no differences in the daily occupancy of Covid-19 patients from different age groups in Pakistan for the month of December 2020. There are three categories of age groups in Covid-19 for the daily occupancy of ICUs (35 years and below, 35 to 55 years, and 55 years and above). For more information see [15]. The data frequency of Covid-19 patients is shown in Table 5.

The neutrosophic null and alternative hypotheses are

$H_{N0}: \theta_{N1} = \theta_{N2} = \theta_{N3}$ vs H_{N1}: At least one median differs.

$Median = [405, 410]$.

$\chi_N^2 = [14.800, 17.753]$.

The neutrosophic form of the proposed test-χ_N^2 can be expressed as $\chi_N^2 = 14.800 + 17.753I_N;\ I_N \in [0,0.166]$.

$p_N - value = [0.001, 0.000] < 0.05$. Then we reject the null hypothesis H_{N0} and accept H_{N1} at the level $\alpha = 0.05$. That is, there is a difference in the medians of the samples.

5. COMPAISON ANALYSIS USING THE NEUTROSOPHIC FORM OF MOOD'S MEDIAN TEST

In this section, we numerically assess the performance of our proposed test data analysis through the previous examples. In order to determine the efficiency of the proposed test, we calculate and compare the proposed Mood's median test in terms of a measure of uncertainty with the existing Mood's median test under classic statistics.

TABLE 4. Daily ICU occupancy data representing Corona-positive patients from Pakistan

Day	**Age**		
	35 and below	**35-55**	**55 and above**
1	[460, 465]	[359, 361]	[443, 450]
2	[427, 429]	[352, 365]	[421, 426]
3	[407, 410]	[445, 455]	[436, 450]
4	[378, 380]	410	[376, 385]
5	[364, 368]	[458, 464]	458
6	[345, 349]	[410, 415]	[408, 420]
7	[342, 346]	[463, 470]	[422, 425]
8	345	[580, 584]	[431, 440]
9	[313, 318]	[432, 440]	[459, 462]
10	[277, 280]	379	369
11	[268, 271]	370	360
12	[259, 262]	[584, 589]	[431, 445]
13	[256, 260]	[410, 416]	[403, 415]
14	251	[587, 590]	[436, 445]
15	249	415	376
16	[233, 227]	[419, 422]	370
17	[209, 211]	357	443
15	[187, 191]	[467, 472]	445
19	[173, 175]	[415, 418]	[355, 365]
20	168	358	450
Median	[272.5, 275.5]	[415, 417]	[426.5, 433]

TABLE 5. Data frequencies of Covid-19 patients

	35 and below	35-55	55 and above	Total
>Median	[13, 14]	[14, 13]	[3, 2]	[30, 29]
≤Median	[7, 6]	[6, 7]	[17, 18]	[30, 31]
Total	[20, 20]	[20, 20]	[20, 20]	[60, 60]

According to what was mentioned earlier, the proposed test represents a generalization of Mood's median test. The neutrosophic logic literature has indicated that a method based on data in an indeterminate interval is more effective and suitable for use in uncertainty than determined values under classical statistics. The neutrosophic form of χ_N^2-test is $\chi_N^2 = \chi_L^2 + \chi_U^2 I_N;\ I_N \in [I_L, I_U]$, where the first part χ_L^2 is known as the determined part and presents the value of χ^2-test under CS. The second part $\chi_U^2 I_N;\ I_N \in [I_L, I_U]$ is known as the indeterminate part. Note that the χ_N^2-test reduces to χ^2-test under classical statistics if $I_{F_N} = 0$. This means that the NS approach provides the χ_N^2-test values in an interval with the measure of indeterminacy which is more general and includes the determinate part of the CS. For example, in the Example 1, the neutrosophic form of χ_N^2-test is $\chi_N^2 = 7.556 + 11.806 I_N;\ I_N \in [0, 0.360]$. It means that the proposed χ_N^2-test indicate that χ_N^2 ranges between 7.556 and 11.806 with the degree of indeterminacy 0.360. Therefore, in light of the comparisons, it is concluded that the proposed test under NS is more informative than the existing test under CS.

6. CONCLUSIONS

The use of classical statistics in analysing indeterminate data due to uncertain experimental measurements may result in incorrect conclusions regarding Mood's median test. The Mood's median test with indeterminate observations was considered in light of this study, and a neutrosophic statistical approach was suggested for analysing the data. We formulated neutrosophic hypotheses, a neutrosophic decision rule, and a test statistic. An illustration of the proposed test application was provided through real-life examples. This study indicates that the proposed approach outperforms the classical approach in terms of accuracy, flexibility, and generality of analysis under uncertain circumstances. We hope the proposed test will be used in numerous future scientific experiments.

Conflicts of Interest: The authors declare that there are no conflicts of interest regarding the publication of this paper.

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Jonckheere Trend Test under Indeterminacy with Applications

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Abstract. The classical Jonckheere trend test is a non-parametric statistical tool usually employed to compare the medians of multiple independent groups, especially when there is a natural ordering or trend among the groups. This paper aims to develop a more comprehensive and adaptable version of the Jonckheere trend test, called the neutrosophic Jonckheere trend test (NJT), which can be used to analyze different types of uncertainty data. This paper discusses neutrosophic hypotheses and decision rules pertaining to the NJT test. Furthermore, the practical uses of the NJT test have been discussed in the context of real-world applications with COVID-19 data. Lastly, a simulation study is carried out to evaluate the effectiveness of the proposed test in terms of Type I error and test power. The results validate that the proposed test is more effective and adaptable than the existing test in uncertain environments.

1. Introduction

Classical statistical tests often assume that data conform to a specific distribution, such as the normal distribution. However, this assumption may not hold true in many real-world situations, which can make it difficult to conduct parametric tests. Non-parametric tests provide a useful alternative to analyzing data when normality assumptions are violated, rendering them more versatile and applicable across various fields. The Jonckheere trend test is one of the most commonly used non-parametric tests for testing differences in medians in cases where there is an anticipated order to group medians.

To perform the Jonckheere trend test, it is imperative to gather sample elements from each group randomly and independently. It is also critical to ensure that samples within each group are independent of each other. Additionally, it is necessary to assume that groups' distributions have similar shapes and variability. The test involves ordering the observations in ascending order

across all groups and finding the sum of signs for each group. The differences between these sums of signs can be used to test whether there is a significant difference in medians between the groups. One of the main advantages of the Jonckheere trend test over other non-parametric tests, such as the Mann-Whitney U test or Kruskal-Wallis test, is that it takes into account the expected order of the group medians. This makes it particularly useful in situations where there is a natural ordering to the groups, such as in medical studies where treatments may have a graded response. In this light, this test was first introduced by Jonckheere and Terpstra [1,2]. Vock and Balakrishnan [3] suggested an extension to the Jonckheere trend test used to identify a perfect ranking in balanced ranked set sampling. Ali et al. [4] proposed a non-parametric test called the Jonckheere trend test for ordered medians and discussed its application in medical research. Murakami and Lee [5] investigated the unbiasedness and biasedness of two statistical tests, Jonckheere trend and Kruskal-Wallis under different sample sizes and distributions. Joutard [6] proposed large deviation approximations for Mann-Whitney and Jonckheere trend statistics in nonparametric statistics. Magel [7] suggested an extension to the Jonckheere trend test for ranked-set sample data to improve its sensitivity and performance in identifying variations among treatment groups. A collection of articles and books discussing the Jonckheere trend test can be found in [8–11]. As mentioned above, the Jonckheere trend test has been considered and analyzed under classical statistics; however, experimenters are sometimes confronted with indeterminate data under uncertain conditions. Therefore, it is, therefore, necessary to look for an appropriate generalization of this test so that we may use indeterminate data in an uncertain environment.

Smarandache introduced neutrosophic statistics (NS) [12]. Aslam [13] provided an explanation of the distinctions among fuzzy statistics, NS, and classical statistics. According to Smarandache [14], NS encompasses a wider scope compared to interval statistics. It embraces various forms of indeterminacy such as uncommon sample sizes, neutrosophic random variables, and neutrosophic probability distributions. The neutrosophic sign test was applied to COVID-19 data by Sherwani, et al. [15]. The neutrosophic Kruskal Wallis H test was used to analyze COVID-19 data by Sherwani, et al. [16]. Aslam and Aldosari [17] discussed alloy melting point data using the Mann-Whitney test. Miari, et al. [18] suggested single-valued neutrosophic Kruskal-Wallis and Mann Whitney tests. Aslam [19] discussed the neutrosophic ANOVA method. Ullah, et al. [20] presented a comprehensive method for calculating the neutrosophic k-factor analysis of variance. AlAita and Talebi [21] provided an exact neutrosophic analysis of the missing value issue in an augmented randomized complete block design. AlAita and Aslam [22] introduced the application of neutrosophic analysis of covariance in three different types of experimental designs: neutrosophic completely random designs, neutrosophic randomized complete block designs, and neutrosophic split-plot designs. AlAita, et al [23] presented neutrosophic statistical analysis for split-plot designs. Aslam and Albassam [24] suggested post-hoc multiple comparison tests under NS. Salama, et al. [25] discussed simple linear regression and correlation. Nagarajan, et al [26] suggested neutrosophic multiple regression. Alomair and Shahzad [27] introduced a method known

as neutrosophic mean estimation to estimate both sensitive and non-sensitive variables using robust Hartley-Ross-Type estimators. Aslam and Saleem [28] presented a discussion that focused on the application of the neutrosophic test of linearity. Aslam [29] used analysis of means under neutrosophic statistics to examine wind power data. In recent years, numerous neutrosophic statistical studies have been discussed [30–32].

We have considered the classic Jonckheere trend test and raised some ambiguities in data, such as indeterminacy and imprecise test statistic results. The probable inadequacy of the classic approach to dealing with such complexity motivated us to utilize NS. In this paper, the indeterminacies are tackled by examining the Jonckheere trend test in the NS framework, which provides additional information about uncertainty levels. This is the first time the Jonckheere trend test has been discussed in a neutrosophic environment.

In the following section, we delve into some fundamental NS-related concepts. Section 3 provides an overview of the proposed Jonckheere trend test methodology. Section 4 presents and explains a numerical example and simulation study. In Section 5, we introduce the discussion, followed by a presentation of future research directions in Section 6. Finally, we present our conclusions in Section 7.

2. Preliminaries

The following offers some basic concepts regarding neutrosophic random variables that will be useful in the subsequent sections.

The $X_N \in [X_L, X_U]$ is a neutrosophic random variable with indeterminacy interval, I_N, and is written as $X_N = X_L + X_U I_N$, where X_L is determinate part and $X_U I_N$ is indeterminate part, where $I_N \in [I_L, I_U]$ is measure of uncertainty. Clearly, X_N is reduced to the classical random variable at $I_L = 0$.

Assume that we have a population of size N with indeterminate observations, and take a neutrosophic random sample of size n from it. The neutrosophic population median is denoted by M_N and defined as the value that splits the population into two equal halves. In other words,

If n is odd, $M_N(x) = x_{N\left(\frac{n+1}{2}\right)}$.

If n is even, $M_N(x) = \frac{1}{2}(x_{N\left(\frac{n}{2}\right)} + x_{N\left(\frac{n}{2}+1\right)})$.

3. Computational method of the Jonckheere trend test under uncertainty

In neutrosophic statistics, neutrosophic nonparametric tests refer to methods of statistical analysis that require no particular distribution (especially when data are not normally distributed). The neutrosophic nonparametric tests can be used as an alternative to neutrosophic parametric tests, such as the neutrosophic T-test or NANOVA, if the underlying neutrosophic data fits certain criteria and assumptions. According to the literature on neutrosophic nonparametric tests, many statistical neutrosophic tests have been studied (e.g., neutrosophic Kruskal-Wallis H test [16], neutrosophic sign test [15], etc.). In this section, we delve into the application of the neutrosophic Jonckheere

trend test as a substitute for the neutrosophic Kruskal-Wallis H test in cases where the medians of each group conform to an anticipated pattern.

3.1. *Neutrosophic hypotheses*

Under neutrosophic statistics, the neutrosophic Jonckheere trend test is used to test the null hypothesis that all k samples have equal medians against the alternative hypothesis that at least one population differs from the rest. In other words; Suppose M_{Ni}, $i = 1, 2, \ldots, k$ represents the population median for the ith population, the neutrosophic null, and alternative hypotheses are formulated as follows;

$$H_{N0}: M_{N1} = M_{N2} = \cdots = M_{Nk},$$
$$H_{N1}: M_{N1} \leq M_{N2} \leq \ldots \leq M_{Nk}, \text{ with at least a strict inequality.}$$

3.2. *Neutrosophic Jonckheere trend test statistic*

The following steps are taken to perform the test:

Step 1: Organize the sample data in the anticipated sequence of group medians. In other words, the first column should consist of the data points with the lowest expected median, followed by the second column, which contains the data elements with the second-lowest expected median, and so on.

Step 2: The JT_N-test is computed under H_{N0}.

$$U_{Nij} = \sum_{s<r} \sum_{ij} I\left(X_{Nis} < X_{Njr}\right) + 0.5I\left(X_{Nis} = X_{Njr}\right); U_{Nij} \in [U_{Lij},\ U_{Uij}],$$

where i, j are observations in groups s and r respectively. The neutrosophic standardized test statistic is $JT_N = |Z_N|$, where

$$Z_N = \frac{U_{Nij} - \mu_{NU}}{\sigma_{NU}}; Z_N \in [Z_L,\ Z_U].$$

Assuming that there are no ties, the neutrosophic mean and the variance are, respectively

$$\mu_{NU} = \frac{N_N^2 - \sum_{i=1}^{k} N_{Ni}^2}{4}; \mu_{NU} \in [\mu_{LU},\ \mu_{UU}], \text{ and}$$
$$\sigma_{NU}^2 = \frac{N_N^2 (2N_N + 3) - \sum_{i=1}^{k} N_{Ni}^2 (2N_{Ni} + 3)}{72}; \sigma_{NU}^2 \in [\sigma_{LU}^2,\ \sigma_{UU}^2],$$

where N_N is neutrosophic total sample size and N_{Ni} is neutrosophic sample number in each group.

Step 3: Calculation of the p_N-value at the level $\alpha = 0.05$.
When dealing with a significant amount of data, Z tends to have an approximate normal distribution. By making this assumption, it becomes feasible to calculate the 1-tailed p-value as

$$p_N = \int_u^{\infty} f_U(u_N)\, du_N, (u_N > 0)$$

$$= \frac{1}{2}\left(\int_{z}^{\infty} f_Z\left(z_N\right) dz_N + \int_{-\infty}^{-z} f_Z\left(z_N\right) dz_N\right), (z_N > 0)$$
$$= \frac{1}{2}\left(1 - \operatorname{erf}\left(\frac{z_N}{\sqrt{2}}\right)\right), (z_N > 0)$$
$$= 1 - \frac{1}{2}\operatorname{erfc}\left(-\frac{JT_N}{\sqrt{2}}\right), \left(JT_N = |z_N|\right).$$

Smarandache [12] outlined that the neutrosophic decision rule can be summarized as follows:

If $min\{p_N{-}value\} > \alpha$, then we accept the null hypothesis H_{N0} at the level α.
If $max\{p_N{-}value\} \leq \alpha$, then we reject the null hypothesis H_{N0} at the level α.
If $min\{p_N{-}value\} < \alpha < max\{p_N{-}value\}$, then there is indeterminacy.
Thus $\frac{\alpha - min\{p_N - value\}}{max\{p_N - value\} - min\{p_N - value\}}$ represents the chance to reject H_{N0} at the level α, and
$\frac{max\{p_N - value\} - \alpha}{max\{p_N - value\} - min\{p_N - value\}}$ represents the chance to accept H_{N0} at the level α.

4. Application of the neutrosophic Jonckheere trend test

This section aims to evaluate the numerical performance of the proposed NS in handling the Jonckheere trend test. To achieve this, a set of real neutrosophic data with uncertain observations for the NJT test will be analyzed. In addition, a simulation study will be conducted for further evaluation. In order to assess the effectiveness of the NJT test, the JT_N-test is calculated and compared with the existing test under classical statistics in terms of uncertainty.

4.1. *Real data example*

To apply the proposed neutrosophic Jonckheere trend test, daily ICU occupancy data representing Corona-positive patients from Pakistan have been analyzed. In this study, the hypothesis being tested is whether there is a statistically significant difference in ICU occupancy of COVID-19 patients according to their age group. The data shown in Table 1 is uncertainty data [16]. This study uses the neutrosophic Jonckheere trend test to test the null hypothesis that there are no differences in the daily occupancy of COVID-19 patients from different age groups in Pakistan for December 2020. There are three categories of age groups in COVID-19 for daily ICU occupancy (35 years and below, 35 to 55 years, and 55 years and above).

As previously mentioned, the proposed test is a generalization of JT test. The neutrosophic logic literature has indicated that a method based on data in an indeterminate interval is more effective and suitable for use in uncertainty than determining values under classical statistics. The neutrosophic form of the JT_N-test is $JT_N{=}JT_L{+}JT_U I_N; I_N{\in}[I_L, I_U]$, where the first part JT_L is known as the determined part and presents the value of JT-test under CS. The second part $JT_U I_N; I_N{\in}[I_L, I_U]$ is known as the indeterminate part. Note that the JT_N-test reduces to JT-test under classical statistics if $I_N{=}\,0$. This means that the NS approach provides the JT_N-test values in an interval with the measure of indeterminacy which is more general and includes the determinate

Table 1. Daily ICU occupancy data representing COVID-19 positive patients from Pakistan.

Day	Age		
	35 and below	**35-55**	**55 and above**
1	[460, 465]	[359, 361]	[443, 450]
2	[427, 429]	[352, 365]	[421, 426]
3	[407, 410]	[445, 455]	[436, 450]
4	[378, 380]	410	[376, 385]
5	[364, 368]	[458, 464]	458
6	[345, 349]	[410, 415]	[408, 420]
7	[342, 346]	[463, 470]	[422, 425]
8	345	[580, 584]	[431, 440]
9	[313, 318]	[432, 440]	[459, 462]
10	[277, 280]	379	369
11	[268, 271]	370	360
12	[259, 262]	[584, 589]	[431, 445]
13	[256, 260]	[410, 416]	[403, 415]
14	251	[587, 590]	[436, 445]
15	249	415	376
16	[233, 227]	[419, 422]	370
17	[209, 211]	357	443
15	[187, 191]	[467, 472]	445
19	[173, 175]	[415, 418]	[355, 365]
20	168	358	450

part of the CS. As an example, the neutrosophic form of the JT_N-test for groups in Table 2 is $4.116 - 4.198I_N;\ I_N \in [0,\ 0.020]$. It means that the proposed JT_N-test ranges between 4.116 and 4.198 with the degree of indeterminacy 0.020. After evaluating the comparisons, it can be inferred that the proposed test conducted under NS provides more comprehensive information compared to the current test carried out under CS.

Table 2. Neutrosophic Jonckheere trend test for COVID 19 data

Observed JT_N Statistic U_N	[898.000, 904.000]
μ_{NU}	[595.500, 595.500]
σ_{NU}	[73.488, 73.484]
JT_N-test$= \lvert Z_N \rvert$	[4.116, 4.198]
$p_N - value$ (1-tailed)	[0.000, 0.000]

4.2. *Simulation study*

The following simulation study compares the proposed test with the existing test under classic statistics using neutrosophic statistics. To do so, the empirical neutrosophic type I error and the power of the test for neutrosophic treatment effects at a given significant nominal level were calculated. The simulation was conducted on a given number of groups. The data are generated from the neutrosophic normal distribution, neutrosophic gamma distribution, and neutrosophic t distribution which has a non-centrality parameter (neutrosophic noncentral t-distribution). Then, by assuming significant nominal levels at 0.05 and 0.01, the MC method was employed to compute both the empirical neutrosophic type I error and the power of the test. The simulation was replicated 10000 times.
To calculate the neutrosophic empirical Type I error rate and the test power for an MC experiment, the following steps need to be completed:

MC simulation to compute $\alpha_{\text{Empirical}}$
Step 1: The random sample $x_{N1}^{(i)}, x_{N2}^{(i)}, ..., x_{Nn}^{(i)}$ is generated from a neutrosophic continuous distribution under H_{N0}, $i= 1, 2, \ldots, 10000$.
Step 2: The JT_N-test is computed under H_{N0}.
Step 3: The results are recorded by assigning a value of $I_{Ni}= 1$ when the H_{N0} is rejected, and $I_{Ni}= 0$ otherwise.
Step 4: The ratio $\frac{1}{10000}\sum_{i=1}^{10000} I_{Ni}$ is computed and take it as $\alpha_{\text{Empirical}}$.

MC simulation to compute $Power_{Empirical}$
Step 1: The random sample $x_{N1}^{(i)}, x_{N2}^{(i)}, ..., x_{Nn}^{(i)}$ is generated from a neutrosophic continuous distribution under H_{N1}, $i = 1, 2, \ldots, 10000$.
Step 2: The JT_N-test is computed under H_{N1}.
Step 3: The results are recorded by assigning a value of $I_{Ni} = 1$ when the H_{N1} is rejected, and $I_{Ni} = 0$ otherwise.
Step 4: The ratio $\frac{1}{10000}\sum_{i=1}^{10000} I_{Ni}$ is computed and take it as $Power_{Empirical}$.

5. Discussion

This study enhanced the analysis of uncertain and indeterminate data within NS for a Jonckheere trend test. The study established the fundamental analytical requirements for the Jonckheere trend test in a neutrosophic context, which can also apply to classical statistics. Subsequently, the proposed method was evaluated using COVID-19 data and verified through a simulation study to determine its accuracy. This test has been evaluated using empirical type I error and test power. Results indicated that the neutrosophic approach outperformed the existing approach, with better compatibility between empirical and nominal α values. Furthermore, the proposed test exhibited

TABLE 3. Simulation results for the NJT test in neutrosophic normal distribution with means $\delta = (\mu_{N1}, \mu_{N2}\ ,\ \mu_{N3},\ \mu_{N4})$ and $\sigma_N^2 = 1$ at the nominal levels of 0.01 and 0.05.

Sample Size	α	$\alpha_{Empirical}$	$Power_{Empirical}$					
			$\delta_1 = (\ \mathbf{0,0,0,0.1})$	$\delta_2 = (\mathbf{0.1,0.1,0.3,0.4})$	$\delta_3 = (\mathbf{0.1,0.2,0.4,0.5})$	$\delta_4 = (\mathbf{0,0.2,0.4,0.6})$	$\delta_5 = (\mathbf{0,0.3,0.5,0.7})$	$\delta_6 = (\mathbf{0,0.2,0.8,0.9})$
(5,5,5,5)	0.01	[0.0088, 0.0097]	[0.0129, 0.0139]	[0.0271, 0.0296]	[0.0394, 0.0432]	[0.0655, 0.0760]	[0.0820, 0.0939]	[0.1626, 0.1884]
	0.05	[0.0504, 0.0502]	[0.0631, 0.0647]	[0.1219, 0.1268]	[0.1494, 0.1585]	[0.2173, 0.2292]	[0.2600, 0.2850]	[0.4121, 0.4517]
(20,20,20,20)	0.01	[0.0111, 0.0100]	[0.0194, 0.0197]	[0.0881, 0.0977]	[0.1403, 0.1609]	[0.2922, 0.3346]	[0.3891, 0.4462]	[0.7346, 0.7982]
	0.05	[0.0492, 0.0501]	[0.0826, 0.0871]	[0.2520, 0.2780]	[0.3531, 0.3820]	[0.5638, 0.6085]	[0.6604, 0.7152]	[0.9034, 0.9353]
(4,6,7,5)	0.01	[0.0088, 0.0093]	[0.0128, 0.0146]	[0.0266, 0.0308]	[0.0371, 0.0416]	[0.0577, 0.0666]	[0.0709, 0.0799]	[0.1614, 0.1849]
	0.05	[0.0451, 0.0462]	[0.0588, 0.0598]	[0.1173, 0.1229]	[0.1413, 0.1499]	[0.1949, 0.2135]	[0.2244, 0.2472]	[0.3918, 0.4292]
(14,11,17,12)	0.01	[0.0106, 0.0097]	[0.0170, 0.0175]	[0.0566, 0.0640]	[0.0879, 0.1020]	[0.1890, 0.2183]	[0.2454, 0.2809]	[0.5183, 0.5830]
	0.05	[0.0479, 0.0494]	[0.0686, 0.0707]	[0.2014, 0.2172]	[0.2560, 0.2843]	[0.4169, 0.4571]	[0.5070, 0.5569]	[0.7734, 0.8210]

TABLE 4. Simulation results for the NJT test in neutrosophic gamma distribution with shape parameter $\alpha = (\alpha_{N1},\ \alpha_{N2},\ \alpha_{N3},\ \alpha_{N4})$ and inverse scale parameter $\beta_N = 1$ at the nominal levels of 0.01 and 0.05.

Sample Size	α	$\alpha_{Empirical}$	$Power_{Empirical}$					
			$\alpha_1 = (\ \mathbf{0,0,0,0.1})$	$\alpha_2 = (\mathbf{0.1,0.1,0.3,0.4})$	$\alpha_3 = (\mathbf{0.1,0.2,0.4,0.5})$	$\alpha_4 = (\mathbf{0,0.2,0.4,0.6})$	$\alpha_5 = (\mathbf{0,0.3,0.5,0.7})$	$\alpha_6 = (\mathbf{0,0.2,0.8,0.9})$
(5,5,5,5)	0.01	[0.0081, 0.0097]	[0.0137, 0.0134]	[0.0281, 0.0320]	[0.0360, 0.0444]	[0.0601, 0.0783]	[0.0725, 0.0928]	[0.1376, 0.1706]
	0.05	[0.0481, 0.0507]	[0.0650, 0.0677]	[0.1217, 0.1372]	[0.1456, 0.1639]	[0.2139, 0.2529]	[0.2452, 0.2851]	[0.3683, 0.4366]
(20,20,20,20)	0.01	[0.0123, 0.0099]	[0.0206, 0.0264]	[0.0874, 0.1155]	[0.1281, 0.1726]	[0.2732, 0.3652]	[0.3604, 0.4617]	[0.6565, 0.7616]
	0.05	[0.0520, 0.0502]	[0.0871, 0.0962]	[0.2526, 0.3014]	[0.3353, 0.4032]	[0.5482, 0.6506]	[0.6239, 0.7206]	[0.8701, 0.9230]
(4,6,7,5)	0.01	[0.0091, 0.0099]	[0.0109, 0.0124]	[0.0266, 0.0312]	[0.0322, 0.0434]	[0.0563, 0.0704]	[0.0642, 0.0827]	[0.1310, 0.1661]
	0.05	[0.0427, 0.0480]	[0.0598, 0.0646]	[0.1105, 0.1264]	[0.1314, 0.1531]	[0.1979, 0.2262]	[0.2199, 0.2532]	[0.3506, 0.4104]
(14,11,17,12)	0.01	[0.0090, 0.0101]	[0.0146, 0.0171]	[0.0614, 0.0776]	[0.0865, 0.1113]	[0.1720, 0.2301]	[0.2237, 0.2889]	[0.4546, 0.5548]
	0.05	[0.0487, 0.0503]	[0.0729, 0.0805]	[0.1950, 0.2274]	[0.2511, 0.2992]	[0.3965, 0.4781]	[0.4802, 0.5609]	[0.7179, 0.8000]

TABLE 5. Simulation results for the NJT test in neutrosophic t distribution with non-centrality parameter (ncp) ranging from 1 to 1.9 at the nominal levels of 0.01 and 0.05.

Sample Size and df	α	$\alpha_{Empirical}$	$Power_{Empirical}$					
			$ncp_1 = (\ \mathbf{1,1,1,1.1})$	$ncp_2 = (\mathbf{1.1,1.1,1.3,1.4})$	$ncp_3 = (\mathbf{1.1,1.2,1.4,1.5})$	$ncp_4 = (\mathbf{1,1.2,1.4,1.6})$	$ncp_5 = (\ \mathbf{1,1.3,1.5,1.7})$	$ncp_6 = (\ \mathbf{1,1.2,1.8,1.9})$
(5,5,5,5) and $df = \mathbf{3}$	0.01	[0.0112, 0.0107]	[0.0105, 0.0117]	[0.0255, 0.0271]	[0.0315, 0.0330]	[0.0543, 0.0580]	[0.0677, 0.0690]	[0.1265, 0.1376]
	0.05	[0.0523, 0.0511]	[0.0626, 0.0636]	[0.1069, 0.1103]	[0.1280, 0.1326]	[0.1873, 0.1938]	[0.2187, 0.2243]	[0.3468, 0.3622]
(20,20,20,20) and $df = \mathbf{10}$	0.01	[0.0108, 0.0099]	[0.0186, 0.0199]	[0.0855, 0.0925]	[0.1294, 0.1479]	[0.2718, 0.3081]	[0.3621, 0.4071]	[0.6839, 0.7314]
	0.05	[0.0475, 0.0495]	[0.0821, 0.0873]	[0.2379, 0.2516]	[0.3313, 0.3531]	[0.5413, 0.5753]	[0.6253, 0.6689]	[0.8819, 0.9087]
(4,6,7,5) and $df = \mathbf{3}$	0.01	[0.0086, 0.0091]	[0.0112, 0.0119]	[0.0265, 0.0275]	[0.0319, 0.0323]	[0.0485, 0.0538]	[0.0595, 0.0622]	[0.1179, 0.1294]
	0.05	[0.0435, 0.0442]	[0.0545, 0.0590]	[0.1050, 0.1121]	[0.1178, 0.1271]	[0.1668, 0.1780]	[0.1934, 0.2017]	[0.3145, 0.3300]
(14,11,17,12) and $df = \mathbf{10}$	0.01	[0.0098, 0.0101]	[0.0156, 0.0163]	[0.0540, 0.0584]	[0.0838, 0.0957]	[0.1710, 0.1906]	[0.2248, 0.2557]	[0.4741, 0.5262]
	0.05	[0.0518, 0.0509]	[0.0732, 0.0780]	[0.1848, 0.1953]	[0.2477, 0.2685]	[0.3930, 0.4302]	[0.4700, 0.5131]	[0.7408, 0.7823]

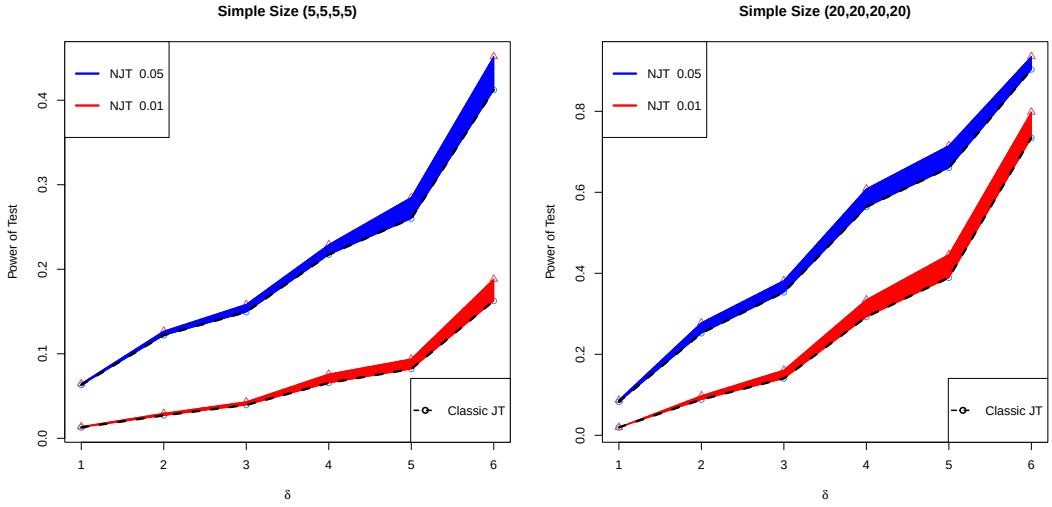

FIGURE 1. Power curves for the NJT test in neutrosophic normal distribution with sample sizes of $(5, 5, 5, 5)$ and $(20, 20, 20, 20)$ at the nominal levels of 0.01 and 0.05.

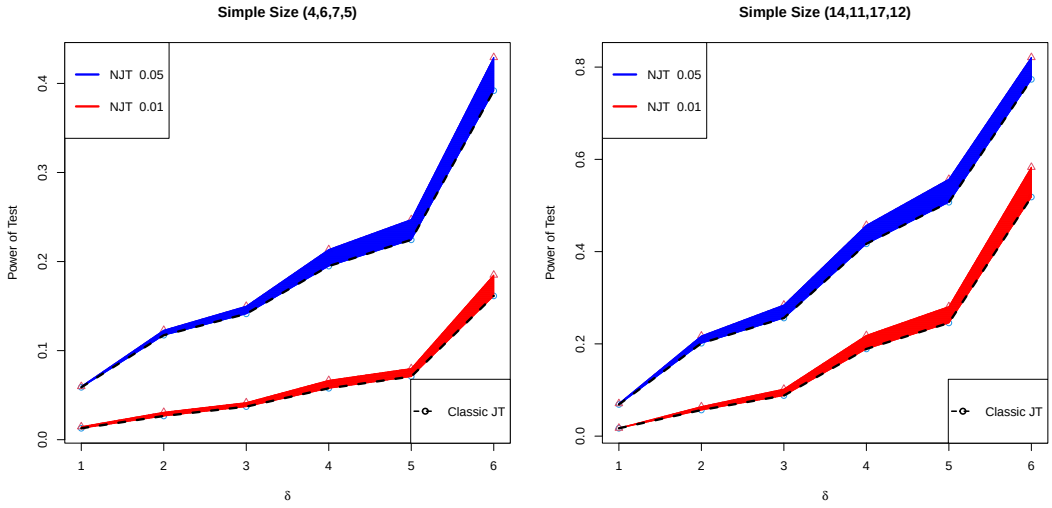

FIGURE 2. Power curves for the NJT test in neutrosophic normal distribution with sample sizes of $(4, 6, 7, 5)$ and $(14, 11, 17, 12)$ at the nominal levels of 0.01 and 0.05.

higher testing power than the existing test, as demonstrated by Tables 3-5 and Figures 1-6. The JT_N-test offers superior flexibility, applicability, and information than the current JT-test in uncertain environments.

6. Future research directions

This research paper focuses on the Jonckheere trend test and its application in uncertain environments. However, this test can also be adapted to address more complicated scenarios, such as missing data. Missing cells can create various situations that require further investigation. Future

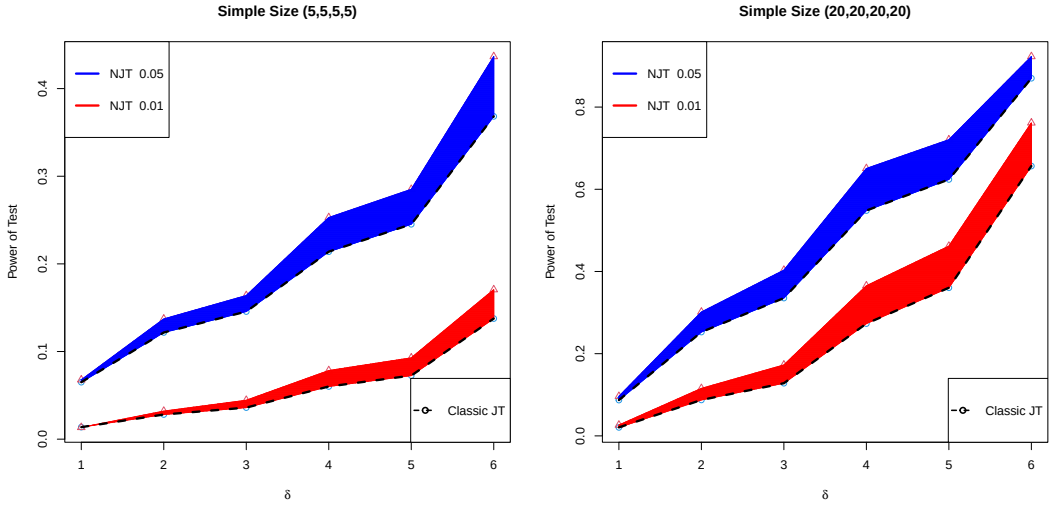

FIGURE 3. Power curves for the NJT test in neutrosophic gamma distribution with sample sizes of $(5, 5, 5, 5)$ and $(20, 20, 20, 20)$ at the nominal levels of 0.01 and 0.05.

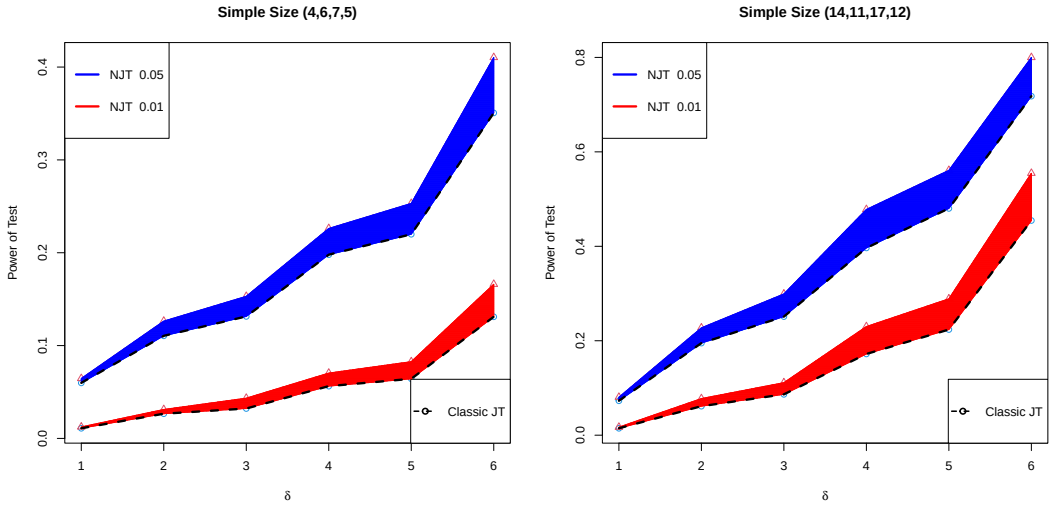

FIGURE 4. Power curves for the NJT test in neutrosophic gamma distribution with sample sizes of (4, 6, 7, 5) and (14, 11, 17, 12) at nominal levels of 0.01 and 0.05.

studies could explore this issue and expand upon the results to incorporate missing values into the test. The presence of missing data can introduce bias, making it crucial to select the most effective method for minimizing bias and obtaining accurate outcomes.

7. Conclusions

Using a neutrosophic approach is necessary when dealing with data uncertainties. However, traditional methods can be limited by errors and vagueness in the environment, making a neutrosophic

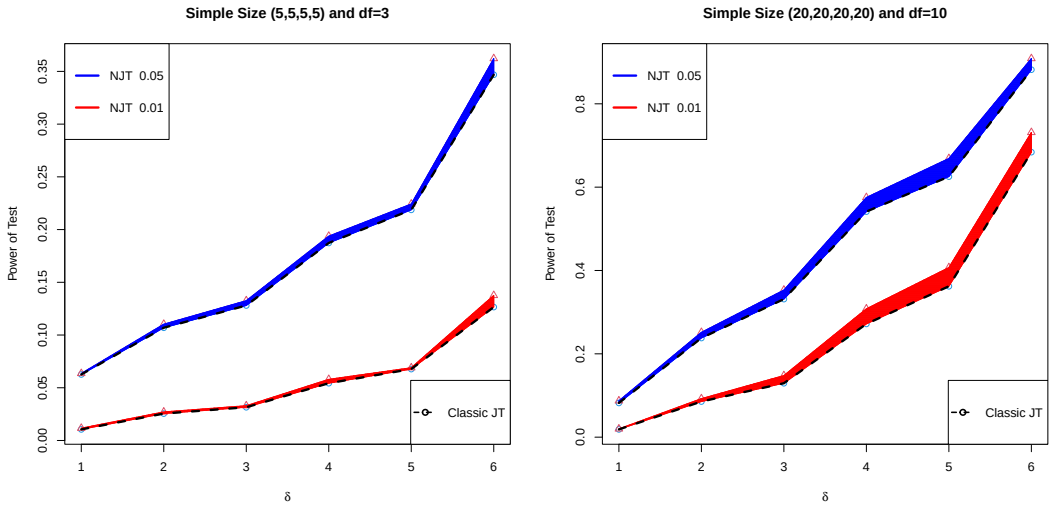

FIGURE 5. Power curves for the NJT test in neutrosophic noncentral t-distribution with sample sizes of $(5, 5, 5, 5)$ and $(20, 20, 20, 20)$ at the nominal levels of 0.01 and 0.05.

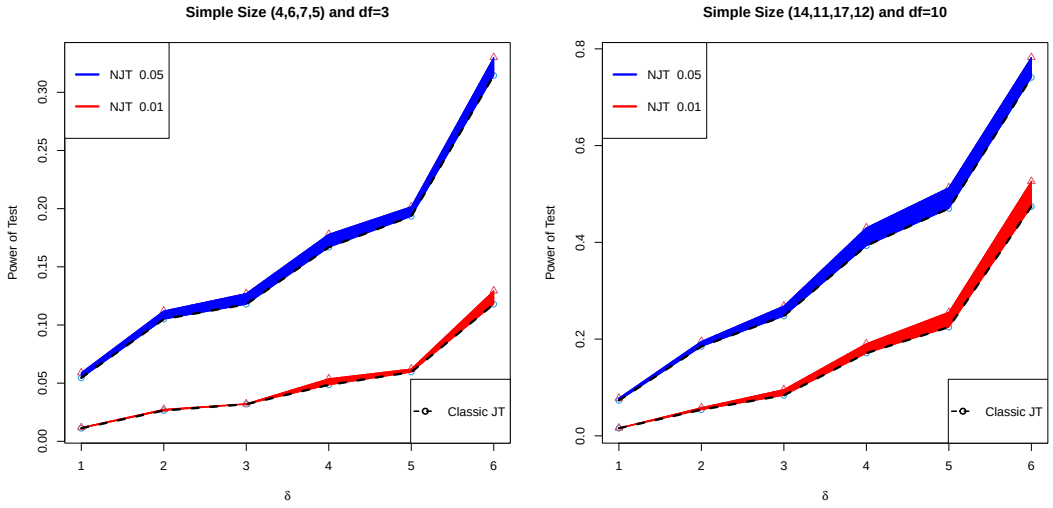

FIGURE 6. Power curves for the NJT test in neutrosophic noncentral t-distribution with sample sizes of $(4, 6, 7, 5)$ and $(14, 11, 17, 12)$ at the nominal levels of 0.01 and 0.05.

model a more effective option. It is worthwhile to note that this represents a methodological advancement, making the improvement significant. It was found that the proposed test significantly impacted the handling of uncertainties in this study. Importantly, this study also acknowledges computational progress. This was the first attempt to propose the neutrosophic Jonckheere trend test to address these issues while also considering the intrinsic indeterminacy of the data. As such, this study provides an appropriate framework for analyzing data under such conditions. The proposed neutrosophic test offers superior precision, flexibility, applicability, and information than existing tests in uncertain environments.

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ADVANCES IN PLITHOGENIC SETS

An Overview of Plithogenic Set and Symbolic Plithogenic Algebraic Structures

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Abstract

This paper is devoted to Plithogeny, Plithogenic Set, and its extensions. These concepts are branches of uncertainty and indeterminacy instruments of practical and theoretical interest. Starting with some examples, we proceed towards general structures. Then we present definitions and applications of the principal concepts derived from plithogeny, and relate them to complex problems.

Keywords: Plithogeny, Neutrosophy, Dialectics, Plithogenic set, Plithogenic logic, Plithogenic probability, Plithogenic statistics, Multivariate analysis, Symbolic Plithogenic algebraic structures.

1 | Etymology of Plithogeny

Plitho-geny etymologically comes from: (Gr.) πλήθος (plithos) = crowd, large number of, multitude, plenty of, and -geny < (Gr.) -γενιά (-geniá) = generation, the production of something & γένεια (géneia) = generations, the production of something < -γένεση (-génesi) = genesis, origination, creation, development, according to Translate Google dictionaries and Webster's new world dictionary of American English, third college edition, Simon & Schuster, Inc.

Therefore, Plithogeny is the genesis or origination, creation, formation, development, and evolution of new entities from dynamics and organic fusions of contradictory and/or neutrals and/or non-contradictory multiple old entities.

Plithogeny is a dynamics of many pairs of opposites ($<A_1>$, $<antiA_1>$), …, ($<A_n>$, $<antiA_n>$), and their neutralities (indeterminacies) $<neutA_1>$, …, $<neutA_n>$.

While Neutrosophy, as a particular case of Plithogeny, is the dynamics of only one pair of opposites (<A>, <antiA>) and its neutral (indeterminacy) <neutA>.

Dialectics and Yin Yang are the dynamics of one pair of opposites <A> and <antiA>, without taking into consideration their neutral (indeterminacy) <neutA>.

Plithogenic means what is pertaining to Plithogeny.

All Plithogeny, Plithogenic set, Plithogenic logic, Plithogenic probability, Plithogenic statistics, and symbolic Plithogenic algebraic structures were [29]-[32].

This article is a synthesis of evolution of Plithogenic domain during a six-year span, with articles published by many authors around the world [1]-[28].

2 | Plithogenic Set

A Plithogenic set P is a set whose elements are characterized by one or more attributes, and each attribute may have many values. Each attribute's value v has a corresponding degree of appurtenance d(x,v) of the element x to the set P, with respect to some given criteria.

In order to obtain a better accuracy for the Plithogenic aggregation operators, a contradiction (dissimilarity) degree is defined between each attribute value and the dominant (most important) attribute value. The contradiction degrees may similarly be: either fuzzy, intuitionistic fuzzy, neutrosophic, or any fuzzy extension, yet for simplicity we'll be using the fuzzy ones.

However, there are cases when such dominant attribute value may not be taking into consideration or may not exist [therefore it is considered zero by default], or there may be many dominant attribute values. In such cases, either the contradiction degree function is suppressed, or another relationship function between attribute values should be established.

The Plithogenic aggregation operators (intersection, union, complement, inclusion, equality) are based on contradiction degrees between attributes' values, and the first two operators (intersection and union) are linear combinations of the fuzzy operators' tnorm ($\wedge_F$) and tconorm ($\vee_F$) where 'F' stands for 'fuzzy'.

Let $(\overset{0}{a_o}, \overset{c_1}{a_1}, ..., \overset{c_i}{a_i}, ..., \overset{c_n}{a_n})$ and $(\overset{0}{b_o}, \overset{c_1}{b_1}, ..., \overset{c_i}{b_i}, ..., \overset{c_n}{b_n})$ be the attribute value degrees of a generic element x from a Plithogenic set *P*, assigned by an Expert A, and respectively an Expert B, where the contradiction degrees of the attribute values are: $0 \le c_1 \le ... \le c_i \le ... \le c_n \le 1$, i.e. $c(a_o, a_i) = c_i$ for $1 \le i \le n$. Let $\wedge_P, \vee_P$ represent respectively the Plithogenic intersection and Plithogenic union. Then,

The Plithogenic Intersection:

$$x(\overset{0}{a_o}, \overset{c_1}{a_1}, ..., \overset{c_i}{a_i}, ..., \overset{c_n}{a_n}) \wedge_P x(\overset{0}{b_o}, \overset{c_1}{b_1}, ..., \overset{c_i}{b_i}, ..., \overset{c_n}{b_n}) = x(a_0 \wedge_F b_o, (1-c_1)\cdot(a_1 \wedge_F b_1) +$$
$$c_1 \cdot (a_1 \vee_F b_1), ..., (1-c_i)\cdot(a_i \wedge_F b_i) + c_i \cdot (a_i \vee_F b_i), ..., (1-c_n)\cdot(a_n \wedge_F b_n) + c_n \cdot (a_n \vee_F b_n)).$$

The Plithogenic Union:

$$x(\overset{0}{a_o}, \overset{c_1}{a_1}, ..., \overset{c_i}{a_i}, ..., \overset{c_n}{a_n}) \vee_P x(\overset{0}{b_o}, \overset{c_1}{b_1}, ..., \overset{c_i}{b_i}, ..., \overset{c_n}{b_n}) = x(a_0 \vee_F b_o, (1-c_1)\cdot(a_1 \vee_F b_1) +$$
$$c_1 \cdot (a_1 \wedge_F b_1), ..., (1-c_i)\cdot(a_i \vee_F b_i) + c_i \cdot (a_i \wedge_F b_i), ..., (1-c_n)\cdot(a_n \vee_F b_n) + c_n \cdot (a_n \wedge_F b_n)).$$

Plithogenic set is a generalization of the crisp set, fuzzy set, intuitionistic fuzzy set, neutrosophic set, since these these types of sets are characterized by a single attribute value (appurtenance): which has one value (membership), for the crisp set and fuzzy set, two values (membership, and nonmembership), for intuitionistic fuzzy set, or three values (membership, nonmembership and indeterminacy), for neutrosophic set.

2.1 | Example of Plithogenic Fuzzy Set

Let P be a Plithogenic set, representing the students from a college. Let x belonging to P be a generic student that is characterized by three attributes:

I. altitude (a), whose values are {tall, short} = {a_1, a_2}.
II. weight (w), whose values are {obese, fat, medium, thin} = {w_1, w_2, w_3, w_4}.
III. hair color (h), whose values are {blond, reddish, brown} = {h_1, h_2, h_3}.

The uni-dimensional attribute contradiction degrees are:

$$c(a_1, a_2) = 1,$$
$$c(w_1, w_2) = 1/3; c(w_1, w_3) = 2/3; c(w_1, w_4) = 1,$$
$$c(h_1, h_2) = 1/2; c(h_1, h_3) = 1.$$

Dominant attribute values are: a_1, w_1, and h_1 respectively for each corresponding uni-dimensional attribute a, w, and h respectively.

The multi-attribute dimension is 9 attribute values.

P = { John(tall, short; obese, fat, medium, thin; blond, reddish, brown),

John(tall, short; obese, fat, medium, thin; blond, reddish, brown) }.

For example:

P = { John(0.8, 0.2; 0.1, 0.2, 0.4, 0.6; 0.9, 0.6, 0.0),

Richard (0.5, 0.4; 0.3, 0.21, 0.4, 0.7; 0.8, 0.5, 0.01) }.

2.2 | Example of Plithogenic Neutrosophic Set

P = { John((0.8, 0.1, 0.3), (0.2, 0.0, 0.1); (0.1, 0.5, 0.5), (0.2, 0.6, 0.6), (0.4, 0.4, 0.4),

(0.6, 0.7, 0.2); (0.9, 0.8, 0.8), (0.6, 0.3, 0.3), (0.0, 0.2, 0.2)),

Richard ((0.5, 0.2, 0.0), (0.6, 0.8, 0.8); (0.7, 0.5, 0.4), (0.1, 0.6, 0.1), (0.8, 0.4, 0.7),

(0.3, 0.4 0.2); (0.7, 0.8, 0.9), (0.5, 0.0, 0.3), (0.2, 0.8, 0.1) }.

2.3 | A Plithogenic Application to Images

A pixel x may be characterized by colors k_1, k_2, ..., k_n. We write x(k_1, k_2, ..., k_n), where $n \geq 1$.

We may consider the degree of each color either fuzzy, intuitionistic fuzzy, or neutrosophic, or any fuzzy extension. For example:

Fuzzy degree:

$$x(0.4, 0.6, 0.1, ..., 0.3).$$

Intuitionistic fuzzy degree:

$$x\big(\ (0.1,\ 0.2),\ (0.3,\ 0.5),\ (0.0,\ 0.6),\ ...,\ (0.8,\ 0.9)\ \big).$$

Neutrosophic degree:

$$x\big(\ (0.0,\ 0.3,\ 0.6),\ (0.2,\ 0.8,\ 0.9),\ (0.7,\ 0.4,\ 0.2),\ ...,\ (0.1,\ 0.1,\ 0.9)\ \big).$$

Then, we can use the Plithogenic (fuzzy, intuitionistic fuzzy, neutrosophic, other fuzzy extension) intersection operator to combine them.

But, we first establish the degrees of contradictions $c(k_i, k_j)$ between the colors k_i and k_j in order to find the linear combinations of t-norm and t-conorm that one applies to each color (similar to the indeterminacy above). The contradiction degrees may also be: either fuzzy, intuitionistic fuzzy, neutrosophic, or other fuzzy extension. In majority of cases we choose, for simplicity, the fuzzy one.

3 | Plithogenic Logic

A Plithogenic logic proposition P is a proposition that is characterized by degrees of many truth-values with respect to the corresponding attributes' values. For each attribute's value v there is a corresponding degree of truth-value d(P, v) of P with respect to the attribute value v. Plithogenic logic is a generalization of the classical logic, fuzzy logic, intuitionistic fuzzy logic, and neutrosophic logic, since these four types of logics are characterized by a single attribute value (truth-value): which has one value (truth), for the classical logic and fuzzy logic, two values (truth, and falsehood), for intuitionistic fuzzy logic, or three values (truth, falsehood, and indeterminacy), for neutrosophic logic. A Plithogenic logic proposition P, in general, may be characterized by more than four degrees of truth-values resulted from under various attributes.

3.1 | Example of Plithogenic Fuzzy Logic

Let P = "John is a knowledgeable person" be a logical proposition. It is evaluated in a fuzzy way (one component for each attribute value) by multiple experts (two) and under multiple attribute values (six), that's why it is called Plithogenic .

The three attributes under which this proposition has to be evaluated about-according to the experts-are: Science (whose attribute values are: mathematics, physics, anatomy), literature (whose attribute values are: poetry, novel), and arts (whose only attribute value is: sculpture).

According to Expert A (lexander), the fuzzy truth-values of Plithogenic proposition P are:

$P_A(0.7, 0.6, 0.4; 0.9, 0.2; 0.5)$, which means that John's fuzzy degree of truth (knowledge) in mathematics is 0.7, degree of truth (knowledge) in physics is 0.6, degree of truth (knowledge) in anatomy is 0.4;

Degree of truth (knowledge) in poetry is 0.9, degree of truth (knowledge) in novels is 0.2;

Degree of truth (knowledge) in sculpture is 0.5.

But, according to Expert B (arbara), the truth-values of Plithogenic proposition P are:

$$P_{B\,=}\big(0.9,\ 0.6,\ 0.2;\ 0.8,\ 0.7;\ 0.3\big).$$

Assume that the experts consider that the attributes' values contradiction (dissimilarity) degrees are:

0	0.3	0.8	0	0.9	0
mathematics	physics	anatomy	poetry	novels	sculpture

And taking the fuzzy $t_{norm}(a, b) = a \wedge_F b = a \cdot b$

And fuzzy $t_{conorm}(a, b) = a \vee_F b = a + b - a \cdot b$

One gets:

$P_A(0.7, 0.6, 0.4; 0.9, 0.2; 0.5) \wedge_P P_B(0.9, 0.6, 0.2; 0.8, 0.7; 0.3) =$

$P_{A \wedge_P B}(0.7 \wedge_P 0.9, 0.6 \wedge_P 0.6, 0.4 \wedge_P 0.2; 0.9 \wedge_P 0.8, 0.2 \wedge_P 0.7; 0.5 \wedge_P 0.3) =$

$P_{A \wedge_P B}(0.7 \wedge_F 0.9, 0.6 \wedge_P 0.6, 0.4 \wedge_P 0.2; 0.9 \wedge_F 0.8, 0.2 \wedge P 0.7; 0.5 \wedge_F 0.3) =$

$P_{A \wedge_P B}(0.7 \cdot 0.9, (1-0.3) \cdot [0.6 \wedge_F 0.6] + 0.3 \cdot [0.6 \vee_F 0.6], (1-0.8) \cdot [0.4 \wedge_F 0.2] + 0.8 \cdot [0.4 \vee_F 0.2];$

$0.9 \cdot 0.8, (1-0.9) \cdot [0.2 \wedge_F 0.7] + 0.9 \cdot [0.2 \vee_F 0.7]; 0.5 \cdot 0.3) =$

$P_{A \wedge_P B}(0.630, 0.7 \cdot [0.6 \cdot 0.6] + 0.3 \cdot [0.6 + 0.6 - 0.6 \cdot 0.6],$

$0.2 \cdot [0.4 \cdot 0.2] + 0.8 \cdot [0.4 + 0.2 - 0.4 \cdot 0.2];$

$0.720, 0.1 \cdot [0.2 \cdot 0.7] + 0.9 \cdot [0.2 + 0.7 - 0.2 \cdot 0.7]; 0.150) =$

$P_{A \wedge_P B}(0.630, 0.504, 0.432; 0.720, 0.698; 0.150),$

Where $/\backslash_P$, $/\backslash_F$, $\backslash/_P$, $\backslash/_P$ represent the fuzzy intersection, fuzzy union, Plithogenic intersection, and Plithogenic union respectively.

3.2 | Example of Plithogenic Neutrosophic Logic

Let P = "John is a knowledgeable person" be a logical proposition. It is evaluated in a neutrosophic way (three components for each attribute value) by multiple attribute values (six), that's why it is called Plithogenic .

P((0.7, 0.1, 0.5), (0.6, 0.0, 0.9), (0.4, 0.4, 0.4); (0.9,0.2, 0.3), (0.2, 0.6, 0.7); (0.5, 0.3, 0.1)), which means that John's neutrosophic degrees of truth (knowledge), indeterminate-truth, and falsehood (nonknowledge) in mathematics is 0.7, 0.1, and 0.5 respectively, and similarly for his neutrosophic degrees in physics, and anatomy.

In the same way for Richard's neutrosophic degrees.

4 | Plithogenic Probability

Since in Plithogenic probability each event E from a probability space U is characterized by many chances of the event to occur [not only one chance of the event E to occur: as in classical probability, imprecise probability, and neutrosophic probability], a Plithogenic probability distribution function, PP(x), of a random variable x, is described by many Plithogenic probability distribution sub-functions, where each sub-function represents the chance (with respect to a given attribute value) that value x occurs, and these chances of occurrence can be represented by classical, imprecise, or neutrosophic probabilities (depending on the type of degree of a chance).

4.1 | Example of Plithogenic Probabilistic

What is the Plithogenic probability that Jennifer will graduate at the end of this semester in her program of electrical engineering, given that she is enrolled in and has to pass two courses of mathematics (non-

linear differential equations, and stochastic analysis), and two courses of Mechanics (Fluid Mechanics, and Solid Mechanics)?

We have a 4 attribute-values of Plithogenic probability.

According to her adviser, Jennifer's Plithogenic single-valued fuzzy probability of graduating at the end of this semester is:

$$J(0.5, 0.6; 0.8, 0.4).$$

Which means 50% chance of passing the non-linear differential equations class,

60% chance of passing the Stochastic Analysis class (as part of Mathematics), and 80% of passing the Fluid Mechanics class, and 40% of passing the Solid Mechanics class (as part of Physics).

Therefore, the Plithogenic probability in this example is composed from 4 classical probabilities.

Also, according to her adviser, Jennifer's Plithogenic single-valued neutrosophic probability of graduating at the end of this semester is:

$$J((0.5, 0.1, 0.3), (0.6, 0.2, 0,7); (0.8, 0.5, 0.4), (0.4, 0.0, 0.9)).$$

Which means 50% chance of passing, 10% indeterminate-chance, and 30% chance of non-passing the non-linear differential equations class; similarly for the others.

While the Plithogenic Probability of an event E to occur is calculated with respect to many chances of the event E to occur (it is calculated with respect to each event's attribute/parameter chance of occurrence).

Therefore, the Plithogenic Probability is a Multi-Probability (i.e. multi-dimensional probability), unlike the classical, imprecise and neutrosophic probabilities that are uni-dimensional probabilities.

The Neutrosophic Probability (and similarly for classical probability, and for the imprecise probability) of an event E to occur is calculated with respect to the chance of the event E to occur (i.e. it is calculated with respect to only ONE chance of occurrence).

5 | Plithogenic Statistics

As a generalization of classical statistics and neutrosophic statistics, the Plithogenic Statistics is the analysis of events described by the Plithogenic probability.

Plithogenic statistics is a multivariate statistics, characterized by multiple random variables, whose degrees may be: classical, fuzzy, intuitioninstic fuzzy, neutrosophic, or any other fuzzy extension.

In neutrosophic statistics we have some degree of indeterminacy, incompleteness, inconsistency into the data or into the statistical inference methods.

5.1 | Example of Plithogenic Statistics

Let's consider the previous example of Plithogenic Probability that Jenifer will graduate at the end of this semester in her program of electrical engineering.

We now graph four probability distribution functions, instead of one as in the case when considering the neutrosophic distribution as a uni-dimensional neutrosophic function. Therefore, Plithogenic Statistics is an extension of the multi-variate statistics.

6 | Symbolic Plithogenic Algebraic Structures

Definitions of symbolic Plithogenic set and symbolic Plithogenic algebraic structures

Let SPS be a non-empty set, included in a universe of discourse U, defined as follows:

SPS = {x| $x = a_0 + a_1p_1 + a_2P_2 + ... + a_nP_n$, $n \geq 1$, all a_i belong to R, or to C, or belong to some given algebraic structure space, for $0 \leq i \leq n$}.

where R = the set of real numbers, C = the set of complex numbers, and all P_i are symbols (letters, or variables), and are called Symbolic (Literal) Plithogenic Components (Variables)}, where 1, P_1, P_2, …, P_n act like a base for the elements of the above set SPS. a_0, a_1, a_2, …, a_n are called coefficients.

SPS is called a Symbolic Plithogenic Set. And the algebraic structures defined on this set are called Symbolic Plithogenic Algebraic Structures.

In general, Symbolic (or Literal) Plithogenic theory is referring to the use of abstract symbols {i.e. the letters/parameters) P_1, P_2, …, P_n, representing the Plithogenic components (variables) as above} in some theory.

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Introduction to the Symbolic Plithogenic Algebraic Structures (revisited)

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Abstract: In this paper, we recall and study the new type of algebraic structures called Symbolic Plithogenic Algebraic Structures. Their operations are given under the Absorbance Law and the Prevalence Order.

Keywords: Absorbance Law; Prevalence Order; Neutrosophic Quadruple Numbers; Plithogenic Set; Type-k Neutrosophic Set; Type-k Plithogenic Set; Hybridization of Classical, Fuzzy and Fuzzy Extension Sets; Symbolic Plithogenic Components; Symbolic Plithogenic Operations; Plithogenic Numbers; Symbolic Plithogenic Algebraic Structures; Symbolic Plithogenic Group; Symbolic Plithogenic Ring.

1. Introduction

The plithogeny, plithogenic set, plithogenic logic, plithogenic probability, plithogenic statistics, and the symbolic plithogenic algebraic structures were introduced in 2018-2019 by Smarandache [1, 2, 3, 4, 5].

Plithogeny is the genesis or origination, creation, formation, development, and evolution of new entities from dynamics and organic fusions of contradictory and/or neutrals and/or non-contradictory multiple old entities.

And **plithogenic** means what is pertaining to plithogeny.

Plithogeny is an extension of neutrosophy, which is an extension of paradoxism.

While **paradoxism** [6] is based on using opposite ideas, contradictions, paradoxes in arts, letters, and science creations,

neutrosophy is based on the dynamics of a pair of opposites (<A> ,<antiA>) and their neutral (indeterminacy) <neutA>,

but **plithogeny** on the dynamics of many pairs of opposites (<A_1>, <antiA_1>), …, (<A_k>, <antiA_k>) and their neutralities <neutA_1>, …, <neutA_k>, for $k \geq 2$ ["plitho" means "many" in Greek language].

Plithogenic Set was extended to Type-n Plithogenic Set, for integer $n \geq 1$.

Symbolic Plithogenic Numbers are generalizations of Neutrosophic Quadruple Numbers, Refined Neutrosophic Quadruple Numbers, and Symbolic Turiyam Numbers.

Consequently, the Symbolic Plithogenic Algebraic Structures (semigroup, group, ring, etc.) are generalization of the corresponding algebraic structures built on these particular cases described above.

2. Informal Definition of Plithogenic Set

A plithogenic set (*PS*) is a set whose elements are characterized, as in our real world, by many attributes (parameters): $P_1, P_2, \ldots, P_n$.

$PS = \{x(P_1, P_2, ..., P_n), x \in U\}$, where *U* is a universe of discourse.

A generic element *x* belongs to the plithogenic set *PS* in a certain degree $d(P_i)$ with respect to each attribute (parameter) P_i. The degree of appurtenance of an element to the plithogenic set may be: classical, fuzzy, intuitionistic fuzzy, neutrosophic, refined neutrosophic, and any other type of extended fuzzy.

In a better descriptive way, emphasizing the degrees, we may re-write it as:

$$PS = \{x(d(P_1), d(P_2), ..., d(P_n)), x \in U\}$$

The attributes (parameters) $P_1, P_2, \ldots, P_n$ may be independent, dependent, or partially independent and partially dependent of each other - according to each application.

This is also called Type-1 Plithogenic Set.

3. Type-k Plithogenic Set

The Type-k Neutrosophic Set [13] has been extended to Type-k Plithogenic Set.

If the parameters $P_i, 1 \le i \le n,$ depend on sub-parameters $P_{i1}, P_{i2}, ..., P_{im_i}$ for $m_i \ge 1$ then one gets a Type-2 Plithogenic Set.

Afterward, if the sub-parameters $P_{ij}, 1 \le i \le n, 1 \le j \le m_i$ are formed by sub-sub-parameters $P_{ij1}, P_{ij2}, ..., P_{ijm_j}$ for $m_j \ge 1$ then one gets a Type-3 Plithogenic Set.

And so on, up to Type-k Plithogenic Set.

Passing to degrees, one may write:

$$PS_1 = \{x(d_1(P_1), d_1(P_2), ..., d_1(P_n)), x \in U\}$$

Type-2 Plithogenic Set

$$PS_2 = \{x(d_2(d_1(P_1)), d_2(d_1(P_2)), ..., d_2(d_1(P_n))), x \in U\}$$

In general, Type-n Plithogenic Set

$$PS_k = \{x(d_k(...d_2(d_1(P_1))...), d_k(...d_2(d_1(P_2))...), ..., d_k(...d_2(d_1(P_n))...), x \in U\}.$$

4. Hybridization of Classical, Fuzzy, and Fuzzy Extension Sets

The real applications require many times to deal with multiple types of classical, fuzzy, and fuzzy extension sets.

Assume that, starting from a neutrosopohic element of the form x(T, I, F), with $0 \le T + I + F \le 3$, one has be combined it with a Picture_Fuzzy form (T, N, F), with $0 \le T + N + F \le 1$, then one gets: the neutrosophic-picture_fuzzy hybrid form: $((TT, TN, TF), (IT, IN, IF), (FT, FN, FF))$,

with $0 \le TT + TN + TF \le 1, 0 \le IT + IN + IF \le 1, 0 \le FT + FN + FF \le 1,$

where T was split into TT, TN, TF representing the confidence in T, neutral-confidence in T, and nonconfidence in T respectively; similarly for I and F.

Further on, let's combine the result with the Spherical_Fuzzy Set, where the sum of squares of components is between 0 and 1, then one obtains a neutrosophic-picture_fuzzy-spherical_fuzzy hybrid form: $((TT, TN, TF), (IT, IN, IF), (FT, FN, FF))$, with

$$0 \le TT^2 + TN^2 + TF^2 \le 1, 0 \le IT^2 + IN^2 + IF^2 \le 1, 0 \le FT^2 + FN^2 + FF^2 \le 1.$$

The hybridization chain may be as long as needed, and may deal with various types of classical, fuzzy, and fuzzy extension sets – including repeated types.

5. Definitions of Symbolic Plithogenic Set & Symbolic Plithogenic Algebraic Structures

Let *SPS* be a non-empty set, included in a universe of discourse *U*, defined as follows:

$SPS = \{x \mid x = a_0 + a_1P_1 + a_2P_2 + ... + a_nP_n, n \ge 1, a_i \in R$ or $a_i \in C$ or a_i belong to some given algebraic structure, for $0 \le i \le n$,

where R = the set of real numbers, C = the set of complex numbers, and all P_i are letters (or variables) and are called *Symbolic (Literal) Plithogenic Components (Variables)*}, where *1, P1, P2, …, Pn* act like a base for the elements of the above set *SPS*.

$a_0, a_1, a_2, \ldots, a_n$ are called coefficients.

SPS is called a Symbolic Plithogenic Set. And the algebraic structures defined on this set are called *Symbolic Plithogenic Algebraic Structures.*

In general, Symbolic (or Literal) Plithogenic Theory is referring to the use of abstract symbols {i.e. the letters/parameters) *P1, P2, …, Pn*, representing the plithogenic components (variables) as above} in some theory.

6. Definition of Plithogenic Numbers *(PN)*

The numbers of the form $PN = a_0 + a_1P_1 + a_2P_2 + ... + a_nP_n$ defined as above are called Plithogenic Numbers, where a_nP_n is called the *leading (strongest) term*.

7. Prevalence Order *(PO)*

The experts establish a *prevalence order* [1], or total order, according to the importance of each attribute/parameter (P_i) into the application. To obtain a total order among the symbolic plithogenic components $\{P_1, P_2, ..., P_n\}$, one defines some relationships (laws) between them.

The most used one is the absorbance law.

8. Absorbance Law

We recall and use now our 2015 Absorbance Law [1], simply defined as:

the greater absorbs the smaller [the bigger fish eats the smaller fish].

9. Multiplication and Power of Symbolic Plithogenic Components under the Absorbance Law

We assume that in the above definition of the plithogenic numbers, the symbolic plithogenic components are ranked increasingly, or

$$P_1 < P_2 < ... < P_n \quad (\textit{prevalence order})$$

where "<" may signify: smaller, less important, under, inferior, etc.

Whence, the multiplication and power of symbolic plithogenic components are:

$P_i \cdot P_j = P_{\max\{i,j\}}$, whence $(P_i)^2 = P_i$.

In general, $P_{i_1} \cdot P_{i_2} \cdot \ldots \cdot P_{i_k} = P_{\max\{i_1, i_2, \ldots, i_k\}}$ and $(P_i)^m = P_i$ for integer $m \geq 1$.

Negative powers of Symbolic Plithogenic Components do not exist, $\left(P_i\right)^{-m} = \dfrac{1}{\left(P_i\right)^m}$ does not exist. For example, $\left(P_i\right)^{-1} = \dfrac{1}{P_i}$ does not exist.

And P_i to the power zero is equal to 1 by definition: $\left(P_i\right)^0 \triangleq 1$.

10. m-th Root of the Symbolic Plithogenic Components

$\sqrt[m]{P_i} = P_i, 1 \leq i \leq n$, for integer $m \geq 2$, because $\left(\sqrt[m]{P_i}\right)^m = (P_i)^m$, or $P_i = P_i$.

$\left(\sqrt[m]{P_i}\right)^m$ cannot be equal to P_{i-1} or lower, nor P_{i+1} or upper, because the last two raised to the power *m* would not give P_i.

Examples: $\sqrt{P_1} = P_1$, $\sqrt[3]{P_7} = P_7$, $\sqrt[4]{16P_9} = \sqrt[4]{16} \cdot \sqrt[4]{P_9} = 2P_9$.

11. Example of Plithogenic Set

Let's have a classical set

$$S = \{John, George, Mary\},$$

and each element is characterized with respect to the attributes: *Weight (W), Tallness (T), Oldness (()), Beauty (B), Health (H).*

Each person/element has some (classical, fuzzy, or any fuzzy extension) degree (*d*) with respect to each attribute (parameter): d(Weight), d(Tallness), d(Oldness), d(Beauty), d(Health).

And thus one transforms the classical set into a plithogenic set:

$$PS = \{John[d(Weight), d(Tallness), d(Oldness), d(Beauty), d(Health)],$$
$$George[d(Weight), d(Tallness), d(Oldness), d(Beauty), d(Health)],$$
$$Mary[d(Weight), d(Tallness), d(Oldness), d(Beauty), d(Health)]\}.$$

As a numerical example, see below, as evaluated by Expert 1:

$PS_1 = \{John(0.5, 0.6, 0.3, 0.1, 0.7), George(0.1, 0.8, 0.3, 0.1, 0.4), Mary(0.9, 0.4, 0.6, 0.1, 0.2)\}.$

PS_1 is a *Type-1 Plithogenic Set.*

Which means that on some corresponding scales, John's fuzzy degree of Weight is 0.5, fuzzy degree of Tallness 0.6, fuzzy degree of Oldness 0.3, fuzzy degree of Beauty 0.1, and fuzzy degree of Health 0.7. {Of course, one may consider all kind of degrees: not only fuzzy, but also: classical, intuitionistic fuzzy, neutrosophic, refined neutrosophic, and other fuzzy extensions.}

Similarly for George's and Mary's degrees.

12. Example of Type-2 Plithogenic Set

Assume that Expert 2 is not totally confident on the evaluation of the Expert 1 in the above example. Thus, he decides to evaluate the first evaluation. Expert 2 may, as well, use any types of degrees – according to the expert desire and tools, not necessarily the same as in the previous evaluation.

For the sake of simplicity, let's consider that Expert 2 also uses fuzzy degrees. Now one gets a Type-2 Plithogenic Set:

$$PS_2 = \{John[0.5(0.9), 0.6(0.4), 0.3(1.0), 0.1(0.0), 0.7(0.8)],$$
$$George[0.1(0.3),\ 0.8(0.4),\ 0.3(0.5),\ 0.1(0.7),\ 0.4(0.9)],$$
$$Mary[0.9(0.1),\ 0.4(0.5),\ 0.6(0.6),\ 0.1(0.8),\ 0.2(0.9)]\}$$

Which are interpreted as follows:

0.5(0.9) means that Expert 2 is 0.9 (90%) confident in John's fuzzy degree of Weight of 0.5 assigned by Expert 1;

0.6(0.4) means that Expert 2 is 0.4 (40%) confident in John's fuzzy degree of Tallness of 0.6 assigned by Expert 1;

0.3(1.0) means that Expert 2 is 1.0 (100%) confident in John's fuzzy degree of Oldness of 0.3 assigned by Expert 1;

0.1(0.0) means that Expert 2 is 0.0 (0%) confident in John's fuzzy degree of Beauty of 0.1 assigned by Expert 1;

0.7(0.8) means that Expert 2 is 0.8 (80%) confident in John's fuzzy degree of Health of 0.7 assigned by Expert 1.

And similarly for George's and Mary's second round of degrees.

13. Example of Type-3 Plithogenic Set

The process may go on and have an Expert 3 evaluate the Expert 2. Assume Expert 3 uses neutrosophic degrees.

$$PS_3 = \{John\{0.5[0.9(0.6,0.7,0.3)],0.6[0.4(0.6,0.7,0.3)],$$
$$0.3[1.0(0.6,0.7,0.3)],0.1[0.0(0.6,0.7,0.3)],0.7[0.8(0.6,0.7,0.3)]\},$$
$$George[0.1[0.3(0.4,0.4,0.06)],\ 0.8[0.4(0.9,0.1,0.03)],\ 0.3[0.5(0.9,1.0,0.2)],$$
$$0.1[0.7(0.7,0.3,0.6)],\ 0.4[0.9(0.1,0.0,0.4)],$$
$$Mary\{0.9[0.1(0.2,0.3,0.4)],\ 0.4[0.5(0.7,0.8,0.7)],$$
$$0.6[0.6(1.0,0.0,0.0)],\ 0.1[0.8(0.1,0.4,0.6)],\ 0.2[0.9(0.0,0.0,0.0)]\}$$

Therefore,
$0.5[0.9(0.6,0.7,0.3)]$
means that Expert 3 assigns the neutrosophic degrees of truth = 0.6, indeterminacy = 0.7, and falsehood = 0.3, to the Expert 2's evaluation of 0.9 (90%) confidence on Expert 1's evaluation of 0.5 degree of John's Weight.
And so on for all others.

One may generalize to **Type-k Plithogenic Set**, recurrently going on from a type to the next type, but it becomes more sophisticated and not usable in practice.

14. Example of Symbolic Plithogenic Numbers

The corresponding Symbolic Plithogenic Algebraic Structure is based on the symbolic (or literal) plithogenic components *W, T, O, B, H*, and we get the **plithogenic numbers** (*PN*) of the form:

$$PN = a + bW + cT + dO + eB + fH,$$

where *a, b, c, d, e, f* are real, or complex numbers, or they may belong to a set of a given classical algebraic structure. As a particular example, let $PN_1 = 2 - 3W + 5T - O + 6B - 4H$.

In this example, let's assume that the *prevalence order* is:

$$W < T < O < B < H, \text{ where "<" means "less important",}$$

or *W* is less important than *T*, which is less important than *O*, which is less important that B, which is less important than *H*.

The *absorbance law* is defined as follows: the most important absorbs the less important in the multiplication operation, for example $W \cdot T = T$, since *T* absorbs *W* because *T* is more important (bigger) than *W*. Similarly for the other multiplications.

15. Operations with Plithogenic Numbers

Let's consider two plithogenic numbers:

$$PN_1 = a_0 + a_1P_1 + a_2P_2 + ... + a_nP_n$$
$$PN_2 = b_0 + b_1P_1 + b_2P_2 + ... + b_nP_n.$$

15.1. Addition of Plithogenic Numbers

$$PN_1 + PN_2 = (a_0 + b_0) + \sum_{i=1}^{n}(a_i + b_i)P_i$$

15.2. Subtraction of Plithogenic Numbers

$$PN_1 - PN_2 = (a_0 - b_0) + \sum_{i=1}^{n}(a_i - b_i)P_i$$

(SPS, +) is a Symbolic Plithogenic Commutative Group

15.3. Scalar Multiplication of Plithogenic Numbers

$$c \cdot PN_1 = c \cdot (a_0 + a_1P_1 + a_2P_2 + ... + a_nP_n) = c \cdot a_0 + c \cdot a_1P_1 + c \cdot a_2P_2 + ... + c \cdot a_nP_n$$

15.4. Multiplication of and Ppower of Plithogenic Numbers

$PN_1 \cdot PN_2 = (a_0 + a_1P_1 + a_2P_2 + ... + a_nP_n) \cdot (b_0 + b_1P_1 + b_2P_2 + ... + b_nP_n)$

and then one multiplies them, term by term $(a_iP_i) \cdot (a_jP_j) = a_i \cdot a_j \cdot P_{\max\{i,j\}}$, where $\cdot$ is the classical multiplication, as in classical algebra, using the above multiplication of symbolic plithogenic components.

As particular case: $0 \cdot P_i = 0$.

$(SPS, +, \cdot)$ is a Symbolic Plithogenic Commutative Ring, with the plithogenic unitary element: $1_* \equiv 1 + 0 \cdot P_1 + 0 \cdot P_2 + ... + 0 \cdot P_n$.

The symbolic plithogenic components P_i's are not inversible, therefore the elements of SPS are non-inversible (except the plithogenic unitary element 1_*).

$(PN_1)^m = \underbrace{PN_1 \cdot PN_1 \cdot ... \cdot PN_1}_{m-times}$ for integer $m \geq 1$;

The negative power of a plithogenic number $(PN_1)^{-m}$ does not exist.

15.5. Alternative Multiplication of Plithogenic Numbers

$PN_1 \otimes PN_2 = a_0 \cdot b_0 + a_1 \cdot b_1 \cdot P_1 + a_2 \cdot b_2 \cdot P_2 + ... + a_n \cdot b_n \cdot P_n$.

$(SPS, +, \otimes)$, is a Symbolic Plithogenic Commutative Ring, with the unitary element: $1_\otimes \equiv 1 + 1 \cdot P_1 + 1 \cdot P_2 + ... + 1 \cdot P_n$.

The plithogenic numbers that have coefficients equal to zero do not have an inverse, for example: $2 + 3P_1 - 5P_3 = 2 + 3P_1 + 0P_2 - 5P_3$ is not inversible.

15.6. Division of Symbolic Plithogenic Components

$$\frac{P_i}{P_j} = \begin{Bmatrix} x_0 + x_1P_1 + x_2P_2 + ... + x_jP_j + P_i & x_0 + x_1 + ... + x_j = 0 & i > j \\ x_0 + x_1P_1 + x_2P_2 + ... + x_iP_i & x_0 + x_1 + ... + x_i = 1 & i = j \\ \phi & & i < j \end{Bmatrix}$$

where all coefficients $x_0, x_1, x_2, ...x_i, ..., x_j, ... \in SPS$.

There are j-tuple infinities of quotients when i > j,
also i-tuple infinities of quotients when i = j,
and no quotient (indeterminate division) when i < j.
Therefore, the operation of division d(. , .) of symbolic plithogenic components
$d(P_i, P_j) : \{P_1, P_2, ..., P_n\}^2 \to SPS$
is a NeutroOperation, because:
it is well-defined (inner-defined) for no elements, since one never gets a single quotient, or $d(P_i, P_j) \notin SPS$;

it is indeterminate (cannot be calculated) for some elements (when $P_i < P_j$) with $d(P_i, P_j)$ being indeterminate;

and outer-defined (when $P_i = P_j$ and $P_i > P_j$) with $d(P_i, P_j) \notin SPS$

but $d(P_i, P_j) \in P(SPS)$ the powerset of *SPS*.

15.6.1. Example 1 of Division of Symbolic Plithogenic Components

$i > j$ [j-tuple infinities of quotients]
Let's divide P_5 by P_2.
$\frac{P_5}{P_2} = x$

where $x = x_0 + x_1P_1 + x_2P_2 + ... + x_nP_n \in SPS$.

$P_5 = x \cdot P_2$

Since the multiplication $x \cdot P_2$ should not exceed P_5 we take $n = 5$ into the formula of x, or

$$x \cdot P_2 = (x_0 + x_1P_1 + x_2P_2 + x_3P_3 + x_4P_4 + x_5P_5) \cdot P_2$$
$$= x_0P_2 + x_1P_1P_2 + x_2P_2P_2 + x_3P_3P_2 + x_4P_4P_2 + x_5P_5P_2$$
$$= (x_0 + x_1 + x_2)P_2 + x_3P_3 + x_4P_4 + x_5P_5$$
$$\equiv P_5 \equiv 0 + 0P_1 + 0P_2 + 0P_3 + 0P_4 + 1P_5$$

Therefore, $x_5 = 1$, $x_4 = 0$, $x_3 = 0$, $x_0 + x_1 + x_2 = 0$.

Whence, $\frac{P_5}{P_2} = x_0 + x_1P_1 + x_2P_2 + P_5$, with $x_0 + x_1 + x_2 = 0$, and the coefficients $x_0, x_1, x_2 \in SPS$ [2-tuple infinities of quotients].

15.6.2. Example 2 of Division of Symbolic Plithogenic Components

$i = j$ [i-tuple infinities of quotients]

$\frac{P_3}{P_3} = x$ or

$$P_3 = P_3 \cdot x = P_3 \cdot (x_0 + x_1P_1 + x_2P_2 + x_3P_3) = x_0P_3 + x_1P_1P_3 + x_2P_2P_3 + x_3P_3P_3$$
$$= x_0P_3 + x_1P_3 + x_2P_3 + x_3P_3 = (x_0 + x_1 + x_2 + x_3)P_3 \equiv 1 \cdot P_3$$

whence $x_0 + x_1 + x_2 + x_3 = 1$.

Thus, $\frac{P_3}{P_3} = x_0 + x_1P_1 + x_2P_2 + x_3P_3$,

where $x_0 + x_1 + x_2 + x_3 = 1$, and the coefficients $x_0, x_1, x_2, x_3 \in SPS$

15.6.3. Example 3 of Division of Symbolic Plithogenic Components

$i < j$ [indeterminate, no quotient]

$\frac{P_2}{P_4} = x$ or $P_2 = P_4 \cdot x \geq P_4 > P_2$ or $P_2 > P_2$, which is impossible.

This multiplication, P_4 times any of $1, P_1, P_2, \ldots, P_n$, will give a result that is greater than or equal to P_4 according to the absorbing law.

This division is undefined (indeterminate).

15.7. Division of Symbolic Plithogenic Numbers

Let consider two symbolic plithogenic numbers as below:

$PN_3 = a_0 + a_1P_1 + a_2P_2 + ... + a_rP_r$ and $PN_4 = b_0 + b_1P_1 + b_2P_2 + ... + b_sP_s$

where $r, s \leq n$ are positive integers, and the leading coefficients (the coefficients of the highest/largest symbolic plithogenic components P_r and respectively P_s) are nonnull, $a_r \neq 0, b_s \neq 0$. The division is also based on the absorbance law.

$$\frac{PN_r}{PN_s} = \frac{a_0 + a_1P_1 + a_2P_2 + ... + a_rP_r}{b_0 + b_1P_1 + b_2P_2 + ... + b_sP_s} = x$$

We need to find $x \in SPS$.

$$a_0 + a_1P_1 + a_2P_2 + ... + a_rP_r \equiv x \cdot (b_0 + b_1P_1 + b_2P_2 + ... + b_sP_s)$$

We are focusing first on the division of their leading symbolic plithogenic components: $\frac{P_r}{P_s}$ as we did on the previous section. Of course the leading coefficients $a_r \neq 0, b_s \neq 0$.

$\frac{PN_r}{PN_s} = \begin{cases} none, one, many & r \geq s \\ \phi & r < s \end{cases}$ This is also a NeutroOperation since one has indeterminate cases.

For $r \geq s$ there may be: none, one, or many (including infinitely many) quotients.

For r < s no quotient. Indeterminacy.

We prove these through several examples:

15.7.1. Example 1 (no quotient)

$$\frac{P_1+1}{P_1} = ?$$

$$\frac{P_1+1}{P_1} = x = (x_0 + x_1P_1), \text{ we need to solve for } x \text{ (actually for } x_0 \text{ and } x_1).$$

$$P_1+1 = (x_0 + x_1P_1)\cdot P_1 = x_0P_1 + x_1P_1P_1 = x_0P_1 + x_1P_1 = (x_0+x_1)P_1$$

We may set $x_0 + x_1 = 1$, but we are not able to catch the free coefficient 1 from the left-hand side, i.e.

$$P_1 + 1 \neq P_1$$

15.7.2. Example 3 (one quotient only)

$$\frac{P_1+2}{P_1+1} = ?$$

$$\frac{P_1+2}{P_1+1} = x = x_0 + x_1P_1$$

whence

$$P_1 + 2 = (x_0 + x_1P_1)\cdot(P_1+1) = x_0P_1 + x_1P_1P_1 + x_0 + x_1P_1$$

$$= x_0P_1 + x_1P_1 + x_0P_1 + x_1P_1 = x_0 + (x_0 + 2x_1)P_1$$

we get

$x_0 = 2, x_0 + 2x_1 = 1,$ then $x_1 = -0.5$.

There is only one quotient (solution): $x_0 + x_1P_1 = 2 - 0.5P_1 = -0.5P_1 + 2$.

Let's check the result:

$$(P_1+1)\cdot(-0.5P_1+2) = -0.5P_1P_1 + 2P_1 - 0.5P_1 + 2 = -0.5P_1 + 2P_1 - 0.5P_1 + 2$$

$$= P_1 + 2.$$

15.7.3. Example 3 (double infinities of quotients (solutions))

$$\frac{5P_3}{P_3 - 2P_2} = ?$$

$$\frac{5P_3}{P_3 - 2P_2} = x = x_0 + x_1P_1 + x_2P_2 + x_3P_3.$$

We need to find the coefficients x_0, x_1, x_2, x_3.

$$5P_3 = (x_0 + x_1P_1 + x_2P_2 + x_3P_3)\cdot(P_3 - 2P_2)$$

$$= (x_0 + x_1P_1 + x_2P_2 + x_3P_3)\cdot P_3 + (x_0 + x_1P_1 + x_2P_2 + x_3P_3)\cdot(-2P_2)$$

$$= (x_0 + x_1 + x_2 + x_3)P_3 - (2x_0 + 2x_1 + 2x_2)P_2 - 2x_3P_3$$

$$= (x_0 + x_1 + x_2 - x_3)P_3 - (2x_0 + 2x_1 + 2x_2)P_2 \equiv 5P_3 + 0P_2$$

Whence we get two equations:

$$x_0 + x_1 + x_2 - x_3 = 5$$

$$2x_0 + 2x_1 + 2x_2 = 0$$

Hence $2(x_0 + x_1 + x_2) = 0$, or $x_0 + x_1 + x_2 = 0$.

Replace it into the first equation:

$0 - x_3 = 5$, then $x_3 = -5$.

$\frac{5P_3}{P_3 - 2P_2} = x_0 + x_1P_1 + x_2P_2 + x_3P_3 = x_0 + x_1P_1 + x_2P_2 - 5P_3,$

where $x_0 + x_1 + x_2 = 0$.

15.8. **m-th Root of the Plithogenic Number**

$\sqrt[m]{PN_1} = \sqrt[m]{a_0 + a_1P_1 + a_2P_2 + ... + a_nP_n} = x_0 + x_1P_1 + x_2P_2 + ... + x_nP_n$.

We need to find the coefficients $x_0, x_1, x_2, ..., x_n$.

Raising to the power *m* both sides, one gets:

$a_0 + a_1P_1 + a_2P_2 + ... + a_nP_n = (x_0 + x_1P_1 + x_2P_2 + ... + x_nP_n)^m$, where $x_0, x_1, x_2, ..., x_n$ are coefficients that we need to find out. After raising to the power *k* the right-hand side, we identify the coefficients two by two.

The *m-root* of a plithogenic number may have: no solution, several solutions, or infinitely many solutions.

Example 1 of *m-th* Root of the Plithogenic Number with real coefficients (several solutions)

$\sqrt{4 - 3P_1} = ?$

$\sqrt{4 - 3P_1} = x_0 + x_1P_1$, where we need to find x_0 and x_1.

Raise both sides to the second power:

$(\sqrt{4 - 3P_1})^2 = (x_0 + x_1P_1)^2$ or

$4 - 3P = (x_0)^2 + 2x_0x_1P_1 + (x_1)^2(P_1)^2 = (x_0)^2 + 2x_0x_1P_1 + (x_1)^2P_1$

$= (x_0)^2 + [2x_0x_1 + (x_1)^2]P_1 \equiv 4 - 3P_1$

Identify the coefficients:

$$\begin{cases} (x_0)^2 = 4 \\ 2x_0x_1 + (x_1)^2 = -3 \end{cases}$$

Whence $x_0 = 2, -2$ from the first equation. Replaced into the second equation one gets:

$\pm 4x_1 + (x_1)^2 = -3$, or two quadratic equations $(x_1)^2 \pm 4x_1 + 3 = 0$ that we need to solve.

For $x_0 = 2$, $(x_1)^2 + 4x_1 + 3 = 0$, has the solutions $x_1 = -1, -3$,

thus $(x_0, x_1) = (2, -1)$ or $(2, -3)$.

For $x_0 = -2$, $(x_1)^2 - 4x_1 + 3 = 0$, has the solutions $x_1 = 1, 3$,

thus $(x_0, x_1) = (-2, 1)$ or $(-2, 3)$.

Final answer:

$\sqrt{4 - 3P_1} = x_0 + x_1P_1 = 2 - P_1, 2 - 3P_1, -2 + P_1, -2 + 3P_1$ (four solutions).

Example 2 of m-th Root of the Plithogenic Number with real coefficients (no solution)

$\sqrt{-4 - 3P_1}$ has no solution since one gets, in the above calculation $(x_0)^2 = -4$, which does not work in the set of real numbers.

15.9. Remark 1

Other operations may be constructed on the Symbolic Plithogenic Set (SPS), giving birth to various symbolic plithogenic algebraic structures.

15.10. Remark 2

All previous operations are valid for the absorbance law and prevalence order defined above. If different law and order are defined by the experts, then different operations and results one gets.

16. Particular Cases of Symbolic Plithogenic Algebraic Structures

16.1. Neutrosophic Quadruple Numbers

Let's consider an entity (i.e. a number, an idea, an object, etc.) which is represented by a known part (a) and an unknown part ($bT + cI + dF$).

Numbers of the form

$NQ = a + bT + cI + dF$,

where *a, b, c, d* are real (or complex) numbers (or intervals, or in general subsets), and

T = truth / membership / probability,

I = indeterminacy / neutrality,

F = false / membership / improbability,

are called Neutrosophic Quadruple (Real respectively Complex) Numbers (or Intervals, or in general Subsets) [1].

"a" is called the known part of NQ,

while "$bT + cI + dF$" is called the unknown part of NQ.

Neutrosophic Quadruple Numbers [1] are particular case of the Plithogenic Numbers, since one takes $n = 3$, and P_1, P_2, P_3 are more general than *T, I,* and *F* respectively.

16.2. Refined Neutrosophic Quadruple Numbers

The Refined Neutrosophic Quadruple Numbers [1, 7] have the form:

$$RQN = a + \sum_{j=1}^{p} b_j T_j + \sum_{k=1}^{r} c_k I_k + \sum_{l=1}^{s} d_l F_l$$

where a, all b_i, all c_j, and all d_k are real (or complex) numbers, intervals, or, in general, subsets, while $T_1, T_2, \dots, T_p$ are refinements of T;

$I_1, I_2, \dots, I_r$ are refinements of I;

and $F_1, F_2, \dots, F_s$ are refinements of F,

for integers $p, r, s \geq 0$ and at least one of them be ≥ 2, with $p + r + s = n$.

Refined Neutrosophic Quadruple Numbers are also particular case of the Plithogenic Numbers, since instead of symbolic sub-truths / sub-indeterminacies / sub-falsehoods T_j, I_k, F_l one may use all kinds of symbolic plithogenic components $P_1, P_2, \dots, P_n$.

All, Neutrosophic Quadruple Numbers and Refined Neutrosophic Quadruple Numbers, together with the Prevalence Order and Absorbance Law, were introduced by Smarandache [1] in 2015.

16.3. (Symbolic) Turiyam Set

Turiyam Set (*TS*) was introduced by P. K Singh [9] in 2021, who added to the neutrosophic components *T (Truth), I (Indeterminacy), F(Falsehood),* another component *Y* (called state of *awareness*).

According to him, Turiyam component (*Y*) means: "Rejection of both acceptation and rejection of attribute at the given time i.e. unknown region (*l*). It needs Turiyam consciousness to explore it" [9].

Turiyam Set is very similar to Belnap's Logic, based on: True (*T*), False (*F*), Unknown (*U*), and Contradiction (*C*), where *T, F, U, C* are taken as symbols, not numbers. Belnap's Logic is a particular case of Refined Neutrosophic Logic [10].

Turiyam Set was defined as:

$TS = \{(a_0, a_1T, a_2F, a_3I, a_4Y), a_i \in A\}$, where *A* is a given set, or it is the set of a given classical algebraic structure.

The Symbolic Turiyam Numbers have the form:

$STN = a_0 + a_1T + a_2F + a_3I + a_4Y$

where $a_i \in A$.

It is clear that Turiyam Set (2021) is a particular case of the Plithogenic Set, because one replaces $n = 4$, and P_1, P_2, P_3, P_4 by T, F, I, Y respectively, since the symbolic plithogenic components may be either independent, or dependent, or partially independent/dependent as we desire.

The operations on *TS* were defined as particular cases to Smarandache's 2015 neutrosophic quadruple numbers and absorbance law [1] and 2019 symbolic plithogenic numbers [5].

Let

$x = (a_0, a_1T, a_2F, a_3I, a_4Y) = a_0 + a_1T + a_2F + a_3I + a_4Y$

$y = (b_0, b_1T, b_2F, b_3I, b_4Y) = b_0 + b_1T + b_2F + b_3I + b_4Y$

be two STNs, and c be a scalar.

Then the addition

$x + y = (a_0 + b_0, (a_1 + b_1)T, (a_2 + b_2)F, (a_3 + b_3)I, (a_4 + b_4)Y)$

$= (a_0 + b_0) + (a_1 + b_1)T + (a_2 + b_2)F + (a_3 + b_3)I + (a_4 + b_4)Y$

The multiplication of the symbolic components T, F, I, Y were more complicated listed in [12], as:

$T \cdot T = T^2 = T, F \cdot F = F^2 = F, I \cdot I = I^2 = I, Y \cdot Y = Y^2 = Y, T \cdot Y = Y \cdot T = Y,$

$T \cdot F = F \cdot T = F, T \cdot I = I \cdot T = I, I \cdot Y = Y \cdot I = I, F \cdot Y = Y \cdot F = Y, F \cdot I = I \cdot F = I.$

While using the absorbance law (the stronger absorbs the weaker) and the prevalence order T < F < I < Y (as chosen by author Singh [12]) it would have been much simpler.

Multiplication of STNs:

$x \cdot y = (a_0 + a_1T + a_2F + a_3I + a_4Y) \cdot (b_0 + b_1T + b_2F + b_3I + b_4Y)$

Then similarly multiply them term by term, taking into consideration the multiplication of symbolic components T, F, I, Y as explained above.

Scalar Multiplication in the similar way:

$c \cdot x = c \cdot (a_0 + a_1T + a_2F + a_3I + a_4Y) = c \cdot a_0 + c \cdot a_1T + c \cdot a_2F + c \cdot a_3I + c \cdot a_4Y$.

Consequently, the Symbolic Turiyam Group [11] and Symbolic Turiyan Ring [12], as algebraic structures, are particular cases of the **Symbolic Plithogenic Commutative Group** (defined above in sections 15.1 & 15.2), and respectively **Symbolic Plithogenic Commutative Ring** (defined above in sections 15.4 or 15.5).

17. Practical Application

Since the cases $n = 3$ and 4 of Symbolic Plithogenic Algebraic Structures have been investigated, the reader may try to develop it for the case when $n = 5$, using Hexagonal Plithogenic Numbers (*HPN*), hexa since the dimension of *HPN* is *5 + 1 = 6* because one has 6 vectors into the base: $1, P_1, P_2, P_3, P_4, P_5$.

$HPN = a_0 + a_1P_1 + a_2P_2 + a_3P_3 + a_4P_4 + a_5P_5$, where all coefficients a_i belong to a given set.

As practical application, for example, assume that the parameters represent various colors, C_1, C_2, C_3, C_4, C_5, then we denote it as:

$HPN = a_0 + a_1C_1 + a_2C_2 + a_3C_3 + a_4C_4 + a_5C_5$.

As multiplication law of the symbolic plithogenic components C_i with C_j one adopts a law from the real world. For example, if C_1 = *yellow*, and C_2 = *red*, then it makes sense to consider $C_1 \cdot C_2$ = *pink* (because *yellow* mixed with *red* give *pink*), and so on.

In this practical application, the absorbance law does not work, that's why one designs a new law in order to be able to multiply the components.

18. Open Question

Future possible study for researchers would be to investigate the *infinite-case*, we mean when each element in the plithogenic set (section 2 above) is characterized by infinitely many attributes (parameters), and similarly the symbolic plithogenic numbers (section 3 above) have infinitely many symbolic plithogenic components $P_1, P_2, ..., P_\infty$ and, eventually, their applications.

19. Conclusion

In this paper, the new types of algebraic structures from 2018-2019, called Symbolic Plithogenic Algebraic Structures, were revisited, and afterwards compared to other related structures.

We proved that the Symbolic Plithogenic Numbers are generalizations of Neutrosophic Quadruple Numbers, Refined Neutrosophic Quadruple Numbers, and Symbolic Turiyam Numbers.

Consequently, the Symbolic Plithogenic algebraic structures (semigroup, group, ring, etc.) are generalization of the corresponding algebraic structures built on these particular cases described above. We recalled the Symbolic Plithogenic Group and Ring.

Many examples and practical applications were also revealed.

Any future application may require a special multiplication law of the components and of plithogenic numbers that the experts should design themselves.

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Study for General Plithogenic Soft Expert Graphs

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Abstract

Graph theory, a branch of mathematics, focuses on studying graphs that model relationships between objects through vertices and edges [29]. The Turiyam Neutrosophic Graph handles uncertainty by assigning four values to vertices and edges: truth, indeterminacy, falsity, and liberal state. In contrast, the Plithogenic Graph is a general structure where vertices and edges are defined by degrees of appurtenance and contradiction across multiple attributes [45]. The General Plithogenic Graph extends the Plithogenic Graph, offering a broader framework [45]. This study presents new concepts of the General Plithogenic Soft Expert Graph and the Turiyam Neutrosophic Soft Expert Graph, with the former expanding the existing Soft Expert Graph model into a more generalized structure.

Keywords: Neutrosophic Graph; Fuzzy Graph; Plithogenic Graph; Soft Expert Graph.

1 | Introduction

1.1 | Uncertain Graph

Graph theory, a branch of mathematics, explores the study of graphs that model relationships between objects using vertices and edges [29]. It has found extensive applications in both mathematical and real-world contexts [38, 64, 74, 76, 94]. Recently, graph theory has also been widely utilized in artificial intelligence research [23 96, 114, 119].

To address real-world uncertainty, concepts like Fuzzy Sets [116] were introduced, later extended to Neutrosophic Sets [98] and other frameworks, leading to various applications across multiple fields. This paper examines several models of uncertain graphs-namely, Fuzzy, Intuitionistic Fuzzy, Neutrosophic, Turiyam, and Plithogenic Graphs-designed to handle uncertainty in diverse contexts. These models, collectively referred to as uncertain graphs, extend classical graph theory by integrating various levels of uncertainty. Due to their significance, numerous related graph classes and applications have been developed [37, 39, 41–44, 48, 50, 52–56]. In addition to graph models, foundational concepts like Fuzzy Sets and Neutrosophic Sets have been extensively explored and are widely recognized in the literature [10–14, 26, 30–32, 34, 73, 75, 83, 98, 116, 117].

This paper specifically focuses on the Turiyam Neutrosophic Graph and the General Plithogenic Graph. The Turiyam Neutrosophic Graph represents uncertainty using four values for vertices and edges: truth, indeterminacy, falsity, and liberal state. Conversely, the Plithogenic Graph is a general structure in which

vertices and edges are characterized by degrees of appurtenance and contradiction across multiple attributes [45]. The General Plithogenic Graph is an extension of the Plithogenic Graph, providing a broader framework [45]. Note that the Turiyam Neutrosophic Set is actually a particular case of the quadripartitioned Neutrosophic Set, by replacing "Contradiction" with "Liberal" [97]. The corresponding graph concept known as quadripartitioned neutrosophic graphs is well-documented [67,68].

1.2 | Soft Expert Graph

A Soft Expert Graph combines graph theory with expert evaluations, where vertices and edges are associated with fuzzy sets, and experts' inputs define uncertainty and relationships within the graph. Related concepts include the Fuzzy Soft Expert Graph [92], Intuitionistic Fuzzy Soft Expert Graph [109], and Neutrosophic Soft Expert Graph [111]. These models have been studied for applications such as multi-criteria decision-making (e.g., [15, 33, 88, 106, 108]). Additionally, related concepts such as Soft Expert Sets are also well-known [6, 9, 84, 91].

1.3 | Contributions

Building upon the research of Uncertain Graphs and Soft Expert Graphs, this study introduces and analyzes new concepts of the General Plithogenic Soft Expert Graph and the Turiyam Neutrosophic Soft Expert Graph. The General Plithogenic Soft Expert Graph extends the existing Soft Expert Graph model into a more generalized framework.

2 | Preliminaries and Definitions

In this section, we present a brief overview of the definitions and notations used throughout this paper.

2.1 | Basic Graph Concepts

Here are a few basic graph concepts listed below. For more foundational graph concepts and notations, please refer to [27, 28-29, 64, 113].

Definition 1 (Graph). [29] A graph G is a mathematical structure consisting of a set of vertices $V(G)$ and a set of edges $E(G)$ that connect pairs of vertices, representing relationships or connections between them. Formally, a graph is defined as $G = (V, E)$, where V is the vertex set and E is the edge set.

Definition 2 (Degree). [29] Let $G = (V, E)$ be a graph. The degree of a vertex $v \in V$, denoted $\deg(v)$, is the number of edges incident to v. Formally, for undirected graphs:

$$\deg(v) = |\{e \in E \mid v \in e\}|$$

In the case of directed graphs, the in-degree $\deg^-(v)$ is the number of edges directed into v, and the out-degree $\deg^+(v)$ is the number of edges directed out of v.

2.2 | Uncertain Graph

This paper addresses Fuzzy, Intuitionistic Fuzzy, Neutrosophic, Turiyam, and Plithogenic concepts within the framework of Unified Uncertain Graphs. Note that Turiyam Neutrosophic Set is actually a particular case of the Quadruple Neutrosophic Set, by replacing "Contradiction" with "Liberal" 97].

Definition 3 (Unified Uncertain Graphs Framework). (cf.[51]) Let $G = (V, E)$ be a classical graph with a set of vertices V and a set of edges E. Depending on the type of graph, each vertex $v \in V$ and edge $e \in E$ is assigned membership values to represent various degrees of truth, indeterminacy, falsity, and other nuanced measures of uncertainty.

1. Fuzzy Graph [18,40,60,61,72,77,80,89,90,104,112]:
    - Each vertex $v \in V$ is assigned a membership degree $\sigma(v) \in [0,1]$.

- Each edge $e = (u, v) \in E$ is assigned a membership degree $\mu(u, v) \in [0,1]$.

2. Intuitionistic Fuzzy Graph (IFG) 1 1, 17, 24, 70,78,107,110,118:
   - Each vertex $v \in V$ is assigned two values: $\mu_A(v) \in [0,1]$ (degree of membership) and $v_A(v) \in [0,1]$ (degree of non-membership), such that $\mu_A(v) + v_A(v) \leq 1$.
   - Each edge $e = (u, v) \in E$ is assigned two values: $\mu_B(u, v) \in [0,1]$ and $v_B(u, v) \in [0,1]$, with $\mu_B(u, v) + v_B(u, v) \leq 1$.
3. Neutrosophic Graph [4,5,22,35,36,45,47,49,53,56,65,66,71,93,101,102]:
   - Each vertex $v \in V$ is assigned a triplet $\sigma(v) = (\sigma_T(v), \sigma_I(v), \sigma_F(v))$, where $\sigma_T(v), \sigma_I(v), \sigma_F(v) \in [0,1]$ and $\sigma_T(v) + \sigma_I(v) + \sigma_F(v) \leq 3$.
   - Each edge $e = (u, v) \in E$ is assigned a triplet $\mu(e) = (\mu_T(e), \mu_I(e), \mu_F(e))$.
4. Turiyam Neutrosophic Graph [57-59]:
   - Each vertex $v \in V$ is assigned a quadruple $\sigma(v) = (t(v), iv(v), fv(v), lv(v))$, where each component is in $[0,1]$ and $t(v) + iv(v) + fv(v) + lv(v) \leq 4$.
   - Each edge $e = (u, v) \in E$ is similarly assigned a quadruple.
5. Vague Graph [2, 3, 19, 21, 86, 87, 95]:
   - Each vertex $v \in V$ is assigned a pair $(\tau(v), \phi(v))$, where $\tau(v) \in [0,1]$ is the degree of truthmembership and $\phi(v) \in [0,1]$ is the degree of false-membership, with $\tau(v) + \phi(v) \leq 1$.
   - The grade of membership is characterized by the interval $[\tau(v), 1 - \phi(v)]$.
   - Each edge $e = (u, v) \in E$ is assigned a pair $(\tau(e), \phi(e))$, satisfying:

$$\tau(e) \leq \min\{\tau(u), \tau(v)\},\ \phi(e) \geq \max\{\phi(u), \phi(v)\}$$

6. Hesitant Fuzzy Graph [16, 63, 81, 82,115]:
   - Each vertex $v \in V$ is assigned a hesitant fuzzy set $\sigma(v)$, represented by a finite subset of $[0,1]$, denoted $\sigma(v) \subseteq [0,1]$.
   - Each edge $e = (u, v) \in E$ is assigned a hesitant fuzzy set $\mu(e) \subseteq [0,1]$.
   - Operations on hesitant fuzzy sets (e.g., intersection, union) are defined to handle the hesitation in membership degrees.
7. Single-Valued Pentapartitioned Neutrosophic Graph [25, 67, 69, 85]:
   - Each vertex $v \in V$ is assigned a quintuple $\sigma(v) = (T(v), C(v), R(v), U(v), F(v))$, where:
     - $T(v) \in [0,1]$ is the truth-membership degree.
     - $C(v) \in [0,1]$ is the contradiction-membership degree.
     - $R(v) \in [0,1]$ is the ignorance-membership degree.
     - $U(v) \in [0,1]$ is the unknown-membership degree.
     - $F(v) \in [0,1]$ is the false-membership degree.
     - $T(v) + C(v) + R(v) + U(v) + F(v) \leq 5$.

- Each edge $e = (u, v) \in E$ is assigned a quintuple $\mu(e) = (T(e), C(e), R(e), U(e), F(e))$, satisfying:

$$\begin{cases} T(e) \leq \min\{T(u), T(v)\} \\ C(e) \leq \min\{C(u), C(v)\} \\ R(e) \geq \max\{R(u), R(v)\} \\ U(e) \geq \max\{U(u), U(v)\} \\ F(e) \geq \max\{F(u), F(v)\} \end{cases}$$

Definition 4. [62, 99, 100, 103, 105] Let $G = (V, E)$ be a crisp graph where V is the set of vertices and $E \subseteq V \times V$ is the set of edges. A Plithogenic Graph PG is defined as:

$$PG = (PM, PN)$$

where:

1. Plithogenic Vertex Set $PM = (M, l, Ml, adf, aCf)$:
   - $M \subseteq V$ is the set of vertices.
   - l is an attribute associated with the vertices.
   - Ml is the range of possible attribute values.
   - adf : $M \times Ml \to [0,1]^s$ is the Degree of Appurtenance Function (DAF) for vertices.
   - $aCf: Ml \times Ml \to [0,1]^t$ is the Degree of Contradiction Function (DCF) for vertices.
2. Plithogenic Edge Set $PN = (N, m, Nm, bdf, bCf)$:
   - $N \subseteq E$ is the set of edges.
   - m is an attribute associated with the edges.
   - Nm is the range of possible attribute values.
   - bdf: $N \times Nm \to [0,1]^s$ is the Degree of Appurtenance Function (DAF) for edges.
   - bCf: Nm $\times Nm \to [0,1]^t$ is the Degree of Contradiction Function (DCF) for edges.

The Plithogenic Graph PG must satisfy the following conditions:

1. Edge Appurtenance Constraint: For all $(x, a), (y, b) \in M \times Ml$:

$$bdf((xy), (a, b)) \leq \min\{adf(x, a), adf(y, b)\}$$

where $xy \in N$ is an edge between vertices x and y, and $(a, b) \in Nm \times Nm$ are the corresponding attribute values.

2. Contradiction Function Constraint: For all $(a, b), (c, d) \in Nm \times Nm$:

$$bCf((a, b), (c, d)) \leq \min\{aCf(a, c), aCf(b, d)\}$$

3. Reflexivity and Symmetry of Contradiction Functions:

$$\begin{aligned} aCf(a, a) &= 0, && \forall a \in Ml \\ aCf(a, b) &= aCf(b, a), && \forall a, b \in Ml \\ bCf(a, a) &= 0, && \forall a \in Nm \\ bCf(a, b) &= bCf(b, a), && \forall a, b \in Nm \end{aligned}$$

Example 5. (cf. 45 51]) The following examples are provided.

- When $s = t = 1, PG$ is called a Plithogenic Fuzzy Graph.
- When $s = 2, t = 1, PG$ is called a Plithogenic Intuitionistic Fuzzy Graph.
- When $s = 3, t = 1, PG$ is called a Plithogenic Neutrosophic Graph.
- When $s = 4, t = 1, PG$ is called a Plithogenic Turiyam Neutrosophic Graph.

The General Plithogenic Graph is a generalization of the Plithogenic Graph (cf.[35, 45, 79]).

Definition 6 (General Plithogenic Graph). [45] Let $G = (V, E)$ be a classical graph, where V is a finite set of vertices, and $E \subseteq V \times V$ is a set of edges.

A General Plithogenic Graph $G^{GP} = (PM, PN)$ consists of:

1. General Plithogenic Vertex Set PM:

$$PM = (M, l, Ml, adf, aCf)$$

Where:

- $M \subseteq V$: Set of vertices.
- l : Attribute associated with the vertices.
- Ml : Range of possible attribute values.
- $adf: M \times Ml \to [0,1]^s$: Degree of Appurtenance Function (DAF) for vertices.
- $aCf: Ml \times Ml \to [0,1]^t$: Degree of Contradiction Function (DCF) for vertices.

2. General Plithogenic Edge Set PN:

$$PN = (N, m, Nm, bdf, bCf)$$

Where:

- $N \subseteq E$: Set of edges.
- m : Attribute associated with the edges.
- Nm: Range of possible attribute values.
- $bdf: N \times Nm \to [0,1]^s$: Degree of Appurtenance Function (DAF) for edges.
- $bCf: Nm \times Nm \to [0,1]^t$: Degree of Contradiction Function (DCF) for edges.

The General Plithogenic Graph G^{GP} Only needs to satisfy the following Reflexivity and Symmetry properties of the Contradiction Functions:

- Reflexivity and Symmetry of Contradiction Functions:

$$\begin{aligned} aCf(a,a) &= 0, && \forall a \in Ml \\ aCf(a,b) &= aCf(b,a), && \forall a,b \in Ml \\ bCf(a,a) &= 0, && \forall a \in Nm \\ bCf(a,b) &= bCf(b,a), && \forall a,b \in Nm \end{aligned}$$

2.3 | Soft Expert Graph

The definitions of the Intuitionistic Fuzzy Soft Expert Graph and the Neutrosophic Soft Expert Graph are provided below.

Definition 7. [109] An Intuitionistic Fuzzy Soft Expert Graph (IFSEG) is defined over a simple graph $G^* = (\mathcal{V}, \mathcal{E})$, where:

- $\mathcal{V}$ is a set of vertices,
- $\mathcal{E}$ is a set of edges,
- y is a set of parameters,
- X is a set of experts,
- $O = \{1 = \text{agree}, 0 = \text{disagree}\}$ is a set of opinions, and
- $Z = \mathcal{Y} \times \mathcal{X} \times O$ is the Cartesian product of the sets.

Let $A \subseteq Z$ and let $\text{IFSE}(\mathcal{V})$ denote the set of all intuitionistic fuzzy sets in $\mathcal{V}$. The IFSEG is represented as a 4-tuple:

$$G = (G^*, A, f, g)$$

where:

- $f: A \to \text{IFSE}(\mathcal{V})$ is a function mapping each parameter in A to an intuitionistic fuzzy set of vertices,
- $g: A \to \text{IFSE}(\mathcal{V} \times \mathcal{V})$ is a function mapping each parameter in A to an intuitionistic fuzzy set of edges.

The mappings f and g are defined as:

$$f(\alpha) = f_\alpha = \{\langle x, \mu_{f_\alpha}(x), v_{f_\alpha}(x)\rangle : x \in \mathcal{V}\}$$
$$g(\alpha) = g_\alpha = \{\langle (x,y), \mu_{g_\alpha}(x,y), v_{g_\alpha}(x,y)\rangle : (x,y) \in \mathcal{V} \times \mathcal{V}\}$$

where:

- $\mu_{f_\alpha}(x)$ and $v_{f_\alpha}(x)$ represent the membership and non-membership degrees of vertex x under parameter α, respectively.
- $\mu_{g_\alpha}(x,y)$ and $v_{g_\alpha}(x,y)$ represent the membership and non-membership degrees of the edge (x,y) under parameter α, respectively.

These mappings satisfy the following conditions for all $(x,y) \in \mathcal{V} \times \mathcal{V}$ and $\alpha \in A$:

$$\mu_{g_\alpha}(x,y) \leq \min\{\mu_{f_\alpha}(x), \mu_{f_\alpha}(y)\}$$
$$v_{g_\alpha}(x,y) \leq \min\{v_{f_\alpha}(x), v_{f_\alpha}(y)\}$$

The IFSEG can also be denoted as:

$$G = (G^*, A, f, g) = \{\text{IFSE}(\alpha) : \alpha \in A\}$$

where $\text{IFSE}(\alpha)$ represents a family of parameterized intuitionistic fuzzy soft expert graphs.

Definition 8. [111] A Neutrosophic Soft Expert Graph (NSEG) is defined over a simple graph $G^* = (V, E)$, where V is the set of vertices, E is the set of edges, A is a set of parameters, and X is a set of experts. Let $N(V)$ denote the set of all neutrosophic sets in V. The NSEG is represented as a 4-tuple:

$$G = (G^*, A, f, g)$$

Where:

- $f: A \to N(V)$ is a function mapping each parameter in A to a neutrosophic set of vertices,
- $g: A \to N(V \times V)$ is a function mapping each parameter in A to a neutrosophic set of edges.

The mappings f and g are defined as:

$$f(\alpha) = \{x, \mu_f(x), v_f(x), \pi_f(x): x \in V\}$$
$$g(\alpha) = \{(x,y), \mu_f(x,y), v_f(x,y), \pi_f(x,y): (x,y) \in V \times V\}$$

Where:

- $\mu_f(x), v_f(x)$, and $\pi_f(x)$ represent the truth, indeterminacy, and falsity membership degrees of vertex x, respectively.
- $\mu_f(x,y), v_f(x,y)$, and $\pi_f(x,y)$ represent the truth, indeterminacy, and falsity membership degrees of the edge (x,y), respectively.

These mappings satisfy the following conditions for all $(x,y) \in V \times V$ and $\alpha \in A$:

$$\mu_g(x,y) \leq \min\{\mu_f(x), \mu_f(y)\}$$
$$v_g(x,y) \leq \min\{v_f(x), v_f(y)\}$$
$$\pi_g(x,y) \geq \max\{\pi_f(x), \pi_f(y)\}$$

The NSEG can also be denoted as:

$$G = (G^*, A, f, g) = \{N(\alpha): \alpha \in A\}$$

Where $N(\alpha)$ represents a family of parameterized neutrosophic graphs.

3 | Result in this Paper

In this section, we present the results of this paper.

3.1 | Turiyam Neutrosophic Soft Expert Graph

Definition 9. A Turiyam Neutrosophic Soft Expert Graph (TSEG) is defined over a simple graph $G^* = (V, E)$, where:

- V is a set of vertices,
- E is a set of edges,
- Y is a set of parameters,
- X is a set of experts,
- $O = \{1 = \text{agree}, 0 = \text{disagree}\}$ is a set of opinions,
- $Z = Y \times X \times O$ is the Cartesian product of the sets.

Let $A \subseteq Z$ and let $T(V)$ denote the set of all Turiyam Neutrosophic sets in V. The TSEG is represented as a 4-tuple:

$$G = (G^*, A, f, g)$$

where:

- $f: A \to T(V)$ is a function mapping each parameter in A to a Turiyam Neutrosophic set of vertices,
- $g: A \to T(V \times V)$ is a function mapping each parameter in A to a Turiyam Neutrosophic set of edges.

The mappings f and g are defined as:

$$f(\alpha) = \{x, t_f(x), iv_f(x), fv_f(x), lv_f(x) \mid x \in V\}$$
$$g(\alpha) = \{(x,y), t_f(x,y), iv_f(x,y), fv_f(x,y), lv_f(x,y) \mid (x,y) \in V \times V\}.$$

where:

- $t_f(x), iv_f(x), fv_f(x)$, and $lv_f(x)$ represent the truth, indeterminacy, falsity, and liberal state membership degrees of vertex x, respectively.
- $t_f(x,y), iv_f(x,y), fv_f(x,y)$, and $lv_f(x,y)$ represent the corresponding membership degrees of the edge (x,y).

These mappings satisfy the following conditions for all $(x,y) \in V \times V$ and $\alpha \in A$:

$$t_g(x,y) \leq \min\{t_f(x), t_f(y)\}$$
$$iv_g(x,y) \leq \min\{iv_f(x), iv_f(y)\}$$
$$fv_g(x,y) \geq \max\{fv_f(x), fv_f(y)\}$$
$$lv_g(x,y) \leq \min\{lv_f(x), lv_f(y)\}$$

The TSEG can also be denoted as:

$$G = (G^*, A, f, g) = \{T(\alpha) \mid \alpha \in A\}$$

Where $T(\alpha)$ represents a family of parameterized Turiyam Neutrosophic soft graphs.

Theorem 10. The Turiyam Neutrosophic Soft Expert Graph (TSEG) can be transformed into the following graphs under appropriate parameter settings:

i). Turiyam Neutrosophic Graph (TG)

ii). Neutrosophic Soft Expert Graph (NSEG)

iii). Neutrosophic Graph (NG)

iv). Intuitionistic Fuzzy Soft Expert Graph (IFSEG)

Proof. We will demonstrate that by adjusting the parameters of a TSEG, it can be reduced to each of the specified graph types.

We consider Transformation to Turiyam Neutrosophic Graph (TG). By considering a single parameter and a single expert, and ignoring the soft expert structure, the TSEG reduces to a Turiyam Neutrosophic Graph.

- Let A be a singleton set: $A = \{\alpha\}$.
- Let X be a singleton set: $X = \{x_0\}$.
- The opinion set $O = \{1\}$, indicating agreement only.

Under these settings, the set $Z = Y \times X \times O$ simplifies, and the TSEG becomes:

$$G = (G^*, A, f, g)$$

Where:

- $f: A \to T(V)$ assigns Turiyam Neutrosophic membership degrees to vertices.
- $g: A \to T(V \times V)$ assigns Turiyam Neutrosophic membership degrees to edges.

Since there's only one parameter and one expert, the soft expert aspect is eliminated, resulting in a standard Turiyam Neutrosophic Graph.

We consider Transformation to Neutrosophic Soft Expert Graph (NSEG). By setting the liberal state membership degree $lv_f(x) = 0$ for all vertices and edges, the TSEG reduces to an NSEG.

- For all $x \in V$ and $(x, y) \in E$, set $lv_f(x) = 0$ and $lv_g(x, y) = 0$.
- The Turiyam Neutrosophic membership degrees become $t_f(x), iv_f(x)$, and $fv_f(x)$.
- These correspond to the truth-membership, indeterminacy-membership, and falsity-membership degrees in a neutrosophic set.

Therefore, the TSEG simplifies to a Neutrosophic Soft Expert Graph with mappings:

$$f(\alpha) = \{x, \mu_f(x), v_f(x), \pi_f(x) \mid x \in V\}$$
$$g(\alpha) = \{(x, y), \mu_g(x, y), v_g(x, y), \pi_g(x, y) \mid (x, y) \in E\}$$

Where:

- $\mu_f(x) = t_f(x)$,
- $v_f(x) = iv_f(x)$,
- $\pi_f(x) = fv_f(x)$.

We consider Transformation to a Neutrosophic Graph (NG). By considering a single parameter and a single expert in an NSEG, it reduces to an NG.

- Let $A = \{\alpha\}$ and $X = \{x_0\}$.

  The NSEG becomes:

$$G = (G^*, A, f, g),$$

With f and g mapping to neutrosophic sets over V and E, respectively. Without the soft expert framework, this structure aligns with the definition of a Neutrosophic Graph.

We consider transformation to Intuitionistic Fuzzy Soft Expert Graph (IFSEG). By setting $iv_f(x) = 0$ and $lv_f(x) = 0$ for all vertices and edges, the TSEG reduces to an IFSEG.

- For all $x \in V$ and $(x, y) \in E$, set $iv_f(x) = 0, lv_f(x) = 0, iv_g(x, y) = 0$, and $lv_g(x, y) = 0$.
- The remaining membership degrees are $t_f(x)$ and $fv_f(x)$.
- Define:

$$\mu_f(x) = t_f(x)$$
$$v_f(x) = fv_f(x)$$

  satisfying $\mu_f(x) + v_f(x) \leq 1$.

- Similarly for edges.

  This conforms to the definition of an Intuitionistic Fuzzy Soft Expert Graph.

3.2 | General Plithogenic Soft Expert Graph

Definition 11. A General Plithogenic Soft Expert Graph (GPSEG) is defined over a simple graph $G^* = (V, E)$, where:

- V is a set of vertices,
- E is a set of edges,

- Y is a set of parameters,
- X is a set of experts,
- $O = \{1 = \text{agree}, 0 = \text{disagree}\}$ is a set of opinions,
- $Z = Y \times X \times O$ is the Cartesian product of the sets.

Let $A \subseteq Z$ and let $P(V)$ denote the set of all Plithogenic sets in V. The GPSEG is represented as a 4-tuple:

$$G = (G^*, A, f, g),$$

Where:

- $f: A \to P(V)$ maps each parameter to a Plithogenic set of vertices,
- $g: A \to P(V \times V)$ maps each parameter to a Plithogenic set of edges.

The mappings f and g are defined as:

$$f(\alpha) = \{x, \text{DAF}(x), \text{DCF}(x) \mid x \in V\},$$
$$g(\alpha) = \{(x, y), \text{DAF}(x, y), \text{DCF}(x, y) \mid (x, y) \in V \times V\}.$$

The GPSEG can also be denoted as:

$$G = (G^*, A, f, g) = \{P(\alpha) \mid \alpha \in A\},$$

Where $P(\alpha)$ represents a family of parameterized Plithogenic soft graphs.

Theorem 12. The General Plithogenic Soft Expert Graph (GPSEG) can be transformed into the following graphs under appropriate parameter settings:

i). General Plithogenic Graph (GPG)

ii). Turiyam Neutrosophic Soft Expert Graph (TSEG)

iii). Turiyam Neutrosophic Graph (TG)

iv). Neutrosophic Soft Expert Graph (NSEG)

v). Intuitionistic Fuzzy Soft Expert Graph (IFSEG)

Proof. We will demonstrate that by adjusting the parameters and settings of a GPSEG, it can be reduced to each of the specified graph types.

We consider Transformation to be a General Plithogenic Graph (GPG). By considering a single parameter and a single expert, and eliminating the soft expert structure, the GPSEG reduces to a General Plithogenic Graph.

- Let A be a singleton set: $A = \{\alpha\}$.
- Let X be a singleton set: $X = \{x_0\}$.
- The opinion set $O = \{1\}$, indicating agreement only.

Under these settings, the set $Z = Y \times X \times O$ simplifies, and the GPSEG becomes:

$$G = (G^*, A, f, g),$$

Where:

- $f: A \to P(V)$ assigns Plithogenic membership degrees to vertices.
- $g: A \to P(V \times V)$ assigns Plithogenic membership degrees to edges.

Since there's only one parameter and one expert, and opinions are fixed, the soft expert aspect is eliminated, resulting in a standard General Plithogenic Graph.

We consider Transformation to Turiyam Neutrosophic Soft Expert Graph (TSEG). By setting the Degree of Appurtenance Function (DAF) to correspond to Turiyam Neutrosophic membership degrees and adjusting the Degree of Contradiction Function (DCF) accordingly, the GPSEG reduces to a TSEG.

- Set $s = 4$ and $t = 1$ in the GPSEG, where $[0,1]^s$ Corresponds to the Turiyam Neutrosophic membership degrees.
- The DAF for vertices and edges becomes:

$$\text{DAF} : M \times Ml \to [0,1]^4$$

which aligns with the quadruple membership degrees $(t(v), iv(v), fv(v), lv(v))$ in the Turiyam Neutrosophic set.

- The DCF is adjusted to match the Turiyam Neutrosophic logic.

Therefore, the GPSEG becomes a Turiyam Neutrosophic Soft Expert Graph.

We consider Transformation to Turiyam Neutrosophic Graph (TG). By considering a single parameter and a single expert, and eliminating the soft expert structure from the TSEG obtained in step 2, we get a Turiyam Neutrosophic Graph.

- Let $A = \{\alpha\}$ and $X = \{x_0\}$.

The TSEG reduces to a Turiyam Neutrosophic Graph with Turiyam Neutrosophic membership degrees assigned directly to vertices and edges.

We consider Transformation to a Neutrosophic Soft Expert Graph (NSEG). By setting the liberal state membership degree to zero in the GPSEG configured as a TSEG, the GPSEG reduces to an NSEG.

- In the GPSEG, set $s = 4, t = 1$, and for all vertices and edges, set the fourth component of the DAF to zero:

$$lv_f(x) = 0,\ lv_g(e) = 0$$

- The remaining components correspond to the truth, indeterminacy, and falsity membership degrees of the Neutrosophic set.

Adjust the DCF accordingly.

Thus, the GPSEG reduces to a Neutrosophic Soft Expert Graph.

We consider Transformation to Intuitionistic Fuzzy Soft Expert Graph (IFSEG). By setting the indeterminacy and liberal state membership degrees to zero in the GPSEG, we can reduce it to an IFSEG.

- In the GPSEG, set $s = 4, t = 1$, and for all vertices and edges, set:

$$iv_f(x) = 0,\ lv_f(x) = 0,\ iv_g(e) = 0,\ lv_g(e) = 0$$

- The remaining components $t_f(x)$ and $fv_f(x)$ correspond to the membership and non-membership degrees in an intuitionistic fuzzy set.
- Adjust the DCF accordingly.

  Therefore, the GPSEG reduces to an Intuitionistic Fuzzy Soft Expert Graph.

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Data Availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that there is no conflict of interest in the research.

Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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An Overview of Neutrosophic and Plithogenic Theories and Applications

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Abstract

We present this research to all researchers and scholars who have realized the existence of indeterminacy in all data, through the results they obtain and the values that are not accurate enough and that may cause loss to the systems and facilities under study, and we will present through it the emergence, foundations and development of Neutrosophic theories and their applications for more than two decades (1995- 2023) since it was defined and studied, along with its applications, in order to be able to present new studies and research that keep pace with the great scientific development that our contemporary world is witnessing, through the use of research that has been published by the professionals and found on the attached open links.

Keywords: Neutrosophic; origin of the word Neutrosophic; stages of development of neutrosophic science; applications of neutrosophic science; research in neutrosophic science; neutrosophic journals

1. Introduction

The beginning of the great revolution in all fields of science was in the year 1995 when the American scientist Florentin Smarandache introduced the degree of indeterminacy/neutrality (I) as independent component in 1995 (published in 1998) and he defined the neutrosophic set on three components:(T, I, F) = (Truth, Indeterminacy, Falsehood), where in general T, I, F are subsets of the interval [0, 1]; in particular T, I, F may be intervals, hesitant sets, single-values, etc. Indeterminacy (or Neutrality), as independent component from the truth and from the falsehood, is the main distinction between Neutrosophic Theories and other classical and fuzzy theory or fuzzy extension theories [1]
Neutrosophic Set is a Generalization of Intuitionist Fuzzy Set, Inconsistent Intuitionist Fuzzy Set (Picture Fuzzy Set, Ternary Fuzzy Set), Pythagorean Fuzzy Set (Atanassov's Intuitionist Fuzzy Set of second type), q-Rung Orthopair Fuzzy Set, Spherical Fuzzy Set, and n-HyperSpherical Fuzzy Set, while Neutrosophication is a Generalization of Regret Theory, Grey System Theory, and Three-Ways Decision.[2,3]

Zadeh introduced the degree of membership/truth (T) in 1965 and defined the fuzzy set. **Atanassov** introduced the degree of nonmembership/falsehood (F) in 1986 and defined the intuitionistic fuzzy set. **Smarandache** introduced the degree of indeterminacy/neutrality (I) as independent component in 1995 (published in 1998) and he defined the neutrosophic set on three components:(T, I, F) = (Truth, Indeterminacy, Falsehood), where in general T, I, F are subsets of the interval [0, 1]; in particular T, I, F may be intervals, hesitant sets, single-values, etc. See F. Smarandache, Neutrosophy / Neutrosophic probability, set, and logic", Proquest, Michigan, USA, 1998,[4,5] reviewed in Zentralblatt für Mathematik (Berlin, Germany)[6], and cited by Denis Howe in The Free Online Dictionary of Computing, England, 1999.Neutrosophic Set and Logic are generalizations of classical, fuzzy, and intuitionist fuzzy set and logic[7,8]

2. Etymology

a. Neutrosophy" and "Neutrosophic:

The words "neutrosophy" and "neutrosophic" were coined/invented by F. Smarandache in his 1998 book.
Neutrosophy: A branch of philosophy, introduced by F. Smarandache in 1980, which studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra. Neutrosophy considers a proposition, theory, event, concept, or entity <A> in relation to its opposite <antiA>, and with their neutral <neutA>.
Neutrosophy (as dynamic of opposites and their neutrals) is an extension of the Dialectics and Yin Yang (which are the dynamic of opposites only).
Neutrosophy is the basis of neutrosophic set, neutrosophic logic, neutrosophic measure, neutrosophic probability, neutrosophic statisticsetc[9]

b. **Neutrosophic Logic:**

Neutrosophic Logic is a general framework for unification of many existing logics, such as fuzzy logic (especially intuitionistic fuzzy logic), paraconsistent logic, intuitionist logic, etc. The main idea of NL is to characterize each logical statement in a 3D-Neutrosophic Space, where each dimension of the space represents respectively the truth (T), the falsehood (F), and the indeterminacy (I) of the statement under consideration, where T, I, F are standard or non-standard real subsets of]-0, 1+[with not necessarily any connection between them.
For software engineering proposals the classical unit interval [0, 1] may be used.
While Neutrosophic Probability and Statistics are generalizations of classical and imprecise probability and classical statistics respectively.

c. **The Most Important Books and Papers on the Advancement of Neutrosophics:**
1980s - Foundation of Paradoxism that is an international movement in science and culture **based on excessive use of contradictions, antitheses, oxymoron, and paradoxes** [Smarandache]. During three decades (1980-2020) hundreds of authors from tens of countries around the globe contributed papers to 15 international paradoxist anthologies [10]
1995-1998: Smarandache extended the paradoxism (based on opposites) to a new branch of philosophy called Neutrosophy (based on opposites and their neutral), that gave birth to many scientific branches, such as: neutrosophic logic, neutrosophic set, neutrosophic probability and statistics, neutrosophic algebraic structures, and so on with multiple applications in all fields. Neutrosophy is also an extension of the Dialectics (which is a particular case of Yin-Yang ancient Chinese philosophy),[11]
introduced the neutrosophic set/logic/probability/statistics;
introduces the single-valued neutrosophic set (pp. 7-8) [12,13]
1998-2001: Introduction of Nonstandard Neutrosophic Logic, Set, Probability,
https://arxiv.org/ftp/arxiv/papers/1903/1903.04558.pdf
2002: Introduction of corner cases of sets / probabilities / statistics / logics, such as:- Neutrosophic intuitionistic set (different from intuitionist fuzzy set), neutrosophic paraconsistent set, neutrosophic fallibilists set, neutrosophic paradoxist set, neutrosophic pseudo-paradoxist set, neutrosophic tautological set, neutrosophic nihilist set, neutrosophic dialetheist set, neutrosophic trivialist set;
- Neutrosophic intuitionistic probability and statistics, neutrosophic paraconsistent probability and statistics, neutrosophic fallibilists probability and statistics, neutrosophic paradoxist probability and statistics, neutrosophic pseudo-paradoxist probability and statistics, neutrosophic tautological probability and statistics, neutrosophic nihilist probability and statistics, neutrosophic dialetheist probability and statistics, neutrosophic trivialist probability and statistics;
- Neutrosophic paradoxist logic (or paradoxism), neutrosophic pseudo-paradoxist logic (or neutrosophic pseudo-paradoxism), neutrosophic tautological logic (or neutrosophic tautologism)
https://arxiv.org/ftp/math/papers/0301/0301340.pdf
http://fs.unm.edu/DefinitionsDerivedFromNeutrosophics.pdf
2003 – Introduction by Kandasamy and Smarandache of Neutrosophic Numbers (a+bI,) where I = literal indeterminacy, $I^2 = I$, which is different from the numerical indeterminacy that is a real set), I-Neutrosophic Algebraic Structures and Neutrosophic Cognitive Maps
2005 - Introduction of Interval Neutrosophic Set/Logic , https://arxiv.org/pdf/cs/0505014.pdf
http://fs.unm.edu/INSL.pdf

2006 – Introduction of Degree of Dependence and Degree of Independence between the Neutrosophic Components T, I, F. for single valued neutrosophic logic, the sum of the components is:
$0 \leq t+i+f \leq 3$ when all three components are independent;
$0 \leq t+i+f \leq 2$ when two components are dependent, while the third one is independent from them;
$0 \leq t+i+f \leq 1$ when all three components are dependent.
When three or two of the components T, I, F are independent, one leaves room for incomplete information (sum < 1), paraconsistent and contradictory information (sum > 1), or complete information (sum = 1).
If all three components T, I, F are dependent, then similarly one leaves room for incomplete information (sum < 1), or complete information (sum = 1).
In general, the sum of two components x and y that vary in the unitary interval [0, 1] is:
$0 \leq x + y \leq 2 - d°(x, y)$, where $d°(x, y)$ is the degree of dependence between x and y, while
$d°(x, y)$ is the degree of independence between x and y.
Degrees of Dependence and Independence between Neutrosophic Components T, I, F are independent components, leaving room for incomplete information (when their superior sum < 1), paraconsistent and contradictory information (when the superior sum > 1), or complete information (sum of components = 1).
For software engineering proposals the classical unit interval [0, 1] is used ,
https://doi.org/10.5281/zenodo.571359
http://fs.unm.edu/eBook-Neutrosophics6.pdf (p. 92)
http://fs.unm.edu/NSS/DegreeOfDependenceAndIndependence.pdf

2007:

1- The neutrosophic set was extended [Smarandache, 2007] to neutrosophic overset (when some neutrosophic component is > 1), since he observed that, for example, an employee working overtime deserves a degree of membership > 1, with respect to an employee that only works regular full-time and whose degree of membership = 1;
and to neutrosophic underset (when some neutrosophic component is < 0), since, for example, an employee making more damage than benefit to his company deserves a degree of membership < 0, with respect to an employee that produces benefit to the company and has the degree of membership > 0;
and to and to neutrosophic offset (when some neutrosophic components are off the interval [0, 1], i.e. some neutrosophic component > 1 and some neutrosophic component < 0).then, similarly, the neutrosophic logic/measure/probability/statistics etc. were extended to respectively neutrosophic over-/under-/off- logic / measure / probability / statistics etc, https://arxiv.org/ftp/arxiv/papers/1607/1607.00234.pdf
http://fs.unm.edu/NeutrosophicOversetUndersetOffset.pdf
http://fs.unm.edu/SVNeutrosophicOverset-JMI.pdf
http://fs.unm.edu/IV-Neutrosophic-Overset-Underset-Offset.pdf
http://fs.unm.edu/NSS/DegreesOf-Over-Under-Off-Membership.pdf

2- Smarandache introduced the neutrosophic tripolar set and neutrosophic multipolar set and consequently the Neutrosophic Tripolar Graph and Neutrosophic Multipolar Graph , http://fs.unm.edu/eBook-Neutrosophics6.pdf (p. 93)
http://fs.unm.edu/IFS-generalized.pdf

2009 – Introduction of N-norm and N-conorm https://arxiv.org/ftp/arxiv/papers/0901/0901.1289.pdf
http://fs.unm.edu/N-normN-conorm.pdf

2013:

1- Development of neutrosophic measure and neutrosophic probability
(chance that an event occurs, indeterminate chance of occurrence, chance that the event does not occur)
https://arxiv.org/ftp/arxiv/papers/0901/0901.1289.pdf
http://fs.unm.edu/N-normN-conorm.pdf

2- Smarandache refined / split the neutrosophic components (t, i, f) into neutrosophic subcomponents (T1, T2, ...; I1, I2, ...; F1, F2, ...) https://arxiv.org/ftp/arxiv/papers/1407/1407.1041.pdf
http://fs.unm.edu/n-ValuedNeutrosophicLogic-PiP.pdf

2014:

1- Introduction of the Law of Included Multiple-Middle (as extension of the Law of Included Middle)(<A>; <neutA1>, <neutA2>, …; <antiA>) http://fs.unm.edu/LawIncludedMultiple-Middle.pdf

2- Development of neutrosophic statistics (indeterminacy is introduced into classical statistics with respect to any data regarding the sample / population, probability distributions / laws / graphs / charts etc., with respect to the individuals that only partially belong to a sample / population, and so on),
https://arxiv.org/ftp/arxiv/papers/1406/1406.2000.pdf
http://fs.unm.edu/NeutrosophicStatistics.pdf

2015:

1- Introduction of Neutrosophic Precalculus and Neutrosophic Calculus , https://arxiv.org/ftp/arxiv/papers/1509/1509.07723.pdf
http://fs.unm.edu/NeutrosophicPrecalculusCalculus.pdf
2- Refined Neutrosophic Numbers (a+ b1I1 + b2I2 + … + bnIn), where I1, I2, …, In are SubIndeterminacies of Indeterminacy I,
3- (t,i,f)-neutrosophic graphs
4- Thesis-Antithesis-Neutrothesis, and Neutrosynthesis, Neutrosophic Axiomatic System, neutrosophic dynamic systems, symbolic neutrosophic logic, (t, i, f)-Neutrosophic Structures, I-Neutrosophic Structures, Refined Literal Indeterminacy, Quadruple Neutrosophic Algebraic Structures, Multiplication Law of SubIndeterminacies, and Neutrosophic Quadruple Numbers of the form a + bT + cI + dF , https://arxiv.org/ftp/arxiv/papers/1512/1512.00047.pdf
http://fs.unm.edu/SymbolicNeutrosophicTheory.pdf
5- Introduction of the SubIndeterminacies of the form $I_0^K = \frac{K}{0}$, for k $\in$ {0, 1, 2, …, n-1}, into the ring of modulo integers Zn - called natural Neutrosophic indeterminacies (Vasantha-Smarandache), http://fs.unm.edu/MODNeutrosophicNumbers.pdf

6- Introduction of Neutrosophic Topology (Salama & Smarandache) , http://fs.unm.edu/NeutrosophicCrispSetTheory.pdf

2016:

1- Introduction of Neutrosophic Multisets (as generalization of classical multisets) http://fs.unm.edu/NeutrosophicMultisets.htm
2- Introduction of Neutrosophic Triplet Structures and m-valued refined Neutrosophic triplet structures [Smarandache – Ali] http://fs.unm.edu/NeutrosophicTriplets.htm
3- Introduction of Neutrosophic Duplet Structures, http://fs.unm.edu/NeutrosophicDuplets.htm

2017:

1- Neutrosophic Score, Accuracy, and Certainty Functions form a total order relationship on the set of (single-valued, interval-valued, and in general subset-valued) Neutrosophic triplets (T, I, F); and these functions are used in MCDM (Multi-Criteria Decision Making), http://fs.unm.edu/NSS/TheScoreAccuracyAndCertainty1.pdf
2- In biology Smarandache introduced the Theory of Neutrosophic Evolution: Degrees of Evolution, Indeterminacy or Neutrality, and Involution, http://fs.unm.edu/neutrosophic-evolution-PP-49-13.pdf

3- Introduction by F. Smarandache of Plithogeny (as generalization of Yin-Yang, Dialectics, and Neutrosophy (as generalization of fuzzy, intuitionistic fuzzy, Neutrosophic set/logic/probability/statistics)
4- Enunciation of the Law that: It Is Easier to Break from Inside than from Outside a Neutrosophic Dynamic System (Smarandache - Vatuiu)

2018:

1- Generalization of the Soft Set to Hyper Soft Set, http://fs.unm.edu/NSS/ExtensionOfSoftSetToHypersoftSet.pdf
2- Introduction to Neutrosophic Psychology (Neutropsyche, Refined Neutrosophic Memory: conscious, a conscious, unconscious, Neutropsychic Personality, Eros / Aoristos / Thanatos, Neutropsychic Crisp Personality) , http://fs.unm.edu/NeutropsychicPersonality-ed3.pdf

2019:

1- Theory of Spiral Neutrosophic Human Evolution (Smarandache - Vatuiu), http://fs.unm.edu/SpiralNeutrosophicEvolution.pdf

2- Introduction to Neutrosophic Sociology (Neutrosociology) [neutrosophic concept, or (T, I, F)-concept, is a concept that is T% true, I% indeterminate, and F% false] http://fs.unm.edu/Neutrosociology.pdf

3- Refined Neutrosophic Crisp Set, http://fs.unm.edu/RefinedNeutrosophicCrispSet.pdf

4- Generalized of the classical Algebraic Structures to NeutroAlgebraic Structures (or NeutroAlgebras) {whose operations and axioms are partially true, partially indeterminate, and partially false} as extensions of Partial Algebra, and to AntiAlgebraic Structures (or AntiAlgebras) {whose operations and axioms are totally false}. And, in general, he extended any classical Structure, in no matter what field of knowledge, to a NeutroStructure and an AntiStructure.
5- Extension of HyperGraph to n-SuperHyperGraph

2020 –
Introduction to Neutrosophic Genetics ,http://fs.unm.edu/NeutrosophicGenetics.pdf
2021
1- As alternatives and generalizations of the Non-Euclidean Geometries, Smarandache introduced in 2021 the NeutroGeometry & AntiGeometry. While the Non-Euclidean Geometries resulted from the total negation of only one specific axiom (Euclid's Fifth Postulate), the AntiGeometry results from the total negation of any axiom and even of more axioms from any geometric axiomatic system (Euclid's, Hilbert's, etc.), and the NeutroAxiom results from the partial negation of one or more axioms [and no total negation of no axiom] from any geometric axiomatic system.
2- Real Examples of NeutroGeometry and AntiGeometry .
3- Introduction of Plithogenic Logic as a generalization of MultiVariate Logic.
4- Introduction of Plithogenic Probability and Statistics as generalizations of MultiVariate Probability and Statistics respectively
2022:
1- SuperHyperAlgebra & Neutrosophic SuperHyperAlgebra
2- SuperHyperGraph, Neutrosophic SuperHyperGraph
3- SuperHyperFunction, SuperHyperTopology
4- IndetermSoft Set, IndetermHyperSoft Set
5- TreeSoft Set

2023 - Symbolic Plithogenic Algebraic Structures built on the set of Symbolic Plithogenic Numbers of the form a0 + a1P1 + a2P2 + ... + anPn where the multiplication Pi·Pj is based on the prevalence order and absorbance law.

d. **Applications in:**
Artificial Intelligence, Information Systems, Computer Science, Cybernetics, Theory Methods, Mathematical Algebraic Structures, Applied Mathematics, Automation, Control Systems, Big Data, Engineering, Electrical, Electronic, Philosophy, Social Science, Psychology, Biology, Biomedical, Genetics, Engineering, Medical Informatics, Operational Research, Management Science, Imaging Science, Photographic Technology, Instruments, Instrumentation, Physics, Optics, Economics, Mechanics, Neurosciences, Radiology Nuclear, Medicine, Medical Imaging, Interdisciplinary Applications, Multidisciplinary Sciences etc. [Xindong Peng and Jingguo Dai, A bibliometric analysis of neutrosophic set: two decades review from 1998 to 2017, Artificial Intelligence Review, Springer, 18 August 2018 .

e. Important Neutrosophic Researchers:

There are about 7,000 neutrosophic researchers, within 7489 countries around the globe, that have produced about 2,000 publications within more than two decades. Many neutrosophic researchers got specialized into various fields of neutrosophics:

Xiaohong Zhang & Yingcang Ma (neutrosophic triplet and quadruple algebraic structures), Yanhui Guo (neutrosophic image processing), Jun Ye & Peide Liu & Jianqiang Wang (neutrosophic optimization), Xindong Peng & Jingguo Dai (neutrosophic bibliometrics), Jianqiang Wang, Guiwu Wei, Donghai Liu, Xiaohong Chen, Dan Peng, Jiongmei Mo, Han-Liang Huang, Victor Chang, Hongjun Guan, Shuang Guan, Aiwu Zhao, Wen-Hua Cui, Xiaofei Yang, Xin Zhou, G.L. Tang, W.L. Liu, Wen Jiang, Zihan Zhang, Xinyang Deng, Changxing Fan, Sheng Feng, En Fan, Keli Hu, Xingsen Li, Xin Zhou, Ping Li;
Rajab Ali Borzooei & Young Bae Jun (neutrosophic BCK/BCI-algebras), Arsham Borumand Saeid (neutrosophic structures), Saied Jafari (neutrosophic topology), Prem Kumar Singh (Neutrosophic MCDM), Maikel Leyva-Vazquez (neutrosophic cognitive maps);
Amira S. Ashour, Muhammad Aslam (neutrosophic statistics), Nguyen Xuan Thao (neutrosophic similarity measures), Le Hoang Son, Vakkas Ulucay & Memet Sahin (neutrosophic quadruple structures), Irfan

Deli, Madad Khan (neutrosophic algebraic structures), Said Broumi & Muhammad Akram (neutrosophic graphs), Mohamed Abdel-Baset (neutrosophic linear and non-linear programming), Ahmed Mostafa Khalil, Ahmed Salama (neutrosophic crisp topology), etc.

f. Neutrosophic Journals:
1- Neutrosophic Sets and Systems (NSS) international journal started in 2013 and it is indexed by Scopus, Web of Science (ESCI), DOAJ, Index Copernicus, Redalyc - Universidad Autonoma del Estado de Mexico

(IberoAmerica), Publons, CNKI (Beijing, China), Chinese Baidu Scholar, etc, Submit papers on neutrosophic set/logic/probability/statistics, Simulation, etc. and their applications through our OJS, [14 – 19]

1- International Journal of Neutrosophic Science (IJNS, in SCOPUS) , Submit papers on neutrosophic, inventory management, Linear Programming , dynamic programming,
Decision – Making , Machine learning, Queue Theory , etc , [20-29]

2- Neutrosophic Computing and Machine Learning (NCML), in Spanish
http://fs.unm.edu/NCML/

3- Neutrosophic Knowledge (NK), in English and Arabic: http://fs.unm.edu/NK/

4- Journal of Neutrosophic and Fuzzy Systems (JNFS), Submit papers on neutrosophic Machine learning, Queue Theory , etc , [30 – 33]

5- Prospects for Applied Mathematics and Data Analysis (PAMDA) , Submit papers on neutrosophic Machine learning , Neutrosophic Simulation , etc, [34-36]

6- Encyclopedia of Neutrosophic Researchers, To certify the names of authors who have published or presented research on Neutrosophic,[37-40]

Note : The authors who have published or presented papers on neutrosophics and are not included in the Encyclopedia of Neutrosophic Researchers (ENR), vols. 1, 2, 3, and 4 are pleased to send their CV, photo, and List of Neutrosophic Publications to smarand@unm.edu in order to be included into the next volume of ENR.

3. Conclusion and results:

There is no absolute truth, there is nothing certain that any data presented to conduct a study in any field of science contains some kind of indeterminacy that affects the desired results of this study, which made neutrosophic science and the basic concepts of neutrosophic science the focus of attention of scholars and researchers who wish to obtain results Perfect and accurate, far from taking risks, they achieve the desired goal

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Applications of Extended Plithogenic Sets in Plithogenic Sociogram

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Abstract

The theory of plithogeny developed by Smarandache is described as a more generalized form of representing sets of different nature such as crisp, fuzzy, intuitionistic and neutrosophic. Plithogenic set comprises degree of appurtenance and contradiction degree with respect only to the dominant attribute. This paper introduces extended plithogenic sets comprising degrees of appurtenance and contradiction with respect to both dominant and recessive attributes. The extension of the 5-tuple Plithogenic sets to a 7-tuple plithogenic sets helps in developing a more comprehensive kind of Plithogenic sociogram. The newly developed plithogenic sets and its implications in Plithogenic sociogram is validated by the decision making problem on food processing industries. The obtained results using extended plithogenic sets are more promising in comparison to the conventional plithogenic sets. The proposed kind of plithogenic sets will benefit the decision makers to make optimal decisions based on both optimistic and pessimistic approaches.

Keywords: Plithogenic Sociogram; Extended Plithogenic Sets; Food Packaging materials; Decision making

1. Introduction

Plithogenic sets introduced by Smarandache in 2018 [1] have generalized the representations of both the conventional crisp sets and contemporary sets such as fuzzy sets, intuitionistic sets and neutrosophic sets. A plithogenic set is a 5-tuple set of the form (P,a,V,d,c) comprising 5 components namely a set P, the attribute a, the attribute values V, the degree of appurtenance d and the contradiction degree c. The classification of plithogenic sets into crisp, fuzzy, intuitionistic and neutrosophic sets is based on the respective crisp, fuzzy, intuitionistic and neutrosophic values of d. Thus the degree of appurtenance serves as the deciding factor of the nature of Plithogenic sets. Smarandache[2,3] have well presented the theoretical framework of Plithogenic sets, Plithogenic logic, Plithogenic probability and also the generalized versions of the above with suitable practical implications.[1-3]

Plithogenic sets are recently applied in several decision making problems especially in multi-criteria decision making problems (MCDM) because of the significant characteristic of Plithogenic sets in handling of attribute and attribute values. Under Plithogenic MCDM, the methods are further extended with Plithogenic arguments. For instance the method of TOPSIS is discussed under Plithogeny in which the plithogenic representations and plithogenic operators are used. Sankar et al [4] have applied Plithogenic TOPSIS (PTOPSIS) to obtain optimal ranking of the alternatives of the decision making problem on COVID 19. Researchers have also integrated Plithogenic TOPSIS with other MCDM methods. To mention a few, PTOPSIS-CRITIC by Basset et al [5] in sustainability risk analysis, SWARA-PTOPSIS by Martin et al [6] in food processing industries. The other MCDM method explored

under plithogeny is Plithogenic MARICA by Abdullah et al [7]. From the literature the applications of Plithogenic sets in diverse fields are very evident. To mention some of the significant contributions of the researchers, Mohamed et al [8] in IOT based supply chain, Shio et al [9] with entropy measures, Basset et al [10] in supplier selection, Gomathy et al [11] in decision making, Abdel et al [12] in choosing sustainability metrics, Priyadharshini et al [13,14] in multi criteria decision making with refined plithogenic neutrosophic sets, Abdel et al [15,16] in assessing financial performance, hospital care system. In addition to MCDM applications the plithogenic sets are also associated with other decision making tools with further extensions and theoretical arguments.

Researchers have integrated the theory of plithogeny with soft and Hypersoft sets to evolve hybrid decision making techniques. Smarandache [17] has given key contributions to the theory of Plithogenic Hypersoft sets by extending soft sets to Hypersoft sets. Alkhazaleh et al [18] introduced Plithogenc softs sets. Rana et al [19] developed Plithogenic fuzzy whole hypersoft set with applications in constructing frequency matrix. Dhivya et al [20] introduced near plithogenic neutrosophic Hypersoft sets with Heronian Mean Aggregation Operators. Muhammad et al [21] have described Plithogenic Hypersoft Sets under Fuzzy Neutrosophic Environment. Shazia et al [22] introduced the notion of Plithogenic Subjective Hyper-Super-Soft Matrices. Martin et al[23-25] conceptualized combined Hypersoft sets, extended plithogenic Hypersoft sets in covid diagnosis and associated with concentric plithogenic hypergrpahs. Plithogenic graphs and hypergraphs are also initiated by the researchers. Vasantha and Smarandache [26] introduced the theory of Plithogenic graphs. Sultana[27] applied plithogenic graphs in disease diagnosis. Gayen et al [28] developed the notion of Plithogenic Hypersoft set subgroup, Martin et al [29] introduced PROMTHEE Plithogenic Pythagorean Hypergraphic Approach in decision making on smart materials, Singh [30-32] has applied Single-valued Plithogenic graph, Single-Valued Neutrosophic Plithogenic, Single-Valued Neutrosophic Plithogenic in data analysis and optimization. Bharathi and Leo [33] used Plithogenic Product Fuzzy Graphs in analysing social networks.

The concept of Plithogeny is also applied to define various algebraic structures and topological spaces. Gayen et al [28,34] introduced and analyzed the properties of plithogenic subgroup, plithogenic Hypersoft subgroup. Priyadharshini and Nirmala [35] have investigated on the theoretical ideas of Plithogenic Neutrosophic Topological Spaces. Bhimraj [36] contributed to the theory of Plithogenic Neutrosophic Hypersoft Almost Topological Group, plithogenic cubic sets and with orders P and R. Singh [37] has developed complex plithogenic set. Martin et al [38] introduced the notion of Plithogenic numbers. Noel et al [39] applied Plithogenic numbers in assessing Entrepreneurship Competence in University Students and Raúl [40] in evaluating the performances. The theory of Plithogenic Cognitive Maps, New Plithogenic Sub Cognitive Maps and Induced Plithogenic Cognitive Maps are introduced by Nivetha and Smarandache [41-43]. These novel concepts are applied in making decisions on disease diagnosis, learning systems. Sujatha et al [44] applied Plithogenic Cognitive Maps in diagnosing novel coronavirus.

The comprehensive nature of the Plithogenic sets has widened its extent of applications to various other fields especially to the domain of Physical sciences. Smarandache [45,46] coined the aspect of Physical Plithogenic sets and have applied aggregate operators in physical filed. Bala [47] applied plithogenic sets and logic in handling information. Walid [48] integrated Plithogenic sets with mathematical programming in making assessments , Ariel et al [49] in the domain of management, Gustavo [50] in the educational filed. The theory of Plithogeny is applied in the field of social science and this has motivated the researchers to develop Plithogenic sociogram (PS) an integrated sociometric technique which is an extended version of neutrosophic sociogram (NS) applied in group analysis. The main objective of neutrosophic sociogram is to make a quantitative measurement of the group dynamics. The possibilities of enhancing the degree of association between the members of the group are determined using NS by considering both positive and negative preferentialism. Based on the framework of NS, PS is constructed only using positive preferentialism by considering the degrees of appurtenances and contradiction degrees only with respect to the dominant attribute values. On observing the theoretical conceptualization and practical implications of NS and PS from the literature, the following findings are determined

(i) In NS, the decision making is dependent on both the positive and negative preferential scores representing their interest and disinterest in associating oneself with the group members. Also the final score values obtained using both the types of preferentialism helped in making interpretations on the possibilities of increasing the relationship between the members of the group

(ii) In PS, the decision making is based only on the positive preferentialism (i.e) it is based only on the dominant attribute values. The final scores based on only positive preferences helped in ranking the persons based on their preferential scores, but the possibility of enhancing the relationship is not able to be determined in Plithogenic sociogram approach of group analysis.

On intense analysis, in Plithogenic sociogram the preferential scores are based only on the degree of appurtenance with respect to only one dominant attribute value i.e the positive preferentialism, whereas the space for negative preferentialism is nil as the components d and c of the plithogenic set are confined only to the dominant attribute value. This shortcoming of PS has motivated the authors to extend the 5-tuple plithogenic set to 7-tuple plithogenic set by considering the degree of appurtenance and contradiction degree with respect to recessive attribute value in addition to dominant attribute value to calculate the negative preferential score. The main outcome of such extended kind of plithogenic sets will make PS better than NS in the following contexts

(i) The members of the group shall be ranked based on both positive and negative preferentialism

(ii) The possibilities of enhancing the relationship between the members shall also be determined based on both dominant and recessive attribute values.

To bridge this research gap, an extended form of plithogenic set (ExPlS) is introduced in this research work. The theoretical construction of ExPlS is presented in section 2, the conceptualization and the algorithm of Plithogenic sociogram with extended plithogenic sets is presented in section 3, the application of the proposed PS with ExPlS is presented in section 4 with respect to a decision making problem on food processing industries. Section 5 presents the comparative analysis and the results. The last section concludes the work.

2. Conceptualization of Extended Plithogenic Sets (ExPlS)

This section presents the theoretical developments of extended plithogenic sets which is the extension of the existing plithogenic sets developed by Smarandache.

Based on the previous theoretical genesis of plithogeny, the plithogenic set of the form (P,a,V,d,c) is extended to(P,a,V,d_D,c_D,d_R,c_R) where

d_D – the degree of appurtenance is with respect to dominant attribute value

c_D-the contradiction degree with respect to dominant attribute value

d_R – the degree of appurtenance is with respect to recessive attribute value

c_R - the contradiction degree with respect to recessive attribute value

The differences between the extended plithogenic sets and the conventional plithogenic sets are the inclusions of degree of appurtenance and contradiction with respect to recessive attribute value in addition to that of the dominant attribute value.

Let us consider an example to explain the representation of ExPlS.

Let P be a set be representing the products {P1,P2,P3,P4,P5}

Generally, the expected attributes considered in purchasing the products are price, quality

The attribute values of price are {cheap, budgetary, expensive} = {P11,P12,P13}

The attribute values of the quality are {low, moderate, high} ={Q11,Q12,Q13}

In this case, let us consider the attribute price with attribute values as P11,P12 and P13. Let the dominant attribute value be cheap and recessive attribute value be expensive. The ExPlS considers the degrees of appurtenances and contradiction degrees with respect to both dominant and recessive attribute value pertinent to the attribute of price.

2.1 Classification of ExPlS

The ExPlS shall be classified based on the degrees of appurtenance with respect to both dominant and recessive attribute values.

2.1.1 Crisp ExPlS

AExPls is said to be crisp if dD and dR are crisp in nature.

$d_D : P \times V \to \{0,1\}$ & $d_R : P \times V \to \{0,1\}$ and $c_D : V \times V \to \{0,1\}$ & $c_R : P \times V \to \{0,1\}$

2.1.2 Fuzzy ExPlS

If dD and dR are fuzzy sets, A ExPls is described to as a Fuzzy ExPlS.

$d_D : P \times V \to [0,1]$ & $d_R : P \times V \to [0,1]$and $c_D : V \times V \to [0,1]$ & $c_R : P \times V \to [0,1]$

2.1.3 Intuitionistic ExPlS

If dD and dR are Intuitionistic fuzzy sets, then ExPls is refered to as anIntuitionistic fuzzy ExPlS.

$d_D : P \times V \to [0,1]^2$& $d_R : P \times V \to [0,1]^2$and $c_D : V \times V \to [0,1]$ & $c_R : P \times V \to [0,1]$

2.1.4 Neutrosophic ExPlS

If dD and dR are Neutrosophic fuzzy values ,then an ExPls is referred to as a Neutrosophic ExPlS

$d_D : P \times V \to [0,1]^3$& $d_R : P \times V \to [0,1]^3$and $c_D : V \times V \to [0,1]$ & $c_R : P \times V \to [0,1]$

The different degrees of appurtenance with respect to both dominant and recessive attribute values of the attribute price and quality are presented in the following Table 2.1. Let us consider the dominant attribute value of price is cheap and the recessive attribute value as expensive. Let us also consider the dominant attribute value of quality as high and the recessive attribute value as low

Table 1: represents Contradiction Degree with respect to Dominant attribute value and recessive attribute value for Price

Attribute: Price								
Contradiction Degree with respect to Dominant attribute value		0	1/3	2/3	Contradiction Degree with respect to Recessive attribute value		2/3	1/3	0
Attribute values		**P11**	P12	P13	Attribute values		P11	P12	**P13**
Crisp degree of appurtenance	P1	1	0	0	Crisp degree of appurtenance	P1	0	0	1
	P2	0	1	0		P2	0	1	0
	P3	0	0	1		P3	1	0	0
	P4	1	0	0		P4	0	0	1
	P5	0	1	0		P5	0	1	0
Fuzzy degree of appurtenance	P1	0.8	0.2	0.1	Fuzzy degree of appurtenance	P1	0.3	0.4	0.7
	P2	0.1	0.9	0.2		P2	0.2	0.8	0.3
	P3	0.2	0.3	0.8		P3	0.9	0.2	0.1
	P4	0.9	0.2	0.1		P4	0.1	0.3	0.9
	P5	0.2	0.7	0.2		P5	0.3	0.8	0.1
Intuitionistic degree of	P1	(0.7, 0.2)	(0.2, 0.8)	(0.1, 0.9)	Intuitionistic degree of	P1	(0.3,0.9)	(0.4,0.8)	(0.7, 0.2)

appurtenance	P2	(0.1, 0.8)	(0.9, 0.1)	(0.2, 0.7)	appurtenance	P2	(0.2,0.7)	(0.8, 0.1)	(0.3, 0.6)
	P3	(0.2, 0.7)	(0.3, 0.7)	(0.8, 0.2)		P3	(0.9,0.1)	(0.2,0.7)	(0.1, 0.7)
	P4	(0.8, 0.3)	(0.2, 0.6)	0.1, 0.7)		P4	(0.1,0.6)	(0.3,0.7)	(0.9, 0.2)
	P5	(0.2, 0.7)	(0.7, 0.1)	(0.2, 0.8)		P5	(0.3,0.8)	(0.8,0.1)	(0.1, 0.9)
Neutrosophic degree of appurtenance	P1	(0.7, 0.2, 0.1)	(0.2, 0.8, 0.1)	(0.1, 0.9, 0.3)	Neutrosophic degree of appurtenance	P1	(0.3,0.9,0 .2)	(0.4, 0.8,0.1)	(0.7, 0.2,0.2)
	P2	(0.1, 0.8, 0.2)	(0.9, 0.1, 0.1)	(0.2, 0.7, 0.3)		P2	(0.2,0.7,0 .3)	(0.8, 0.1,0.2)	(0.3, 0.6, 0.2)
	P3	(0.2, 0.7, 0.3)	(0.3, 0.8, 0.1)	(0.9, 0.2, 0.2)		P3	(0.9,0.1,0 .3)	(0.2,0.7, 0.1)	(0.1, 0.7, 0.3)
	P4	(0.8, 0.1, 0.2)	(0.2, 0.7, 0.1)	0.1, 0.7, 0.3)		P4	(0.1,0.6,o .2)	(0.3,0.7, 0.1)	(0.9, 0.2, 0.3)
	P5	(0.1, 0.7, 0.3)	(0.7, 0.1, 0.2)	(0.2, 0.8, 0.2)		P5	(0.3,0.8,0 .2)	(0.8,0.1, 0.3)	(0.1, 0.9, 0.2)

According to the dominant attribute value and recessive attribute value for Quality,

Table 2: shows different forms of contradiction degrees.

Attribute: Quality									
Contradiction Degree with respect to Dominant attribute value		0	1/3	2/3	Contradiction Degree with respect to Recessive attribute value		2/3	1/3	0
Attribute values		**Q11**	Q12	Q13	Attribute values		Q11	Q12	**Q13**
Crisp degree of appurtenance	P1	1	0	0	Crisp degree of appurtenance	P1	0	0	1
	P2	0	1	0		P2	0	1	0
	P3	0	0	1		P3	1	0	0
	P4	1	0	0		P4	0	0	1
	P5	0	1	0		P5	0	1	0
Fuzzy degree of appurtenance	P1	0.85	0.2	0.15	Fuzzy degree of appurtenance	P1	0.3	0.3	0.75
	P2	0.1	0.75	0.2		P2	0.2	0.85	0.1

	P3	0.2	0.2	0.8		P3	0.9	0.1	0.1
	P4	0.95	0.2	0.2		P4	0.1	0.2	0.8
	P5	0.1	0.7	0.1		P5	0.3	0.8	0.1
Intuitionistic degree of appurtenance	P1	(0.7,0.2)	(0.2,0.9)	(0.1,0.85)	Intuitionistic degree of appurtenance	P1	(0.3,0.7)	(0.3, 0.8)	(0.7, 0.1)
	P2	(0.1,0.7)	(0.8,0.1)	(0.2,0.8)		P2	(0.2,0.6)	(0.8, 0.2)	(0.3, 0.75)
	P3	(0.2,0.9)	(0.3,0.8)	(0.8,0.15)		P3	(0.9,0.3)	(0.2,0.8)	(0.2, 0.7)
	P4	(0.95, 0.3)	(0.2,0.65)	0.2, 0.7)		P4	(0.1,0.8)	(0.2,0.75)	(0.8, 0.1)
	P5	(0.1,0.7)	(0.7,0.1)	(0.1,0.9)		P5	(0.3,0.9)	(0.8,0.1)	(0.1, 0.7)
Neutrosophic degree of appurtenance	P1	(0.7,0.2, 0.2)	(0.2,0.9, 0.1)	(0.1,0.9, 0.1)	Neutrosophic degree of appurtenance	P1	(0.3,0.8,0.1)	(0.2, 0.9,0.1)	(0.75, 0.2,0.1)
	P2	(0.2,0.85, 0.2)	(0.8,0.1, 0.1)	(0.2,0.75, 0.2)		P2	(0.1,0.7,0.1)	(0.8, 0.1,0.1)	(0.3, 0.7, 0.1)
	P3	(0.2,0.8, 0.1)	(0.2,0.8, 0.2)	(0.8,0.2, 0.1)		P3	(0.9,0.1,0.1)	(0.1,0.75, 0.1)	(0.1, 0.75, 0.1)
	P4	(0.9,0.2, 0.2)	(0.2,0.7, 0.2)	(0.1, 0.8, 0.2)		P4	(0.1,0.7,o.1)	(0.2,0.7, 0.2)	(0.8, 0.2, 0.1)
	P5	(0.1,0.7, 0.15)	(0.7,0.15, 0.2)	(0.1,0.8, 0.1)		P5	(0.1,0.8,0.1)	(0.8,0.1, 0.2)	(0.1, 0.8, 0.2)

Thus in an extended plithogenic set, both the dominant and recessive attribute values are considered for each of the attribute to make comprehensive decisions. Also in the table 2.1 & 2.2, the nature of both the dominant and recessive attribute values are of same type (either crisp, fuzzy, intuitionistic or neutrosophic) but in realistic perspective, the values of dD and dR may occur in combination with different representations giving a new look of mixed type of ExPlS. It is illustrated in Table 2.3.

Table 3: Illustration of Mixed Extended Plithogenic Sets

Attribute: Price									
Contradiction Degree with respect to Dominant attribute value		0	1/3	2/3	Contradiction Degree with respect to Recessive attribute value		2/3	1/3	0
Attribute values		**P11**	P12	P13	Attribute values		P11	P12	**P13**
Crisp degree of appurtenance	P1	1	0	0	Fuzzy degree of appurtenance	P1	0.1	0.1	0.75

	P2	0	1	0		P2	0.25	0.8	0.15
	P3	0	0	1		P3	0.9	0.15	0.1
	P4	1	0	0		P4	0.1	0.1	0.85
	P5	0	1	0		P5	0.2	0.8	0.2
Fuzzy degree of appurtenance	P1	0.8	0.2	0.2	Intuitionistic degree of appurtenance	P1	(0.1,0.7)	(0.2, 0.7)	(0.7, 0.2)
	P2	0.15	0.65	0.2		P2	(0.2,0.65)	(0.8, 0.3)	(0.2, 0.7)
	P3	0.2	0.25	0.7		P3	(0.9,0.2)	(0.2,0.7)	(0.2, 0.7)
	P4	0.9	0.1	0.1		P4	(0.1,0.9)	(0.2,0.85)	(0.8, 0.2)
	P5	0.1	0.9	0.15		P5	(0.2,0.8)	(0.8 ,0.1)	(0.1, 0.75)
Intuitionistic degree of appurtenance	P1	(0.8,0.2)	(0.2,0.9)	(0.2, 0.8)	Neutrosophic degree of appurtenance	P1	(0.1,0.7,0 .1)	(0.2, 0.75,0.1)	(0.75, 0.1,0.1)
	P2	(0.15, 0.7)	(0.65, 0.1)	(0.2, 0.8)		P2	(0.1,0.7,0 .1)	(0.8, 0.3,0.1)	(0.2, 0.7, 0.1)
	P3	(0.2,0.9)	(0.3,0.8)	(0.7, 0.1)		P3	(0.9,0.1,0 .1)	(0.1,0.7, 0.1)	(0.1, 0.75, 0.1)
	P4	(0.9,0.1)	(0.2,0.7)	(0.2, 0.7)		P4	(0.1,0.8,o .1)	(0.2,0.85, 0.2)	(0.8, 0.2, 0.1)
	P5	(0.1,0.75)	(0.8,0.1)	(0.1, 0.8)		P5	(0.2,0.7,0 .1)	(0.8,0.1, 0.2)	(0.1, 0.8, 0.2)
Neutrosophic degree of appurtenance	P1	(0.8,0.2, 0.2)	(0.2,0.9, 0.1)	(0.1, 0.9, 0.2)	Crisp degree of appurtenance	P1	0	0	1
	P2	(0.2,0.8, 0.2)	(0.8,0.1, 0.1)	(0.1, 0.7, 0.2)		P2	0	1	0
	P3	(0.2,0.8, 0.1)	(0.2,0.8, 0.1)	(0.6, 0.2, 0.1)		P3	1	0	0
	P4	(0.9,0.1, 0.2)	(0.2,0.7, 0.2)	(0.1, 0.8, 0.2)		P4	0	0	1
	P5	(0.1,0.7, 0.1)	(0.8,0.1, 0.2)	(0.1, 0.8, 0.1)		P5	0	1	0

One may question on the need of such a kind of extended plithogenic sets. Generally the dominant attribute values of the attributes are considered which is referred to the most desirable attribute value by the decision makers. In multi-criteria decision making scenario, this kind of approach will facilitate to make decisions on the best of the best alternatives. But at the same time, at certain instances the decision makers need to find the worst of the worst and best of the worst alternatives to avoid them from entering the choice list of decision making. The determination of the worst alternatives shall be found easily by considering recessive attribute values. Henceforth the extended plithogenic sets are developed in this

section with the above substantiating arguments. The applications of extended plithogenic sets are presented in the following sections.

3. Extended Plithogenic Sociogram

Plithogenic Sociogram introduced by Nivetha and Smarandache [38] is an extension of neutrosophic sociogram. To investigate the magnitude of the association between the members of the group, the affirmative and indeterminate nature of establishing relationship between the members are considered. Following the same fashion, in Plithogenic sociogram the affirmative kind of relationship is considered only with respect to dominant attribute value of each of the attribute. To obtain the non-affirmative nature of relationship between the members, the recessive attribute values must be identified and included in the representation for which the extended plithogenic sets highly facilitate. The algorithm of Extended Plithogenic Sociogram and the applications with respect to group dynamics are presented in this section with an example.

3.1 Algorithm of Extended Plithogenic Sociogram

This sub section elucidates the steps involved in computing the ranking preferences using Plithogenic sociogram with ExPls. The above step wise procedure is consolidated and presented in the form of an algorithm for better understanding of the readers.

Step 1: The decision making problem is well defined and the set of preferences (alternatives) say {M1,M2,M3,..Mn}, set of attributes say {Q1,Q2,Q3,…Qm} and set of attribute values say {Q11, Q12,Q13,..Q1p} for the attribute Q11, {Q21,Q22,Q23,..Q2p} for the attribute Q2 and so on are decided based on the expert's opinion along with degree of appurtenance (either crisp, fuzzy, intuitionistic or neutrosophic) and the contradiction degree.

Step 2: The decisions on choosing both the dominant and recessive attribute values with respect to each of the attributes are made.

Step 3: The table comprising the Plithogenic representations of preferential and non-preferential choice making of the alternatives with respect to the dominant and recessive attribute values is constructed as follows

Alternatives	**Preferential choice making based on the dominant attribute values**	**Non-preferential choice making based on the recessive attribute values**
M_1	{M2(Q12(M_1x_{12}),Q21($M1x_{21}$)….,Qmp($M1x_{mp}$)), M3(Q12($M1y_{12}$),Q21($M1y_{21}$)…,Qmp($M1y_{mp}$)),…. Mn(Q12(M1v12),Q21(M1v21),Qmp(M1vmp))}	{M4(Q11(M1z11),Q23(M1z23),….,Qm-1p-1 ($M1z_{(m-,1)(p-1)}$), …… Mn-1(Q11(M1w11),Q23(M1w23),….,Q m-1p-1 ($M1w_{(m-,1)(p-1)}$))}
. . . . .	… … … … … … … … … … … … …. …. …. …. …. ….	… … … … … … … … … … …. …. …. …. ….
M_n	M1(Q12(Mnf12),Q21(Mnf21),….,Qmp(Mnfmp)), M4(Q12(Mnz12),Q21(Mnz21),….,Qmp(Mnzmp))}… Mn-3(Q12(Mne12),Q21(Mne21),Qmp(Mnemp))}	{ M1(Q11(Mnf11),Q23(Mnf23),..,Qm-2p-1($Mnf_{(m-2)(p-1)}$)), M2(Q11(Mnx11),Q23(Mnx23),…, Qm-2p-1($Mnx_{(m-2)(p-1)}$)),……, Mn-2(Q11(Mng11),Q23(Mng23),),…, Qm-2p-1($Mng_{(m-2)(p-1)}$))}

The expression in the first row representing the preferential choice making is
$M1\rightarrow\{M2(Q12(M_1x_{12}),Q21(M1x_{21})\ldots,Qmp(M1x_{mp})),M3(Q12(M1y_{12}),Q21(M1y_{21})\ldots,Qmp(M1y_{mp})),\ldots..,$
$Mn(Q12(M1v12),Q21(M1v21),Qmp(M1vmp))\}$ shall be understood as follows,

The alternative M1 prefers M2 with respect to the dominant attribute values Q12,Q21,.. , Qmp and the respective preferential values are $M1x12$, $M1x_{21\ldots.,}$ $M1x_{mp.}$

Similarly, the expression representing the non-preferential choice making is
$M1\rightarrow\{M4(Q11(M1x11),Q23(M1x23),\ldots,Qm\text{-}1p\text{-}1\ (M1x_{(m-,1)(p-1)}),\ \ldots\ldots$

$Mn\text{-}1(Q11(M1w11),Q23(M1w23),\ldots..,Q\ m\text{-}1p\text{-}1\ (M1w_{(m-,1)(p-1)}))\}$ shall be understood as follows,

The alternative M1 do not prefers M4 with respect to the recessive attribute values Q11,Q23,.. , Q(m-1)(p-1) and the respective non-preferential values are M1x11, $M1x_{23\ldots.,}$ M1x(m-1)(p-1). The same kind of interpretations shall be made from the above representations for other alternative preferences.

Step 4: The Evaluation matrix of the below form is constructed for each of the dominant attribute values and recessive attribute values. Let us consider for instance the attribute values Q12 and Q11 are dominant and recessive respectively.

	M_1	M_2	M_3	M_4	…..	Mn-3	……..	M_n
M_1	0	M_1x_{12}	M_1y_{12}	0	……	0	…..	M_1v_{12}
.	…	…	…	…	…	…	…	…
.	…	…	…	…	…	…	…	…
.	….	….	….	….	….	….	….	….
Mn	M_nf_{12}	0	0	M_nz_{12}	…..	M_ne_{12}	……	0

	M_1	M_2	M_3	M_4	…..	M_{n-2}	M_{n-1}	M_n
M_1	0	0	0	M_1z11	……	0	M1w11	0
.	…	…	…	…	…	…	…	…
.	…	…	…	…	…	…	…	…
.	….	….	….	….	….	….	….	….
Mn	Mnf11	Mnx11	0	0	……	Mng11	0	0

Step 5: The combined evaluation matrix with respect to the dominant attribute values is determined first by multiplying the weights w_k of the dominant attribute values with the corresponding evaluation matrix obtained with respect to each of the dominant attribute values and then the evaluation matrices are summed up together. The combined evaluation matrix with respect to the recessive attribute values is obtained similarly.

Step 6: The plithogenic amicability degree pD_{ij} with respect to dominant attribute values is obtained using $\frac{2}{pD_{ij}} = \frac{1}{u_{ij}} + \frac{1}{u_{ji}}$, where u_{ij} stands for the degree of compatibility between members i and j, indicating that member i favours member j and vice versa. The plithogenic amicability degree pR_{ij} is also calculated with respect to recessive attribute values.

Step 7: The score values for the alternatives are obtained from the plithogenic amicability degree values with respect to both the dominant and recessive attribute values using $F_D(S_g) = \frac{\sum_j pD_{ij}}{\sum_i \sum_j pD_{ij}}$ and $F_R(S_g) = \frac{\sum_j pR_{ij}}{\sum_i \sum_j pR_{ij}}$ respectively

Step 8: The alternatives are ranked separately both with respect to dominant and recessive attribute values. In the context of dominant attribute values, the alternatives with highest scores are given first priorities and in the context of recessive attribute values, the alternatives with least scores are given first priorities.

Step 9: The overall ranking of the alternatives using both the dominant and recessive attribute values is determined by finding the deviations between $F_D(S_g)$ and $F_R(S_g)$. The alternatives with smallest magnitude of deviations are given priorities.

3.2 Example of Plithogenic Sociogram

The example presented in this section is based on [38] is considered with few modifications. Let us consider a situation where the students are asked to group themselves and they are also asked to give their preferential choices of grouping.

Let M = {M1, M2, M3, M4, M5} be the students who are subjected to the following questions. Their preferential choices are collected based on not just with the questions but also with the attribute values.

Write your friends with whom you wish to group with respect to the

Q1: Type of personality

Q2: Social skills

Q3: Degree of Compassion

Here in this case the attributes are the Type of personality, social skills and degree of compassion. The attribute values of the attributes are as follows

Type of Personality = {introvert (Q11), extrovert (Q12), ambivert (Q13)}.

Social skills = {Excellent (Q21), good (Q22), average (Q23)}.

Degree of compassion = {low (Q31), moderate (Q32), high (Q33)}.

In the above example, the dominant attribute values are Q12, Q21, Q33 and the recessive attribute values are Q11, Q23, Q31.

In the case of affirmative preferential choice making, the dominant attribute values Q12, Q21,Q33 are considered, whereas in the case of non-affirmative or non-preferential choice making, the recessive attribute values Q11, Q23, Q31 are considered. Based on these assumptions, the following table 3.1 presents both the affirmative and non-affirmative fuzzy preferential choice making based on respective dominant and recessive attribute values among the five students is presented.

Table 4: Preferential choice making based on the dominant attribute values and Non-preferential choice making based on the recessive attribute values

Students	**Preferential choice making based on the dominant attribute values**	**Non-preferential choice making based on the recessive attribute values**
M_1	{ M2(Q12(0.5),Q21(0.6),Q33(0.8)), M3(Q12(0.5),Q21(0.65),Q33(0.7)), M5(Q12(0.4),Q21(0.3),Q33(0.45))}	{M4(Q11(0.6),Q23(0.7),Q31(0.8)), M2(Q11(0.7),Q23(0.6),Q31(0.65))}
M_2	{M1(Q12(0.8),Q21(0.8),Q33(0.8)), M3(Q12(0.6),Q21(0.5),Q33(0.56)),	{M1(Q11(0.6),Q23(0.7),Q31(0.8)),

	M5(Q12(0.6),Q21(0.7),Q33(0.8))}	M3(Q11(0.75),Q23(0.6),Q31(0.6)), M5(Q11(0.7),Q23(0.6),Q31(0.65))}
M_3	M1(Q12(0.8),Q21(0.7),Q33(0.75)), M4(Q12(0.65),Q21(0.5),Q33(0.65))}	{ M1(Q11(0.75),Q23(0.5),Q31(0.65)), M2(Q11(0.65),Q23(0.7),Q31(0.9)), M5(Q11(0.45),Q23(0.65),Q31(0.75))}
M_4	M1(Q12(0.55),Q21(0.75),Q33(0.8)), M5(Q12(0.6),Q21(0.45),Q33(0.7))}	M1(Q11(0.8),Q23(0.7),Q31(0.7)), M3(Q11(0.7),Q23(0.5),Q31(0.85)), M5(Q11(0.5),Q23(0.6),Q31(0.8))}
M_5	M1(Q12(0.5),Q21(0.75,Q33(0.8)), M3(Q12(0.75),Q21(0.7),Q33(0.55)), M4(Q12(0.7),Q21(0.6),Q33(0.7))}	M2(Q11(0.85),Q23(0.65),Q31(0.75)), M4(Q11(0.7),Q23(0.5),Q31(0.85))}

In the above table the preferential choice making of the student m1 is based on dominant attribute values of extrovert type of personality, excellent social skills and high degree of compassion. According to the student M1, the preferential score values given to the student M3 based on the respective dominant attribute values are 0.5, 0.6, and 0.8. Similarly the student M1 has given the non-preferential score value in grouping with the student M4 based on the recessive attribute values of introvert type of personality, average social skills and low degree of compassion. The respective score values are 0.6, 0.7 and 0.8.

The evaluation matrix for each of the attribute values is framed based on the approach of Plithogenic sociogram as described by Nivetha and Smarandache [52] and for better understanding the authors shall refer the same. The above problem is discussed under two cases.

Case (1a)

3.2.1 Ranking the alternatives based on equal weightage of dominant attribute values

Table 3.2,3.3, 3.4 present the evaluation matrix for the dominant attribute value Q12,Q21andQ 33.

Table 5: Evaluation matrix M_{D1} for the dominant attribute value Q12

	M_1	M_2	M_3	M_4	M_5
M_1	0	0.5	0.5	0	0.4
M_2	0.8	0	0.6	0	0.6
M_3	0.8	0	0	0.65	0
M_4	0.55	0	0	0	0.6
M_5	0.5	0.75	0	0.7	0

Table 6: Evaluation matrix M_{D2} for the dominant attribute value Q21

	M_1	M_2	M_3	M_4	M_5
M_1	0	0.6	0.65	0	0.3
M_2	0.8	0	0.5	0	0.7
M_3	0.7	0	0	0.5	0
M_4	0.75	0	0	0	0.45

M_5	0.75	0.7	0	0.6	0

Table 7: Evaluation matrix M_{D3} for the dominant attribute value Q33

	M_1	M_2	M_3	M_4	M_5
M_1	0	0.8	0.7	0	0.45
M_2	0.8	0	0.56	0	0.8
M_3	0.75	0	0	0.65	0
M_4	0.8	0	0	0	0.7
M_5	0.8	0.55	0	0.7	0

The combined matrix comprising the aggregate dominant attribute values with equal weightage (0.33 each) is present in Table 8

Table 8: Combined Matrix with equal weightage to Dominant Attribute Values

	M_1	M_2	M_3	M_4	M_5
M_1	0	0.627	0.611	0	0.3795
M_2	0.792	0	0.548	0	0.693
M_3	0.7425	0	0	0.594	0
M_4	0.693	0	0	0	0.5775
M_5	0.677	0.66	0	0.66	0

The plithogenic amicability degree p_{ij} is obtained using Eq.3.1

$$\frac{2}{pD_{ij}} = \frac{1}{u_{ij}} + \frac{1}{u_{ji}} \qquad \text{---------- (3.1)}$$

where u_{ij} stands for the degree of compatibility between members i and j, indicating that member i favours member j and vice versa.

Table 9: Measure of Amicability Degree with equal weightage to Dominant Attribute Values

	M_1	M_2	M_3	M_4	M_5
M_1	0	0.699	0.6704	0	0.4864
M_2	0.699	0	0	0	0.6761
M_3	0.6704	0	0	0	0
M_4	0	0	0	0	0.616
M_5	0.4864	0.6761	0	0.616	0

The score values for the alternatives are obtained using Eq.3.2 and it is represented in Table 3.7

$$F_D(Sg) = \frac{\sum_j pD_{ij}}{\sum_i \sum_j pD_{ij}} \qquad \text{----------(3.2)}$$

Table 10: Final Ranking of the Alternatives with equal weightage to Dominant Attribute Values

Alternatives	**Score Value**	**Ranking of Alternatives**
M_1	0.2948	1
M_2	0.2184	3
M_3	0.1065	4
M_4	0.0978	5
M_5	0.2825	2

The alternatives are ranked based on their score values. The alternatives with maximum score values are highly preferred.

Case (1b)

3.2.2 Ranking the alternatives based on unequal weightage of dominant attribute values

The combined matrix shall be obtained by considering unequal weights to the dominant attribute values. In this case, the weights are assigned as follows W (Q12) =0.5, W(Q21) =0.25, W(Q33)=0.25 and Table 3.8 represents the aggregate values .

Table 11: Combined Matrix with unequal weightage to Dominant Attribute Values

	M_1	M_2	M_3	M_4	M_5
M_1	0	0.6	0.5875	0	0.3875
M_2	0.8	0	0.565	0	0.675
M_3	0.763	0	0	0.6125	0
M_4	0.6625	0	0	0	0.5875
M_5	0.6375	0.6875	0	0.675	0

The plithogenic amicability degree obtained using Eq.3.1 is presented in Table 3.9

Table 12: Measure of Amicability Degree with unequal weightage to Dominant Attribute Values

	M_1	M_2	M_3	M_4	M_5
M_1	0	0.6857	0.6638	0	0.482
M_2	0.6857	0	0	0	0.6812
M_3	0.6638	0	0	0	0
M_4	0	0	0	0	0.6282
M_5	0.482	0.6812	0	0.6282	0

The score values for the alternatives obtained using Eq. 3.2 are shown in the Table 3.10

Table 13: Final Ranking of the Alternatives with unequal weightage to Dominant Attribute Values

Alternatives	Score Value	Ranking of Alternatives
M_1	0.2916	1
M_2	0.2176	3

M_3	0.1057	4
M_4	0.1	5
M_5	0.2852	2

In this case also the alternatives are ranked based on the score values, the alternatives with higher score values are given first priorities. The ranking of the alternatives using the score values based on dominant attribute values facilitate ranking of best of the best alternatives. Now

Case (2a)

3.2.3 Ranking the alternatives based on equal weightage of recessive attribute values

In this case the alternatives are ranked based on the recessive attribute values by following the same procedure as in case (1a)

Table 3.11, 3.12 and 3.13 presents the evaluation matrix for the recessive attribute value Q11,Q23 and Q31.

Table 14: Evaluation matrix M_{R1} for the recessive attribute value Q11

		M_1	M_2	M_3	M_4	M_5
M_1		0	0.7	0	0.6	0
M_2		0.6	0	0.75	0	0.7
M_3		0.75	0.65	0	0	0.45
M_4		0.8	0	0.7	0	0.5
M_5		0	0.85	0	0.7	0

Table 15: Evaluation matrix M_{R2} for the recessive attribute value Q23

	M_1	M_2	M_3	M_4	M_5
M_1	0	0.6	0	0.7	0
M_2	0.7	0	0.6	0	0.6
M_3	0.5	0.7	0	0	0.65
M_4	0.7	0	0.5	0	0.6
M_5	0	0.65	0	0.5	0

Table 16: Evaluation matrix M_{R3} for the recessive attribute value Q31

	M_1	M_2	M_3	M_4	M_5
M_1	0	0.65	0	0.8	0
M_2	0.8	0	0.6	0	0.65
M_3	0.65	0.9	0	0	0.75
M_4	0.7	0	0.85	0	0.8
M_5	0	0.75	0	0.85	0

The combined matrix comprising the aggregate recessive attribute values with equal weightage (0.33 each) is present in Table 3.14

Table 17: Combined Matrix with equal weightage to Recessive Attribute Values

	M_1	M_2	M_3	M_4	M_5
M_1	0	0.6435	0	0.693	0
M_2	0.726	0	0.6435	0	0.6435
M_3	0.627	0.7425	0	0	0.6105
M_4	0.726	0	0.6765	0	0.627
M_5	0	0.7425	0	0.6765	0

The plithogenic amicability degree is obtained using Eq.3.3

$$\frac{2}{pR_{ij}} = \frac{1}{u_{ij}} + \frac{1}{u_{ji}} \quad \text{---------- (3.3)}$$

Table 18: Measure of Amicability Degree with equal weightage to Recessive Attribute Values

	M_1	M_2	M_3	M_4	M_5
M_1	0	0.6823	0	0.7091	0
M_2	0.6823	0	0.6895	0	0.6895
M_3	0	0.6895	0	0	0
M_4	0.7091	0	0	0	0.6508
M_5	0	0.6895	0	0.6508	0

The score values of the alternatives are obtained using Eq.3.4.

$$F_R(Sg) = \frac{\sum_j pR_{ij}}{\sum_i \sum_j pR_{ij}} \quad \text{---------- (3.4)}$$

The ranking of the alternatives based on the score values is presented in Table 3.16.

Table 19: Final Ranking of the Alternatives with equal weightage to Recessive Attribute Values

Alternatives	**Score Values**	**Ranking of Alternatives**
M_1	0.2034	4
M_2	0.3013	5
M_3	0.1008	1
M_4	0.1988	3
M_5	0.1959	2

Case (2b)

3.2.4 Ranking the alternatives based on unequal weightage of recessive attribute values

The combined matrix in Table 3.17 is also determined by considering unequal weightage to all the recessive attribute values as follows: W(Q12)=0.5, W(Q21)=0.25 W(Q33)=0.25

Table 20: Combined Matrix with unequal weightage to Recessive Attribute Values

	M_1	M_2	M_3	M_4	M_5
M_1	0	0.6375	0	0.7	0
M_2	0.7	0	0.6375	0	0.6375
M_3	0.6	0.7375	0	0	0.625
M_4	0.725	0	0.6325	0	0.625
M_5	0	0.725	0	0.6375	0

The plithogenic amicability degree is obtained using Eq.3.3

Table 21: Measure of Amicability Degree with unequal weightage to Recessive Attribute Values

	M_1	M_2	M_3	M_4	M_5
M_1	0	0.6673	0	0.7123	0
M_2	0.6673	0	0.6817	0	0.6784
M_3	0	0.6817	0	0	0
M_4	0.7123	0	0	0	0.6311
M_5	0	0.6784	0	0.6311	0

The score values of the alternatives are obtained using Eq.3.4. The ranking of the alternatives based on the score values is presented in Table 22.

Table 22: Final Ranking of the Alternatives with unequal weightage to Recessive Attribute Values

Alternatives	**Score Value**	**Ranking of Alternatives**
M_1	0.2046	4
M_2	0.3007	5
M_3	0.1011	1
M_4	0.1993	3
M_5	0.1947	2

The score values of the alternatives are generated using equation (4), and they are then ranked from highest to lowest based on their score values for the dominating attribute values. This kind of ranking facilitate in choosing best out of worst alternatives

The ranking results obtained in four cases namely case 1a,1b,2a,2b are compared in Table 3.20

Table 23: Comparison of Ranking Results

Alternatives	**Ranking with respect**	**Ranking with**	**Ranking with**	**Ranking with**

	to Dominant Attribute Values with equal weightage	respect to Dominant Attribute Values with unequal weightage	respect to Recessive Attribute Values with equal weightage	respect to Recessive Attribute Values with unequal weightage
M_1	1	1	4	4
M_2	3	3	5	5
M_3	4	4	1	1
M_4	5	5	3	3
M_5	2	2	2	2

Thus the Extended Plithogenic Sets facilitate in choosing the best of the best alternatives using dominant attribute values and best of the worst alternatives using recessive attribute values in decision making. In this case the alternative M5 is chosen to be best as in both the cases of dominant and recessive attribute values, the ranking is promising. The ranking results in Table 3.20 are more specific with respect to the decision maker's perception and the nature of the decision making situation. If the objective of the decision maker is to maximize the profit then the concern on the dominant attribute values will be very high. But at the same time if the decision maker is intended to minimize his loss, then he is more concerned on the recessive attribute values. In the former case M1 is preferred and in the latter M3 is preferred. Thus the ranking results in Table 3.20 are more specific and one can make choices of their own based on the constraints of profit maximization and loss minimization.

On other hand, it is not possible to obtain an overall ranking of the alternatives from the ranking results of Table 3.20 and henceforth the score values of the alternatives which are obtained under different cases are combined to arrive at global ranking. The score values obtained in case 1a and 2a are combined to determine the overall ranking of the alternatives (i.e. with respect to both dominant and recessive attribute values by considering equal weightages). Similarly, the score values obtained in case 1b and 2b are aggregated together to find the overall ranking of the alternatives (i.e. with respect to both dominant and recessive attribute values by considering unequal weightages). The overall ranking of the alternatives obtained from the deviations in the score values is presented in Table 3.21.

Table 24: Overall Ranking of the Alternatives

Alternatives	Deviations in the score values with respect to Dominant and Recessive Attribute Values with equal weightage	Ranking of the Alternatives	Deviations in the score values with respect to Dominant and Recessive Attribute Values with unequal weightage	Ranking of the Alternatives
M_1	0.0914	4	0.087	3
M_2	0.0829	2	0.0831	2
M_3	0.0057	1	0.0046	1
M_4	0.101	5	0.0993	5
M_5	0.0866	3	0.0905	4

From Table 3.21, the overall ranking of the alternatives is obtained by giving preference to the alternative with small differences between the score values. The score values obtained by considering dominant attribute values indicate positive score of the alternatives and the score values obtained using recessive attribute values indicate the negative scores of the alternatives. Therefore the differences between the positive and negative score values are considered in ranking the alternatives.

If equal weightages are considered then the ranking result obtained are M3 > M2 >M5>M1 > M4and if unequal weightages are considered then the ranking results obtained are M3 > M2 > M1> M5 >M4. Thus the ranking results differs accordingly to the assignment of weightages to the attribute values.

4. Applications of Extended Plithogenic Sociogram in Decision making on Food Packaging materials

In this section the extended plithogenic sociogram approach is applied in making decisions on food packaging materials. Food industries are supplying different varieties of food to the populace across this globe. The reputation of these industries is dependent on quality conservation. The aspect of quality and taste shall be conserved by using suitable food packaging materials. These industries must be cautious in making optimal selection of packaging materials and also must measure the potency of these materials using certain standard criteria or attributes. The packaging materials used in the food industry are available in a wide range of materials, forms, and colours that have distinct purposes in relation to maintaining the characteristics of the food item that they hold. In general the food industries are using different types of food packaging materials made up of metals, paper, plastic and glass. Table 4.1 presents the literature review of the nature of materials used in the Food packaging industries and the attributes considered

Table 25: State of Art of Research on the nature and attributes of the Food Packaging Materials

Author & Year	**Nature of the material**	**Attributes to be Considered**
Thakur.P et al., (2020) [51]	Food packaging colours for identification	Quality, storage, safety, reusable, easy to use and identification
Kenneth Marsh. ,et al (2007) [52]	Protection of food products and environmental issues	Shelf-life.
Alamri M.S [53]	Food safety perspective	Safety, quality,
Isabelle Maillet., et al (2013) [54]	Safety analysis and decision-making tool	Economic, safety, eco-system
Dikky Indrawan.,et al., (2019) [55]	Priority choice for ecofriendly bio-based food packaging	Quality , cost of material , cost of new investment , cost of human resource training , environmental friendly
Norton, V., et al.,(2022) [56]	Examining how consumers in the UK understand and view sustainable food packaging	Quality and shelf life
K. Cooksey., et al (2005) [57]	Antimicrobial packaging	Safety
M. Asgher., et al (2020) [58]	Bio-based food packaging materials	Safety.
Karina Petersen.,et al (1999) [59]	Bio-based food packaging materials	Safety, eco-system., quality
Michael G. K.,et al (2020) [60]	Preservation of food packaging technology	Quality ,safety, eco-system
Dele Raheem et al., (2012) [61]	Plastics and paper as food packaging materials	Quality ,safety, eco-system, shelf life
Ioannis S. A., et al (2004)[62]	Antimicrobial food packaging	Quality ,safety, eco-system
Jung H. Han.,et al (2005) [63]	Preservation of foods	Quality ,safety, eco-system
Md. Wadud Ahmed .,et al (2022) [64]	Current trends, applications, prospects and challenges	Quality and safety, shelf life
Hao Cheng .,et al (2022) [65]	Principles and preparations of food packaging materials	Maturity, quality, and safety.
Porta R.,et al(2020)[66]	Biopolymer based Food Packaging Materials	Cost, eco-friendly, storage and transport, shelf life
Saurabh Sid., et al (2021)[67]	Bio-sourced polymers in food packaging materials	Quality, storage, cost, health and environment

Based on the above Table 4.1, the following attributes and attribute values are considered for making optimal decisions are presented in Table 4.2

Table 26: Attribute & Attribute Values of Decision Making

Attribute	**Attribute values**

Price F1	Expensive (F11), affordable (F12), cheap (F13)
Resistance F2	High (F21), average (F22), low (F23)
Shell Life Longevity F3	Long (F31), moderate(F32), short(F33)
Eco-friendly F4	Reusable(F41), recyclable(F42) , non-biodegradable(F43)

4.1 Decision making based on Dominant Attribute Values

The dominant attribute values considered for decision making are Cheap (F13), High (F21), Long shell life (F31) and Reusable (F41). The preferential choice –making with respect to compatibility from the viewpoint of the experts is presented in Table 4.3

Table 27: Expert's opinion on Preferential Choice-Making over Compatibility

Alternatives/ Machines	**Preferential Choice-Making over Compatibility based on Dominant Attribute Values**	
	Expert-I	**Expert-II**
M_1	{M2(F13(0.5),F21(0.6),F31(0.7),F41(0.5)),M4(F13(0.6),F21(0.8),F31(0.6),F41(0.4))}	{M3(F13(0.4),F21(0.5),F31(0.7),F41(0.7)),M5(F13(0.5),F21(0.6),F31(0.6), F41(0.8))}
M_2	M1(F13(0.7),F21(0.6),F31(0.6), F41(0.4)), M5(F13(0.8),F21(0.6),F31(0.5), F41(0.7))}	{M3(F13(0.6),F21(0.4),F31(0.7),F41(0.8)), M4(F13(0.9),F21(0.8),F31(0.7), F41(0.6))}
M_3	{M1(F13(0.4),F21(0.5),F31(0.6),F41(0.7)),M2(F13(0.5),F21(0.6),F31(0.6),F41(0.8)),M4(F13(0.6),F21(0.8),F31(0.6), F41(0.5))}	M2(F13(0.6),F21(0.6),F31(0.7),F41(0.8)), M4(F13(0.4),F21(0.5),F31(0.6), F41(0.5))}
M_4	{M2(F13(0.7),F21(0.5),F31(0.6),F41(0.8)),M3(F13(0.4),F21(0.6),F31(0.5), F41(0.7))}	{M1(F13(0.5),F21(0.6),F31(0.6),F41(0.8)), M3(F13(0.5),F21(0.5),F31(0.7),F41(0.6)),M5(F13(0.8),F21(0.8),F31(0.7), F41(0.4))}
M_5	{M1(F13(0.5),F21(0.5),F31(0.7),F41(0.7)),M2(F13(0.8),F21(0.6),F31(0.8),F41(0.4)),M3(F12(0.7),F23(0.7),F31(0.6), F41(0.5))}	{M1(F13(0.5),F21(0.4),F31(0.6),F41(0.7)), M4(F13(0.6),F21(0.8),F31(0.5), F41(0.5))}

Based on the procedural explained in section 3, the combined matrix considering equal weightage of 0.25 to each of the dominant attribute values is presented in Table 4.4

Table 28: Combined Matrix with equal weightage to Dominant Attribute Values

	M_1	M_2	M_3	M_4	M_5
M_1	0	0.2875	0.285	0.3	0.3125
M_2	0.2875	0	0.3125	0.375	0.325
M_3	0.275	0.65	0	0.3125	0
M_4	0.3125	0.325	0.5625	0	0.3375
M_5	0.575	0.325	0.3125	0.3	0

The plithogenic amicability degree is obtained using Eq.3.1 and the values are presented in Table 4.5

Table 29:Measure of Amicability Degree with equal weightage to Dominant Attribute Values

	M_1	M_2	M_3	M_4	M_5
M_1	0	0.2875	0.2799	0.3061	0.405
M_2	0.2875	0	0.4221	0.3482	0.325
M_3	0.2799	0.4221	0	0.4018	0
M_4	0.3061	0.3482	0.4018	0	0.3176
M_5	0.405	0.325	0	0.3176	0

The score values of the alternatives are obtained using Eq.3.2. The ranking of the alternatives based on the score values is presented in Table 4.6

Table 30: Final Ranking of the Alternatives with equal weightage to Dominant Attribute Values

Alternatives	**Score Value**	**Ranking of Alternatives**
M_1	0.2067	3
M_2	0.2235	1
M_3	0.1784	4
M_4	0.2221	2
M_5	0.1693	5

The above procedure is repeated with unequal weights to the dominant attribute values namely F13 = 0.4, F21 = 0.3, F31 = 0.2 and F41 = 0.1.

Table 31: Combined Matrix with unequal weightage to Dominant Attribute Values

	M_1	M_2	M_3	M_4	M_5
M_1	0	0.285	0.26	0.32	0.29
M_2	0.31	0	0.29	0.4	0.335
M_3	0.25	0.61	0	0.565	0
M_4	0.29	0.315	0.53	0	0.37
M_5	0.535	0.35	0.33	0.315	0

The plithogenic amicability degree is obtained using Eq.3.1 and the values are presented in Table 4.8

Table 32: Measure of Amicability Degree with unequal weightage to Dominant Attribute Values

	M_1	M_2	M_3	M_4	M_5
M_1	0	0.297	0.255	0.304	0.376
M_2	0.297	0	0.393	0.352	0.342
M_3	0.255	0.393	0	0.547	0

M_4	0.304	0.352	0.547	0	0.34
M_5	0.376	0.342	0	0.34	0

The score values of the alternatives are obtained using Eq.3.2. The ranking of the alternatives based on the score values is presented in Table 4.9

Table 33: Final Ranking of the Alternatives with unequal weightage to Dominant Attribute Values

Alternatives	Score Value	Ranking of Alternatives
M_1	0.1921	3
M_2	0.2158	2
M_3	0.1864	4
M_4	0.2406	1
M_5	0.165	5

4.2 Decision making based on Recessive Attribute Values

The recessive attributive values are Expensive (F11), low (F23), short (F33), non-biodegradable(F43). The opinion of the experts in the context of non-preferential choice making is presented in Table 4.10.

Table 34: Expert's opinion on Non-Preferential Choice-Making over Compatibility

Alternatives/ Machines	**Non-Preferential Choice-Making over Compatibility based on Recessive Attribute Values**	
	Expert-I	Expert-II
M_1	{M_3(F11(0.5),F23(0.4),F33(0.4),F43(0.5)), M_5((F11(0.6),F23(0.6),F33(0.4),F43(0.6)))}	{M2(F11(0.4),F23(0.7),F33(0.7),F43(0.6)), M_4((F11(0.4),F23(0.7),F33(0.6),F43(0.7)))}
M_2	{M3(F11(0.6),F23(0.4),F33(0.5),F43(0.5)), M_4((F11(0.7),F23(0.6),F33(0.5),F43(0.4)))}	{M1(F11(0.6),F23(0.6),F33(0.8),F43(0.6)), M_5((F11(0.7),F23(0.6),F33(0.5),F43(0.4)))}
M_3	{M2(F11(0.3),F23(0.6),F33(0.8),F43(0.6)), M_5((F11(0.5),F23(0.6),F33(0.7),F43(0.5)))}	{M1(F11(0.6),F23(0.9),F33(0.5),F43(0.4)), M_4((F11(0.5),F23(0.6),F33(0.7),F43(0.5)), M_5((F11(0.5),F23(0.5),F33(0.4),F43(0.5)))}
M_4	M2(F11(0.5),F23(0.4),F33(0.7),F43(0.4)), M_1((F11(0.6),F23(0.7),F33(0.7),F43(0.6)))}	{M3(F11(0.8),F23(0.6),F33(0.6),F43(0.7)), M_5((F11(0.7),F23(0.6),F33(0.7),F43(0.6)))}
M_5	M2(F11(0.5),F23(0.7),F33(0.8),F43(0.7)), M_4((F11(0.7),F23(0.4),F33(0.6),F43(0.5)), M_3((F11(0.5),F23(0.6),F33(0.7),F43(0.5)))}	{M1(F11(0.5),F23(0.7),F33(0.6),F43(0.4)), M_3((F11(0.6),F23(0.7),F33(04),F43(0.5)))}

Based on the procedural explained in section 3, the combined matrix considering equal weightage of 0.25 to each of the recessive attribute values is presented in Table 4.11

Table 35: Combined Matrix with equal weightage to Recessive Attribute Values

	M_1	M_2	M_3	M_4	M_5
M_1	0	0.3	0.225	0.3	0.275
M_2	0.325	0	0.25	0.275	0.275
M_3	0.3	0.2875	0	0.2875	0.525
M_4	0.325	0.25	0.3375	0	0.325
M_5	0.275	0.3375	0.5625	0.275	0

The plithogenic amicability degree is obtained using Eq.3.3 and the values are presented in Table 4.12

Table 36: Measure of Amicability Degree with equal weightage to Recessive Attribute Values

	M_1	M_2	M_3	M_4	M_5
M_1	0	0.312	0.2571	0.312	0.275
M_2	0.312	0	0.2674	0.2619	0.3031
M_3	0.2571	0.2674	0	0.3105	0.5431
M_4	0.312	0.2619	0.3105	0	0.2979
M_5	0.275	0.3031	0.5431	0.2979	0

The score values of the alternatives are obtained using Eq.3.4. The ranking of the alternatives based on the score values is presented in Table 4.13

Table 37: Final Ranking of the Alternatives with equal weightage to Recessive Attribute Values

Alternatives	Score Value	Ranking of Alternatives
M_1	0.1841	2
M_2	0.1822	1
M_3	0.2194	4
M_4	0.1883	3
M_5	0.226	5

The above procedure is repeated with unequal weights to the Recessive attribute values namely F11 = 0.5, F23 = 0.2, F33 = 0.2 and F43 = 0.1.

Table 38: Combined Matrix with unequal weightage to Recessive Attribute Values

	M_1	M_2	M_3	M_4	M_5
M_1	0	0.27	0.23	0.109	0.28
M_2	0.32	0	0.265	0.305	0.305
M_3	0.31	0.245	0	0.28	0.52
M_4	0.32	0.26	0.36	0	0.335
M_5	0.275	0.31	0.565	0.3	0

The plithogenic amicability degree is obtained using Eq.3.3 and the values are presented in Table 4.15

Table 39: Measure of Amicability Degree with unequal weightage to Recessive Attribute Values

	M_1	M_2	M_3	M_4	M_5
M_1	0	0.293	0.264	0.162	0.278
M_2	0.293	0	0.255	0.28	0.307
M_3	0.264	0.255	0	0.315	0.541
M_4	0.162	0.28	0.315	0	0.317
M_5	0.278	0.307	0.541	0.317	0

The score values of the alternatives are obtained using Eq.3.4. The ranking of the alternatives based on the score values is presented in Table 4.16

Table 40: Final Ranking of the Alternatives with unequal weightage to Recessive Attribute Values

Alternatives	**Score Value**	**Ranking of Alternatives**
M_1	0.166	1
M_2	0.1884	3
M_3	0.2283	4
M_4	0.1783	2
M_5	0.2395	5

5. Discussion

In this section the ranking results obtained using the conditions of equal and unequal weightages to both of the dominant and recessive attribute values are compared and presented in Table 5.1

Table 41: Comparison of Ranking Results

Alternatives	**Ranking with respect to Dominant Attribute Values with equal weightage**	**Ranking with respect to Dominant Attribute Values with unequal weightage**	**Ranking with respect to Recessive Attribute Values with equal weightage**	**Ranking with respect to Recessive Attribute Values with unequal weightage**
M_1	3	3	2	1
M_2	1	2	1	3
M_3	4	4	4	4
M_4	2	1	3	2
M_5	5	5	5	5

Based on the dominant attribute values, the ranking results of the alternative packaging materials obtained is M4>M2>M1>M3>M5.In this case, the alternatives having greater score values are given high priorities. Based on the recessive attribute values, the ranking results of the alternative packaging materials obtained is M1>M4>M2>M3>M5. The ranking of the materials is based on the least scores of the alternatives. The ranking results shall also be interpreted as follows, based on the score values, the alternative M5 has given least preference then followed by M3, M2, M4 and M1. From both the ranking

on the suitability and non-suitability. If the decision makers have to make decisions based on feasibility then the ranking results obtained using dominant attribute values shall be considered, also if the decision has to be made based on infeasibility, then the ranking result obtained using recessive attribute values shall be taken into account. The first set of ranking results is centered on acceptance of the alternatives and the latter is based on denial of the alternatives. Also the second set of ranking facilitates us to decide the alternatives based on the principle of best out of worst. On the other hand, the student M5 has the lowest score, which indicates that the member is not much preferred as the attributes of M5 may not seem to be influential. It is very evident that the member M1 has the highest score, which represents the significance of the member M1 in the group and how his influencing attributes have made M2 more preferable.

Also the overall ranking of the alternatives shall be determined by considering the differences in the respective score values of dominant and recessive attribute values with equal and unequal weightages. The ranking results are presented in Table 5.2

Table 42: Overall Ranking of the Alternatives

Alternatives	**Deviations in the score values with respect to Dominant and Recessive Attribute Values with equal weightage**	**Ranking of the Alternatives**	**Deviations in the score values with respect to Dominant and Recessive Attribute Values with unequal weightage**	**Ranking of the Alternatives**
M_1	0.0226	1	0.0261	1
M_2	0.0413	4	0.0274	2
M_3	0.041	3	0.0419	3
M_4	0.0338	2	0.0623	4
M_5	0.0567	5	0.0745	5

From Table 5.2, it is observed that the alternative M1 gets the first preference and the alternative M5 gets the last preference in both the cases. The combined ranking results are also in consensus with the ranking results of Table 5.1. The decision maker can make use of the ranking results in both Table 5.1 and 5.2 based on the requirements of specificity or a comprehensive ranking.

6. Conclusion

This article explains the idea of developing of a more extensive form of Plithogenic sets. The conceptualization of 7-tuple plithogenic set plays a significant role in Plithogenic sociogram. The decision making based on Plithogenic sociogram with extended plithogenic sets is a novel approach of this research work. The consideration of dominant and recessive attribute values facilitates in obtaining the most feasible alternatives. The ranking results obtained using dominant attribute values are more closely associated with the intention of acceptance of the alternatives by the decision makers, on other hand the ranking results obtained using recessive attribute values are more closely associated with the intention of avoidance of the alternatives by the decision makers. On intensive analysis and comparison with the overall framework of multi-criteria decision making, the ranking approach based on dominant attribute values is in consensus with the concept of positive ideal solution and preferential choice making, also the ranking approach based on recessive attribute values is in consensus with the concept of negative ideal solution and non-preferential choice making. Thus the applications of extended plithogenic sets in Plithogenic sociogram discloses new approaches of making optimal decisions in specific to multi-criteria decision making methods.

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Plithogenic Forest Hypersoft Sets in Plithogenic Contradiction Based Multi-Criteria Decision Making

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Sathya, P., et al. "Plithogenic Forest Hypersoft Sets in Plithogenic Contradiction Based Multi-Criteria Decision Making." *Neutrosophic Sets and Systems*, vol. 73, 2024, pp. 668-693.

Abstract: Plithogenic based decision making approaches are more versatile and accommodative with regard to attribute handling This paper introduces the concepts of Plithogenic Forest hypersoft set (PFHS) and develops a Plithogenic centered decision-making model with PFHS representations. The Plithogenic method of devising decisions based on contradictions is integrated with the newly introduced representations of PFHS to develop a robust decision-making technique to deal with attributes and sub-attributive values at a larger scale. The integrated method proposed in this work is applied to a decision-making problem of site selection for establishing manufacturing plants. The core attributes are identified and the respective Plithogenic forest hypersoft sets are constructed with the possible sub-attributes. In this case, each of the core attributes itself forms a Plithogenic tree hypersoft set representations with several sub-attribute values, the alternatives are subjected to each of the criteria to determine the optimal ranking in specific to the criteria. Also, aggregate score values are determined to obtain a more comprehensive ranking. The concept of PFHS shall be integrated with other decision-making methods to evolve novel methods of decision-making.

Keywords: Plithogenic Forest Hypersoft sets, attributes, sub-attribute values, decision-making, site selection.

1. Introduction

Smarandache [1] introduced the concept of hypersoft sets as an extension of soft sets to deal with attributes values subjected to each of the attributes say A1, A2,..An. The hypersoft set representations of the form Ai1, Aj2,..Akn are more compatible and comprehensive as they deal with various attribute values at a time whereas the soft sets deal with only a single attribute value at an instance.

The hypersoft sets shall be primarily classified into fuzzy, intuitionistic, neutrosophic based on the nature of the values indicating the magnitude of association between the attribute values and the elements of universe of discourse. The hypersoft sets are applied in several decision-making circumstances especially in ranking kind of multi-criteria decision-making problems. Researchers prefer these hypersoft sets of different types to handle the scenario involving several criteria or the attribute in designing optimal solution. Researchers have also discussed different forms of hypersoft sets such as Single-valued, multi-valued hypersoft sets[2], Bipolar Hypersoft sets[3], Picture Hypersoft sets[4], convex and concave hypersoft sets[5], N-hypersoft sets[6], bijective hypersoft sets[7] and many other as a means of extending the efficacy of hypersoft sets in data handling.
Smarandache[8] extended hypersoft sets to Plithogenic hypersoft sets (PHS) in which the degree of appurtenance is presented to each of the attribute values with respect to the elements of the universe of discourse. Smarandache[20] has sketched out a vivid picture between Plithogenic soft sets and Plithogenic hypersoft sets. Plithogenic hypersoft sets are also applied in decision making to develop a more comprehensive solution to the decision-making problems. Researchers have developed PHS based decision models in different domains, To mention a few significant works, Martin and Smarandache[9] presented the applications of combined plithogenic hypersoft sets. Martin et al [10, 13] explored the applications of extended plithogenic hypersoft sets in Covid-19 decision making. Priya et al[11,12] induced the plithogenic cognitive analysis with combined connection. Martin and Smarandache[14] leveraged the notion of concentric plithogenic hypergraph embedded with Plithogenic hypersoft sets in decision making. Ahmad et al[15,17] formulated a multi-criteria decision-making model using plithogenic hypersoft sets. Rana et al[16,18] introduced plithogenic fuzzy whole hypersoft sets and generalized plithogenic whole hypersoft sets and applied multi-attribute decision making diagnostic models. Majid et al[19] formulated a decision model for site selection using plithogenic multipolar fuzzy hypersoft sets. These recent contributions substantiate the proficiency of plithogenic hypersoft sets in making optimal decisions.

Smarandache[21] also introduced few types of soft sets such as indeterm soft sets, indeterm hypersoft sets and tree soft sets. In the indeterm hypersoft sets, either any of the attributes deal with indeterminate values. In case of tree soft sets, the attributes that are considered form a tree structure with root attribute at level 0 and branches indicating the attribute values and attribute sub-values in the subsequent levels. However, in considering the attribute values, the tree soft set representations reflect tree hypersoft sets. This uniqueness of tree soft set has motivated the authors to evolve the concept of Plithogenic forest hypersoft sets (PFHS) which is a union of several Plithogenic tree soft sets (PTSS). The proposed notion of PFHS based representations is employed in making decisions on site selection on integrating with contradiction based Plithogenic decision method.

The remaining contents of the paper are structured into different sections as follows. Section 2 presents the conceptualization of Plithogenic Forest Hypersoft sets. Section 3 describes the decision-making model framework applied in this research work. Section 4 applies the model to the decision-making problem of site selection of manufacturing plants. Section 5 discusses the results and the last section concludes the work.

2. Theoretical Development of Plithogenic Forest Hypersoft Sets (PFHS)

This section presents the conceptualization of Forest Hypersoft sets and then describes the extension of the same to PFHS.

Let U be the universe of discourse, H be the non-empty subset of U, A be the set of attributes
Each of the attributes has different levels
Level 1 be the sub attribute values
Level 2 be the sub-sub attribute values:
Level n be the n-sub attribute values
Each of the attributes forms a Tree soft sets and all these tree soft sets together form a forest hypersoft sets.
The Forest Hypersoft Set shall be defined as G : P(Forest (A)) → P(H)
Where Forest (A) = {Tree (A)} and Tree (A) = {Ai1 | i1= 1, 2, ... }

Let us first discuss the construction of forest hypersoft sets with a simple example. Let us consider a decision-making problem on supplier selection based on different attributes say A1,A2,A3. In this case, the attributes A1, A2 and A3 forms the root level. Each of the attributes has different levels where each level indicates the sub attribute values. Thus, each attribute with its respective sets of sub attribute values together forms a tree soft set. In this case three tree soft sets are obtained for three different attributes. The union of these three tree soft sets together form a forest hypersoft sets.

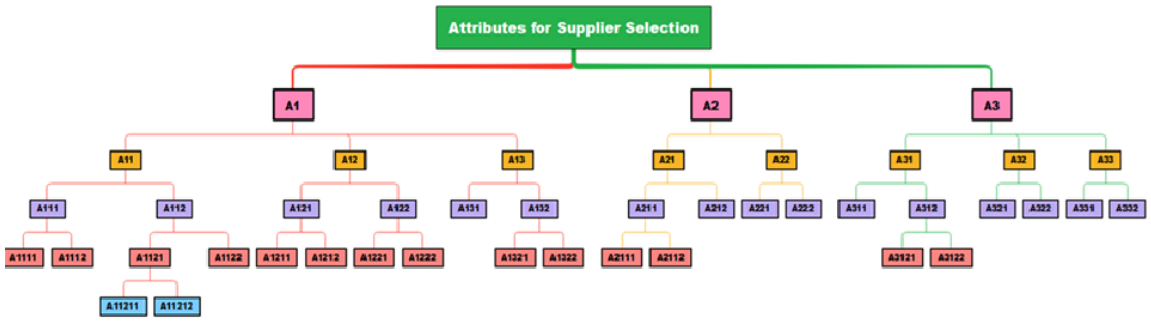

Fig.1. Plithogenic Forest Hypersoft sets

From the above fig.1., the attributes A1,A2 and A3 form the root level say L0. The next level L1 is subjected to each of the attributes which consists of the attribute values say for $A1$, the attribute sub values are A11, A12, A13 and for A2 it is, A21, A22 are the attribute values and then for A3 it is A31, A32 and A33. The level L2, consists of sub - sub – attribute values (sub_2 – attribute values) say for A11, the A111 and A1112 are the respective sub-attribute values however they are the sub_2 – attribute values of A1. Similarly, the attribute values A1111 and A1112 are the sub attribute values of A111 and sub_2 – attribute values of A11 and sub_3 – attribute values of A1. Also, A11211 and A11212 are the sub-attribute values of A1121, sub_2 – attribute values of A112, sub_3 – attribute values of A11 and sub_4 – attribute values of A1. A similar kind of discussion shall be made for the attributes A2 and A3.

If suppose the attribute A1 is considered for ranking the suppliers then, the attribute values from each of the sub_n attribute values has to be chosen and this will be of the form
F (A1112 × A11212 × A1211 × A1221 × A131 × A1322), this expression is derived from the representations of tree soft sets. If the decision maker considers the attribute A1, then the attribute values say A11, A12 and A13 have to be considered. In this case, each of the attribute values form a tree structure and hence ultimately at the end only the attribute values represented in last levels shall be taken for decision-making.

3. Classification of Forest Hypersoft sets

The forest hypersoft sets shall be classified generally as fuzzy forest hypersoft sets, intuitionistic forest hypersoft sets and neutrosophic hypersoft sets based on the membership values representing the degree of satisfaction, the alternatives make with the attribute values taken for consideration.

Example

Let the set of suppliers be {S1,S2,..Sn} and let us consider a set say H = {S1,S2,S3,S4}. Then let us consider the attribute values A1112 ,A11212 , A1211, A1221, A131 and A1322. Then Forest Hypersoft set is of the form F (A1112 × A11212 × A1211 × A1221 × A131 × A1322) = { S1 (0.2),S2 (0.5), S3 (0.5), S4 (0.7)} is termed as fuzzy forest hypersoft set.

F (A1112 × A11212 × A1211 × A1221 × A131 × A1322) = { S1 (0.2,0.7),S2 (0.5,0.4), S3 (0.5,0.3), S4 (0.7,0.2)} is termed as intuitionistic forest hypersoft set.

F (A1112 × A11212 × A1211 × A1221 × A131 × A1322) = { S1 (0.2,0.1,0.6),S2 (0.5,,0.2,0.4), S3 (0.5,0.3,0.4), S4 (0.7,0.2,0.1)} is termed as neutrosophic forest hypersoft set.

However, in case of Plithogenic forest hypersoft sets, the dominant attribute values are initially assigned and the contradiction degrees between the attribute values are determined. Then the degree of appurtenance of the alternative with respect to the dominant attribute values are determined and the contradiction degree are also considered in making decisions. For simplification, the plithogenic forest hypersoft sets are represented in a tabular form. Also, the Plithogenic forest hypersoft sets shall be classified based on their degree of appurtenance.

Table 1. Classification of Plithogenic forest hypersoft sets

Crisp	**A1112**	**A11212**	**A1211**	**A1221**	**A131**	**A1322**
S1	1	1	1	1	1	1
S2	1	1	1	1	1	1
S3	1	1	1	1	1	1
S4	1	1	1	1	1	1
S5	1	1	1	1	1	1
Fuzzy	**A1112**	**A11212**	**A1211**	**A1221**	**A131**	**A1322**
S1	0.2	0.4	0.5	0.3	0.7	0.8
S2	0.1	0.7	0.9	0.4	0.6	0.4
S3	0.5	0.6	0.3	0.2	0.7	0.1

S4	0.4	0.6	0.7	0.6	0.4	0.5
S5	0.6	0.4	0.2	0.7	0.5	0.8
Intuitionistic	**A1112**	**A11212**	**A1211**	**A1221**	**A131**	**A1322**
S1	(0.2, 0.8)	(0.4, 0.6)	(0.5, 0.5)	(0.3, 0.7)	(0.7, 0.3)	(0.8, 0.2)
S2	(0.1, 0.9)	(0.7, 0.3)	(0.9, 0.1)	(0.4, 0.6)	(0.6, 0.4)	(0.4, 0.6)
S3	(0.1, 0.9)	(0.7, 0.3)	(0.9, 0.1)	(0.4, 0.6)	(0.6, 0.4)	(0.4, 0.6)
S4	(0.4, 0.6)	(0.6, 0.4)	(0.7, 0.3)	(0.6, 0.4)	(0.4, 0.6)	(0.5, 0.5)
S5	(0.6, 0.4)	(0.4, 0.6)	(0.2, 0.8)	(0.7, 0.3)	(0.5, 0.5)	(0.8, 0.2)
Neutrosophic	**A1112**	**A11212**	**A1211**	**A1221**	**A131**	**A1322**
S1	(0.2,0.2,0.7)	(0.4,0.1,0.5)	(0.5,0.1,0.4)	(0.3,0.2,0.6)	(0.7,0.1,0.2)	(0.8,0.01,0.1)
S2	(0.1,0.15,0.8)	(0.7,0.20,0.2)	(0.9,0.1,0.0)	(0.4,0.1,0.5)	(0.6,0.2,0.3)	(0.4, 0.1, 0.5)
S3	(0.5, 0.1, 0.4)	(0.6, 0.2, 0.3)	(0.3,0.1,0.6)	(0.2,0.1,0.7)	(0.7,0.1,0.2)	(0.1, 0.1, 0.8)
S4	(0.4, 0.1, 0.5)	(0.6, 0.1, 0.3)	(0.7,0.2,0.2)	(0.6,0.1,0.3)	(0.4,0.1,0.5)	(0.5, 0.1, 0.4)
S5	(0.6, 0.1, 0.3)	(0.4, 0.1, 0.5)	(0.2,0.1,0.7)	(0.7,0.1,0.2)	(0.5,0.1,0.4)	(0.8, 0.1, 0.1)

In the above table 1., the values in each cell indicate the degrees of appurtenance of the suppliers with each of the dominant attribute values. The above table demonstrates the classifications of Plithogenic forest hypersoft sets into crisp, fuzzy, intuitionistic and neutrosophic based on the appurtenance values. In other representations of Forest Hypersoft sets the appurtenance degree reflect all the attribute values whereas in case of Plithogenic Forest Hypersoft sets, the appurtenance degree is considered for each of the attribute values.

Modelling Framework of Plithogenic Forest Hypersoft sets based Decision Making [22]

This section outlines the working procedure of the Plithogenic contradiction-based decision method with the representations of Plithogenic Forest Hypersoft sets.

Step 1: The decision making ecosystem is well defined by determining the alternatives say S1, S2,..Sn and attributes with sub_n attribute values. The dominant attribute values are identified and classified as benefit and non-benefit.

Step 2: The initial contradiction matrix considering the alternatives and the dominant attribute values is constructed from the attribute matrix with each cell representing the attribute value of the alternatives. The qualitative attribute matrix is then converted to the contradiction matrix by calculating the degree of contradiction existing between the attribute value of the alternative and the dominant attribute value.

	A_{1i}	A_{2j}	…..	…..	A_{nh}
S_1	A_{11}	A_{23}	…..	…..	A_{n4}
S_2	…..	…..	…..	…..	…..
:	…..	…..	…..	…..	…..
:	…..	…..	…..	…..	…..
S_n	A_{14}	A_{21}	…..	…..	A_{n2}

The above matrix representation is the attribute matrix which is constructed by considering the attribute values possessed by the alternatives in par with the dominant attribute values. The respective contradiction matrix is drawn by considering the degree of contradiction between the attribute values in the matrix and the dominant attribute values.

	A_{1i}	A_{2j}	…..	…..	A_{nh}
S_1	$C(A_{11}, A_{1i})$	$C(A_{23}, A_{2j})$	…..	…..	$C(A_{n4}, A_{nh})$
S_2	…..	…..	…..	…..	…..
:	…..	…..	…..	…..	…..
:	…..	…..	…..	…..	…..
S_n	$C(A_{14}, A_{1i})$	$C(A_{21}, A_{2j})$	…..	…..	$C(A_{n2}, A_{nh})$

The above is the contradiction matrix obtained from the above attribute matrix, where each of the cell values represents the contradiction degrees between the dominant attribute values and the actual attribute value of the alternatives

Step 3: The weighted contradiction matrix is calculated by multiplying the attribute value weights with the contradiction degrees.

	A_{1i}	A_{2j}	…..	…..	A_{nh}
S_1	$w_1C(A_{11},A_{1i})$	$w_2C(A_{23}, A_{2j})$	…..	…..	$w_nC(A_{n4},A_{nh})$
S_2	…..	…..	…..	…..	…..
:	…..	…..	…..	…..	…..
:	…..	…..	…..	…..	…..
S_n	$w_1C(A_{14},A_{1i})$	$w_2C(A_{21}, A_{2j})$	…..	…..	$w_nC(A_{n2},A_{nh})$

Step 4: The cumulative score values of the alternatives with respect to the benefit nature of attribute values and the cost nature of attribute values are determined. The cumulative score values of the benefit nature of attribute values are denoted by B_q and the cumulative score values of the cost nature of attribute values are denoted by C_k.

Step 5: The alternatives are ranked based on the differences between these benefit and non-benefit attribute values i.e B_q - $C_k$and the highest rankings are assigned to the alternatives with maximum differences of values.

The above working procedure shall be characterized into three major phased process. The first phase comprises defining decision making ecosystem, construction of attribute matrix and contradiction matrix. The second phase consists of steps 3 and 4 and the last phase consists of step 5.

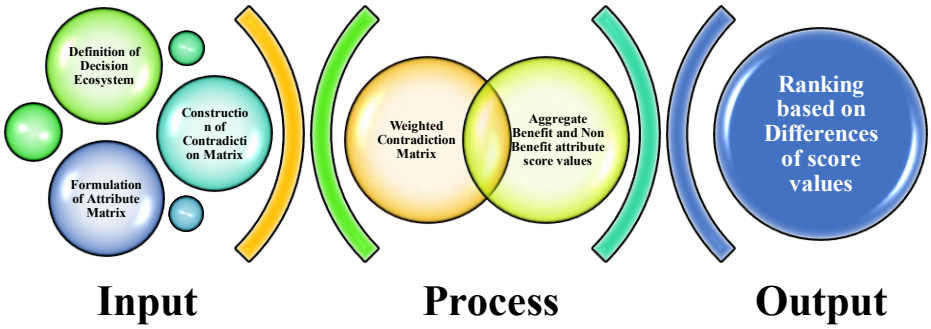

Fig.2. Process of three major phased

4. Application of the Proposed Modeling Framework in site selection

This section presents the modality of applying Plithogenic contradictions-based decision making with Plithogenic hypersoft sets in site selection for establishing manufacturing plants say P1,P2,P3,P4,P5. The core attributes considered in general are presented in Fig.3.

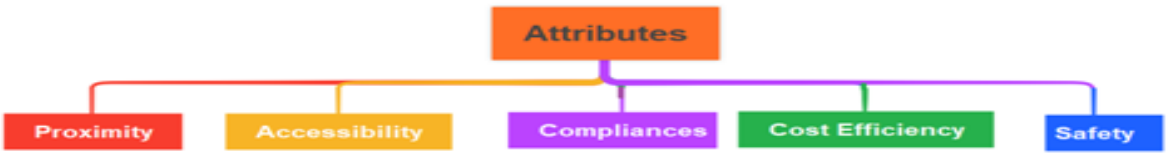

Fig.3. Classification of Attributes

The core attributes occupy the root level of the forest and each of the attributes takes a form of tree hypersoft sets and it is presented as follows.

Firstly, the attribute Proximity is considered. The following figure 4, represents the tree representations of the attribute Proximity and its sub attribute values at different levels.

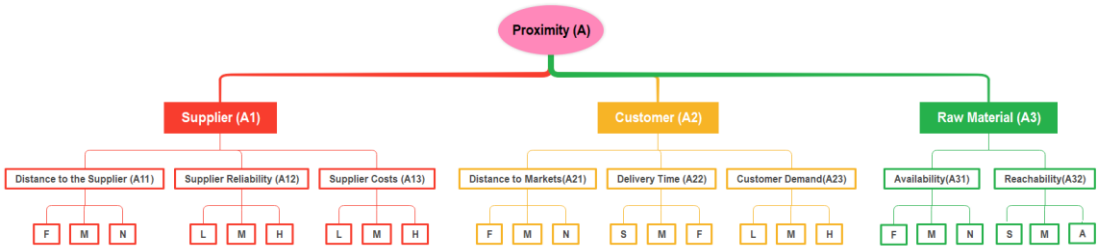

Fig.4.Proximity

The description of the attributes in the above fig.4. is sketched out in the following table 2.

Table 2. Description of the attributes

Attribute	**Sub-Attribute values**	**Sub_2 Attribute Values (Sub-sub Attribute Values)**	**Sub_3Attribute Values (Sub-sub – sub–Attribute Values)**
Proximity (A)	Supplier (A1)	Distance to the Supplier (A11)	Far (A111) Moderate (A112) **Near (A113)**
		Supplier Reliability (A12)	Low (A121) Moderate (A122) **High (A123)**
		Supplier Costs (A13)	**Low (A131)** Moderate (A132) High (A133)
	Customer (A2)	Distance to Markets (A21)	Far (A211) Moderate (A212) **Near (A213)**
		Delivery Time (A22)	Slow (A221) Moderate (A222) **Fast (A223)**
		Customer Demand (A23)	Low (A231) Moderate (A232) **High (A233)**
	Raw Material (A3)	Proximity to Sources (A31)	Far (A311) Moderate (A312) **Near (A313)**
		Raw material Availability (A32)	Scare (A321) Moderate (A322) **Abundant (A323)**

If the criteria proximity is alone considered the dominant sub-sub-sub attribute values have to be considered which are {Near, High, Low, Near, Fast, High, Near, Abundant}
Let us label it as P(A1111× A1123×A1131×A1213×A1223×A1233×A1313× A1323)

Table 3. Attribute matrix

Attributive Values	**P1**	**P2**	**P3**	**P4**	**P5**
A1133	Far	Far	Moderate	Near	Near
A1233	Low	Moderate	High	Low	Low
A1311	Moderate	Low	Moderate	High	Low
A2133	Near	Moderate	Fast	Moderate	Moderate
A2233	Moderate	Fast	Fast	Slow	Slow
A2333	High	Low	Low	Moderate	High
A3133	Scarce	Abundant	Moderate	Abundant	Scarce
A3233	Moderate	Near	Far	Far	Far

The above table 3, represents the attribute matrix.
The contradiction degrees with respect to the dominant attribute values are presented in Table 4.

Table 4. Contradiction value

Dominant Attribute Value	**Contradiction Degree**
A113	C(A111,A113) =2/3, C(A112,A113) =1/3
A123	C(A121,A123) =2/3, C(A122,A123) =1/3
A131	C(A132,A131) =1/3, C(A133,A131) =2/3
A213	C(A211,A213) =2/3, C(A212, A213)= 1/3
A223	C(A221,A223) =2/3, C(A222,A223) =1/3
A233	C(A231,A233) =2/3, C(A232, A233) =1/3
A313	C(A311, A313)=2/3, C(A312,A313)=1/3
A323	C(A321,A323) =2/3, C(A322,A323)= 1/3

Based on the contradiction values presented in the above Table 4, the respective contradiction matrix is as follows.

Table 5. Contradiction matrix

Attributive Values	**P1**	**P2**	**P3**	**P4**	**P5**
A1133 (C)	2/3	2/3	1/3	0	0

Attributive Values	**P1**	**P2**	**P3**	**P4**	**P5**
A1233 (B)	2/3	1/3	0	2/3	2/3
A1311 (C)	1/3	0	1/3	2/3	0
A2133 (C)	0	1/3	2/3	1/3	1/3
A2233 (C)	1/3	0	0	2/3	2/3
A2333 (B)	0	2/3	2/3	1/3	0
A3133 (C)	2/3	0	1/3	0	2/3
A3233 (B)	1/3	0	2/3	2/3	2/3

In the above matrix, the contradiction values of the attribute values with respect to the dominant attribute values are presented in the contradiction matrix. The weighted contradiction matrix is determined by considering equal weightage to all the dominant attribute values.

Table 6. Weighted contradiction matrix

Alternatives	**A1133 (C)**	**A1233 (B)**	**A1311 (C)**	**A2133 (C)**	**A2233 (C)**	**A2333 (B)**	**A3133 (C)**	**A3233 (B)**
P1	0.08333	0.08333	0.041667	0	0.041667	0	0.08333	0.041667
P2	0.08333	0.041667	0	0.041667	0	0.08333	0	0
P3	0.041667	0	0.041667	0.08333	0	0.08333	0.04167	0.08333
P4	0	0.08333	0.08333	0.041667	0.08333	0.04167	0	0.08333
P5	0	0.08333	0	0.041667	0.08333	0	0.08333	0.08333

By using step 4 and step 5, the score values of the alternatives are determined and presented in Table 7.

Table 7. Differences in Score values

Alternatives	**B_A**	**C_A**	**B_A- C_A**
P1	0.124997	0.249994	-0.124997
P2	0.125	0.124997	3E-06
P3	0.16666	0.208337	-0.04168
P4	0.20833	0.208327	3E-06
P5	0.16666	0.208327	-0.04167

Secondly, the attribute Accessibility is considered. The following figure 5. represents the tree representations of the attribute Accessibility and its sub attribute values at different levels.

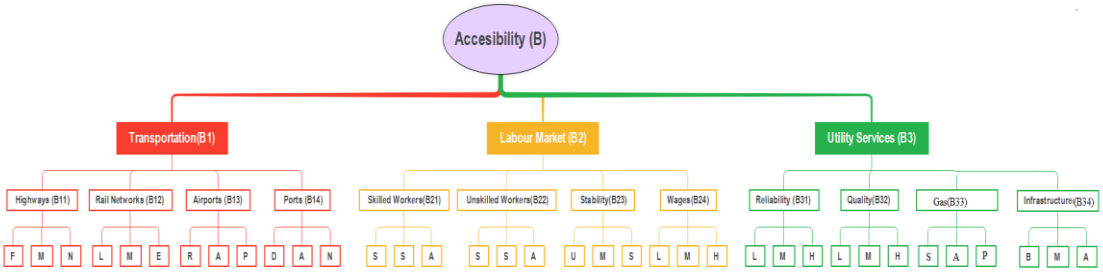

Fig.5. Accessibility

The description of the attributes in the above fig.5. is sketched out in the following table 8.

Table 8. Description of the attributes

Attribute	**Sub-Attribute values**	**Sub_2 Attribute Values (Sub-sub–Attribute Values)**	**Sub_3Attribute Values (Sub-sub – sub–Attribute Values)**
Accessibility (B)	Transportation (B1)	Proximity to Highways (B11)	Far (B111) Moderate (B112) **Near (B113)**
		Access to Rail Networks (B12)	Limited (B121) Moderate (B122) **Extensive (B123)**
		Access to Airports (B13)	Remote (B131) Accessible (B132) **Proximate (B133)**
		Access to Ports (B14)	Distant (B141) Accessible (B142) **Nearby (B143)**
	Labour Market (B2)	Availability to Skilled Workers (B21)	Scare (B211) Sufficient (B212) **Abundant (B213)**
		Availability to Unskilled Workers (B22)	Scare (B221) Sufficient (B222) **Abundant (B223)**
		Labor Market Stability (B23)	Unstable (B231) Moderate (B232) **Stable (B233)**
		Wage Labour (B24)	**Low (B241)** Moderate (B242) High (B243)

	Utility Services (B3)	Reliability of Electricity (B31)	Unreliable (B311) Moderate (B312) **Reliable (B313)**
		Availability and Quality of Water (B32)	Limited (B321) Adequate (B322) **Excellent (B323)**
		Availability of Gas (B33)	Scare(B331) Available (B332) **Plentiful (B333)**
		Telecommunications Infrastructure (B34)	Basic (B341) Moderate (B342) **Advanced(B343)**

If the criteria Accessibility is alone considered the dominant (sub_3) sub-sub-sub attribute values to be considered are {Near, Extensive, Proximate, Nearby, Abundant, Stable, Low, Reliable, Excellent, Plentiful, Advanced}

Let us consider the representation of the form

P(B1133xB1233xB1333xB1433xB2133xB2233xB233xB2411xB313xB3233xB3333xB3433)

The attribute matrix with respect to the attribute Accessibility is presented in Table 9.

Table 9. Attribute matrix

Attributive Values	**P1**	**P2**	**P3**	**P4**	**P5**
B1133	Near	Far	Moderate	Near	Far
B1233	Limited	Moderate	Limited	Extensive	Moderate
B1333	Accessible	Proximate	Proximate	Remote	Accessible
B1433	Nearby	Accessible	Distant	Nearby	Accessible
B2133	Scare	Abundant	Sufficient	Abundant	Scare
B2233	Sufficient	Scare	Scare	Sufficient	Abundant
B2333	Stable	Unstable	Stable	Unstable	Moderate
B2411	Moderate	Low	High	Moderate	High
B3133	Reliable	Moderate	Unreliable	Moderate	Unreliable
B3233	Adequate	Excellent	Limited	Adequate	Excellent
B3333	Plentiful	Available	Scare	Plentiful	Scare
B3433	Basic	Advanced	Basic	Moderate	Advanced

The contradiction degrees with respect to the dominant attribute values are presented in Table 10.

Table 10. Contradiction value

Dominant Attribute Value	**Contradiction Degree**
B113	C(B111, B113) = 2/3, C(B112, B113) =1/3
B123	C(B121, B123) = 2/3, C(B122, B123) =1/3
B133	C(B131, B133) =2/3, C(B132, B133) =1/3
B143	C(B141, B143) = 2/3, C(B142, B143) =1/3
B213	C(B211, B213) = 2/3, C(B212, B213) =1/3
B223	C(B221, B223) = 2/3, C(B222, B223) =1/3
B233	C(B231, B233) = 2/3, C(B232, B233) =1/3
B241	C(B242, B241) =1/3, C(B243, B241) =2/3
B313	C(B311, B313) =2/3, C(B312, B313) =1/3
B323	C(B321, B323) =2/3, C(B322, B323) =1/3
B333	C(B331, B333) = 2/3, C(B332, B333) =1/3
B343	C(B341, B343) =2/3, C(B412, B343) =1/3

The respective contradiction matrix is as follows and presened in Table 11.

Table 11. Contradiction matrix

Attributive Values	**P1**	**P2**	**P3**	**P4**	**P5**
B1133 (C)	0	2/3	1/3	0	2/3
B1233(C)	2/3	1/3	2/3	0	1/3
B1333 (C)	1/3	0	0	2/3	1/3
B1433 (C)	0	1/3	2/3	0	1/3
B2133(B)	2/3	0	1/3	0	2/3
B2233(B)	1/3	2/3	2/3	1/3	0
B2333(B)	0	2/3	0	2/3	1/3
B2411(C)	1/3	0	2/3	1/3	2/3
B3133(B)	0	1/3	2/3	1/3	2/3
B3233(B)	1/3	0	2/3	1/3	0
B3333(B)	0	1/3	2/3	0	2/3
B3433(B)	2/3	0	2/3	1/3	0

The weighted contradiction matrix is as follows

Table 12. Weighted contradiction matrix

Alternatives	B113 (C)	B123 (C)	B133 (C)	B143 (C)	B213 (B)	B223 (B)	B233 (B)	B241 (C)	B313 (B)	B323 (B)	B333 (B)	B343 (B)
P1	0	0.055556	0.027778	0	0.055556	0.027778	0	0.027778	0	0.027778	0	0.055556
P2	0.055556	0.027778	0	0.027778	0	0.055556	0.055556	0	0.027778	0	0.027778	0
P3	0.027778	0.055556	0	0.055556	0.027778	0.055556	0	0.055556	0.055556	0.055556	0.055556	0.055556
P4	0	0	0.055556	0	0	0.027778	0.055556	0.027778	0.027778	0.027778	0	0.027778
P5	0.055556	0.027778	0.027778	0.027778	0.055556	0	0.027778	0.055556	0.055556	0	0.055556	0

By following the similar procedure, the score values are presented in Table 13

Table 13. Difference in Score values

Alternatives	B_B	C_B	B_B- C_B
P1	0.111116	0.166676	0.05556
P2	0.111116	0.166676	0.05556
P3	0.194458	0.305578	0.11112
P4	0.083338	0.166672	0.083334
P5	0.194454	0.194458	4E-06

Thirdly the attribute Compliances is taken into account. Fig.6., represents the tree representations of the attribute

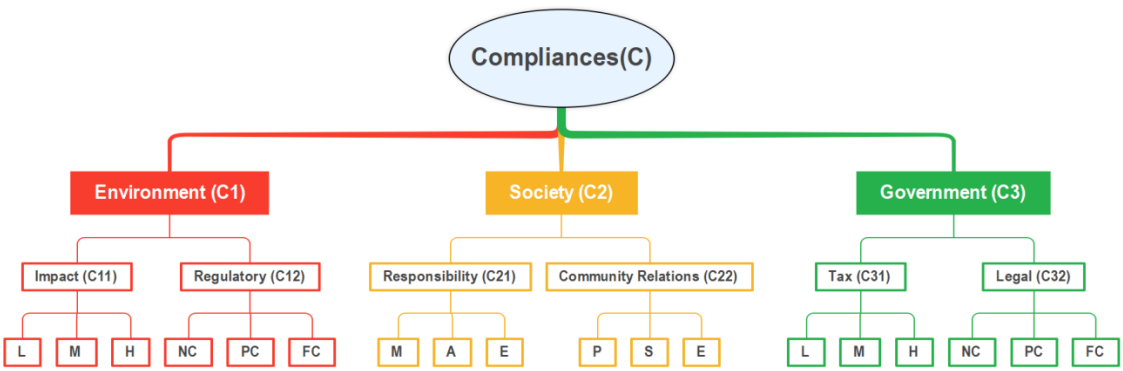

Fig.6. Compliances

The description of the attributes in the above fig.6. is sketched out in the following table 14.

Table 14. Description of attributes

Attribute	**Sub-Attribute values**	**Sub$_2$ Attribute Values (Sub-sub Attribute Values)**	**Sub$_3$Attribute Values (Sub-sub – sub–Attribute Values)**
Compliances (C)	Environment (C1)	Environmental Impact (C11)	**Low (C111)** Moderate (C112) High (C113)
		Regulatory Compliance (C12)	Non-Compliant (C121) Partially Compliant (C122) **Fully Compliant (C123)**
	Society (C2)	Social Responsibility (C21)	Minimal (C211) Adequate (C212) **Extensive(C213)**
		Community Relations (C22)	Poor (C221) Satisfactory (C222) **Excellent (C223)**
	Government (C3)	Tax Compliance (C31)	Low(C311) Moderate (C312), **High (C313)**
		Legal Compliance (C32)	Non- Compliant (C321) Partially Compliant (C322) **Fully Compliant (C323)**

If the criteria Compliances is alone considered the dominant sub-sub-sub attribute values to be considered are {Low, Fully Compliant, Extensive, Excellent, High, Fully Compliant}

Let us consider the representation of the form P(C1111xC1233xC2133xC2233xC3133xC3233)

The attribute matrix with respect to the attribute Compliances is presented in Table 15.

Table 15. Attribute matrix

Attributive Values	**P1**	**P2**	**P3**	**P4**	**P5**
C1111	High	Low	Moderate	High	Moderate
C1233	Partially Compliant	Non-Compliant	Non-Compliant	Partially Compliant	Fully Compliant
C2133	Extensive	Adequate	Minimal	Adequate	Minimal
C2233	Poor	Satisfactory	Excellent	Satisfactory	Excellent

Attributive Values	P1	P2	P3	P4	P5
C3133	Moderate	High	Low	Moderate	High
C3233	Non-Compliant	Partially Compliant	Fully Compliant	Non-Compliant	Fully Compliant

The contradiction degrees with respect to the dominant attribute values are presented in Table 16.

Table 16. Contradiction value

Dominant Attribute Value	Contradiction Degree
C111	C(C112, C111) = 1/3, C(C113, C111) = 2/3
C123	C(C121, C123) = 2/3, C(C122, C123) = 1/3
C213	C(C211, C213) = 2/3, C(C212 ,C213) = 1/3
C223	C(C221, C223)= 2/3, C(C222, C223) = 1/3
C313	C(C311,C313) = 2/3, C(C312, C313) = 1/3
C323	C(C321, C323) = 2/3 , C(C322, C323) = 1/3

The respective contradiction matrix is as follows and presented in Table 17.

Table 17. Contradiction matrix

Attributive Values	P1	P2	P3	P4	P5
C1111(C)	2/3	0	1/3	2/3	1/3
C1233(B)	1/3	2/3	2/3	1/3	0
C2133(B)	0	1/3	2/3	1/3	2/3
C2233(B)	2/3	1/3	0	1/3	0
C3133(C)	1/3	0	2/3	1/3	0
C3233(C)	2/3	1/3	0	2/3	0

The weighted contradiction matrix is as follows,

Table 18. Weighted contradiction matrix

Alternatives	C1111(C)	C1233(B)	C2133(B)	C2233(B)	C3133(C)	C3233(C)
P1	0.11111	0.055556	0	0.11111	0.055556	0.11111
P2	0	0.11111	0.055556	0.055556	0	0.055556

Alternatives	C1111(C)	C1233(B)	C2133(B)	C2233(B)	C3133(C)	C3233(C)
P3	0.055556	0.11111	0.11111	0	0.11111	0
P4	0.11111	0.055556	0.055556	0.055556	0.055556	0.11111
P5	0.055556	0	0.11111	0	0	0

By following the similar procedure, the score values are obtained and presented in Table 19.

Table 19. Difference in Score values

Alternatives	B_C	C_C	B_C- C_C
P1	0.166666	0.277776	-0.11111
P2	0.222222	0.055556	0.166666
P3	0.22222	0.166666	0.055554
P4	0.166668	0.277776	-0.11111
P5	0.11111	0.055556	0.055554

Fourthly the attribute Cost Efficiency is taken into account. Fig.7. represents the tree representations of the attribute

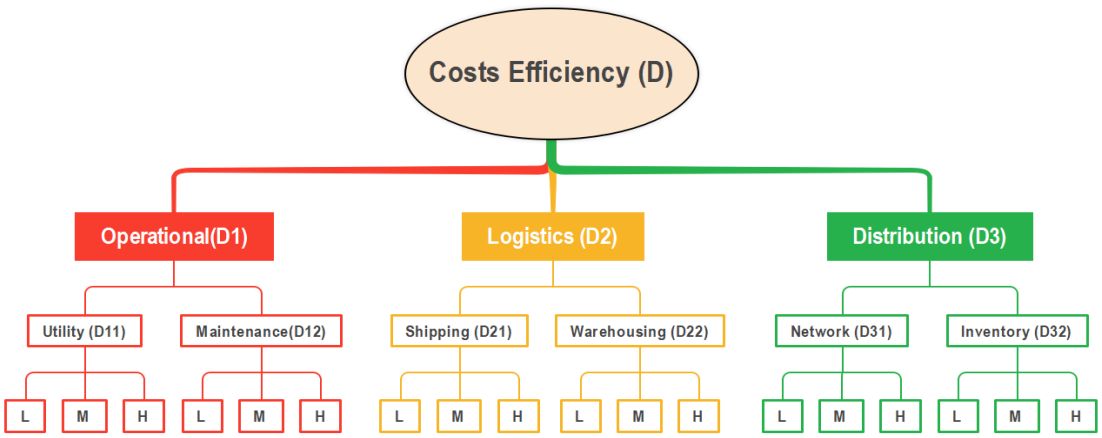

Fig.7. Cost Efficiency

The description of the attributes in the above fig.7. is sketched out in the following table 20.

Table 20. Description of Attributes

Attribute	**Sub-Attribute values**	**Sub$_2$ Attribute Values (Sub-sub Attribute Values)**	**Sub$_3$Attribute Values (Sub-sub – sub–Attribute Values)**
Costs Efficiency (D)	Operation (D1)	Utility Costs(D11)	**Affordable (D111)** Moderate (D112) Expensive (D113)
		Maintenance Costs (D12)	**Low (D121)** Moderate (D122) High (D123)
	Logistics (D2)	Shipping and Freight Costs (D21)	**Economical (D211)** Moderate (D212) Expensive (D213)
		Ware housing Costs (D22)	**Low (D221)** Moderate (D222) High (D223)
	Distribution (D3)	Distribution Network Costs (D31)	**Low-Cost (D311)** Moderate (D312) High-cost (D313)
		Inventory Holding Costs (D32)	**Low (D321)** Moderate (D322) High (D323)

If the criteria Cost Efficiency is alone considered the dominant sub-sub-sub attribute values to be considered are {Affordable, Low, Economical, Low, Low- Cost, Low}

Let us consider the representation of the form P(D1111xD1211xD2111xD2211xD3111xD3211)
The attribute matrix with respect to the attribute Cost Efficiency is presented in Table 21.

Table 21. Attribute matrix

Attributive Values	**P1**	**P2**	**P3**	**P4**	**P5**
D1111	Affordable	Moderate	Expensive	Affordable	Moderate
D1211	Moderate	High	Low	High	Low
D2111	Expensive	Economical	Moderate	Moderate	Expensive
D2211	Low	Moderate	High	Moderate	High
D3111	Moderate	Low-cost	Moderate	High-cost	Low-cost
D3211	High	Moderate	Low	Low	High

The contradiction degrees with respect to the dominant attribute values are presented in Table 22.

Table 22. Contradiction value

Dominant Attribute Value	Contradiction Degree
D111	C(D112, D111) = 1/3, C(D113, D111) = 2/3
D121	C(D122,D121) = 1/3 , C(D123, D121) =2/3
D211	C(D212, D211) = 1/3, C(D213, D211) =2/3
D221	C(D222, D221) = 1/3 , C(D223, D221) =2/3
D311	C(D312, D311) = 1/3 , C(D313, D311) = 2/3
D321	C(D322, D321) = 1/3 , C(D323, D321) = 2/3

The respective contradiction matrix obtained as follows

Table 23. Contradiction matrix

Attributive Values	P1	P2	P3	P4	P5
D1111(C)	0	1/3	2/3	0	1/3
D1211(B)	1/3	2/3	0	2/3	0
D2111(C)	2/3	0	1/3	1/3	2/3
D2211(C)	0	1/3	2/3	1/3	2/3
D3111(C)	1/3	0	1/3	2/3	0
D3211(B)	2/3	1/3	0	0	2/3

The weighted contradiction matrix is

Table 24. Weighted contradiction matrix

Alternatives	D1111(C)	D1211(B)	D2111(C)	D2211(C)	D3111(C)	D3211(B)
P1	0	0.05556	0.11111	0	0.05556	0.11111
P2	0.05556	0.11111	0	0.05556	0	0.05556
P3	0.11111	0	0.05556	0.11111	0.05556	0
P4	0	0.11111	0.05556	0.05556	0.11111	0
P5	0.05556	0	0.11111	0.11111	0	0.11111

By following the similar procedure, the score values are obtained and presented in Table 25.

Table 25. Difference in Score values

Alternatives	B_D	C_D	B_D- C_D
P1	0.16667	0.16667	0
P2	0.16667	0.11112	0.05555
P3	0	0.33334	-0.33334
P4	0.11111	0.22223	-0.11112
P5	0.11111	0.27778	-0.16667

Finally, the attribute Safety is considered. The diagrammatic representation is presented in Fig.8.

Fig.8. Safety

The description of the attributes in the above fig.8. is sketched out in the following table 26.

Table 26. Description of Attributes

Attribute	Sub-Attribute values	Sub_2 Attribute Values (Sub-sub Attribute Values)	Sub_3 Attribute Values (Sub-sub – sub–Attribute Values)
Safety (E)	Community (E1)	Community Safety (E11)	Low(E111) Moderate (E112) **High (E113)**
		Natural Disaster Risk (E12)	**Low (E121)** Moderate (E122) High (E123)
	Risk (E2)	Risk Assessment (E21)	**Low (E211)** Moderate (E212)

			High (E213)
		Emergency Response (E22)	Inadequate (E221) Adequate (E222) **Robust (E223)**
	Quality (E3)	Work Environment quality (E31)	Poor (E311) Acceptable (E312) **Excellent (E313)**
		Health and Safety (E32)	Basic (E321) Safety (E322) **Comprehensive (E323)**

If the criteria Compliances is alone considered the, respective dominant sub-sub-sub attribute values to be considered are {Affordable, Low, Economical, Low, Low- Cost, Low}

Let us consider a representation of the form P(E1133xE1211xE2111xE2233xE3133xE3233)

Table 27. Attribute matrix

Attributive Values	**P1**	**P2**	**P3**	**P4**	**P5**
E1133	Moderate	High	Low	High	Low
E1211	Low	Moderate	High	Low	Moderate
E2111	High	High	Low	Moderate	Moderate
E2233	Adequate	Robust	Adequate	Inadequate	Robust
E3133	Excellent	Acceptable	Excellent	Poor	Acceptable
E3233	Basic	Safety	Basic	Safety	Comprehensive

The contradiction degrees with respect to the dominant attribute values are presented in Table 28.

Table 28. Contradiction value

Dominant Attribute Value	**Contradiction Degree**
E113	C(E111, E113) = 2/3, C(E112, E113) = 1/3
E121	C(E122, E121) = 1/3, C(E123, E121) = 2/3
E211	C(E212,E211) = 1/3 , C(E213, E211) = 2/3
E223	C(E221, E223) = 2/3, C(E222, E223) = 1/3
E313	C(E311, E313) = 2/3, C(E312, E313) = 1/3
E323	C(E321, E323) = 2/3, C(E322, E323) = 1/3

By using the contradiction values, the contradiction matrix is as follows and presented in Table 29.

Table 29. Contradiction matrix

Attributive Values	**P1**	**P2**	**P3**	**P4**	**P5**
E1133(B)	1/3	0	2/3	0	2/3
E1211(C)	0	1/3	2/3	0	1/3
E2111(C)	2/3	2/3	0	1/3	1/3
E2233(C)	1/3	0	1/3	2/3	0
E3133(C)	0	1/3	0	2/3	1/3
E3233(B)	2/3	1/3	2/3	1/3	0

The respective weighted contradiction matrix is as follows,

Table 30. Weighted contradiction matrix

Alternatives	**E1133(B)**	**E1211(C)**	**E2111(C)**	**E2233(C)**	**E3133(C)**	**E3233(B)**
P1	0.05556	0	0.1111	0.05556	0	0.1111
P2	0	0.05556	0.1111	0	0.05556	0.05556
P3	0.1111	0.1111	0	0.05556	0	0.1111
P4	0	0	0.05556	0.1111	0.1111	0.05556
P5	0.1111	0.05556	0.05556	0	0.05556	0

By following the similar procedure, the score values are obtained and presented in Table 31.

Table 31. Difference in Score values

Alternatives	**B_E**	**C_E**	**B_E- C_E**
P1	0.16666	0.16666	0
P2	0.05556	0.22222	-0.16666
P3	0.2222	0.16666	0.05554
P4	0.05556	0.27776	-0.2222
P5	0.1111	0.16668	-0.05558

From the Table [7,13,19,25,31] the overall score values shall be determined and presented in Table 32.

Table 32. Overall score value

Alternatives	**Cumulative Score Values**
P1	-0.18057

Alternatives	Cumulative Score Values
P2	0.111119
P3	-0.1528
P4	-0.36109
P5	-0.20836

5. Results and Discussion

The final score values of the alternatives with respect to each of the core attributes and the aggregate score values of the alternatives are determined from the Tables respectively.

The ranks of the alternatives are presented in the Table 33.

Table 33. Ranks of the alternatives

Alternatives	Ranking Results Based on the Core Attributes and Aggregate Score Values					
	Proximity	Accessibility	Compliance	Cost Efficiency	Safety	Aggregate
P1	5	4	5	3	3	5
P2	1	4	1	1	5	1
P3	4	1	3	5	2	2
P4	1	2	5	4	4	4
P5	3	3	3	2	1	3

The graphical representation of the scores of the alternatives with respect to each of the attributes and the aggregate measures is presented as follows

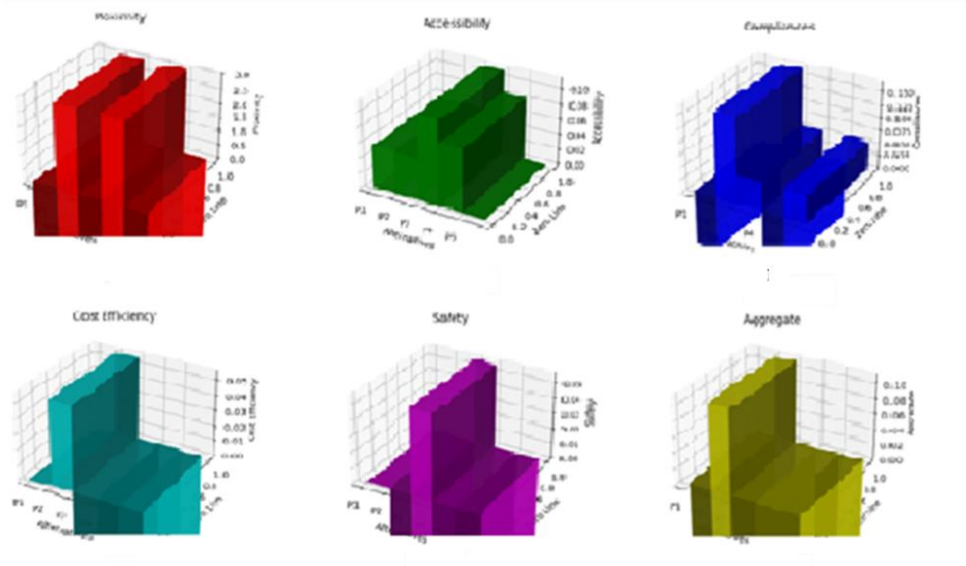

Fig.9. Graphical representation of Scores

From the ranking results presented in the table 33, the following inferences are obtained,

- P2 consistently performs well across most attributes, especially in compliance, cost efficiency, and overall aggregate score.
- P1 consistently ranks low in most attributes, particularly in proximity and aggregate score.
- P3 shows a strong performance in accessibility but is the worst in cost efficiency.
- P4 ranks well in proximity and accessibility but poorly in compliance and aggregate score.
- P5 excels in safety and has a balanced performance across other attributes, making it a solid all-around option.

Thus Plithogenic Forest Hypersoft sets are effectively applied in ranking the alternatives and these sets facilitates in choosing the alternatives based on attributes and sub_3 attribute values. The intervention of PFHS assists in making intense decisions by laying deep examination of the attributes. In other decision methods, the attributes are considered in shallow sense, however in this research work, the attributes are considered in a deeper manner.

Conclusion

This research work introduces the notion of Forest Hypersoft sets and Plithogenic Forest Hypersoft sets. The conceptualizations and classifications are well presented with suitable illustrations. The applications of Plithogenic Forest Hypersoft sets are sketched out with special reference to site selection of the manufacturing plants. The results are comprehensive in nature with the implications of Plithogenic based Forest Hypersoft sets. This decision approach shall be extended to different decision circumstances by augmenting with other decision methods.

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A Novel Method of Decision Making Based on Plithogenic Contradictions

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Abstract: Plithogenic decision-making models are evolved integrating the Plithogenic modelling approach with various methods of multi-criteria decision-making (MCDM). The earlier Plithogenic based decision methods are primarily based on the degrees of appurtenance. This paper introduces a novel Plithogenic ranking genre of decision-making paradigm based on degrees of contradiction. The method of Decision Making on Plithogenic Contradictions (DMPC) developed in this research work is indigenous and unique as the modeling procedure doesn't resemble any of the decision methods. This simple and logical approach proposed in this paper is applied in making optimal decisions on supplier selection. The proposed contradiction based Plithogenic model shall be integrated with other decision methods and this will certainly create a breakthrough in framing contradictions based combined Plithogenic decision-making models.

Keywords: Plithogenic Sets; Plithogenic Contradiction; MCDM; Decision Making on Plithogenic Contradictions.

1. Introduction

The everlasting conflict of choosing the optimal alternatives satisfying all the criteria to the expected extent is motivating the researchers to develop new methods. This has led to the expansion of the theoretical aspects of decision-making with the development of scientific and algorithmic approaches to decision-making methods. The construction of any decision-making problem comprises certainly an elementary decision-making matrix with values matching the alternatives and criteria. The two prime objectives of the decision methods are to find the criterion weights and ranking of the alternatives. The decision-making methods are classified based on information availability, decision timeline, domain, level, structure, outcome, approach, and process.

The circumstances of making decisions are influenced by several factors affecting the deterministic nature of decision-making. The representations using crisp sets are replaced with the extension of fuzzy sets developed by Zadeh [1] to handle impreciseness and uncertainty. These fuzzy sets are further extended to intuitionistic sets [2] and neutrosophic sets to deal the situations of decision-making with hesitancy and indeterminacy. The decision-making methods developed in crisp sense are discussed by the researchers in the extended version of sets. However, these different representations of set are unified under one roof of Plithogeny by Smarandache [3] in the year 2018. The origin and development of Plithogenic sets has made novel plithogenic decision-making methods to evolve. Smarandache has contributed a lot to the field of Plithogeny, especially to the development of fundamental concepts of the Plithogenic sets [4-6]. Smarandache has also contributed

to Plithogenic algebraic structures [7-8]. Nivetha and Smarandache have together initialized the conceptualization of Plithogenic based hypergraphs and super hypergraphs [9-10].

A plithogenic set is basically a 5-tuple set that deals with attributes. This set comprises attribute values, degrees of appurtenance, and contradiction. The degrees of appurtenance decide the nature of the Plithogenic sets and it assumes any of the set representations such as crisp, fuzzy, intuitionistic, and neutrosophic. The Plithogenic decision-making methods primarily involve plithogenic operators to obtain a unified decision-making matrix based on the expert's opinion. The literature on Plithogenic based multi criteria decision making (MCDM) methods is limited. Some of the most commonly applied conventional decision-making methods are discussed in Plithogenic environment only with the inclusion of the Plithogenic operators of union and intersection and degrees of appurtenance. This has motivated the authors to develop a new genre of decision-making method based on the degrees of contradiction. The method of making decisions with a contradiction degree is proposed as a method of ranking the alternatives. This method is very simple in its formulation and the logical approach makes the method more rational.

The paper is organized as follows: section 2 sketches out the contributions in the domain of Plithogenic decision-making. Section 3 presents the proposed method of Decision Making on Plithogenic Contradictions. Section 4 applies the proposed method to the supplier selection problem. Section 5 discusses the results under different cases and section 6 concludes the work with future directions.

2. Literature review

The theory of Plithogeny is applied in MCDM integrating a wide range of different concepts of soft sets, Hypersoft sets, cognitive maps, hypergraphs, and many others. Plithogenic decision-making models are developed based on these concepts to design solutions to real-life problems. Plithogenic based MCDM are either the extensions or the generalizations of the existing mathematical concepts. The Plithogenic logic, probability statistics, and optimization assist in obtaining optimal solutions to decision-making problems. The contributions of researchers towards the formulation of Plithogenic decision-making models are presented in Table 1.

Table 1. Contributions of Plithogenic based decision making.

Authors & Year	Plithogenic Decision Making Method	Domain of Application	Highlights of the contribution
Ozcil et al. [11]	MAIRCA	Green Supplier selection	• Minimization of the gap between ideal and empirical values • Plithogenic aggregation operators
Abdel-Basset et al. 12]	VIKOR	Hospital medical care systems	• Plithogenic contradiction degree for dominant attribute
Abdel-Basset et al. [13]	QFD	Selecting supply chain sustainability	• Plithogenic aggregation operators
Rana et al. [14]	Plithogenic Hypersoft set, Plithogenic Whole Hypersoft set	Selecting faculty for the Engineering department	• Frequency matrix for final ranking

Abdel-Basset, & Mohamed, [15]	TOPSIS-CRITIC	Sustainable supply chain risk management	• Plithogenic aggregation operators
Abdel-Basset et al. [16]	BWM	Supply chain problem	• Plithogenic aggregation operators
Abdel-Basset et al. [17]	AHP,VIKOR, TOPSIS	Financial performance evaluation in manufacturing industries	• Plithogenic aggregation operators
Gómez et al. [18]	VIKOR	Pedagogical performance.	• Plithogenic aggregation operators
Grida et al. [19]	VIKOR,BWM	IoT based supply chain	• Plithogenic aggregation operators
Abdel-Basset et al. [20]	MABAC; BWM	Supplier selection	• Plithogenic aggregation
Ahmad et al. [21]	PHSS based TOPSIS	Parking spot choice problem	• Plithogenic aggregation
Smarandache, &Martin [22]	Plithogenic n-super hypergraph, Dominant enveloping vertex	E-learning system of education (Work from Home During Covid-19)	• Classification of Dominant Enveloping Vertex • Plithogenic Connectors
Gomathy et al. [23]	Plithogenic operator laws (fuzzy tnorm & tconorm)	Medical field	• Plithogenic aggregate operators
Martin et al. [24]	Plithogenic sociogram & Plithogenic number	Food processing industry	• Preferential ordering based on attributes
Öztaş et al. [25]	Plirhogeny, DEA	Tourist travelers performance (Accommodation for touristic travelers)	• Plithogenic aggregation operations
Korucuk et al. [26]	CRITIC	logistics sector	• Plithogenic aggregation operations
Sujatha, et al. [27]	FCM, Plithogenic operators	Corona virus (Covid-19)	• Plithogenic aggregation of weights
Martin et al. [28]	PHS,DM	Covid- 19	• Extended combined plithogenic hypersoft sets

Hernández et al. [29]	Plithogenic logic, SWOT	Entrepreneurship competence in university students	• Plithogenic aggregation operators
Martin et al. [30]	PSCM	Factors in COVID-19 diagnostic model	• Degree of contradiction with respect to the factors
Ulutaş et al. [31]	PIPRECIA	Prioritization of logistics sector	• Plithogenic aggregation operators
Ulutaş et al. [32]	SWARA	Logistics sector	• Plithogenic aggregation operators
Singh. [33]	Plithogenic graph; Plithogenic set	Olympic Players performance	• Plithogenic aggregation operators
Ansari & Kant. [34]	AHP	Supply chain	• Plithogenic aggregation operators
Martin et al. [35]	PROMTHEE	Smart materials selection	• Plithogenic aggregation operators
Singh [36]	Plithogenic graphs	Dark data analysis (Performance of players in crickets)	• Conflict situation
Singh [37]	Plithogenic graphs	Air Quality Index Analysis(Impact on human health)	• Single-valued Neutrosophic Plithogenic data visualization
Priyadharshini & Irudayam [38]	MCDM	Agriculture field	• Plithogenic aggregate operators
Rodríguez et al. [39]	Plithogenic number, MCDM	Education and Society	• Representations using Plithogenic number
Priya & Martin [40]	PCM, IPCM, CCM	online learning system	• Plithogenic sets in determining the association between the factors
Fernández et al. [41]	AHP, TOPSIS	Selection of Investment Projects	• Plithogenic aggregate operators
Castro Sánchez et al. [42]	Plithogenic logics	Educational Development	• Plithogenic aggregate operators
Priyadharshini & Irudayam [43]	RPNS	Candidate's selection in interview.	• RPNS Operators • Correlation measures and its properties
Bharathi & Leo [44]	PPfuzzy graph	Social Network	• To discover the network's most outgoing, gregarious, powerful, and key figures.

Villacrés et al. [45]	AHP	Ergonomic Occupational Health Risks for teachers	• Instant solutions
Moncayo et al. [46]	SWOT, Plithogeny	Ecuadorian Hospital environment	• Plithogenic operators
Pai & Prabhu Gaonkar [47]	Plithogenic set	Risk Assessment due to accident	• Assessing risk and ranking of the criteria in a complex system
Romero et al. [48]	AHP,TOPSIS	Investment Projects selection problem	• Plithogenic aggregate operators
Antonio et al. [49]	Plithogenic logic	Electronic payment methods/Mechanism	• Plithogenic aggregate operators
Sultana et al. [50]	Plithogenic graphs	Spreading coronavirus disease (COVID-19)	• Plithogenic aggregate operators
Ahmad & Afzal [51]	PDM,PHSS,PSM	Mathematical modeling and AI (COVID-19 suspect)	• Plithogenic aggregate operators
Martin, N [52]	SWARA-TOPSIS	Food Processing Methods	• Plithogenic operators
Liang et al. [53]	CRITIC, Game theory, TOPSIS-GRA	Air traffic flow problem	• Plithogenic aggregation
Abdelfattah, W. [54]	DEA	University in Saudi Arabia	• Plithogenic aggregation
Wang et al. [55]	COPRAS, PNRN	Sustainable Financing Enterprise selection	• Extended Similarity Measures
Sudha & Martin [56]	BWM	Teaching methods	• Plithogenic Pythagorean set.
Sudha., Martin, &Broumi [57]	CRITIC-MAIRCA	Livestock Feeding Stuff problem	• Plithogenic aggregation
Ulutaş, & Topal [58]	PIPRECIA	Renewable energy industry	• Plithogenic aggregation
Seby, & Ravi [59]	Plithogeny	Supply chain	• Plithogenic aggregation
Priya,, Martin, & Kishore [60]	Plithogeny	Human's cognitive domain	• Contradiction degree in PCM
Zuñiga et al. [61]	Plithogenic number	Classifications of clays	• Representations using Plithogenic numbers
Tayal et al. [62]	TOPSIS, WSM	Business	• Plithogenic aggregation

Wang et al. [63]	VIKOR	Supply Chain Financial risk evaluation	• Probabilistic Linguistic MAGDM
Sudha, & Martin [64]	PIPRECIA, AHP	Logistics selection sector	• Plithogenic Operators

In the above mentioned Plithogeny based decision-making methods, the following research gaps are identified.

- The plithogenic operators based on degree of appurtenance are widely applied and only in few instances the contradiction degree is used.
- The plithogenic oriented decision-making methods lack the use of the aspect of contradiction degree in handling the alternatives and criteria.

Hence this research work designs a decision-making method purely based on the contradiction degrees with respect to the dominant attribute value of the alternatives. The novel attributes of this paper are as follows:

- A distinctive decision making approach based on contradictions degree.
- Simple and compatible method of finding the optimal alternatives.
- Flexible method which accommodates several alternatives and criteria.

3. Proposed Method of Decision making based on Plithogenic Contradictions

This section consists of the steps involved in the method of Decision Making on Plithogenic Contradictions (DMPC). The elementary steps of this method are similar to the general working principle of an MCDM method. Figure 1 presents the overall framework of the proposed method of DMPC.

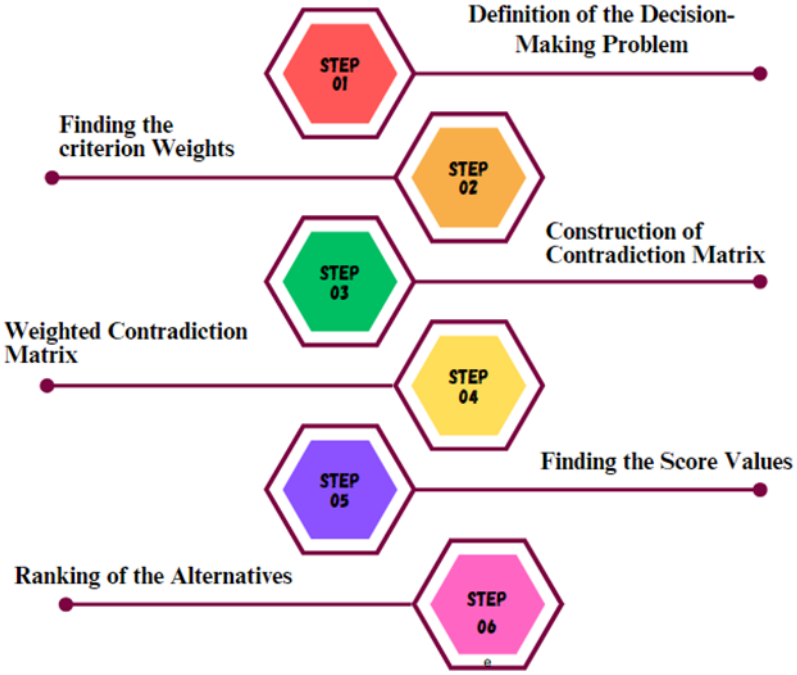

Figure 1. Overall framework of DMPC.

Step 1: *Definition of the decision-making problem*

It is the initial step in which the problem is well defined with alternatives and criteria. The criteria are classified into benefit and non-benefit based on the nature of the problem. Each of the criterion has

sub-values. The decision making matrix with initial values is constructed especially with linguistic variables.

$$D_L = \begin{bmatrix} x_{L11} & \cdots & x_{L1n} \\ \vdots & \ddots & \vdots \\ x_{Lm1} & \cdots & x_{Lmn} \end{bmatrix}$$

The decision making matrix is with m alternatives and n criteria.

Step 2: *Finding the Criterion Weights*

The criterion weights say Wk are determined using any of the methods. Each of the criterion has criterion values say Cki.

Step 3: *Construction of contradiction matrix*

The dominant criterion value say CkD among the criterion values of each criteria is identified. The contradiction degree among the criterion values is determined. Based on the contradiction degree, the contradiction matrix is constructed with contradiction degrees pertaining to the dominant criterion value with respect to the values assumed by each alternative with respect to the criterion value in the initial matrix.

$$C_D = \begin{bmatrix} c_{D11} & \cdots & c_{D1n} \\ \vdots & \ddots & \vdots \\ c_{Dm1} & \cdots & c_{Dmn} \end{bmatrix}$$

Step 4: *Weighted contradiction matrix*

The weighted contradiction matrix $[WC_D]$is obtained by multiplying the criterion weights with the values of contradiction matrix.

Step 5: *Finding the score values*

The score values of each of the alternative with respect to both benefit and cost criteria say BSj and CSh is first calculated. The difference between the values is determined, say BSj - CSh = Df

Step 6: *Ranking of the alternatives*

The alternatives are ranked based on the difference values Df. The alternative with maximum difference value is ranked first and so on.

4. Application of DMPC in supplier selection

In this section, a decision-making problem is solved using the proposed method of DMPC. Let us consider a logistic supplier selection problem with five alternatives and four criteria say C1 – Price, C2 – Time span of delivery, C3 – Flexibility, and C4 – Reliability.

The criteria C1 and C2 are considered to be cost criteria and the criteria C3 and C4 are considered as benefit criteria.

Each criteria presumed to be the attribute possess the attribute values of {L, M, H} i.e. {Low, Moderate, High}.

For the cost criteria, the dominant attribute value is certainly LOW & for the benefit criteria it is HIGH.

Contradiction degree with respect to dominant attribute value (LOW) of the cost criteria (C1 & C2).

C(L,L) = 0
C(L,M) = 1/3
C(L,H) = 2/3

Contradiction degree with respect to dominant attribute value (HIGH) of the benefit criteria.

C(H,H) = 0
C(H,M) = 1/3
C(H,L) = 2/3
C(M,M) =0

The initial decision making matrix with linguistic values is presented in Table 2.

Table 2. Initial decision making matrix.

Alternatives /Criteria	C1	C2	C3	C4
	Cost Criteria		Benefit Criteria	
A1	L	H	L	M
A2	H	M	L	M
A3	M	L	H	M
A4	L	L	M	H
A5	L	H	M	L

The assumed criterion weights and the dominant attribute value with respect to each of criterion are presented as follows in Table 3.

Table 3. Decision matrix with criterion description.

Alternatives/ Criteria	C1	C2	C3	C4
	Cost Criteria		Benefit Criteria	
	0.35	0.25	0.20	0.20
A1	L	H	L	M
A2	H	M	L	M
A3	M	L	H	M
A4	L	L	M	H
A5	L	H	M	L
Dominant Value	L	L	H	H

The contradiction matrix with the contradiction degree of each criterion values with respect to the dominant criterion value is presented as follows in Table 4 using step 3.

Table 4. Contradiction matrix.

Alternatives/ Criteria	C1	C2	C3	C4
	Cost Criteria		Benefit Criteria	
	0.35	0.25	0.20	0.20
A1	0	2/3	2/3	1/3
A2	2/3	1/3	2/3	1/3
A3	1/3	0	0	1/3
A4	0	0	1/3	0
A5	0	2/3	1/3	2/3

The weighted contradiction matrix is computed using step 4 as follows in Table 5.

Table 5. Weighted contradiction matrix.

Alternatives /Criteria	C1	C2	C3	C4
	Cost Criteria		Benefit Criteria	
A1	0.00	0.17	0.13	0.07
A2	0.23	0.08	0.13	0.07
A3	0.12	0.00	0.00	0.07
A4	0.00	0.00	0.07	0.00
A5	0.00	0.17	0.07	0.13

The score values of the benefit and cost criteria with respect to each alternative are calculated as presented in Table 6.

Table 6. Score values of criteria.

Alternatives	Cost Criteria	Benefit Criteria
A1	0.17	0.20
A2	0.31	0.20
A3	0.12	0.07
A4	0.00	0.07
A5	0.17	0.20

The differences between the benefit and the cost criteria score values are presented in Table 7.

Table 7. Difference in score values.

Alternatives	Differences in the score values
A1	0.03
A2	-0.11
A3	-0.05
A4	0.07
A5	0.03

Based on the difference values the alternatives are ranked as follows as in Table 8.

Table 8. Ranking of the alternatives.

Alternatives	Ranking
A1	2
A2	4
A3	3
A4	1
A5	2

5. Discussion

The above ranking of the alternatives is obtained with assumed criterion weights. The same ranking procedure based on contradictions is repeated with different criterion weights obtained using various methods such as the Analytical Hierarchy Process (AHP), Entropy, and the method of CRITIC (CRiteria Importance through Intercriteria Correlation). Table 9 and Figure 2 represent the rankings of the alternatives using different criterion weights.

Table 9. Ranking of alternatives based on different criterion weights.

Alternatives	Rankings based on diverse criterion weights		
	AHP	CRITIC	Entropy
A1	2	2	2
A2	5	4	5
A3	4	5	4
A4	1	1	1
A5	3	3	3

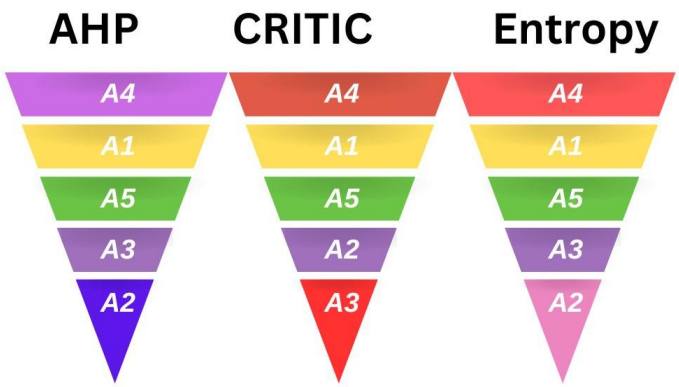

Figure 2. Graphical representation of diverse ranking of the alternatives.

6. Conclusions

This research work proposes a new genre of Plithogenic based decision-making method based on contradictions. The proposed method stands distinct in comparison with other methods as it streamlines a new modality of making optimal decisions. This method will definitely lessen the hurdles in choosing the alternatives based on cost and benefit criteria. The ranking obtained using the Plithogenic method based on contradictions is compared with different criterion weights. This method shall be dealt with extended Plithogenic sets. Also, the method of Plithogenic Cognitive Maps shall be associated with the proposed method as a means of developing several hybrid decision-making methods. This method is highly adaptable and flexible in nature and hence it shall be blended with other decision-making models to evolve new hybrid decision-making systems.

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Data availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflict of interest

The authors declare that there is no conflict of interest in the research.

Ethical approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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Generalized plithogenic whole hypersoft set, PFHSS-Matrix, operators and applications as COVID-19 data structures

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Abstract. This article is a preliminary draft for initiating and commencing a new pioneer dimension of expression. To deal with higher-dimensional data or information flowing in this modern era of information technology and artificial intelligence, some innovative super algebraic structures are essential to be formulated. In this paper, we have introduced such matrices that have multiple layers and clusters of layers to portray multi-dimensional data or massively dispersed information of the plithogenic universe made up of numerous subjects their attributes, and sub-attributes. For grasping that field of parallel information, events, and realities flowing from the micro to the macro level of universes, we have constructed hypersoft and hyper-super-soft matrices in a Plithogenic Fuzzy environment. These Matrices classify the non-physical attributes by accumulating the physical subjects and further sort the physical subjects by accumulating their non-physical attributes. We presented them as Plithogenic Attributive Subjectively Whole Hyper-Super-Soft-Matrix (PASWHSS-Matrix) and Plithogenic Subjective Attributively Whole-Hyper-Super-Soft-Matrix (PSAWHSS-Matrix). Several types of views and level-layers of these matrices are described. In addition, some local aggregation operators for Plithogenic Fuzzy Hypersoft Set (PPFHS-Set) are developed. Finally, few applications of these matrices and operators are used as numerical examples of COVID-19 data structures.

Keywords: Plithogenic, Hyper-Super-Soft-Matrix, matrix layers, parallel-universes, events, realities, aggregation operators, non-physical classifications, as COVID-19, data structures

1. Introduction

Classical mathematics and its applications are based on certain laws and results, while they can be observed in everyday life. When the human mind makes decisions about scientific, philosophical, or economic facts, it is not 100% sure of its results. The natural human brain exhibits a certain uncertainty factor and precariousness in its judgments and conclusions due to different opinions about attributes, events, and information. Therefore, to manage this vagueness in the study of mathematics In 1965 Zadeh [1] introduced fuzzy mathematics. In the theory of fuzzy mathematics, all laws and results are discussed by considering some degree of certainty or truth (membership) and some degree of uncertainty or the opposite of truth (non-membership), so that the combined effect of membership and non-membership is considered complete. That means if membership value of any element $x \in X$ with

respect some attribute A is represented by, $\mu_A(x) \in [0, 1], \forall x \in X$ and non-membership is represented by $\upsilon_A(x) \in [0, 1], \forall x \in X$ such that $\mu_A(x) + \upsilon_A(x) = 1, \forall x \in X$, then the general representation of a fuzzy set is $\{x : \mu_A(x)\}$.

In 1986 Atanassov developed the Intuitionistic Fuzzy Set Theory (IFS) [2, 3]. In IFS theory, Atanassov expanded the vagueness of the human mind by addressing the doubt which arises in assigning membership and non-membership. The doubt which is evoked in decision making was quantified by using linguistic scales and expressed by assigning a numeric value between "0" and "1". Atanassov named this doubt the level of hesitation that was measured by assigning a degree to the hesitation denoted by $\iota_A(x) \in [0, 1] \forall x \in X$. The elements of IFS are expressed as $\{x : (\mu_A(x), \upsilon_A(x))\}$ with a modified condition $\mu_A(x) + \upsilon_A(x) + \iota_A(x) = 1, \forall x \in X$. It is observed that the degree of hesitation of IFS is a dependent factor. Smarandache further expanded the Cloud of vagueness by introducing Neutrosophic Set [4–6]. He introduced indeterminacy and considered the degrees of membership, non-membership, and hesitation/indeterminacy as independent factors. These three factors are represented in a unit cube with the non-standard unit interval $]0^-, 1^+[$. The neutrosophic set was represented as $\{x : (\mu_A(x), \upsilon_A(x), \iota_A(x))\}$ with modified condition $0 \leq (\mu_A(x) + \upsilon_A(x) + \iota_A(x) \leq 3$. Some further latest dilation and modernization of neutrosophic set are portrayed in [7–12]. In 1999, Molodtsov introduced the soft set [13], where he represented the elements of this set as a parameterized family of the subset of the universal set and thereafter some of the further extensions of the soft set were discussed in, [14–16].

Later, in 2018, Smarandache [17, 18] introduced Hypersoft set and plithogenic hypersoft set. In these sets he transformed the function of a single attribute into a multi-attribute/sub-attribute function and assigned a combined membership $\mu_{A_1 \times A_2 \times \ldots \times A_N}(x)$, non-membership $\upsilon_{A_1 \times A_2 \times \ldots \times A_N}(x)$, and Indeterminacy $\iota_{A_1 \times A_2 \times \ldots \times A_N}(x), \forall x \in X$ with condition $A_i \cap A_j = \phi$ for the case of hypersoft set. Whereas individual memberships non-memberships and indeterminacies were assigned for each given attribute for the case of the plithogenic Hypersoft-Set. And introduced hybrids of Crisp/Fuzzy/Intuitionistic Fuzzy and Neutrosophic Hypersoft-Set and Plithogenic Hypersoft-Set. By introducing these sets he raised many open problems for the development of new literature, such as the development of appropriate modern algebraic structures for expressing such widely dispersed higher-dimensional information and the formulation of MADM techniques.

In 2019 Rana et al. [19] extended the Plithogenic Hyper-Soft Set to Plithogenic Whole-Hyper-Soft Set and introduced a new path of organized expression as hypersoft-matrix and Hyper-super-soft-matrix and formulated some local aggregation operators. When these local operators were applied to the Plithogenic Fuzzy Hyper-Soft Set (PFHSS), a new type of soft set emerged called the Plithogenic Fuzzy Whole Hyper-Soft Set (PFWHSS). In addition, the Plithogenic Fuzzy Hyper-Soft Set and the Plithogenic Fuzzy Whole Hyper-Soft Set were presented as a modern extended matrix in the fuzzy environment called Plithogenic Fuzzy Hyper-Soft Matrix (PFHS-Matrix) and Plithogenic Fuzzy Whole Hyper-Soft Matrix. These Hypersoft Matrices are made up of the parallel layers of ordinary matrices that represent parallel universes, parallel realities, parallel events, or parallel information.

In the next phase, Rana et al. [20] further generalized the Plithogenic Whole Hyper-Soft Set to Plithogenic Crisp/Fuzzy/Intuitionistic/Neutrosophic Subjective Hyper-Soft Set and represented it in the expanded version of Soft-Matrix in the fuzzy environment known as the plithogenic subjective hyper-super-soft matrix. Furthermore, these modern matrix expressions were used to develop a new higher dimensional ranking model called the Local-Global Universal Subjective Ranking Model. It is also revealed that the Plithogenic Whole Hyper-Soft Matrix (PWHS Matrix) is a special case of the Plithogenic Subjective Hyper-Super-Soft Matrix (PSHSS Matrix). Additional literature on Hyper-Soft Set and Plithogency can be found in [21–26].

This article, structured as, Section 2, describes materials such as some new concepts of the expanded plithogenic universe, such as universal collective consciousness, clustered data of the accelerated universe, and some Preliminaries. Section 3 contains a brief description of the Generalized Plithogenic Fuzzy Whole HyperSoft and its novelties with some basic concepts and new definitions of Plithogenic Hypersoft Set/Matrix, index-based views of parallel matrix layers. Section 4 describes the development of local aggregation operators for PFHSS matrix. Section 5 presents the construction of hyper-supersoft matrices and their application as COVID-19 data structures. Section 6 mentions conclusions, discussions, and some open questions for future research.

2. Materials, methods, and preliminaries

In this article, on the first level, we have expanded the previously presented Plithogenic Fuzzy Whole HyperSoft Set and Plithogenic Subjective HyperSoft Set [19, 20] to Plithogenic Attributive Subjectively Whole HyperSoft Set (PASWHS Set) and Plithogenic Subjective Attributively Whole HyperSoft Set (PSAWHS Set). To represent these newly introduced hypersoft sets, a new type of matrix structure of connected matrix layers (hypersoft matrix) and connected clusters of matrix layers (hyper-super-soft matrix) is designed. These super-algebraic matrix structures are initially formulated in a fuzzy environment named the Plithogenic Attributive Subjectively Whole Hyper-Super-Soft-Matrix (PASWHSS-Matrix) and the Plithogenic Subjective Attributively Whole-Hyper-Super-Soft-Matrix (PSAWHSS-Matrix). These advanced types of matrices are generated by the hybridization of hyper matrices, super matrices, and soft matrices [13–20, 24–27]. These hypersoft matrices are sets/clusters of connected parallel matrix layers that represent clusters of parallel universes, parallel realities, Parallel events, or parallel information (a combination of attributes and sub-attributes relating to subjects). Furthermore, these modern matrices are rank three and four tensors with three and four variation indices, respectively. Later on, various types of cross-sectional sights are described as parallel layers of hypersoft matrices. These cross-sectional sights are formulated by taking multiple variations in numerous ways and exhibit some symmetries in their structures regarding their number of rows and columns. Furthermore, some aggregation operators are structured. Application of these aggregation operators and Hyper-Super-Soft Matrices are provided as COVID-19 Data structures.

Now comes the query, "Why do we use HyperSoft and Hyper-Super-Soft matrices specifically for the expression of Plithogenic HyperSoft Set, Plithogenic Subjective HyperSoft Set, and Plithogenic Attributive HyperSoft Set?" The answer described in (2.1–2.3), and (3.1.1–3.1.5) could certainly be convincing.

2.1. Expanded plithogenic universe

This Plithogenic Universe is so huge and expanded in its interior (like having Fuzzy, Intuitionistic Fuzzy, Neutrosophic environments with Memberships, Non-Memberships, and Indeterminacies) and its exterior (manages many attributes, sub-attributes and might be sub-sub-attributes concerning to its subjects). To organize and analyze the dispersed information of such an expanded plithogenic universe we are in great need to formulate some super algebraic structures like these hyper-super-soft matrices.

2.2. Universal collective consciousness

Imagine you are a part of a lot of information, events, realities that are constantly flowing all around you. Like everything you see, everything you look at, everything you observe, is in a parallel view. By the way, we are all observing from a different angle so we all are getting a different piece of information, event, or a reality. Because of this connectedness with this universe and its universal attributes, we are actually collectively producing diversity in this flow of information, events, thoughts, and even realities (i.e., Universal Data, Universal Space, Universal Events, Universal Consciousness). That is why this universe is giving different sets of information and numerous interpretations of the same thing (**Subject, Attributes, Sub-Attributes, Sub-Sub-Attributes Collectively All as Consciousness**). We are gathering this information in terms of observing different events, realities, universes as one reality and not as parallel realities, and thus, we are feeding them back to space through our brain structure into the atomic structure and into the vacuum state though they are underlying numerous angles, thoughts, events, visions, etc. In the end, this universal feedback returns and changes those attributes, events a little bit different from each other and makes parallel realities for which we have no expressions. Therefore, these hypersoft and hyper-super-soft matrices are initiatives towards a way of expression for the Universal Collective Consciousness.

2.3. Clustered Data of accelerated universe

In order to find and pave the way for the expression of those parallel realities for such a massively accelerated huge universe. This is, of course, not viable through using regular algebra or matrices therefore, for the handling of this expanded universe along with its clustered and littered data, we are in dire need to construct like super-algebra or hypersoft and hyper-super-soft matrices and likely many more.

2.4. Preliminaries

This section describes some basic definitions of Soft-Sets, Fuzzy-Soft-Sets, Hypersoft-Sets, Fuzzy-Hypersoft-Sets, Plithogen-Hypersoft-Sets, and Plithogen-Fuzzy-Hypersoft-Sets, etc. These definitions are useful in developing literature and widely dispersed data modeling structures.

Definition 2.4.1 [13](Soft Set)

Let U be the initial universe of discourse, and E be a set of parameters or attributes with respect to U let $P(U)$ denote the power set of U, and $A \subseteq E$ is a set of attributes. Then pair (F, A), where $F : A \to P(U)$ is called Soft Set over U. Fore $e \in A$, $F(e)$ may be considered as a set of e elements or e approximate elements

$$(F, A) = \left\{ \begin{array}{c} (F(e) \in P(U) : \\ e \in E, F(e) = \varphi if e \notin A \end{array} \right\} \quad (2.1)$$

Definition 2.4.2 [1] (Fuzzy set)

Let U be the universe. A fuzzy set X over U is a set defined by a membership function μ_X representing a mapping $\mu_X : U \to [0, 1]$

The value of $\mu_X(x)$ for the fuzzy set X is called the membership value of the grade of membership of $x \in U$. The membership value represents the degree of belonging to fuzzy set X. A fuzzy set X on U can be expressed as follows.

$$X = \{(\mu_X(x)/x) : x \in U, \mu_X(x) \in [0, 1]\} \quad (2.2)$$

Definition 2.4.3 [14] (Fuzzy soft set)

Let U be the initial universe of discourse, $F(U)$ be all fuzzy sets over U. E be the set of all parameters or attributes with respect to U and $A \subseteq E$ is a set of attributes. A fuzzy soft set Γ_A on the universe U is defined by the set of ordered pairs as follows,

$$\Gamma_A = \{x, \gamma_A(x) : x \in E, \gamma_A(x) \in F(U)\} \quad (2.3)$$

where $\gamma_A : E \to F(U)$ such that $\gamma_A(x) = \phi$ if $x \notin A$

$$\gamma_A(x) = \left\{\mu_{\gamma_A(x)}(u)/u : u \in U, \mu_{\gamma_A(x)}(u) \in [0, 1]\right\}$$

Definition 2.4.4 [17] (HyperSoft Set)

Let U be the initial universe of discourse $P(U)$ the power set of U let $a_1, a_2, \ldots, a_n$ for $n \geq 1$ be n distinct attributes, whose corresponding attributes values are respectively the sets $A_1, A_2, \ldots, A_n$ with $A_i \cap A_j = \varphi$ for $i \neq j$ and $i, j \in \{1, 2, \ldots, n\}$.

Then the pair $(F, A_1 \times A \times \cdots \times A_n)$ where,

$$F : A_1 \times A \times \cdots \times A_n \to P(U), \quad (2.4)$$

is called a Hypersoft set over U

Definition 2.4.5 [17] (Crisp Universe of Discourse)

A Universe of Discourse U_C is called Crisp if $\forall x \in U_c$ $x \in 100\%$ to U_C or membership of x $T(x)$ with respect to A in M is 1 denoted as $x(1)$.

Definition 2.4.6 [17] (Fuzzy Universe of Discourse)

A Universe of Discourse U_F is called Fuzzy if $\forall x \in U_C$ x partially belongs to U_F or membership of x $T(x) \subseteq [0, 1]$ where $T(x)$ may be a subset, an interval, a hesitant set, a single value set, denoted as x $T(x)$.

Definition 2.4.7 [17] (Crisp Hypersoft-Set set)

Let U_c be the initial universe of discourse $P(U_c)$ the power set of U.

let $a_1, a_2, \ldots, a_n$ for $n \geq 1$ be n distinct attributes, whose corresponding attributes values are respectively the sets $A_1, A_2, \ldots, A_n$ with $A_i \cap A_j = \varphi$ for $i \neq j$ and $i, j \in \{1, 2, \ldots, n\}$, Then the pair, $(F_c, A_1 \times A \times \ldots \times A_n)$ s.t

$$F_c : A_1 \times A \times \cdots \times A_n \to P(U_c)\}, \quad (2.5)$$

is called Crisp Hypersoft set over U_c

Definition 2.4.8 [17] (Fuzzy Hypersoft set)

Let U_F be the initial universe of discourse $P(U_F)$ the power set of U_F.

let $a_1, a_2, \ldots, a_n$ for $n \geq 1$ be n distinct attributes, whose corresponding attributes values are respectively the sets $A_1, A_2, \ldots, A_n$ with $A_i \cap A_j = \varphi$ for $i \neq j$ and $i, j \in \{1, 2, \ldots, n\}$.

Then the pair

$(F_F, A_1, A_2, \ldots, A_n)$, s.t

$$F_F : A_1 \times A \times \cdots \times A_n \to P(U_F)\} \quad (2.6)$$

is called Fuzzy Hypersoft set over U_c.

Special cases of Hypersoft set: Crisp, Fuzzy, Intuitionistic Fuzzy and Neutrosophic sets are special cases of Hypersoft set by taking $N = 1$ in the Combination of N attributes $A_1 \times A_2 \times \cdots \times A_N$.

Definition 2.4.9 [17] (Plithogenic, Crisp, Fuzzy, Intuitionistic Fuzzy, and Neutrosophic Hypersoft Set)

Now instead of assigning combined membership $\mu_{A_1 \times A_2 \times \cdots \times A_N}(x)$ $\forall x \in X$ for Hypersoft sets if each attribute A_j is assigned an individual membership $\mu_{A_j}(x)$, non-membership $\upsilon_{A_j}(x)$ and Indeterminacy $\iota_{A_j}(x) \forall x \in X$ $j = 1, 2, \ldots, n$ in Crisp, Fuzzy, Intuitionistic and neutrosophic Hypersoft set then these generalized Crisp, Fuzzy, Intuitionistic and Neutrosophic Hypersoft sets are called Plithogenic, Crisp,

Fuzzy, Intuitionistic Fuzzy and Neutrosophic Hypersoft Set.

Definition 2.4.10 [27, 28] (Super Matrices)

A Square or rectangular arrangements of numbers in rows and columns are matrices we shall call them simple matrices while a Super-Matrix is one whose elements are themselves matrices with elements that can be either scalars or other matrices.

$a = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$, where

$a_{11} = \begin{bmatrix} 2 & -4 \\ 0 & 1 \end{bmatrix}, a_{12} = \begin{bmatrix} 0 & 40 \\ 21 & -12 \end{bmatrix},$

$a_{21} = \begin{bmatrix} 3 & -1 \\ 5 & 7 \\ -2 & 9 \end{bmatrix}, a_{22} = \begin{bmatrix} 4 & 12 \\ -17 & 6 \\ 3 & 7 \end{bmatrix}$ a is a super-matrix.

Note: The elements of super-matrices are called sub-matrices i.e. $a_{11}, a_{12}, a_{21}, a_{22}$ are submatrices of the super-matrix a in this example, the order of super-matrix a is 2×2, and the order of sub-matrices a_{11} is 2×2, a_{12} is 2×2 a_{21} is 3×2 and order of sub-matrix a_{22} is 3×2, we can see that the order of super-matrix doesn't tell us about the order of its sub-matrices.

Definition 2.4.11 [29, 30] (Hyper-matrices)

For $n_1, \ldots, n_d \in N$, a function,

$$f : (n_1) \times \cdots \times (n_d) \to F \tag{2.7}$$

is a hyper-matrix, also called an order-d hyper-matrix or d-hyper-matrix. We often just write $a_{k_1 \ldots k_d}$

to denote the value $f(k_1 \ldots k_d)$ of f at $(k_1 \ldots k_d)$ and think of f (renamed as A) as specified by a d-dimensional

table of values, writing $A = \left[a_{k_1 \ldots k_d}\right]_{k_1 \ldots k_d}^{n_1, \ldots, n_d}$

A 3-hypermatrix would be written down on a (2-dimensional) piece of paper as a list of usual matrices, called slices. For example,

$$A = \begin{bmatrix} a_{111} & a_{121} & a_{131} & . & a_{112} & a_{122} & a_{132} \\ a_{211} & a_{221} & a_{231} & . & a_{212} & a_{222} & a_{232} \\ a_{311} & a_{321} & a_{331} & . & a_{312} & a_{322} & a_{332} \end{bmatrix} \tag{2.8}$$

3. Generalized plithogenic fuzzy whole hypersoft set

In this section, some new concepts and definitions are described that would be utilized for the development of Generalized Plithogenic Fuzzy Whole Hyper-soft Set as PFASWHSS-Matrix and. PFSAHSS-Matrix. This literature is organized in three subsections in the following manner.

3.1 A brief description of the Generalized Plithogenic Fuzzy Whole Hyper-soft Set is presented and its novelties are discussed.
3.2 Some new concepts/definitions relevant to the Plithogenic hypersoft set are developed.
3.3 The index-based views of PFHS-Matrix are described through variation indices

3.1. Generalized Plithogenic Fuzzy Whole Hyper-Soft Set and its Novelties

3.1.1. Matrix of connected matrix-layers

The clusters of connected matrix layers that we will introduce in this article are the expanded versions of the previously presented [19] Plithogenic Fuzzy Whole HyperSoft Set (PFWHS Set). In PFWHS-Set the aggregation operators and the Plithogenic Frequency Matrix Multi-Attribute Decision Making Technique have been developed for a single required combination of attributes/sub-attributes related to some given subjects. While in the new generalized versions of connected matrix layers, the aggregation operators and ranking models are not only developed for a single combination of attributes/sub-attributes rather, these operators and ranking models are extended to several parallel-connected layers of combinations of attributes/sub-attributes related to the given subjects.

For the development of this ranking model (FMMADM-Model), a novel type of Hyper-Soft and Hyper- Super-Soft Matrices are introduced. These special types of matrices would organize and utilize the widely expanded higher dimensional dispersed information. This information consists of several attributes, sub-attributes, and subjects that are connected by the cluster of matrix layers. These clusters of matrix layers are formulated as Hypersoft and Hyper-Super-Soft matrices. To preserve the expansion of the article inside the required limits of this journal, all of the expanded literature is first presented in the fuzzy environment.

In this version, some new definitions and concepts of the Generalized Plithogenic Fuzzy whole hypersoft set are developed. In addition, their representation is organized as hypersoft, and hyper-super-soft matrix. After the formulation of these hypersoft matrices, some aggregation operators are

developed as the set laws of operations for these matrices. The application of these aggregation operators and the Hyper-Super-Matrix as a ranking model for the organization and classification of the conditions of COVID-19 patients will be presented later. This ranking model is called the classification model for COVID-19 states.

3.1.2. Multi-dimensional mathematical ways of expression

One novelty of this model is modern multi-dimensional mathematical form of expression. It is observed that from the fuzzy set to its extended versions such as intuitionistic, neutrosophic, and other extended fuzzy sets. The membership, non-membership, and indeterminacy were assigned to a specific element or subject concerning to its specific attribute. While in generalized plithogenic Whole-Hypersoft-Set, the fuzzy memberships were assigned to several subjects (elements of the universe) concerning their numerous attributes /sub-attributes separately as individual fuzzy memberships and at the same time collectively as whole fuzzy memberships. If these whole fuzzy memberships are constructed to accumulate information subject-wise for several subjects and being displayed for each attribute, then it is subjectively a whole hype-super-soft matrix. And if these whole fuzzy memberships are constructed to accumulate information attribute-wise for several given attributes while being displayed for each subject it is the case of attributively whole hyper-super-soft-matrix. In addition, this individual and collective information is organized in certain required time levels. Therefore, these hyper-super-soft matrices offer the viewer multiple inspecting expression options by observing many subjects with their numerous attributes/sub-attributes on multiple time levels and expressing their individual and collective states in many environments with different levels of ambiguity. For example, one possibility is to express individual and collective states/information in a crisp environment. This means expressing two opposing states of mind, either true (yes) or false (no). Another expression is the fuzzy environment, which contains some doubts about two opposite states, or an intuitionistic environment an expression with expanded doubts, or a neutrosophic environment that includes indeterminacy. Or some kind of combined environment that expresses different states of mind in one expression. Therefore, this novel form of expression and its mathematical structures are introduced to deal with higher-dimensional data/information/states of subjects by observing them through several angles of vision. The model of this article will first be expressed in the fuzzy environment in order to maintain the length restrictions of the article, further extended expressions will be introduced in future articles. To illustrate these multi-vision expressions, consider that a field is visited with several types of flowers, and it is observed that each type of flower has some specific beneficial properties (attributes) and property levels (sub-attributes) When, for a certain type of flower, its collective level of utility is expressed in a statement taking into account all attributes and expressed as a numerical value using fuzzy linguistic scales. It is a form of the whole expression (case of an attributively whole Hyper-Supersoft-Matrix). When, for a particular type of flower, the utility for each property (attribute) is expressed as a single statement or as a numerical value using fuzzy linguistic scales. It is another form of expression called individual expression (in the case of the Hypersoft Matrix) and the choice of environment will represent the state of mind of the observer or decision-maker.

3.1.3. Whole dilated and compact vision of the shattered information

This mathematical model describes such novel expressions (originally constructed in the plithogenic fuzzy environment) through the formulation and use of accumulated fuzzy memberships. This means that the entire vision of the scattered information/states/universe could be represented as a single numerical value (whole fuzzy membership) that offers a compact view of information/events/universe. Whereas the detailed insight view of the information/events/universe is indicated by certain numerical values that are called individual fuzzy memberships

3.1.4. Higher-dimensional Matrix with multi-layers and clusters of layers

To organize and analyze all relations between several subjects, their attributes (subject states), and sub-attributes (levels of states of subjects) by considering their individual and collective states and then expressing them in a single mathematical structure was not possible by using the classical algebra of ordinary matrices. To represent such higher dimensional information, some super algebraic structures like these hyper super matrices had to be constructed. These hyper-super-matrices can absorb such multi-dimensional information by using their multilayered structures. These higher dimensional matrices with

multiple layers and clusters of layers are another novelty that is being introduced into matrix theory for the first time.

3.1.5. New framework of modeling

In this article, we have developed a new modeling framework that can not only organize the high dimensional numerical data/information, but rather would accept information in any form and transform it into the required mathematical form through linguistic scales. It would also expand or contract the information through the use of established operators. The elements of classical matrices are usually numbers and they follow the classical rules of addition, multiplication, or inversion, rather these modern high-dimensional matrix structures would interact through the modern flexible laws of Set theory such as soft set theory or extended fuzzy set theory, etc. These matrices would interact within the matrix through Local Operators which will be developed in this article. They would also interact outside the matrix through global operators that would be introduced in the next upcoming versions.

3.2. Definitions and concepts of GPFWHSS

In order to develop a better understanding of the literature, some new definitions and concepts are presented below.

Let's consider a brief description of the mathematical terms and expressions used to develop the model.

Definition 3.2.1 (Universe of discourse):

$U_F\ (X) = \{x_i\}$ is the fuzzy universe of discourse where $x_i\ , i = 1, 2, \ldots, M$ represents the number of subjects (elements of the universe). The subject can be perceived as a physical entity that is being discussed.

Definition 3.2.2 (Attributes): $A_j\ j = 1, 2, \ldots, N$ are N number of attributes under consideration. These are the states of subjects i.e. characteristics or behaviors associated with the elements of the set (physical subjects).

Definition 3.2.3 Sub-Attributes: $A_j^k\ j = 1, 2, \ldots, N$ $k = 1, 2, \ldots, L$ where j represents the given number of attributes (states of matter-bodies or subjects) and each attribute is categorized into a number of levels, which are referred to as sub-attributes (substates of subjects). Attributes or sub-attributes are considered to be non-physical aspects of the physical bodies (subjects) under consideration.

Definition 3.2.4 (Plithogenic Fuzzy HyperSoft-Matrix (PFHS-Matrix)):

Let $\mathbf{U_F}$ be the Fuzzy universe of discourse, $P(U_F)$ be the power set of $\mathbf{U_F}$, $\mathbf{A_j^k}$ is a combination of attributes/Sub-Attributes for some $j = 1, 2, 3, \ldots, N$ Attributes, $k = 1, 2, 3, \ldots, L$ Sub-Attributes and $x_i i = 1, 2, 3, \ldots, M$ subjects under consideration then PFHS-Matrix, is a mapping F from the cross product of attributes/sub-attributes on the power set $P(U_F)$ represented in matrix form. This mapping with its matrix form in the plithogenic fuzzy environment is described in Equation 3.1 or Equation 3.2 respectively.

$$F : A_1^k \times A_2^k \times A_3^k \times \cdots \times A_N^k \to P(U_F) \quad (3.1)$$

$$F = \left[\mu_{A_j^k}(x_i)\right] \quad (3.2)$$

s.t $\mu_{A_j^k}(x_i) + \upsilon_{A_j^k}(x_i) = 1,$

$$\mu_{A_j^k}(x_i) \in [0, 1]\,, \upsilon_{A_j^k}(x_i) \in [0, 1]$$

$\mu_{A_j^k}(x_i)$ and $\upsilon_{A_j^k}(x_i)$ are fuzzy memberships and non-memberships for the given x_i subjects regarding each given A_j^k attributes/sub-attributes where, A_j^k is a combination of attributes/sub-attributes for some $j = 1, 2, 3, \ldots, N$ attributes, $k = 1, 2, 3, \ldots, L$ sub-attributes associated to $x_i i = 1, 2, 3, \ldots, M$ subjects under consideration. These fuzzy memberships are assigned by the relevant committees for the piece of universe/reality/event/information.

The expanded form of F is described, in Equation 3.3

$$F = \begin{array}{c} \\ x_1 \\ x_2 \\ \cdot \\ \cdot \\ \cdot \\ x_M \end{array} \begin{array}{c} \begin{array}{ccccc} A_1^k & & A_2^k \ldots & & A_N^k \end{array} \\ \begin{bmatrix} \mu_{A_1^k}(x_1) & \mu_{A_2^k}(x_1) & \ldots & \mu_{A_N^k}(x_1) \\ \mu_{A_1^k}(x_2) & \mu_{A_2^k}(x_2) & \ldots & \mu_{A_N}(x_2) \\ \cdot & \cdot & \cdot\ \cdot\ \cdot\ \cdot & \\ \cdot & \cdot & \cdot\ \cdot\ \cdot\ \cdot & \\ \cdot & \cdot & \cdot\ \cdot\ \cdot\ \cdot & \\ \mu_{A_1^k}(x_M) & \mu_{A_2^k}(x_M) & \ldots & \mu_{A_N^k}(x_M) \end{bmatrix} \end{array} \quad (3.3)$$

Example 1 (PFHS-Set and PFHS-Matrix expressions)

Let mapping F be defined as

$$F : A_1^k \times A_2^k \times A_3^k \times \cdots \times A_N^k \to, P(U_F)$$

(Taking some specific numeric values of A_j^k)

Consider $T = \{x_1, x_2, x_3\}$, is a subset of $P(U_F)$ where x_1, x_2, x_3 represent x_i subjects under con-

sideration with A_j^k attributes/sub-Attributes for $j = 1, 2, 3, 4$ and $k = 3, 1, 1, 2$. The fuzzy memberships would be assigned to x_1, x_2, x_3 for the $A_1^3, A_2^1, A_3^1, A_4^2$, attributes/sub-attributes i.e. a particular α-combination of attributes by the concerned bodies by using the linguistic five-point scale method see Ref. [31–34]. These plithogenic fuzzy memberships are the elements PFHS-Set represented as below,

$$F_\alpha\left(A_1^3, A_2^1, A_3^1, A_4^2\right) = F(\alpha)$$

$$= \left\{ \begin{matrix} x_1\,(0.3, 0.6, 0.5, 0.5)\,, \\ x_2\,(0.4, 0.4, 0.3, 0.1)\,, \\ x_3\,(0.6, 0.3, 0.4, 0.7) \end{matrix} \right\} \tag{3.4}$$

A more organized form of this PFHS-Set is expressed as a single layer of the PFHS-Matrix F_{ij}^α,

$$F_{ij}^\alpha = \begin{matrix} \\ x_1 \\ x_2 \\ x_3 \end{matrix} \begin{matrix} \left[A_1^3, A_2^1, A_3^1, A_4^2\right] \\ \begin{bmatrix} 0.3 & 0.6 & 0.5 & 0.5 \\ 0.4 & 0.4 & 0.3 & 0.1 \\ 0.6 & 0.3 & 0.4 & 0.7 \end{bmatrix} \end{matrix} \tag{3.5}$$

Where $A_1^3 A_2^1\ A_3^1 A_4^2$ is a specific combination of Attributes/Sub-Attributes associated with subjects x_1, x_2, x_3 and F_{ij}^α represents a single layer out of several possible layers of PFHS-Matrix. For further detailed descriptions and applications, see ref [19].

Example 2. Consider the example of Plithogenic Fuzzy HyperSoft matrix $F = \left[\mu_{A_j^k}(x_i)\right]$ for $= 1, 2, 3$, $j = 1, 2, 3, 4$ and, $k = 1$ (the first level layer) $k = 2$, (the second level layer). The fuzzy memberships that are assigned by the concerned bodies for the Part of universe/reality/event for Hypersoft Set are described as, let $T = \{x_1, x_2, x_3\}$ be the set of three Subjects under consideration in PFHS-Set with respect to the given attribute. The fuzzy memberships are assigned by the concerned bodies using the five-point linguistic scale method [19–22]. These fuzzy memberships are assigned regarding each attribute A_j^k at each level and are described underneath as PFHS-Set,

$$F\left(A_1^1, A_2^1, A_3^1, A_4^1\right) = \left\{ \begin{matrix} x_1\,(0.3, 0.6, 0.3, 0.5)\,, \\ x_2\,(0.4, 0.5, 0.2, 0.1)\,, \\ x_3\,(0.6, 0.2, 0.3, 0.7) \end{matrix} \right\} \tag{3.6}$$

$$F\left(A_1^2, A_2^2, A_3^2, A_4^2\right) = \left\{ \begin{matrix} x_1\,(0.5, 0.4, 0.2, 0.6) \\ , x_2\,(0.5, 0.7, 0.8, 0.4)\,, \\ x_3\,(0.7, 0.6, 0.5, 0.9) \end{matrix} \right\} \tag{3.7}$$

This information from the PFHS-Set is organized in the form of PFHS-Matrix F as,

$$\mathbf{F} = \begin{bmatrix} \begin{bmatrix} 0.3 & 0.6 & 0.3 & 0.5 \\ 0.4 & 0.5 & 0.2 & 0.1 \\ 0.6 & 0.2 & 0.3 & 0.7 \end{bmatrix} \\ \begin{bmatrix} 0.5 & 0.4 & 0.2 & 0.6 \\ 0.5 & 0.7 & 0.8 & 0.4 \\ 0.7 & 0.6 & 0.5 & 0.9 \end{bmatrix} \end{bmatrix} \tag{3.8}$$

Detailed descriptions and applications can be found in Ref [20].

Definition 3.2.5 (Individual fuzzy memberships): Elements of PFHS-Matrix $\mu_{A_j^k}(x_i)$ represents Individual fuzzy memberships these memberships are individual fuzzy states of subjects given for each attribute i.e. fuzzy membership of a certain x_i-subject say for $i = \mathrm{m}$, concerning to each A_j^k attribute/sub-attribute. The elements of the Matrix F_{ij}^α and Matrix F in Equations (3.9) and (3.12) represent the individual fuzzy memberships. These Individual fuzzy memberships $\mu_{A_j^k}(x_i)$ are used to represent the inside view of the universe/reality/event/information as individual memberships.

Definition 3.2.6 (Whole fuzzy membership): Whole fuzzy memberships are accumulated states of subjects. Whether accumulated by subject or attribute. For example, if the individual fuzzy memberships $\mu_{A_j^k}(x_i)$ are accumulated for a given combination of attributes $A_j^k\ j = 1, 2, \ldots, N$ and represented with respect to a certain subject x_i for some $i = \mathrm{m}$, then they are called Whole Fuzzy Membership. The individual fuzzy memberships are accumulated to see the combined effect (e.g. average degree of membership) of the elements (subjects) with respect to a particular combination of attributes/sub-attributes. These are two types of whole fuzzy membership i.e.

(a) *Attributively-Whole Fuzzy Memberships.*
(b) *Subjectively-Whole Fuzzy Memberships.*

Definition 3.2.7 (Attributively-Whole Fuzzy Membership): Attributively-Whole Fuzzy Membership represents such fuzzy memberships that are accumulated (attribute-wise) along the rows of PFHS-Matrix

by using any aggregation operator (t) represented by $\Omega_{A^l}(x_i)$. These memberships are obtained by accumulating all given states (attributes) of a certain subject for a fixed sub-attribute level l. and represented with respect for a certain subject. The Whole Fuzzy Memberships $\Omega^t_{\mathbf{A}^k}(x_{\mathrm{i}})$ used to represent the subject-wise exterior view of the universe/reality/event/information revealing the attributive wholeness. For example the whole memberships $\Omega^t_{\mathbf{A}^1}(x_1)$ given in Equation (3.9) represents subjectively-whole fuzzy memberships for the first subject with respect to all given attributes of first level.

$$\left[\left[\mu_{A^1_j}(x_1)\right]\left[\Omega^t_{\mathbf{A}^1}(x_1)\right]\right] \tag{3.9}$$

Definition 3.2.8 (Subjectively-Whole Fuzzy Membership): Subjectively-Whole Fuzzy Memberships representssuch fuzzy memberships that are accumulated (subject-wise) along with the columns of PFHS-Matrix by using any aggregation operator operator (t) represented by $\Omega_{A^l_j}(X)$. The Subjectively-Whole Fuzzy Membership $\Omega^t_{\mathrm{A}^k_j}(X)$ used to represent the attribute-wise exterior view of the universe/reality/event/information revealing the subjective wholeness. For example the whole fuzzy memberships $\Omega^t_{A^1_j}(X)$ given in Equation (3.10) represents attributively-whole fuzzy memberships for first subject with respect to all given attributes of first level.

$$\begin{bmatrix}\left[\mu_{A^1_1}(x_i)\right]\\ \left[\Omega^t_{\mathbf{A}^1_1}(X)\right]\end{bmatrix} \tag{3.10}$$

Definition 3.2.9 (Individual Fuzzy Non-Membership: $\upsilon_{A^k_j}(x_i)$ represents fuzzy non-membership (level of non-belongingness) of a subject (x_i) concerning a certain attribute/sub-attribute given in Equation (3.11)

$$\upsilon_{A^k_j}(x_i) = 1 - \mu_{A^k_j}(x_i) \tag{3.11}$$

Definition 3.2.10 (Attributively-Whole Fuzzy Non-Membership): Attributively-Whole Fuzzy Non-Membership is described as the accumulated level of non-belongingness of a particular subject with respect to a combination of attributes given in Equation (3.12)

$$\Phi_{A^l}(x_i) = 1 - \Omega_{A^l}(x_i) \tag{3.12}$$

If one accumulates individual fuzzy non-memberships along rows of the PFHS matrix, one obtains a combined degree of non-membership with respect to all given attributes for a particular subject, called the attributively-whole fuzzy non-membership.

Definition 3.2.11 (Subjectively-Whole Fuzzy Non-Membership): Subjectively-Whole Fuzzy Non-Membership is described as the accumulated level of non-belongingness of any given A^k_j attribute for fixed $j = \mathrm{n}, k = \mathrm{l}$ regarding a specific combination of subjects. Subjectively whole fuzzy non-memberships are mathematically described as,

$$\Phi_{A^l_j}(X) = 1 - \Omega_{A^l_j}(X) \tag{3.13}$$

If one cumulate these fuzzy non-memberships column-wise for PFHS-Matrix, he gets a combined degree of non-membership with respect to all given subjects for a particular attribute/sub-attribute called Attributively-Whole Non-Membership for the fuzzy environment

Note: For a precise description and application of these terms and mathematical expression a detailed numerical example is constructed as COVID19 data structures in the section-5.

Definition 3.2.12 (Plithogenic Fuzzy Whole HyperSoft-Set)

Let U_F be the initial universe of discourse, in the Fuzzy environment and $P(U_F)$ be the power set of U_F. Let $A^k_1, A^k_2, \ldots, A^k_N$ for $N \geq 1$ be N distinct attributes, $k = 1, 2, \ldots L$ represents attribute values and F is a mapping from cross product of sub-attributes to the power set of U_F $(F : A^k_1 \times A^k_2 \times \cdots \times A^k_{\mathrm{N}} \to P(U_F))$ Then $\left(F, A^k_1 \times A^k_2 \times \cdots \times A^k_{\mathrm{N}}\right)$ represented in the set form is called a Plithogenic Fuzzy Whole-Hypersoft-Set (PFWHS-Set) over U_F if its elements are expressed by both individual fuzzy memberships $\mu_{A^k_j}(x_i)$ and Combined fuzzy memberships $\Omega_{\mathbf{A}_\alpha}(x_i)$, $\forall x_i \in U_F$ for the given α-combination of attributes sub-attributes regarding each subject. In PFWHS-Set F deals with a single specific combination of attributes/sub-attributes out of the complex cross products. It is observed that the individual fuzzy memberships $(\mu_{A^k_j}(x_i))$ of subjects are expressed with respect to each given attribute/sub-attribute and the cumulative fuzzy memberships $\Omega_{\mathrm{A}_\alpha}(x_i)$ are aggregated by some operators and expressed for a specific α-combination of attributes/sub-attributes concerning the given subjects (case of an α universe, that expresses one of the possible reality/event/information in a fuzzy environment).

Definition 3.2.13 (PFSAWHSS-Matrix)

Let U_F be the initial universe of discourse, in the Fuzzy environment and $P(U_F)$ be the power set of U_F. Let $A_1^k, A_2^k, \ldots, A_N^k$ are N distinct attributes sub-attributes for $j = 1, 2, \ldots N, k = 1, 2, \ldots L$ then the mapping F, from the cross product of the attributes sub-attributes to the powerset of U_F ($F : A_1^k \times A_2^k \times \cdots \times A_{\mathrm{N}}^k \to P(U_F)$) represented in the matrix form in such a way that its elements consist of individual fuzzy memberships $\mu_{A_j^k}(x_i)$ as well as attributively aggregated fuzzy memberships $\Omega^t_{\mathrm{A}^k}(x_{\mathrm{i}})$, $\forall x_i \in U_F$, then this matrix is called a Plithogenic Subjective Attributively Whole Hyper-Super-soft-Matrix (PFSAWHSS-Matrix).

The PFSAWHSS-Matrix F_{S_t} is described underneath in Equation (3.14) and its expanded form is presented in Equation (3.15)

$$F_{\mathbf{S}_t} = \left[\left[\mu_{A_j^k}(x_i) \right] \left[\Omega^t_{\mathrm{A}^k}(x_{\mathrm{i}}) \right] \right] \tag{3.14}$$

$$F_{\mathrm{S}_t} = \left[\begin{bmatrix} \mu_{A_1^k}(x_1) & \mu_{A_2^k}(x_1) & \ldots & \mu_{A_N^k}(x_1) \\ \mu_{A_1^k}(x_2) & \mu_{A_2^k}(x_2) & \ldots & \mu_{A_N^k}(x_2) \\ . & . & \ldots & \\ . & . & \ldots & \\ . & . & \ldots & \\ \mu_{A_1^k}(x_M) & \mu_{A_2^k}(x_M) & \ldots & \mu_{A_N^k}(x_M) \end{bmatrix} \begin{bmatrix} \Omega^t_{\mathrm{A}^k}(x_1) \\ \Omega^t_{\mathrm{A}^k}(x_2) \\ . \\ . \\ . \\ \Omega^t_{\mathrm{A}^k}(x_M) \end{bmatrix} \right] \tag{3.15}$$

F_{S_t} is Plithogenic subjective attributively whole hyper-super-soft-matrix in the fuzzy environment, for further details see ref [20].

The elements of the last column of this matrix represented in (Equation 3.16) represent Attributively-Whole Fuzzy Memberships $\Omega^t_{\mathrm{A}^l}(x_{\mathrm{i}})$,

$$\begin{bmatrix} \Omega^t_{\mathrm{A}^k}(x_1) \\ \Omega^t_{\mathrm{A}^k}(x_2) \\ . \\ . \\ . \\ \Omega^t_{\mathrm{A}^k}(x_M) \end{bmatrix} \tag{3.16}$$

t represents an aggregation operator that is used to accumulate the fuzzy memberships i.e. $t = 1$, (Disjunction operator), $t = 2$, (conjunction operator), $t = 3$ (averaging operator). This PFSAWHS-Matrix shows both an inner and outer interpretation of the universe. The inside state of the universe, event, or reality is reflected by individual memberships $\mu_{A_j^k}(x_i)$ of subjects while the outside state of the universe, event, or reality represented by attributively aggregated fuzzy memberships $\Omega_{A^k}(x_i)$. Therefore, the PFSAWHS-Matrix would allow a subjective classification through attributive accumulation. Attributive aggregation is obtained by applying several suitable aggregation operators to $\mu_{A_j^k}(x_i)$ at the index j.

Definition 3.2.14 (PFASWHSS-Matrix)

Let U_F be the initial universe of discourse, in the Fuzzy environment and $P(U_F)$ be the power set of U_F. Let $A_1^k, A_2^k, \ldots, A_N^k$ are N distinct attributes/subattributes for $j = 1, 2, \ldots N, k = 1, 2, \ldots L$ then the mapping F, from the cross product of the attributes sub-attributes to the powerset of the fuzzy universe of discourse $F : A_1^k \times A_2^k \times \cdots \times A_{\mathrm{N}}^k \to P(U_F)$ represented in the matrix form in such a way that its elements consist of individual fuzzy memberships $\mu_{A_j^k}(x_i)$ as well as subjectively aggregated fuzzy memberships $\Omega_{A_{\mathrm{j}}^k}(X)$, $\forall x_i \in U_F$, then this matrix is called a Plithogenic Fuzzy Attributive Subjectively Whole Hyper-Super-soft-Matrix (PFASWHSS-Matrix).

The PFSAWHSS-Matrix F_{A_t} is described underneath in (Equation 3.17)

$$F_{\mathbf{A}_t} = \begin{bmatrix} \left[\mu_{A_j^k}(x_i) \right] \\ \left[\Omega^t_{\mathbf{A}_j^k}(X) \right] \end{bmatrix} \tag{3.17}$$

It is observed that his matrix is represented by both individual fuzzy memberships $\mu_{A_j^k}(x_i)$ (individual fuzzy states of subjects regarding each attribute) and the aggregated fuzzy memberships $\Omega_{A_{\mathrm{j}}^k}(X)$ (subject-wise aggregated states). The expanded form of this matrix is described below.

$$F_{\mathrm{A}_t} = \left[\begin{array}{c} \left[\begin{array}{cccc} \mu_{A_1^k}(x_1) & \mu_{A_2^k}(x_1) & \ldots & \mu_{A_N^k}(x_1) \\ \mu_{A_1^k}(x_2) & \mu_{A_2^k}(x_2) & \ldots & \mu_{A_N^k}(x_2) \\ . & . & .... & \\ . & . & .... & \\ . & . & .... & \\ \mu_{A_1^k}(x_M) & \mu_{A_2^k}(x_M) & \ldots & \mu_{A_N^k}(x_M) \end{array} \right] \\ \\ \left[\Omega^t_{\mathrm{A}_1^k}(X)\ \Omega^t_{\mathrm{A}_2^k}(X) \ldots \Omega^t_{\mathrm{A}_N^k}(X) \right] \end{array} \right] \tag{3.18}$$

The elements of the last column-matrix as given below, in Equation 3.19 are subjectively-Whole Fuzzy Memberships ($\Omega^t_{\mathrm{A}_N^k}(X)$) of PFASWHSS-Matrix.

$$\left[\Omega^t_{\mathrm{A}_1^k}(X)\ \Omega^t_{\mathrm{A}_2^k}(X) \ldots \Omega^t_{\mathrm{A}_N^k}(X) \right] \tag{3.19}$$

Note: The whole fuzzy memberships $\Omega_{\mathbf{A}_\alpha}(x_i)$ and $\Omega^k_{\mathrm{A}_j}(X)$ that are aggregated attribute-wise and subject-wise respectively and whole fuzzy non-memberships $\Phi_{\mathbf{A}_\alpha}(x_i)$ and $\Phi^k_{\mathrm{A}_j}(X)$ are dependent on individual fuzzy memberships $\mu_{A_j^k}(x_i)$, and non-membership $\upsilon_{A_j^k}(x_i)$.

Definition 3.2.15 (Hyper-Super-Matrices): Hyper Super Matrices are clusters of super-matrix-layers. These are such Hyper-matrices that have several layers of super-matrices represented by more than two variation indices ($d \succ 2$) and further their elements are matrices or scalars.

As we know, all ordinary $M \times N$ Matrices on real vector space are tensors of rank 2, i.e., these ordinary matrices are represented by the use of two variation indices. For example $\left[a_{\mathrm{ij}}\right]$ is an ordinary matrix having represents one variation by index i as rows of the Matrix and the second variation by index j represents columns of the Matrix. While the hyper-super-matrices are constructed using four variation indices which represent four types of variations that occur together. The hyper-super-matrices are rank-4 tensors. While the hypermatrices represented by three variation indices that describe three types of variations at a time are rank-3 tensors.

These Hyper Super matrices are formulated by hybridizing hyper-matrices and super-matrices.

$A = \left[a_{ijk}\right]$ is a Hyper-Matrix. This hypermatrix represents three kinds of variations, described by the use of three indices i, j and k where the i, j, k are positive integers.

$\mathrm{B} = \left[\begin{array}{c} [b_{ijk}] \\ [a_{jkt}] \end{array} \right]$ represents a Hyper-Super-Matrix that represents four types of variations described by the use of four indices i, j, k and t.

Definition 3.2.16 (HyperSoft-Matrix): Let U be the initial universe of discourse, and $P(U)$ be the power set of U. Let $A_1^k, A_2^k, \ldots, A_n^k$ for $n \geq 1$ be n distinct attributes, $k = 1, 2, \ldots L$ represents attributes values, a function $F : A_1^k \times A_2^k \times \cdots \times A_{\mathrm{n}}^k \to P(U)$ is a hypersoft matrix, also called an order-$M \times N \times L$ hypersoft matrix or d-hypersoft Matrix ($d = 3$) i.e., a matrix that represents a hypersoft-set is a hypersoft-matrix (HS-Matrix). An HS-Matrix with $d \succ 3$ (More than three variations) and whose elements are matrices is called a Hyper-Super-Soft-Matrices (HSS-Matrix). This HSS-Matrix consisted of clusters of several parallel layers of ordinary matrices, and elements of these matrix layers are matrices or scalars.

Note: All simple $M \times N$ Matrices on real vector space are tensors of rank 2. The new HyperSoft-Matrix (HS- Matrix) with three variation indices is a rank three tensor and the Hyper-Super-Soft matrix (HSS matrix) with four variation indices is a Fourth rank tensor. The HS-Matrix (third rank tensors) and HSS-Matrix (fourth rank tensors) are a generalized version of the ordinary matrices (second rank tensors).

3.3. PFHS-Matrix and index-based views

The Plithogenic Fuzzy HyperSoft Set in matrix form is expressed as, $F = [\mu_{A_j^k}(x_i)]$

Plithogenic Fuzzy HyperSoft-Matrix represents three types of variation indices used to portray the fuzzy memberships $\mu_{A_j^k}(x_i)$. The first variation on index i is with respect to subjects representing M rows of $M \times N \times L$ Matrix while the second variation on index j is concerning to attributes representing N columns of $M \times N \times L$ Matrix, i and j represent arrangements of rows and columns in the form of simple $M \times N$ Matrix-layer.

However, when we consider sub-attributes of respective attributes, the third type of variation arises. This third variation is on the index k that is used to portray sub-attributes (attribute levels). These sub-attributes are displayed in the form of L level layers of an $M \times N$ Matrix. The hyper-matrix is generated by these parallel layers of $M \times N$ ordinary matrices.

These level layers of hyper-matrix are categorized into three types:

1 The front to back and inner Vertical level layers are generated by$L$ number of matrix layers. Where each layer is an ordinary matrix of order $M \times N$.
2 The left to right and inner vertical level layers are generated by N number of matrix layers such that each layer is a Matrix of order $M \times L$.
3 The top to bottom horizontal and inner level layers are generated by M number of matrix layers of order, $L \times N$.

Consider The PFHS-Matrix $F = \left[\mu_{A_j^k}(x_i)\right]$ and its dilated version described in Equation 3.3, the three types of level layers of this PFHS-Matrix F are described as under,

3.3.1. Type-1 level-layers of PFHS-Matrix

If the Equation 3.3 is further expanded with respect to k by varying k from 1 to L, then L number of front to back, and inner level layers of PFHS-Matrix, would be constructed where each layer is a matrix of order $M \times N$. We can describe numerous layers of PFHS-Matrix in three different ways, i.e., parallel layers of type-1 are obtained by vertically cutting a box matrix of order $M \times N \times L$ from front to back. These level cuts are called level-layers of type 1. They can be expressed on a two-dimensional page by giving step-by-step variation to the index k in Equation (3.3) and represented as described below in Equation (3.20).

$$F = \left[\begin{array}{l} \left[\begin{array}{cccc} \mu_{A_1^1}(x_1) & \mu_{A_2^1}(x_1) & \cdots & \mu_{A_N^1}(x_1) \\ \mu_{A_1^1}(x_2) & \mu_{A_2^1}(x_2) & \cdots & \mu_{A_N^1}(x_2) \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \mu_{A_1^1}(x_M) & \mu_{A_2^1}(x_M) & \cdots & \mu_{A_N^1}(x_M) \end{array}\right] \\ \left[\begin{array}{cccc} \mu_{A_1^2}(x_1) & \mu_{A_2^2}(x_1) & \cdots & \mu_{A_N^2}(x_1) \\ \mu_{A_1^2}(x_2) & \mu_{A_2^2}(x_2) & \cdots & \mu_{A_N^2}(x_2) \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \mu_{A_1^2}(x_M) & \mu_{A_2^2}(x_M) & \cdots & \mu_{A_N^2}(x_M) \end{array}\right] \\ \qquad \vdots \\ \left[\begin{array}{cccc} \mu_{A_1^L}(x_1) & \mu_{A_2^L}(x_1) & \cdots & \mu_{A_N^k}(x_1) \\ \mu_{A_1^L}(x_2) & \mu_{A_2^L}(x_2) & \cdots & \mu_{A_N^L}(x_2) \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \mu_{A_1^L}(x_M) & \mu_{A_2^L}(x_M) & \cdots & \mu_{A_N^L}(x_M) \end{array}\right] \end{array}\right] \tag{3.20}$$

Example 3. Consider the PFHS-Matrix F described in Equation (3.8)

$$F = \left[\begin{array}{l} \left[\begin{array}{cccc} 0.3 & 0.6 & 0.3 & 0.5 \\ 0.4 & 0.5 & 0.2 & 0.1 \\ 0.6 & 0.2 & 0.3 & 0.7 \end{array}\right] \\ \left[\begin{array}{cccc} 0.5 & 0.4 & 0.2 & 0.6 \\ 0.5 & 0.7 & 0.8 & 0.4 \\ 0.7 & 0.6 & 0.5 & 0.9 \end{array}\right] \end{array}\right]$$

F represents the front view of PFHS-Matrix and its front to back two Matrix-layers are given bellow,

$$\text{First layer,} \quad \left[\begin{array}{cccc} 0.3 & 0.6 & 0.3 & 0.5 \\ 0.4 & 0.5 & 0.2 & 0.1 \\ 0.6 & 0.2 & 0.3 & 0.7 \end{array}\right] \tag{3.21}$$

for $k = 1$

$$\text{Second layer,} \quad \left[\begin{array}{cccc} 0.5 & 0.4 & 0.2 & 0.6 \\ 0.5 & 0.7 & 0.8 & 0.4 \\ 0.7 & 0.6 & 0.5 & 0.9 \end{array}\right] \tag{3.22}$$

for $k = 2$

3.3.2. Type-2 level layers of PFHS-Matrix level layers of PFHS-Matrix

Similarly, on the other hand, each column of the $M \times N$ matrix when expanded at its rear side creates a matrix of order $M \times L$ which expands N (attribute) into vertical layers (attribute leves) i.e., like creating vertical sections from left to right of the box matrix of the dimension $M \times N \times L$.

These vertical slices from left to right are level layers of type 2 and are expressed on the two-dimensional page by varying the index j as represented in Equation (3.23).

$$F=\left[\begin{array}{l}\left[\begin{array}{cccc}\mu_{A_1^1}(x_1) & \mu_{A_1^2}(x_1) & \cdots & \mu_{A_1^L}(x_1)\\ \mu_{A_1^1}(x_2) & \mu_{A_1^2}(x_2) & \cdots & \mu_{A_1^L}(x_2)\\ \cdot & \cdot & \cdots & \cdot\\ \cdot & \cdot & \cdots & \cdot\\ \cdot & \cdot & \cdots & \cdot\\ \mu_{A_1^1}(x_M) & \mu_{A_1^2}(x_M) & \cdots & \mu_{A_1^L}(x_M)\end{array}\right]\\ \left[\begin{array}{cccc}\mu_{A_2^1}(x_1) & \mu_{A_2^2}(x_1) & \cdots & \mu_{A_2^L}(x_1)\\ \mu_{A_2^1}(x_2) & \mu_{A_2^2}(x_2) & \cdots & \mu_{A_2^L}(x_2)\\ \cdot & \cdot & \cdots & \cdot\\ \cdot & \cdot & \cdots & \cdot\\ \cdot & \cdot & \cdots & \cdot\\ \mu_{A_2^1}(x_M) & \mu_{A_2^2}(x_M) & \cdots & \mu_{A_2^L}(x_M)\end{array}\right]\\ \vdots\\ \left[\begin{array}{cccc}\mu_{A_N^1}(x_1) & \mu_{A_N^2}(x_1) & \cdots & \mu_{A_N^L}(x_1)\\ \mu_{A_N^1}(x_2) & \mu_{A_N^2}(x_2) & \cdots & \mu_{A_N^L}(x_2)\\ \cdot & \cdot & \cdots & \cdot\\ \cdot & \cdot & \cdots & \cdot\\ \cdot & \cdot & \cdots & \cdot\\ \mu_{A_N^1}(x_M) & \mu_{A_N^2}(x_M) & \cdots & \mu_{A_N^L}(x_M)\end{array}\right]\end{array}\right] \tag{3.23}$$

3.3.3. Type-3 level layers of PFHS-Matrix

In a similar manner, level layers of type-3 are top to bottom and middle inner layers. These parallel layers are combined to create M layers of $L \times N$ Matrix.

These top to bottom M number of slices are level layers of type-3 and can be expressed on the two-dimensional page by gradually varying the index i as described below in Equation (3.24).

$$F=\left[\begin{array}{l}\left[\begin{array}{cccc}\mu_{A_1^1}(x_1) & \mu_{A_1^2}(x_1) & \cdots & \mu_{A_1^L}(x_1)\\ \mu_{A_2^1}(x_1) & \mu_{A_2^2}(x_1) & \cdots & \mu_{A_2^L}(x_1)\\ \cdot & \cdot & \cdots & \cdot\\ \cdot & \cdot & \cdots & \cdot\\ \cdot & \cdot & \cdots & \cdot\\ \mu_{A_N^1}(x_1) & \mu_{A_N^2}(x_1) & \cdots & \mu_{A_N^L}(x_1)\end{array}\right]\\ \left[\begin{array}{cccc}\mu_{A_1^1}(x_2) & \mu_{A_1^2}(x_2) & \cdots & \mu_{A_1^L}(x_2)\\ \mu_{A_2^1}(x_2) & \mu_{A_2^2}(x_2) & \cdots & \mu_{A_N^L}(x_2)\\ \cdot & \cdot & \cdots & \cdot\\ \cdot & \cdot & \cdots & \cdot\\ \cdot & \cdot & \cdots & \cdot\\ \mu_{A_N^1}(x_2) & \mu_{A_N^2}(x_2) & \cdots & \mu_{A_N^L}(x_2)\end{array}\right]\\ \vdots\\ \left[\begin{array}{cccc}\mu_{A_1^1}(x_M) & \mu_{A_1^2}(x_M) & \cdots & \mu_{A_1^L}(x_M)\\ \mu_{A_2^1}(x_M) & \mu_{A_2^2}(x_M) & \cdots & \mu_{A_2^L}(x_M)\\ \cdot & \cdot & \cdots & \cdot\\ \cdot & \cdot & \cdots & \cdot\\ \cdot & \cdot & \cdots & \cdot\\ \mu_{A_N^1}(x_M) & \mu_{A_N^2}(x_M) & \cdots & \mu_{A_N^L}(x_M)\end{array}\right]\end{array}\right] \tag{3.24}$$

4. Local aggregation operators for PFHSS-Matrix

In this section, local aggregation operators such as disjunction operators, conjunction operators, averaging operators, and complements for the plithogenic fuzzy hypersoft set/matrix are constructed. These operators would be used to formulate PFASWHSS matrix and PFSAWHSS matrix. The Whole (combined) memberships $\Omega^t_{\mathbf{A}_j^k}(X)$ for Plithogenic HyperSoft-Matrix would be developed by using the following operators.

In PFASWHSS-Matrix $F_{\mathbf{A}_t}=\left[\begin{array}{c}\left[\mu_{A_j^k}(x_i)\right]\\ \left[\Omega^t_{\mathbf{A}_j^k}(X)\right]\end{array}\right]$ the last row of collective memberships $\Omega^t_{\mathbf{A}_j^k}(X)$ is accomplished by using three local aggregation operators i.e $t=1$ represents Max-operator $t=2$ represents the Min-operator, and $t=3$ represents $Averaging$-operator. In addition, $t=4$ stands for Complement.

In PFSAWHS-Matrix, $F_{\mathbf{S}_t}=\left[\left[\mu_{A_j^k}(x_i)\right]\left[\Omega^t_{\mathbf{A}^k}(x_i)\right]\right]$ the last column of cumulative memberships $\Omega_{A^k}(x_i)$ is acquired by using three local aggregation operators, $t=1$, the Max-operator $t=2$, the Min-operator, and $t=3$ the $Averaging$-operator. And $t=4$ represents the Complement.

These four operators are structured as follows,

4.1. Local Disjunction Operator for construction of PFASWHSS-Matrix

$$\cup_i\left(\mu_{A_j^k}(x_i)\right)=\underset{i}{Max}\left(\mu_{A_j^k}(x_i)\right)=\Omega^1_{\mathbf{A}_j^k}(X), \tag{4.1}$$

for some $k=l$

Choose maximum membership from j_{th}col of PFHS-Matrix.

4.2. Local Disjunction Operator for construction of PFSAWHSS-Matrix

$$\cup_j\left(\mu_{A_j^k}(x_i)\right)=\underset{j}{Max}\left(\mu_{A_j^k}(x_i)\right)=\Omega^1_{\mathbf{A}^k}(x_i), \tag{4.2}$$

for some $k=l$

(Choose maximum membership from i_{th} row of PFHS-Matrix)

4.3. Local Conjunction Operator for construction of PFASWHSS-Matrix

$$\cap_i \left(\mu_{A_j^k}(x_i)\right) = \underset{i}{Min}\left(\mu_{A_j^k}(x_i)\right) = \Omega^2_{\mathbf{A}_j^k}(X), \tag{4.3}$$

for some $k = l$

Choose minimum membership from j_{th} col of PFHS-Matrix.

4.4. Local Conjunction Operator for construction of PFSAWHSS-Matrix

$$\cap_j \left(\mu_{A_j^k}(x_i)\right) = \underset{j}{Min}\left(\mu_{A_j^k}(x_i)\right) = \Omega^2_{\mathrm{A}^k}(x_{\mathrm{i}}), \tag{4.4}$$

for some $k = l$

Choose minimum membership from i_{th} row of PFHS-Matrix.

4.5. Local averaging Operator for construction of PFASWHSS-Matrix

$$\Gamma_i \left(\mu_{A_j^k}(x_i)\right) = \frac{\sum\limits_i \left(\mu_{A_j^k}(x_i)\right)}{M} = \Omega^3_{\mathbf{A}_j^k}(X), \tag{4.5}$$

for some $k = l$

Take the average of memberships of j_{th} col) of given specific k_{th}-layer.

4.6. Local averaging Operator for construction of PFSAWHSS-Matrix

$$\Gamma_j \left(\mu_{A_j^k}(x_i)\right) = \frac{\sum\limits_j \left(\mu_{A_j^k}(x_i)\right)}{N} = \Omega^3_{\mathrm{A}^k}(x_{\mathrm{i}}), \tag{4.6}$$

for some $k = l$

Take the average of memberships of i_{th} row of PFHS-Matrix of given specific k_{th}-layer.

In $\mu_{A_j^k}(x_i)$ i offers a row-wise variation of states of subjects, j gives a column-wise variation of states and k produces variations of states as sub-attributes in the form of matrix layers.

4.7. Local Complement for construction of PFASWHSS-Matrix

$$C_{loc}(F) = \left\{ \begin{array}{l} Max_i\left(1 - \mu_{A_j^k}(x_i)\right) \\ Min_i\left(1 - \mu_{A_j^k}(x_i)\right) \\ \sum\limits_{i=1}^{M} \frac{\left(1-\mu_{A_j^k}(x_i)\right)}{M} \end{array} \right\} \tag{4.7}$$

for some $k = l$

4.8. Local Complement for construction of PFSAWHSS-Matrix

$$C_{loc}(F) = \left\{ \begin{array}{l} Max_j\left(1 - \mu_{A_j^k}(x_i)\right) \\ Min_j\left(1 - \mu_{A_j^k}(x_i)\right) \\ \sum\limits_{j=1}^{N} \frac{\left(1-\mu_{A_j^k}(x_i)\right)}{N} \end{array} \right\} \tag{4.8}$$

for some $k = l$

Here C_{loc} represent the local *Complement* of PFHS-Matrix F for a certain level of attributes $k = l$. This Complement is applied across PFHS-Matrix F by taking the Complement of each fuzzy membership $\mu_{A_j^k}(x_i)$ and then choosing either maximum or minimum or an average membership from the established column or row.

Further description and application of these local aggregation operators are presented in Sec-5.

5. Construction of PFASWHSS-Matrix, PFSAWHSS-Matrix, and Application of local aggregation operators as COVID-19 Data Structures

5.1. (Plithogenic Fuzzy Attributive Subjectively-Whole Hyper-Super-Soft-Matrix)

The matrix representation of Plithogenic Attributive Subjectively Whole HyperSoft Set in the fuzzy environment named, PFASWHSS-Matrix. The elements of PFASWHSS-Matrix are matrix-layers, matrices, or scalars, so a PFASWHSS-Matrix is a hybridization of HS-Matrix and HSS-Matrix. The HSS-Matrix has such layers of matrices whose elements are further matrices or scalars. If one considers the HS-Matrix in a fuzzy environment then the subject-wise combined fuzzy memberships $(\Omega^{t}_{\mathrm{A}^{k}_{j}}(\mathrm{X}))$ are constructed by using the aggregation operator, t such that $t = 1, 2, 3, 4$, represents local operators constructed in the previous section see Equation 4.1, to 4.8. The compact form of PFASWHSS-Matrix $F_{\mathbf{A}_t}$ can be described as

$$F_{\mathbf{A}_t} = \begin{bmatrix} \begin{bmatrix} \mu_{A^{k}_{\mathrm{j}}}(x_{\mathrm{i}}) \end{bmatrix} \\ \begin{bmatrix} \Omega^{t}_{\mathbf{A}^{k}_{\mathrm{j}}}(X) \end{bmatrix} \end{bmatrix}$$

In this PFASWHSS-Matrix, four types of diversification are described. These diversification/variations are described as under, the first Variation of $i = 1, 2, \ldots M$ would produce M rows of $F_{\mathbf{A}_t}$. The second variations on $j = 1, 2, \ldots N$ produce N columns, and the third variations on $k = 1, 2, \ldots L$ would generate L combinations of rows and columns as parallel matrix-layers of $M \times N$ matrix. The fourth variation on $t = 1, 2, \ldots P$ would provide P sets, i.e., P numbers of Clusters of L numbers of parallel layers of $M \times N$ matrices. These sets of parallel layers can represent clusters of parallel universes or parallel realities. These clusters in the form of expanded form of PFASWHSS-Matrix $F_{\mathbf{A}_t}$ are described below in Equation 5.1.

$$\mathbf{F}_{\mathbf{A_t}} = \begin{bmatrix} \begin{bmatrix} \begin{bmatrix} \mu_{A^{1}_{1}}(x_1) & \mu_{A^{1}_{2}}(x_1) & \cdots & \mu_{A^{1}_{N}}(x_1) \\ \mu_{A^{1}_{1}}(x_2) & \mu_{A^{1}_{2}}(x_2) & \cdots & \mu_{A^{1}_{N}}(x_2) \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \mu_{A^{1}_{1}}(x_M) & \mu_{A^{1}_{2}}(x_M) & \cdots & \mu_{A^{1}_{N}}(x_M) \end{bmatrix} \\ \begin{bmatrix} \Omega^{1}_{\mathbf{A}^{1}_{1}}(\boldsymbol{X}) & \Omega^{1}_{\mathbf{A}^{1}_{2}}(\boldsymbol{X}) & \cdots & \Omega^{1}_{\mathbf{A}^{1}_{N}}(\boldsymbol{X}) \end{bmatrix} \end{bmatrix} \\ \begin{bmatrix} \begin{bmatrix} \mu_{A^{2}_{1}}(x_1) & \mu_{A^{2}_{2}}(x_1) & \cdots & \mu_{A^{2}_{N}}(x_1) \\ \mu_{A^{2}_{1}}(x_2) & \mu_{A^{2}_{2}}(x_2) & \cdots & \mu_{A^{2}_{N}}(x_2) \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \mu_{A^{2}_{1}}(x_M) & \mu_{A^{2}_{2}}(x_M) & \cdots & \mu_{A^{2}_{N}}(x_M) \end{bmatrix} \\ \begin{bmatrix} \Omega^{1}_{\mathbf{A}^{1}_{2}}(\boldsymbol{X}) & \Omega^{1}_{\mathbf{A}^{2}_{2}}(\boldsymbol{X}) & \cdots & \Omega^{1}_{\mathbf{A}^{2}_{N}}(\boldsymbol{X}) \end{bmatrix} \end{bmatrix} \\ \vdots \\ \begin{bmatrix} \begin{bmatrix} \mu_{A^{L}_{1}}(x_1) & \mu_{A^{L}_{2}}(x_1) & \cdots & \mu_{A^{k}_{N}}(x_1) \\ \mu_{A^{L}_{1}}(x_2) & \mu_{A^{L}_{2}}(x_2) & \cdots & \mu_{A^{L}_{N}}(x_2) \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \mu_{A^{L}_{1}}(x_M) & \mu_{A^{L}_{2}}(x_M) & \cdots & \mu_{A^{L}_{N}}(x_M) \end{bmatrix} \\ \begin{bmatrix} \Omega^{1}_{\mathbf{A}^{L}_{1}}(\boldsymbol{X}) & \Omega^{1}_{\mathbf{A}^{L}_{2}}(\boldsymbol{X}) & \cdots & \Omega^{1}_{\mathbf{A}^{1}_{N}}(\boldsymbol{X}) \end{bmatrix} \end{bmatrix} \\ \vdots \\ \begin{bmatrix} \begin{bmatrix} \mu_{A^{1}_{1}}(x_1) & \mu_{A^{1}_{2}}(x_1) & \cdots & \mu_{A^{1}_{N}}(x_1) \\ \mu_{A^{1}_{1}}(x_2) & \mu_{A^{1}_{2}}(x_2) & \cdots & \mu_{A^{1}_{N}}(x_2) \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \mu_{A^{1}_{1}}(x_M) & \mu_{A^{1}_{2}}(x_M) & \cdots & \mu_{A^{1}_{N}}(x_M) \end{bmatrix} \\ \begin{bmatrix} \Omega^{2}_{\mathbf{A}^{1}_{1}}(\boldsymbol{X}) & \Omega^{2}_{\mathbf{A}^{1}_{2}}(\boldsymbol{X}) & \cdots & \Omega^{2}_{\mathbf{A}^{1}_{N}}(\boldsymbol{X}) \end{bmatrix} \end{bmatrix} \\ \begin{bmatrix} \begin{bmatrix} \mu_{A^{2}_{1}}(x_1) & \mu_{A^{2}_{2}}(x_1) & \cdots & \mu_{A^{2}_{N}}(x_1) \\ \mu_{A^{2}_{1}}(x_2) & \mu_{A^{2}_{2}}(x_2) & \cdots & \mu_{A^{2}_{N}}(x_2) \\ \vdots & \vdots & \cdots & \vdots \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \mu_{A^{2}_{1}}(x_M) & \mu_{A^{2}_{2}}(x_M) & \cdots & \mu_{A^{2}_{N}}(x_M) \end{bmatrix} \\ \begin{bmatrix} \Omega^{2}_{\mathbf{A}^{2}_{1}}(\boldsymbol{X}) & \Omega^{2}_{\mathbf{A}^{2}_{2}}(\boldsymbol{X}) & \cdots & \Omega^{2}_{\mathbf{A}^{2}_{N}}(\boldsymbol{X}) \end{bmatrix} \end{bmatrix} \\ \vdots \\ \begin{bmatrix} \begin{bmatrix} \mu_{A^{L}_{1}}(x_1) & \mu_{A^{L}_{2}}(x_1) & \cdots & \mu_{A^{k}_{N}}(x_1) \\ \mu_{A^{L}_{1}}(x_2) & \mu_{A^{L}_{2}}(x_2) & \cdots & \mu_{A^{L}_{N}}(x_2) \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \mu_{A^{L}_{1}}(x_M) & \mu_{A^{L}_{2}}(x_M) & \cdots & \mu_{A^{L}_{N}}(x_M) \end{bmatrix} \\ \begin{bmatrix} \Omega^{2}_{\mathbf{A}^{L}_{1}}(\boldsymbol{X}) & \Omega^{2}_{\mathbf{A}^{L}_{2}}(\boldsymbol{X}) & \cdots & \Omega^{2}_{\mathbf{A}^{L}_{N}}(\boldsymbol{X}) \end{bmatrix} \end{bmatrix} \end{bmatrix} \tag{5.1}$$

The Hyper-Super-Soft Matrix consists of four clusters associated with four aggregation operators $t = 1, 2, 3, 4$. Every cluster has two matrix-layers associated with attribute-levels. Each matrix-layer has M rows and N columns.

These multiple clusters of parallel layers (parallel universes/realities/events/information) are accomplished by gradually varying the fourth index, which is used to represent multiple aggregation operators to accumulate the memberships of all subjects. These aggregation operators are called local operators.

5.2. (Plithogenic Fuzzy Subjective Attributive-Whole Hyper-Super-Soft-Matrix)

The Plithogenic Fuzzy Subjective Attributively-Whole Hyper-Super-Soft-Matrix is shown in (Equation 5.1a) and is described on a similar basis for further details see Ref. [20]

$$F_{\mathbf{S}_t} = \left[\left[\mu_{A_j^k}(x_i) \right] \left[\Omega^t_{\mathrm{A}^k}(x_{\mathrm{i}}) \right] \right] \tag{5.1a}$$

5.3. Application as COVID-19 Data Structure

Example 4. Let $U = \{x_1, x_2, x_3, x_4, x_5, x_6\}$ be the set of six patients who have visited the hospital with symptoms of COVID-19. They were examined by some doctors. Consider a medical exam case of a doctor who examined three of them by asking some questions about four symptoms (attributes), and each symptom is categorized into two sub-symptoms (sub-attributes). These three patients are considered to be test subjects. The information from their visits was recorded and organized as a plithogenic fuzzy hypersoft matrix and further represented and analyzed with the aid of the plithogenic fuzzy hyper super soft matrix.

Let $T = \{x_1, x_2, x_3\} \subset U$ be the set of these three patients considered by a doctor for examination.

Let the attributes be A_j^k; $j = 1, 2, 3, 4$ with sub-attributes k=1, 2 are described as,

A_1^k = Fever with numeric values, $k = 1, 2$,
A_1^1 = Mild fever, A_1^2 = High-grade fever
A_2^k = Dry cough, with numeric values, $k = 1, 2$
A_2^1 = Dry cough, A_2^2 = Acute dry cough
A_3^k = Breathing difficulty with numeric values, $k = 1, 2$
A_3^1 = Minor breathing difficulty,
A_3^2 = Swear breathing difficulty needs oxygen
A_4^k = Nausea, with numeric values $k = 1, 2$
A_4^1 = Minor nausea A_4^2 = Strong nausea

PFHS-Set representation:

Let the Function F indicate these given attributes/sub-attributes as described below,

$$F : A_1^k \times A_2^k \times A_3^k \times A_4^k \to P(U) \tag{5.1}$$

$$\text{S.t } F\left(A_1^1, A_2^1, A_3^1, A_4^1\right) = \{x_1, x_2, x_3\}$$

be a hypersoft set. let we consider $A_1^1, A_2^1, A_3^1, A_4^1$ a combination (α combination) represents the first level of disease on the first visit)

$F\left(A_1^2, A_2^2, A_3^2, A_4^2\right) = \{x_1, x_2, x_3\}$,
$A_1^2, A_2^2, A_3^2, A_4^2$ a combination (β combination) represents the second level of disease. The patient with the second level, if necessary, would be admitted to the hospital for special care after a week through observation and analysis of conditions. The Individual fuzzy memberships are assigned using a linguistic scale and then represented in PFHS-Set. Consider the fuzzy memberships of elements of T=$\{x_1, x_2, x_3\}$ as $\mu_{A_j^k}(x_i)$ for $i = 1, 2, 3$ and $j = 1$ to 4. The Plithogenic Fuzzy Memberships that reflect the health-states of x_i subjects associated with α combination of attributes are organized as PFHS-Set shown in Equation (5.2).

$$F(\alpha) = F\left(A_1^1, A_2^1, A_3^1, A_4^1\right) = \left\{ \begin{matrix} x_1\,(0.3, 0.6, 0.4, 0.5), \\ x_2\,(0.4, 0.5, 0.3, 0.1), \\ x_3\,(0.6, 0.3, 0.3, 0.7) \end{matrix} \right\} \tag{5.2}$$

The information from the first visit (organized as a PFHS-Set) (shown in Equation (5.2) would generate the first level of the PFHS-Matrix.

Now regarding the states of patients for the second visit for β combination of attributes. Information is portrayed as PFHS-Set that is given in Equation (5.3).

$$F(\beta) = F\left(A_1^1, A_2^1, A_3^1, A_4^1\right) = \left\{ \begin{matrix} x_1\,(0.5, 0.7, 0.5, 0.8), \\ x_2\,(0.7, 0.5, 0.5, 0.8), \\ x_3\,(0.6, 0.5, 0.6, 0.7) \end{matrix} \right\} \tag{5.3}$$

$F(\beta)$ is a Plithogenic fuzzy hypersoft set represents the second visit information as the second level of Matrix

PFHS-Matrix representation: Let F be the matrix form of PFHS-Set. Here the elements of rows represent states of subjects x_1, x_2, x_3 as fuzzy memberships. And elements of columns

represent (the non-Physical aspect of subjects) A_1^k, A_2^k, A_3^k, A_4^kAttributes.

The information of hypersoft set shown in Equations (5.2) and (5.3) are organized in the form of PFHS-Matrix F as follows,

$$F = \begin{bmatrix} \begin{bmatrix} 0.3 & 0.6 & 0.4 & 0.5 \\ 0.4 & 0.5 & 0.3 & 0.1 \\ 0.6 & 0.3 & 0.3 & 0.7 \end{bmatrix} \\ \begin{bmatrix} 0.5 & 0.7 & 0.5 & 0.8 \\ 0.7 & 0.5 & 0.5 & 0.8 \\ 0.6 & 0.5 & 0.6 & 0.7 \end{bmatrix} \end{bmatrix} \tag{5.4}$$

This PFHS matrix consists of two layers and represents an inner view of the universe

** Online mode **

$$\begin{bmatrix} 0.3 & 0.6 & 0.4 & 0.5 \\ 0.4 & 0.5 & 0.3 & 0.1 \\ 0.6 & 0.3 & 0.3 & 0.7 \end{bmatrix}, \begin{bmatrix} 0.5 & 0.7 & 0.5 & 0.8 \\ 0.7 & 0.5 & 0.5 & 0.8 \\ 0.6 & 0.5 & 0.6 & 0.7 \end{bmatrix} \tag{5.5}$$

Equation (5.5) represents the First and Second level layers of PFHS-Matrix.

Individual fuzzy memberships $\mu_{A_j^k}(x_i)$: The elements of This PFHS-Matrix represent individual fuzzy memberships s $\mu_{A_j^k}(x_i)$.

5.3.1. *Local Disjunction Operator for construction of PFASWHSS-Matrix*

By applying local Disjunction operators i.e. the Max-operator ($t = 1$) described in Equation 4.1 on columns of F, we will get Subject-wise whole Fuzzy Memberships $\Omega^1_{\mathbf{A}_j^k}(X)$. These Subject-wise whole Fuzzy Memberships are represented in the last row of PFASWHS-Matrix F_{A_1} as shown below in Equation 5.6

$$F_{\mathrm{A}_1} = \begin{bmatrix} \begin{bmatrix} \begin{bmatrix} 0.3 & 0.6 & 0.4 & 0.5 \\ 0.4 & 0.5 & 0.3 & 0.1 \\ 0.6 & 0.3 & 0.3 & 0.7 \end{bmatrix} \\ \begin{bmatrix} \mathbf{0.6} & \mathbf{0.6} & \mathbf{0.4} & \mathbf{0.7} \end{bmatrix} \end{bmatrix} \\ \begin{bmatrix} \begin{bmatrix} 0.5 & 0.7 & 0.5 & 0.8 \\ 0.7 & 0.5 & 0.5 & 0.8 \\ 0.6 & 0.5 & 0.6 & 0.7 \end{bmatrix} \\ \begin{bmatrix} \mathbf{0.7} & \mathbf{0.7} & \mathbf{0.6} & \mathbf{0.8} \end{bmatrix} \end{bmatrix} \end{bmatrix} \tag{5.6}$$

$\begin{bmatrix} 0.6 & 0.6 & 0.4 & 0.7 \end{bmatrix}$ and $\begin{bmatrix} 0.7 & 0.7 & 0.6 & 0.8 \end{bmatrix}$ are Subject-wise whole Fuzzy Memberships$\Omega_{A_j^k}(X)$ are represented for each attribute of the first and second level-layer of the given matrix and represent the optimist outer view of the universe through the last row of F_{A_1}.

$\begin{bmatrix} 0.4 & 0.4 & 0.6 & 0.3 \end{bmatrix}$ and $\begin{bmatrix} 0.3 & 0.3 & 0.4 & 0.2 \end{bmatrix}$ are Subject-wise whole fuzzy non-memberships $\Phi_{A_j^l}(X)$ for attributes of the first and second levels.

5.3.2. *Local Disjunction Operator for construction of PFSAWHSS-Matrix:*

When the aggregation operator, $t = 1$ (max-operator described in Equation 4.2) is applied across the rows of PFHS-Matrix, F (Equation 5.4) we will get attribute-wise whole fuzzy Memberships $\Omega^1_{A^k}(x_i)$ that are represented in the last column of PFSAWHS-Matrix F_{S_1} given bellow in Equation 5.7,

$$F_{\mathrm{S}_1} = \begin{bmatrix} \begin{bmatrix} 0.3 & 0.6 & 0.4 & 0.5 \\ 0.4 & 0.5 & 0.3 & 0.1 \\ 0.6 & 0.3 & 0.3 & 0.7 \end{bmatrix} \begin{bmatrix} \mathbf{0.6} \\ \mathbf{0.5} \\ \mathbf{0.7} \end{bmatrix} \\ \begin{bmatrix} 0.5 & 0.7 & 0.5 & 0.8 \\ 0.7 & 0.5 & 0.5 & 0.8 \\ 0.6 & 0.5 & 0.6 & 0.7 \end{bmatrix} \begin{bmatrix} \mathbf{0.8} \\ \mathbf{0.8} \\ \mathbf{0.7} \end{bmatrix} \end{bmatrix} \tag{5.7}$$

F_{S_1} is the Plithogenic Subjective Attributively Whole Hyper-Super Soft-Matrix.

$\begin{bmatrix} 0.6 \\ 0.5 \\ 0.7 \end{bmatrix}, \begin{bmatrix} 0.8 \\ 0.8 \\ 0.7 \end{bmatrix}$ are Attributively Whole Fuzzy Memberships constructed for combination attributes of first and second levels of PFSAWHSS-Matrix F. These Attributively Whole Fuzzy Memberships represent the Subjectively optimist exterior manifestation of the universe.

Attributively Whole Fuzzy Non-Memberships $\Phi_{A^l}(x_i) = (1 - \Omega_{A^l}(x_i))$ for attributes of first and second levels of PFSAWHSS-Matrix are $\begin{bmatrix} \mathbf{0.4} \\ \mathbf{0.5} \\ \mathbf{0.3} \end{bmatrix}, \begin{bmatrix} \mathbf{0.2} \\ \mathbf{0.2} \\ \mathbf{0.3} \end{bmatrix}$. These are attributively whole Fuzzy Non-Memberships for attributes of first and second level of PFSAWHSS-Matrix F_{S_1} (Equation 5.7).

5.3.3. Local Conjunction Operator for construction of PFASWHSS-Matrix

The PFASWHSS-Matrix constructed by applying the local conjunction operator described in Equation 4.3 column-wise ($t = 2$) on each specific layer of PFHS-Matrix F (equation 5.4). By applying this aggregation operator on columns of F, we will get Subjectively whole Fuzzy Memberships $\Omega^2_{\mathbf{A}_j^k}(X)$ that are represented in the last row of PFASWHS-Matrix F_{A_2} see Equation 5.8

$$F_{\mathrm{A}_2} = \begin{bmatrix} \begin{bmatrix} \begin{bmatrix} 0.3 & 0.6 & 0.4 & 0.5 \\ 0.4 & 0.5 & 0.3 & 0.1 \\ 0.6 & 0.3 & 0.3 & 0.7 \end{bmatrix} \\ \begin{bmatrix} \mathbf{0.3} & \mathbf{0.3} & \mathbf{0.3} & \mathbf{0.1} \end{bmatrix} \end{bmatrix} \\ \begin{bmatrix} \begin{bmatrix} 0.5 & 0.7 & 0.5 & 0.8 \\ 0.7 & 0.5 & 0.5 & 0.8 \\ 0.6 & 0.5 & 0.6 & 0.7 \end{bmatrix} \\ \begin{bmatrix} \mathbf{0.5} & \mathbf{0.5} & \mathbf{0.5} & 0.7 \end{bmatrix} \end{bmatrix} \end{bmatrix} \tag{5.8}$$

Where,

$$\begin{bmatrix} \begin{bmatrix} 0.3 & 0.6 & 0.4 & 0.5 \\ 0.4 & 0.5 & 0.3 & 0.1 \\ 0.6 & 0.3 & 0.3 & 0.7 \end{bmatrix} \\ \begin{bmatrix} 0.3 & 0.3 & 0.3 & 0.1 \end{bmatrix} \end{bmatrix}$$

and

$$\begin{bmatrix} \begin{bmatrix} 0.5 & 0.7 & 0.5 & 0.8 \\ 0.7 & 0.5 & 0.5 & 0.8 \\ 0.6 & 0.5 & 0.6 & 0.7 \end{bmatrix} \\ \begin{bmatrix} 0.5 & 0.5 & 0.5 & 0.7 \end{bmatrix} \end{bmatrix}$$

are two front and back vertical level-layers of type 1 of the given matrix F_{A_2} for $k = 1$ and $k = 2$ and in these layers the following matrices,

$$\begin{bmatrix} 0.3 & 0.6 & 0.3 & 0.5 \\ 0.4 & 0.5 & 0.2 & 0.1 \\ 0.6 & 0.2 & 0.3 & 0.7 \end{bmatrix}, \begin{bmatrix} 0.5 & 0.4 & 0.2 & 0.6 \\ 0.5 & 0.7 & 0.8 & 0.4 \\ 0.7 & 0.6 & 0.5 & 0.9 \end{bmatrix}$$

The two matrix-layers given above represent the interior view of the universe by individual memberships while the last rows $[0.3\ 0.3\ 0.3\ 0.1]$ and $[0.5\ 0.5\ 0.5\ 0.7]$ of each layer represent the pessimist attributive exterior perception of the universe. The elements of these given rows represent the Whole memberships $\Omega_{A_j^k}(X)$.

5.3.4. Local Conjunction Operator for construction of PFSAWHSS-Matrix

The PFSAWHS-Matrix is constructed using the local conjunction operator ($t = 2$) row-wise as described in Equation 4.4 on each specific layer of PFHS-Matrix F (Equation 5.4).

The attributively whole memberships $\Omega^2_{\mathrm{A}^k}(x_i)$, represented by the last column of F_{S_2} as shown below in Equation (5.9),

$$F_{\mathrm{S}_2} = \begin{bmatrix} \begin{bmatrix} 0.3 & 0.6 & 0.4 & 0.5 \\ 0.4 & 0.5 & 0.3 & 0.1 \\ 0.6 & 0.3 & 0.3 & 0.7 \end{bmatrix} & \begin{bmatrix} \mathbf{0.3} \\ \mathbf{0.1} \\ \mathbf{0.3} \end{bmatrix} \\ \begin{bmatrix} 0.5 & 0.7 & 0.5 & 0.8 \\ 0.7 & 0.5 & 0.5 & 0.8 \\ 0.6 & 0.5 & 0.6 & 0.7 \end{bmatrix} & \begin{bmatrix} \mathbf{0.5} \\ \mathbf{0.5} \\ \mathbf{0.5} \end{bmatrix} \end{bmatrix} \tag{5.9}$$

F_{S_2} is PFSAWHS-Matrix constructed by using the Local Conjunction Operator.

The elements of the last columns of the matrix are Attributively Whole Fuzzy Memberships for attributes of the first and second level of PFSAWHSS-Matrix F. Furthermore, these two columns are representing the pessimist Subjective exterior perception of the universe whaich is obtained by using the conjunction operator.

5.3.5. Local averaging Operator for construction of PFASWHSS-Matrix

The PFASWHSS-Matrix is constructed using local averaging operators ($t = 3$) (described in equation 4.5) for each specific layer of PFHS-Matrix F (Equation 5.4). This aggregation operator (Averaging-operator) is applied along the columns of PFHS-Matrix to get attributively whole Fuzzy Memberships $\Omega^3_{\mathbf{A}_j^k}(X)$. These Memberships are represented In the last row of the PFASWHSS-Matrix F_{A_3} as described below in Equation 5.10,

$$F_{\mathrm{A}_3} = \begin{bmatrix} \begin{bmatrix} \begin{bmatrix} 0.3 & 0.6 & 0.4 & 0.5 \\ 0.4 & 0.5 & 0.3 & 0.1 \\ 0.6 & 0.3 & 0.3 & 0.7 \end{bmatrix} \\ \begin{bmatrix} \mathbf{0.43} & \mathbf{0.46} & \mathbf{0.33} & \mathbf{0.43} \end{bmatrix} \end{bmatrix} \\ \begin{bmatrix} \begin{bmatrix} 0.5 & 0.7 & 0.5 & 0.8 \\ 0.7 & 0.5 & 0.5 & 0.8 \\ 0.6 & 0.5 & 0.6 & 0.7 \end{bmatrix} \\ \begin{bmatrix} \mathbf{0.6} & \mathbf{0.56} & \mathbf{0.53} & \mathbf{0.76} \end{bmatrix} \end{bmatrix} \end{bmatrix} \tag{5.10}$$

Where,

$$\left[\begin{bmatrix}0.3\ 0.6\ 0.4\ 0.5\\0.4\ 0.5\ 0.3\ 0.1\\0.6\ 0.3\ 0.3\ 0.7\end{bmatrix}\ \begin{bmatrix}\mathbf{0.43\ 0.46\ 0.33\ 0.43}\end{bmatrix}\right],$$

$$\left[\begin{bmatrix}0.5\ 0.7\ 0.5\ 0.8\\0.7\ 0.5\ 0.5\ 0.8\\0.6\ 0.5\ 0.6\ 0.7\end{bmatrix}\ \begin{bmatrix}\mathbf{0.6\ 0.56\ 0.53\ 0.76}\end{bmatrix}\right]$$

are two front and back vertical level-layers of $F_{\mathbf{A}_3}$ for $k=1$ and $k=2$ and in these layers the rows $\begin{bmatrix}\mathbf{0.43\ 0.46\ 0.33\ 0.43}\end{bmatrix}$ and $\begin{bmatrix}\mathbf{0.6\ 0.56\ 0.53\ 0.76}\end{bmatrix}$ are representing the neutral attributive exterior view of the universe through the averaging operator.

5.3.6. *Local averaging Operator for construction of PFSAWHSS-Matrix*

The PFSAWHSS-Matrix is constructed by applying local averaging operators described in Equation (4.6) ($t=3$) on each specific layer of PFHS-Matrix. The aggregation operator (Averaging-operator) is applied along the rows of PFHS-Matrix to get subjectively whole Memberships $\Omega^3_{\mathrm{A}^k}(x_1)$

$$F_{\mathrm{S}_3}=\left[\begin{bmatrix}0.3\ 0.6\ 0.4\ 0.5\\0.4\ 0.5\ 0.3\ 0.1\\0.6\ 0.3\ 0.3\ 0.7\end{bmatrix}\begin{bmatrix}\mathbf{0.45}\\\mathbf{0.32}\\\mathbf{0.47}\end{bmatrix}\ \begin{bmatrix}0.5\ 0.7\ 0.5\ 0.8\\0.7\ 0.5\ 0.5\ 0.8\\0.6\ 0.5\ 0.6\ 0.7\end{bmatrix}\begin{bmatrix}\mathbf{0.62}\\\mathbf{0.75}\\\mathbf{0.6}\end{bmatrix}\right] \tag{5.11}$$

F_{S_3} is Plithogenic Subjective attributively Whole Hyper-Super Soft-Matrix. And the elements of the given last columns of F_{S_3} are Attributively Whole Fuzzy Memberships for attributes of first and second levels of PFSAWHSS-Matrix. These Whole fuzzy memberships represent the neutral Subjective outer view of the universe through the averaging operator.

5.3.7. *Final PFASWHSS-Matrix*

The final PFASWHSS-Matrix that is described in Equation 5.1 is obtained by combining the F_{A_1} (Equation 5.6), F_{A_2} (Equation 5.8) and F_{A_2} (Equation 5.10) in a single PFASWHSS-Matrix which is consisted of three Clusters of Matrices associated to the aggregation operators $t=1$ (Max-operator) $t=2$ (Min-operator), and $t=3$ (*averaging*-operator). This Final PFASWHSS-Matrix is described bellow in Equation 5.12,

$$F_{A_t}=\left[\begin{bmatrix}\begin{bmatrix}0.3\ 0.6\ 0.4\ 0.5\\0.4\ 0.5\ 0.3\ 0.1\\0.6\ 0.3\ 0.3\ 0.7\end{bmatrix}\\\begin{bmatrix}\mathbf{0.6\ 0.6\ 0.4\ 0.7}\end{bmatrix}\end{bmatrix}\begin{bmatrix}\begin{bmatrix}0.5\ 0.7\ 0.5\ 0.8\\0.7\ 0.5\ 0.5\ 0.8\\0.6\ 0.5\ 0.6\ 0.7\end{bmatrix}\\\begin{bmatrix}0.7\ 0.7\ 0.6\ 0.8\end{bmatrix}\end{bmatrix}\right.$$
$$\left.\begin{bmatrix}\begin{bmatrix}0.3\ 0.6\ 0.4\ 0.5\\0.4\ 0.5\ 0.3\ 0.1\\0.6\ 0.3\ 0.3\ 0.7\end{bmatrix}\\\begin{bmatrix}\mathbf{0.3\ 0.3\ 0.3\ 0.1}\end{bmatrix}\end{bmatrix}\begin{bmatrix}\begin{bmatrix}0.5\ 0.7\ 0.5\ 0.8\\0.7\ 0.5\ 0.5\ 0.8\\0.6\ 0.5\ 0.6\ 0.7\end{bmatrix}\\\begin{bmatrix}\mathbf{0.5\ 0.5\ 0.5\ 0.7}\end{bmatrix}\end{bmatrix}\right.$$
$$\left.\begin{bmatrix}\begin{bmatrix}\begin{bmatrix}0.3\ 0.6\ 0.4\ 0.5\\0.4\ 0.5\ 0.3\ 0.1\\0.6\ 0.3\ 0.3\ 0.7\end{bmatrix}\\\begin{bmatrix}\mathbf{0.43\ 0.46\ 0.33\ 0.43}\end{bmatrix}\end{bmatrix}\begin{bmatrix}\begin{bmatrix}0.5\ 0.7\ 0.5\ 0.8\\0.7\ 0.5\ 0.5\ 0.8\\0.6\ 0.5\ 0.6\ 0.7\end{bmatrix}\\\begin{bmatrix}\mathbf{0.6\ 0.56\ 0.53\ 0.76}\end{bmatrix}\end{bmatrix}\end{bmatrix}\right] \tag{5.12}$$

5.3.8. *Final PFSAWHSS-Matrix*

The final PFSAWHSS-Matrix that is described in (Equation 5.1a) is constructed by combining the F_{s_1}(Equation 5.7), F_{s_2}(Equation 5.9) and F_{s_2}

(Equation 5.11) in a single PFSAWHSS-Matrix which is consisted of three Clusters of Matrices associated to the aggregation operators $t = 1$ (Max-operator) $t = 2$ (Min-operator), and $t = 3$ (*averaging*-operator). This Final PFSAWHSS-Matrix is described in Equation 5.13

$$F_{S_t} = \begin{bmatrix} \begin{bmatrix} \begin{bmatrix} 0.3\ 0.6\ 0.4\ 0.5 \\ 0.4\ 0.5\ 0.3\ 0.1 \\ 0.6\ 0.3\ 0.3\ 0.7 \end{bmatrix} & \begin{bmatrix} \mathbf{0.6} \\ \mathbf{0.5} \\ \mathbf{0.7} \end{bmatrix} \\ \begin{bmatrix} 0.5\ 0.7\ 0.5\ 0.8 \\ 0.7\ 0.5\ 0.5\ 0.8 \\ 0.6\ 0.5\ 0.6\ 0.7 \end{bmatrix} & \begin{bmatrix} \mathbf{0.8} \\ \mathbf{0.8} \\ \mathbf{0.7} \end{bmatrix} \end{bmatrix} \\ \begin{bmatrix} \begin{bmatrix} 0.3\ 0.6\ 0.4\ 0.5 \\ 0.4\ 0.5\ 0.3\ 0.1 \\ 0.6\ 0.3\ 0.3\ 0.7 \end{bmatrix} & \begin{bmatrix} \mathbf{0.3} \\ \mathbf{0.1} \\ \mathbf{0.3} \end{bmatrix} \\ \begin{bmatrix} 0.5\ 0.7\ 0.5\ 0.8 \\ 0.7\ 0.5\ 0.5\ 0.8 \\ 0.6\ 0.5\ 0.6\ 0.7 \end{bmatrix} & \begin{bmatrix} \mathbf{0.5} \\ \mathbf{0.5} \\ \mathbf{0.5} \end{bmatrix} \end{bmatrix} \\ \begin{bmatrix} \begin{bmatrix} 0.3\ 0.6\ 0.4\ 0.5 \\ 0.4\ 0.5\ 0.3\ 0.1 \\ 0.6\ 0.3\ 0.3\ 0.7 \end{bmatrix} & \begin{bmatrix} \mathbf{0.45} \\ \mathbf{0.32} \\ \mathbf{0.47} \end{bmatrix} \\ \begin{bmatrix} 0.5\ 0.7\ 0.5\ 0.8 \\ 0.7\ 0.5\ 0.5\ 0.8 \\ 0.6\ 0.5\ 0.6\ 0.7 \end{bmatrix} & \begin{bmatrix} \mathbf{0.62} \\ \mathbf{0.75} \\ \mathbf{0.6} \end{bmatrix} \end{bmatrix} \end{bmatrix} \tag{5.13}$$

5.3.9. The Global and Local Complement of PFSAWHSS-Matrix

The Global complement PFASWHSS-Matrix is constructed by applying Complement-operator ($t = 4$, described in Equation 4.7) on each specific layer of PFHS-Matrix $F_{\mathbf{A}_t}$ (Equation 5.12).

$$\mathbf{F}_{\mathbf{A}_{4_t}} = \begin{bmatrix} \begin{bmatrix} \begin{bmatrix} \begin{bmatrix} 0.7\ 0.4\ 0.6\ 0.5 \\ 0.6\ 0.5\ 0.7\ 0.9 \\ 0.4\ 0.7\ 0.7\ 0.3 \end{bmatrix} \\ \begin{bmatrix} \mathbf{0.7\ 0.7\ 0.7\ 0.9} \end{bmatrix} \end{bmatrix} \\ \begin{bmatrix} \begin{bmatrix} 0.5\ 0.3\ 0.5\ 0.2 \\ 0.3\ 0.5\ 0.5\ 0.2 \\ 0.4\ 0.5\ 0.4\ 0.3 \end{bmatrix} \\ \begin{bmatrix} \mathbf{0.5\ 0.5\ 0.5\ 0.3} \end{bmatrix} \end{bmatrix} \end{bmatrix} \\ \begin{bmatrix} \begin{bmatrix} \begin{bmatrix} 0.7\ 0.4\ 0.6\ 0.5 \\ 0.6\ 0.5\ 0.7\ 0.9 \\ 0.4\ 0.7\ 0.7\ 0.3 \end{bmatrix} \\ \begin{bmatrix} \mathbf{0.4\ 0.4\ 0.6\ 0.3} \end{bmatrix} \end{bmatrix} \\ \begin{bmatrix} \begin{bmatrix} 0.5\ 0.3\ 0.5\ 0.2 \\ 0.3\ 0.5\ 0.5\ 0.2 \\ 0.4\ 0.5\ 0.4\ 0.3 \end{bmatrix} \\ \begin{bmatrix} \mathbf{0.3\ 0.3\ 0.4\ 0.2} \end{bmatrix} \end{bmatrix} \end{bmatrix} \\ \begin{bmatrix} \begin{bmatrix} \begin{bmatrix} 0.7\ 0.4\ 0.6\ 0.5 \\ 0.6\ 0.5\ 0.7\ 0.9 \\ 0.4\ 0.7\ 0.7\ 0.3 \end{bmatrix} \\ \begin{bmatrix} \mathbf{0.4\ 0.4\ 0.6\ 0.3} \end{bmatrix} \end{bmatrix} \\ \begin{bmatrix} \begin{bmatrix} 0.5\ 0.3\ 0.5\ 0.2 \\ 0.3\ 0.5\ 0.5\ 0.2 \\ 0.4\ 0.5\ 0.4\ 0.3 \end{bmatrix} \\ \begin{bmatrix} \mathbf{0.3\ 0.3\ 0.4\ 0.2} \end{bmatrix} \end{bmatrix} \end{bmatrix} \end{bmatrix} \tag{5.14}$$

The Local Attributive Subjectively Whole Complement Matrices are achieved by applying three specific aggregation operators, $t = 1$ (Max-operator) $t = 2$ (Min-operator), and $t = 3$ (*averaging*-operator).

$$F_{A_{4_1}} = \begin{bmatrix} \begin{bmatrix} \begin{bmatrix} 0.7\ 0.4\ 0.6\ 0.5 \\ 0.6\ 0.5\ 0.7\ 0.9 \\ 0.4\ 0.7\ 0.7\ 0.3 \end{bmatrix} \\ \begin{bmatrix} \mathbf{0.7\ 0.7\ 0.7\ 0.9} \end{bmatrix} \end{bmatrix} \\ \begin{bmatrix} \begin{bmatrix} 0.5\ 0.3\ 0.5\ 0.2 \\ 0.3\ 0.5\ 0.5\ 0.2 \\ 0.4\ 0.5\ 0.4\ 0.3 \end{bmatrix} \\ \begin{bmatrix} \mathbf{0.5\ 0.5\ 0.5\ 0.3} \end{bmatrix} \end{bmatrix} \end{bmatrix} \tag{5.14a}$$

$$F_{A_{4_2}} = \begin{bmatrix} \begin{bmatrix} \begin{bmatrix} 0.7 & 0.4 & 0.6 & 0.5 \\ 0.6 & 0.5 & 0.7 & 0.9 \\ 0.4 & 0.7 & 0.7 & 0.3 \end{bmatrix} \\ \begin{bmatrix} \mathbf{0.4} & \mathbf{0.4} & \mathbf{0.6} & \mathbf{0.3} \end{bmatrix} \end{bmatrix} \\ \begin{bmatrix} \begin{bmatrix} 0.5 & 0.3 & 0.5 & 0.2 \\ 0.3 & 0.5 & 0.5 & 0.2 \\ 0.4 & 0.5 & 0.4 & 0.3 \end{bmatrix} \\ \begin{bmatrix} \mathbf{0.3} & \mathbf{0.3} & \mathbf{0.4} & \mathbf{0.2} \end{bmatrix} \end{bmatrix} \end{bmatrix} \quad (5.14b)$$

$$F_{A_{4_3}} = \begin{bmatrix} \begin{bmatrix} \begin{bmatrix} 0.7 & 0.4 & 0.6 & 0.5 \\ 0.6 & 0.5 & 0.7 & 0.9 \\ 0.4 & 0.7 & 0.7 & 0.3 \end{bmatrix} \\ \begin{bmatrix} \mathbf{0.56} & \mathbf{0.53} & \mathbf{0.67} & \mathbf{0.56} \end{bmatrix} \end{bmatrix} \\ \begin{bmatrix} \begin{bmatrix} 0.5 & 0.3 & 0.5 & 0.2 \\ 0.3 & 0.5 & 0.5 & 0.2 \\ 0.4 & 0.5 & 0.4 & 0.3 \end{bmatrix} \\ \begin{bmatrix} \mathbf{0.4} & \mathbf{0.43} & \mathbf{0.46} & \mathbf{0.23} \end{bmatrix} \end{bmatrix} \end{bmatrix} \quad (5.14c)$$

in $F_{A_{4_1}}$ index $t = 4$ represents the complement operator of F, and its subscript 1,2,3 represents the subjective accumulation of complements of fuzzy memberships through the following given three operators. Here the Attributive Complement Hyper-Super-Soft Matrix $F_{A_{4_t}}$ represented by equation 5.14 is a cluster of HSS-matrices $F_{A_{4_1}}$, $F_{A_{4_2}}$, $F_{A_{4_3}}$ represented by Equation 5.14a, Equation 5.14b, Equation 5.14c respectively. these are the three attributive subjectively whole complement matrix-layers of this cluster. These matrices are constructed using the complement operator described in Equation 4.7. The last row of each matrix layer represents the whole complement fuzzy memberships.

5.3.10. The global and local complement of PFSAWHSS-Matrix

The Global Complement PFSAWHSS-Matrix is constructed by applying Complement-operator as described in Equation. 4.8 ($t = 4$) on each specific layer of PFHS-Matrix $F_{\mathbf{S}_t}$ (Equation 5.13) and represented as under,

$$F_{S_{4_t}} = \begin{bmatrix} \begin{bmatrix} \begin{bmatrix} 0.7 & 0.4 & 0.6 & 0.5 \\ 0.6 & 0.5 & 0.7 & 0.9 \\ 0.4 & 0.7 & 0.7 & 0.3 \end{bmatrix} & \begin{bmatrix} 0.7 \\ 0.9 \\ 0.7 \end{bmatrix} \\ \begin{bmatrix} 0.5 & 0.3 & 0.5 & 0.2 \\ 0.3 & 0.5 & 0.5 & 0.2 \\ 0.4 & 0.5 & 0.4 & 0.3 \end{bmatrix} & \begin{bmatrix} 0.5 \\ 0.5 \\ 0.5 \end{bmatrix} \end{bmatrix} \\ \begin{bmatrix} \begin{bmatrix} 0.7 & 0.4 & 0.6 & 0.5 \\ 0.6 & 0.5 & 0.7 & 0.9 \\ 0.4 & 0.7 & 0.7 & 0.3 \end{bmatrix} & \begin{bmatrix} 0.4 \\ 0.5 \\ 0.3 \end{bmatrix} \\ \begin{bmatrix} 0.5 & 0.3 & 0.5 & 0.2 \\ 0.3 & 0.5 & 0.5 & 0.2 \\ 0.4 & 0.5 & 0.4 & 0.3 \end{bmatrix} & \begin{bmatrix} 0.2 \\ 0.2 \\ 0.3 \end{bmatrix} \end{bmatrix} \\ \begin{bmatrix} \begin{bmatrix} 0.7 & 0.4 & 0.6 & 0.5 \\ 0.6 & 0.5 & 0.7 & 0.9 \\ 0.4 & 0.7 & 0.7 & 0.3 \end{bmatrix} & \begin{bmatrix} 0.55 \\ 0.67 \\ 0.52 \end{bmatrix} \\ \begin{bmatrix} 0.5 & 0.3 & 0.5 & 0.2 \\ 0.3 & 0.5 & 0.5 & 0.2 \\ 0.4 & 0.5 & 0.4 & 0.3 \end{bmatrix} & \begin{bmatrix} 0.62 \\ 0.37 \\ 0.4 \end{bmatrix} \end{bmatrix} \end{bmatrix} \quad (5.15)$$

The Local Subjective Attributively-Whole Complement Matrices are achieved by applying three specific aggregation operators, $t = 1$ (Max-operator) $t = 2$ (Min-operator), and $t = 3$ (*averaging*-operator). Are described as under

$$F_{S_{4_1}} = \begin{bmatrix} \begin{bmatrix} 0.7 & 0.4 & 0.6 & 0.5 \\ 0.6 & 0.5 & 0.7 & 0.9 \\ 0.4 & 0.7 & 0.7 & 0.3 \end{bmatrix} & \begin{bmatrix} 0.7 \\ 0.9 \\ 0.7 \end{bmatrix} \\ \begin{bmatrix} 0.5 & 0.3 & 0.5 & 0.2 \\ 0.3 & 0.5 & 0.5 & 0.2 \\ 0.4 & 0.5 & 0.4 & 0.3 \end{bmatrix} & \begin{bmatrix} 0.5 \\ 0.5 \\ 0.5 \end{bmatrix} \end{bmatrix} \quad (5.15a)$$

$$F_{S_{4_2}} = \begin{bmatrix} \begin{bmatrix} 0.7 & 0.4 & 0.6 & 0.5 \\ 0.6 & 0.5 & 0.7 & 0.9 \\ 0.4 & 0.7 & 0.7 & 0.3 \end{bmatrix} & \begin{bmatrix} 0.4 \\ 0.5 \\ 0.3 \end{bmatrix} \\ \begin{bmatrix} 0.5 & 0.3 & 0.5 & 0.2 \\ 0.3 & 0.5 & 0.5 & 0.2 \\ 0.4 & 0.5 & 0.4 & 0.3 \end{bmatrix} & \begin{bmatrix} 0.2 \\ 0.2 \\ 0.3 \end{bmatrix} \end{bmatrix} \quad (5.15b)$$

$$F_{S_{4_3}} = \begin{bmatrix} \begin{bmatrix} 0.7 & 0.4 & 0.6 & 0.5 \\ 0.6 & 0.5 & 0.7 & 0.9 \\ 0.4 & 0.7 & 0.7 & 0.3 \end{bmatrix} & \begin{bmatrix} 0.55 \\ 0.67 \\ 0.52 \end{bmatrix} \\ \begin{bmatrix} 0.5 & 0.3 & 0.5 & 0.2 \\ 0.3 & 0.5 & 0.5 & 0.2 \\ 0.4 & 0.5 & 0.4 & 0.3 \end{bmatrix} & \begin{bmatrix} 0.62 \\ 0.37 \\ 0.4 \end{bmatrix} \end{bmatrix} \quad (5.15c)$$

6. Conclusions and discussions

1. The classical matrices would connect several equations and variables by rows and columns which is a limited approach to organize the higher-dimensional data consists of scattered information in numerous forms and vagueness levels therefore to broaden the approach of organizing the higher-dimensional data this unique Model of hyper-super-soft matrix is constructed. The set operations can be performed at each position (i, j) with a localized computation that is applied within the Matrix layer. This is a huge constructed matrix that first connects rows and columns like the classical matrix and then connects the sets of rows and columns as matrix layers. At this stage, it is called the hyper matrix. In each hyper-matrix layer, more matrices would be connected known as super-matrix and these super-matrices would be further connected as hyper-super-matrix, as described in the presented model. Equations 5.12, 5.13, 5.14, and 5.15 present examples of this Connected Hyper-Super-Matrix.
2. In this article, we developed the Max, Min, Averaging, and Complement operators for the PFASWHSS-Matrix and PFSAWHSS-Matrix. Applying the aggregation operator to a particular layer of the PFHS-Matrix would compress the entire layer of rows and columns into a single column (case of constructing PFSAWHSS matrix) or a single row (in constructing PFASWHSS matrix)
3. The final PFASWHSS-Matrix and the PFSAWHSS-Matrix are designed to organize the COVID-19 data structure and are presented in Equation 5.12. or 5.13. The matrix is made up of three clusters associated with three aggregation operators, and each cluster has two layers, with each layer representing the information from each visit.
4. The final PFSAWHSS-Matrix created for the COVID-19 data structure is shown in Equation 5.12 and would be utilized to classify the non-physical attributes (symptoms of COVID-19). It is observed from subjectively whole fuzzy memberships, which are shown in PFASWHSS-Matrix $F_{\mathbf{A}_1}$, the sub-attributes A_4^1, A_4^2 which represent minor nausea and acute nausea, respectively, being the dominant attributes on the first and second visit levels
5. The final PFSAWHSS-Matrix that is constructed for the COVID-19 data Structure is shown in Equation 5.13 and would be utilized to classify the physical subjects (patients of COVID-19)
   The patient whose membership of the cumulative attributes as fuzzy membership is 80% or more than 80% should be hospitalized for intensive care. Therefore, subjects x_1, x_2 would be admitted on their second visit as their attributively whole Fuzzy memberships are 80%.
6. It is observed that the Global Complement PFASWHSS-Matrix $\mathbf{F}_{A_{4_t}}$ described by Equation 5.14 is a higher-dimensional matrix, consisting of three clusters of matrices each cluster being described in Equation 5.14a, Equation 5.14b. equation 5.14c i.e. represents a local complement. While each cluster described has two matrix layers and each layer has two sub-matrices, the first sub-matrix of the layer represents a detailed attributive view of the local complement. And the last row-matrix of the matrix-layer reflects the compact view of the attributive complement. The Global Complement of PFSAWHSS-Matrix $\mathbf{F}_{S_{4_t}}$ is described by Equation 5.14 and its Clusters are given in (Equation 5.15a, Equation 5.15b. Equation 5.15c)
7. This modern hyper-super-matrix has shown the individual and cumulative effect of universe/event/reality/information through several angles of visions in a fuzzy environment. These angles of vision are described through the use of Max, Min, Averaging, and complement operators. These operators represent optimistic, pessimistic, neutral, and antithetical behavior of the universe in the fuzzy environment.

6.1. Future open problems

1. In this article, we have developed the Max, Min, Averaging, and Complement operators

for PFASW HSS-Matrix and PFSAWHSS-Matrix which represent a Hyper-Neutrosophic approach of considering Optimist, Pessimist, Neutral and Antithetical behavior of the Universe/Event/Reality/Information. In addition, some other local operators can be developed to express a broader approach.

2. The choice of a suitable environment like Crisp, Fuzzy, Intuitionistic, or Neutrosophic would reflect the state of mind of the observer. Moreover, this Model of expression (Hyper-Super-Matrix) has the capability of the adoption of any suitable environment to reflect the worth style nature with its physical and non-physical aspects.
3. The aggregation operators are applied row-wise to compact a certain Matrix-layer into a single column and applied column-wise to compact a certain layer of the Matrix into a single row. This procedure of development and application of aggregation operators was carried out by considering only the front view of the dilated Matrix i.e. Type-1 level-layers of PFHS-Matrix described In equation 3.14. This development and Application would be extended to *Type*-2 *level layers of PFHS-Matrix* described in equation 3.17. This unique construction model is presented, as an example in the Fuzzy environments by considering the front view of the dilated Matrix i.e *Type*-1 *level-layers of PFHS-Matrix* described In Equation 3.14 to preserve the duration of the article within the required limits of this general. later, the proposed unique built examples would be constructed in various suitable environments taking into account other described views of the matrix.

By considering *Type-3 level layers of PFHS-Matrix* described in Equation 3.18. and using the proposed extensions Extended Attributive and Subjective Models would be constructed.

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On Clean and Nil-clean Symbolic 2-Plithogenic Rings

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Abstract. A ring is said to be clean if every element of the ring can be written as a sum of an idempotent element and a unit element of the ring and a ring is said to be nil-clean if every element of the ring can be written as a sum of an idempotent element and a nilpotent element of the ring. In this paper, we generalize these arguments to symbolic 2-plithogenic structure. We introduce the structure of clean and nil-clean symbolic 2-plithogenic rings and some of its elementary properties are presented. Also, we have found the equivalence between classical clean(nil-clean) ring R and the corresponding symbolic 2-plithogenic ring $2 - SP_R$.

Keywords: Clean ring; nil-clean ring; symbolic 2-plithogenic ring; clean symbolic 2-plithogenic ring; nil-clean symbolic 2-plithogenic ring.

1. Introduction

The concept of refined neutrosophic structure was studied by many authors in [1–5]. Symbolic plithogenic algebraic structures are introduced by Smarandache, that are very similar to refined neutrosophic structures with some differences in the definition of the multiplication operation [17].

In [14], the algebraic properties of symbolic 2-plithogenic rings generated from the fusion of symbolic plithogenic sets with algebraic rings, and some of the elementary properties and substructures of symbolic 2-plithogenic rings such as AH-ideals, AH-homomorphisms, and AHS-isomorphisms are studied. In [7], some more algebraic properties of symbolic 2-plithogenic

rings are studied. Further, Taffach [15, 16] studied the concepts of symbolic 2-plithogenic vector spaces and modules.

In [8], the concept of symbolic 2-plithogenic matrices with symbolic 2-plithogenic entries, determinants, eigen values and vectors, exponents, and diagonalization are studied. Hamiyet Merkepci et.al [12], studied the the symbolic 2-plithogenic number theory and integers. Ahmad Khaldi et.al [11], studied the different types of algebraic symbolic 2-plithogenic equations and its solutions.

In [18], H. Suryoto and T. Uidjiani studied the concept of neutrosophic clean ring with many elementary interesting properties. Recently, M. Abobala [6], proved that a neutrosophic ring $R(I)$ is clean if and only if R is clean. Motivated by this works, in this paper we have introduced and studied the notion of clean and nil-clean symbolic 2-plithogenic rings. Also, we proved that a symbolic 2-plithogenic $2-SP_R$ is clean(nil-clean) if and only if R is clean(nil-clean).

2. **Preliminaries**

Definition 2.1. [14] Let R be a ring, the symbolic 2-plithogenic ring is defined as follows:

$$2-SP_R=\left\{a_0+a_1P_1+a_2P_2; a_i\in R, P_j^2=P_j, P_1\times P_2=P_{max(1,2)}=P_2\right\}$$

Smarandache has defined algebraic operations on $2-SP_R$ as follows:

Addition:

$[a_0+a_1P_1+a_2P_2]+[b_0+b_1P_1+b_2P_2]=(a_0+b_0)+(a_1+b_1)P_1+(a_2+b_2)P_2$

Multiplication:

$[a_0+a_1P_1+a_2P_2].[b_0+b_1P_1+b_2P_2]=a_0b_0+a_0b_1P_1+a_0b_2P_2+a_1b_0P_1^2+a_1b_2P_1P_2+a_2b_0P_2+a_2b_1P_1P_2+a_2b_2P_2^2+a_1b_1P_1P_1=(a_0b_0)+(a_0b_1+a_1b_0+a_1b_1)P_1+(a_0b_2+a_1b_2+a_2b_0+a_2b_1+a_2b_2)P_2$.

It is clear that $2-SP_R$ is a ring. If R is a field, then $2-SP_R$ is called a symbolic 2-plithogenic field. Also, if R is commutative, then $2-SP_R$ is commutative, and if R has a unity (1), than $2-SP_R$ has the same unity (1).

Example 2.2. [14] Consider the ring $R=Z_4=\{0,1,2,3,4\}$, the corresponding $2-SP_R$ is:

$$2-SP_R=\{a+bP_1+cP_2; a,b,c\in Z_4\}.$$

If $X=1+2P_1+3P_2, Y=P_1+2P_2$; then, $X+Y=1+3P_1+P_2$, $X-Y=1+P_1+P_2$, $X.Y=3P_1+3P_2$.

Theorem 2.3. *[14] Let $2-SP_R$ be a 2-plithogenic symbolic ring, with unity (1). Let $X=x_0+x_1P_1+x_2P_2$ be an arbitrary element, then:*

(1) *X is invertible if and only if $x_0, x_0+x_1, x_0+x_1+x_2$ are invertible.*

(2) $X^{-1} = x_0^{-1} + [(x_0 + x_1)^{-1} - x_0^{-1}]P_1 + [(x_0 + x_1 + x_2)^{-1} - (x_0 + x_1)^{-1}]P_2$

Definition 2.4. [14] Let $X = a + bP_1 + cP_2 \in 2 - SP_R$, then X is idempotent if and only if $X^2 = X$.

Theorem 2.5. *[14] Let $X = a + bP_1 + cP_2 \in 2 - SP_R$, then X is idempotent if and only if $a, a + b, a + b + c$ are idempotent.*

Theorem 2.6. *[14] Let $2 - SP_R$ be a commutative symbolic 2-plithogenic ring, hence if $X = a + bP_1 + cP_2$, then $X^n = a^n + [(a + b)^n - a^n]P_1 + [(a + b + c)^n - (a + b)^n]P_2$ for every $n \in Z^+$.*

Definition 2.7. [14] X is called nilpotent if there exists $n \in Z^+$ such that $X^n = 0$.

Theorem 2.8. *[14] Let $X = a + bP_1 + cP_2 \in 2 - SP_R$, where R is commutative ring, then X is nilpotent if and only if $a, a + b, a + b + c$ are nilpotent.*

3. Clean Symbolic 2-Plithogenic Rings

We begin with the following definition.

Definition 3.1. Let R be any ring, $2 - SP_R$ be its corresponding symbolic 2-plithogenic ring. An element $x \in 2 - SP_R$ is said to be clean if $x = e + u$, where e is an idempotent and u is a unit element of $2 - SP_R$. If, in addition, the existing idempotent e and the unit u are unique, then x is called uniquely clean element.

In this section, we use the notation $U(2 - SP_R)$ to the set of all units in $2 - SP_R$ and $Id(2 - SR_R)$ to the set of all idempotent elements in $2 - SP_R$.

Example 3.2. Consider the symbolic 2-plithogenic ring

$$\begin{aligned} 2 - SP_{Z_2} &= \{a + bP_1 + cP_2; a, b, c \in Z_2\} \\ &= \left\{0, 1, P_1, P_2, P_1 + P_2, 1 + P_1, 1 + P_2, 1 + P_1 + P_2\right\}. \end{aligned}$$

Here, $U(2 - SP_{Z_2}) = 1$ and $Id(2 - SR_{Z_2}) = \{0, 1, P_1, P_2, P_1 + P_2, 1 + P_1, 1 + P_2, 1 + P_1 + P_2\}$. We can easily verify that every element of $2 - SP_{Z_2}$ can be expressed as a sum of an idempotent and a unit in $2 - SP_{Z_2}$. Hence, all the elements in $2 - SP_{Z_2}$ are clean elements. Since 1 is the only unit element in $2 - SP_{Z_2}$, so all the elements in $2 - SP_{Z_2}$ are uniquely clean elements.

Definition 3.3. A symbolic 2-plithogenic ring in which all elements are clean, then the ring is called a clean symbolic 2-plithogenic ring. Furthermore, if each element of the symbolic 2-plithogenic ring is uniquely clean, then the ring is called a uniquely clean symbolic 2-plithogenic ring.

Example 3.4. By the Example 3.2, the ring $2-SP_{Z_2}$ is a uniquely clean symbolic 2-plithogenic ring.

Example 3.5. Consider the symbolic 2-plithogenic ring

$$\begin{aligned} 2-SP_{Z_3} &= \{a+bP_1+cP_2; a,b,c \in Z_3\} \\ &= \left\{ \begin{array}{c} 0,1,2,P_1,P_2,2P_1,2P_2,P_1+P_2,2P_1+2P_2,P_1+2P_2,2P_1+P_2,1+P_1, \\ 1+P_2,1+P_1+P_2,1+2P_1,1+2P_2,1+2P_1+2P_2,1+P_1+2P_2, \\ 1+2P_1+P_2,2+P_1,2+P_2,2+P_1+P_2,2+2P_1,2+2P_2,2+2P_1+2P_2, \\ 2+P_1+2P_2,2+2P_1+P_2 \end{array} \right\}. \end{aligned}$$

Here, $U(2-SP_{Z_3}) = \{1,2,1+P_1,1+P_2,2+2P_1,2+2P_2,1+P_1+2P_2,2+2P_1+P_2\}$ and $Id(2-SR_{Z_3}) = \{0,1,P_1,P_2,P_1+2P_2,1+2P_1,1+2P_2,1+2P_1+P_2\}$. All the elements of $2-SP_{Z_3}$ are clean elements. Hence $2-SP_{Z_3}$ is a clean symbolic 2-plithogenic ring. Take $2+2P_1+P_2 \in 2-SP_{Z_3}$, clearly $2+2P_1+P_2 = (1+2P_1+P_2)+1$ and also we have $2+2P_1+P_2 = 0+(2+2P_1+P_2)$. Therefore $2+2P_1+P_2$ is not a uniquely clean element in $2-SP_{Z_3}$ and hence $2-SP_{Z_3}$ is not a uniquely clean.

Lemma 3.6. *Let R be a ring. Then the class of clean symbolic 2-plithogenic rings is closed under homomorphic images.*

Proof. It is clear since the homomorphic image of an idempotent element in a symbolic 2-plithogenic ring is again an idempotent. □

Theorem 3.7. *Let R be any ring, $2-SP_R$ be its corresponding symbolic 2-plithogenic ring. $2-SP_R$ is clean if and only if R is clean.*

Proof. Assume that $2-SP_R$ is clean. Since R is a homomorphic image of $2-SP_R$, so R is clean by Lemma 3.6.

Conversely, assume that R is clean, we must prove that $2-SP_R$ is clean. Let $x = a+bP_1+cP_2 \in 2-SP_R$ then $a, a+b, a+b+c \in R$. Since R is clean we have $a = e_1+u_1, a+b = e_2+u_2, a+b+c = e_3+u_3$, where e_i are idempotent elements and u_i are unit elements of R. Now,

$$\begin{aligned} x &= a+bP_1+cP_2 \\ &= a+[(a+b)-a]P_1+[(a+b+c)-(a+b)]P_2 \\ &= (e_1+u_1)+[(e_2+u_2)-(e_1+u_1)]P_1+[(e_3+u_3)-(e_2+u_2)]P_2 \\ &= (e_1+u_1)+[(e_2-e_1)+(u_2-u_1)]P_1+[(e_3-e_2)+(u_3-u_2)]P_2 \\ &= [e_1+(e_2-e_1)P_1+(e_3-e_2)P_2]+[u_1+(u_2-u_1)P_1+(u_3-u_2)P_2] \\ &= x_1+x_2. \end{aligned}$$

where, $x_1 = e_1+(e_2-e_1)P_1+(e_3-e_2)P_2$ and $x_2 = u_1+(u_2-u_1)P_1+(u_3-u_2)P_2$. By Theorem 2.5, $e_1, e_1+(e_2-e_1) = e_2, e_1+(e_2-e_1)+(e_3-e_2) = e_3$ are idempotents in R. Therefore, x_1 is a idempotent element of R. Also, x_2 is a unit element of R by a similar discussion. Hence $2-SP_R$ is clean. □

Definition 3.8. Let $2-SP_R$ be a symbolic 2-plithogenic ring. An idempotent element $e \in 2-SP_R$ is called a central idempotent if $e.x = x.e$ for every $x \in 2-SP_R$. The set of all central idempotents of $2-SP_R$ is denoted by $C(2-SP_R)$.

Example 3.9. In the symbolic 2-plithogenic ring $2-SP_{Z_3}$, we have

$$Id(2-SP_{Z_3}) = \{0, 1, P_1, P_2, P_1+2P_2, 1+2P_1, 1+2P_2, 1+2P_1+P_2\}.$$

As $2-SP_{Z_3}$ is commutative so all the idempotents of $2-SP_{Z_3}$ are central. Hence

$$C(2-SP_{Z_3}) = \{0, 1, P_1, P_2, P_1+2P_2, 1+2P_1, 1+2P_2, 1+2P_1+P_2\} = Id(2-SP_{Z_3}).$$

Lemma 3.10. *If x is an idempotent element of $2-SP_R$, then $1-x$ is also an idempotent element of $2-SP_R$, where 1 is the unit element of $2-SP_R$.*

Proof. If x an idempotent element of $2-SP_R$ then $x^2 = x$. But then, $(1-x)^2 = 1-2x-x^2 = 1-x$ and so $1-x$ an idempotent element of $2-SP_R$. □

Lemma 3.11. *Let $2-SP_R$ be a symbolic 2-plithogenic ring with the identity 1. If $e \in C(2-SP_R)$ then $1-e \in C(2-SP_R)$, where 1 is the unit element of $2-SP_R$.*

Proof. Assume that $e \in C(2-SP_R)$. For any $x \in 2-SP_R$, we have $(1-e).x = (1.x)-(e.x) = (x.1)-(x.e) = x(1-e)$. Hence, $1-e \in C(2-SP_R)$. □

Theorem 3.12. *In any symbolic 2-plithogenic ring $2-SP_R$, every central idempotent is a uniquely clean element.*

Proof. Let $x \in C(2-SP_R)$. Then we have, $x^2 = x$ and $x = (1-x)+(2x-1) = e+u$, where $e = 1-x$ is an idempotent by Lemma 3.10 and $u = 2x-1$ is a unit element by Lemma 3.11. Hence x is a clean element. Also, if $x.y = y.x$ we obtain $e+u = (e+u)^2 = e+2eu+u^2$, so $u = 1-2e$. Hence $e = 1-x$. Thus x is a uniquely clean element. □

Theorem 3.13. *Every idempotent element in a uniquely clean 2-plithogenic ring is a central idempotent.*

Proof. Assume that $2 - SP_R$ is a uniquely clean 2-plithogenic ring. Let $e \in 2 - SP_R$ be an idempotent element and x be any element of $2 - SP_R$. Now, the element $e + (ex - exe)$ is an idempotent and $1 + (ex - exe)$ is a unit and $[e + (ex - exe)] + 1 = e + [1 + (ex - exe)]$. Since, $2 - SP_R$ is a uniquely clean 2-plithogenic ring we have $e + (ex - exe) = e$. Hence $ex = exe$ and $xe = exe$, so $ex = ex$ as required. □

Definition 3.14. A symbolic 2-plithogenic ring $2 - SP_R$ is called a boolean symbolic 2-plithogenic ring if $x^2 = x$ for all $x \in 2 - SP_R$.

Example 3.15. In the symbolic 2-plithogenic ring $2 - SR_{Z_2}$, all the elements are idempotent so $2 - SR_{Z_2}$ is a boolean symbolic 2-plithogenic ring

For any boolean symbolic 2-plithogenic ring, we have the following result.

Theorem 3.16. *Every boolean symbolic 2-plithogenic ring is uniquely clean.*

Proof. If $2 - SP_R$ is a boolean symbolic 2-plithogenic ring, then $2 - SP_R = Id(2 - SP_R)$. Since boolean rings are abelian, we have $Id(2 - SP_R) = C(2 - SP_R)$. This implies that, $2 - SP_R = C(2 - SP_R)$. By Theorem 3.12, every element of the ring $2 - SP_R$ are uniquely clean. Hence $2 - SP_R$ is uniquely clean ring. □

4. Nil-clean Symbolic 2-Plithogenic Rings

We begin with the following definition.

Definition 4.1. Let R be any ring, $2 - SP_R$ be its corresponding symbolic 2-plithogenic ring. An element $x \in 2 - SP_R$ is said to be nil-clean if $x = e + n$, where e is an idempotent and n is a nil-potent element of $2 - SP_R$. If, in addition, the existing idempotent element and nil-potent elements are unique, then x is called uniquely nil-clean element.

Example 4.2. Consider the symbolic 2-plithogenic ring

$$\begin{aligned} 2 - SP_{Z_3} &= \{a + bP_1 + cP_2; a, b, c \in Z_3\} \\ &= \left\{ \begin{array}{c} 0, 1, 2, P_1, P_2, 2P_1, 2P_2, P_1 + P_2, 2P_1 + 2P_2, P_1 + 2P_2, 2P_1 + P_2, 1 + P_1, \\ 1 + P_2, 1 + P_1 + P_2, 1 + 2P_1, 1 + 2P_2, 1 + 2P_1 + 2P_2, 1 + P_1 + 2P_2, \\ 1 + 2P_1 + P_2, 2 + P_1, 2 + P_2, 2 + P_1 + P_2, 2 + 2P_1, 2 + 2P_2, 2 + 2P_1 + 2P_2, \\ 2 + P_1 + 2P_2, 2 + 2P_1 + P_2 \end{array} \right\}. \end{aligned}$$

Since 0 is a nil-potent element in $2 - SP_{Z_3}$, so the idempotent elements $0, 1, P_1, P_2, P_1 + 2P_2, 1 + 2P_1, 1 + 2P_2, 1 + 2P_1 + P_2$ are nil-clean elements of $2 - SP_{Z_3}$. The only nilpotent elements of $2 - SP_{Z_3}$ is 0, so $0, 1, P_1, P_2, P_1 + 2P_2, 1 + 2P_1, 1 + 2P_2, 1 + 2P_1 + P_2$ are uniquely nil-clean elements of $2 - SP_{Z_3}$.

Definition 4.3. A symbolic 2-plithogenic ring in which all elements are nil-clean, then the ring is called a nil-clean symbolic 2-plithogenic ring. Furthermore, if each element of the symbolic 2-plithogenic ring is uniquely nil-clean, then the ring is called a uniquely nil-clean symbolic 2-plithogenic ring.

Example 4.4. $2 - SP_{Z_2} = \{0, 1, P_1, P_2, P_1 + P_2, 1 + P_1, 1 + P_2, 1 + P_1 + P_2\}$ is a nil-clean symbolic 2-plithogenic ring, that is because all the elements in $2 - SP_{Z_2}$ are idempotents and 0 is a nilpotent element in $2 - SP_{Z_2}$.

Lemma 4.5. *If x is a nilpotent element of $2 - SP_R$, then $1 + x$ is a unit in $2 - SP_R$.*

Proof. If x is a nilpotent element of $2 - SP_R$ then $x^k = 0$ for some $k > 0$. But then, $(1 + x)(1 - x + x^2 - x^3 + ... + (-1)^{k-1}x^{k-1}) = 1$ and so $1 + x$ is unit in $2 - SP_R$. □

Theorem 4.6. *Every nil-clean symbolic 2-plithogenic ring is clean symbolic 2-plithogenic ring.*

Proof. Suppose that $2 - SP_R$ is a nil-clean symbolic 2-plithogenic ring, and let $x \in 2 - SP_R$. Then $x-1$ is an element of $2-SP_R$ and hence $x-1 = e+n$, where e is an idempotent element and n is a nilpotent element of $2 - SP_R$.
This implies that, $x = e + (1 + n)$ is a nil-clean element of $2 - SP_R$ because $1 + n$ is a unit element of $2 - SP_R$ by Lemma 4.5. □

The converse of the Theorem 4.6 is not true. See the following example.

Example 4.7. Consider, the clean symbolic 2-plithogenic ring $2 - SP_{Z_3}$. All the elements of $2 - SP_{Z_3}$ are clean elements. The only nilpotent element of $2 - SP_{Z_3}$ is 0 and $P_1 + P_2$ is not an idempotent element in $2 - SP_{Z_3}$ so it is not nil-clean. Hence $2 - SP_{Z_3}$ is not a nil-clean ring.

Lemma 4.8. *Let R be a ring. Then the class of nil-clean symbolic 2-plithogenic rings is closed under homomorphic images.*

Proof. It is clear since the homomorphic image of a nil-potent element of a symbolic 2-plithogenic rings is again a nil-potent. □

Theorem 4.9. *Let R be any ring, $2 - SP_R$ be its corresponding symbolic 2-plithogenic ring. $2 - SP_R$ is nil-clean if and only if R is nil-clean.*

Proof. Assume that $2 - SP_R$ is nil-clean. Since R is a homomorphic image of $2 - SP_R$, so R is nil-clean by Lemma 4.8.
Conversely, assume that R is nil-clean, we must prove that $2 - SP_R$ is nil-clean. Let $x = a + bP_1 + cP_2 \in 2 - SP_R$ then $a, a + b, a + b + c \in R$. Since R is nil-clean we have $a = e_1 + n_1, a + b = e_2 + n_2, a + b + c = e_3 + n_3$, where e_i are idempotent elements and n_i are nilpotent elements of R. Now,

$$\begin{aligned} x &= a + bP_1 + cP_2 \\ &= a + [(a + b) - a]P_1 + [(a + b + c) - (a + b)]P_2 \\ &= (e_1 + n_1) + [(e_2 + n_2) - (e_1 + n_1)]P_1 + [(e_3 + n_3) - (e_2 + n_2)]P_2 \\ &= (e_1 + n_1) + [(e_2 - e_1) + (n_2 - n_1)]P_1 + [(e_3 - e_2) + (n_3 - n_2)]P_2 \\ &= [e_1 + (e_2 - e_1)P_1 + (e_3 - e_2)P_2] + [n_1 + (n_2 - n_1)P_1 + (n_3 - n_2)P_2] \\ &= x_1 + x_2. \end{aligned}$$

where, $x_1 = e_1 + (e_2 - e_1)P_1 + (e_3 - e_2)P_2$ and $x_2 = n_1 + (n_2 - n_1)P_1 + (n_3 - n_2)P_2$. By Theorem 2.5, $e_1, e_1 + (e_2 - e_1) = e_2, e_1 + (e_2 - e_1) + (e_3 - e_2) = e_3$ are idempotents in R. Therefore, x_1 is a idempotent element of R. Also, x_2 is a nilpotent element of R by a similar discussion and by Theoerem 2.8. Hence $2 - SP_R$ is nil-clean. □

Theorem 4.10. *If $2 - SP_R$ is a symbolic 2-plithogenic ring, then every central idempotent of $2 - SP_R$ is uniquely nil-clean element.*

Proof. We know that, every idempotent element of $2 - SP_R$ are nil-clean. Let x be a central idempotent element of $2 - SP_R$. Then $x = (1 - x) + (2x - 1)$. Suppose that $x = e + n$, where e is an idempotent and n is a nilpotent element of $2 - SP_R$. Since $nx = xn$, we obtain $e + n = (e + n)^2 = e + 2en + n^2$. So, we have $n = 1 - 2e$ and hence $e = 1 - x$, as reuired. □

Lemma 4.11. *Let $2 - SP_R$ be uniquely nil-clean symbolic 2-plithogenic ring. Then all idempotents of $2 - SP_R$ are central.*

Proof. Let $e \in 2 - SP_R$ be an idempotent element and x be any element of $2 - SP_R$. Now, the element $e + ex - exe$ can be written as $e + (ex - exe)$ or $(e + (ex - exe)) + 0$ each time as the sum of an idempotent and a nilpotent element of $2 - SP_R$. Since $2 - SP_R$ is uniquely nil clean, we have $e = e + (ex - exe)$. This implies that $ex - exe = 0$ and so $ex = exe$. In the similar way, we can show that $xe = exe$. Hence $ex = xe$ as required. □

Theorem 4.12. *Every boolean symbolic 2-plithogenic ring is uniquely nil-clean.*

Proof. If $2 - SP_R$ is a boolean symbolic 2-plithogenic ring, then $2 - SP_R = Id(2 - SP_R)$. Since boolean rings are abelian, we have $Id(2 - SP_R) = C(2 - SP_R)$. This implies that, $2 - SP_R = C(2 - SP_R)$. By Theorem 4.10, every element of the ring $2 - SP_R$ are uniquely nil-clean. Hence $2 - SP_R$ is uniquely nil-clean ring. □

5. Conclusion

In this article, we have introduced the the new classes of rings called, clean symbolic 2-plithogenic rings and nil-clean symbolic 2-plithogenic rings and we have studied various properties of clean and nil-clean symbolic 2-plithogenic rings with proper examples. Also, we have determined necessary and sufficient condition for a symbolic 2-plithogenic ring to be clean and nil-clean.

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Solution of System of Symbolic 2-Plithogenic Linear Equations using Cramer's Rule

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Abstract. In this article, the concept of system of symbolic 2-plithogenic linear equations and its solutions are introduced and studied. The Cramer's rule was applied to solve the system of symbolic 2-plithogenic linear equations. Also, provided enough examples for each case to enhance understanding.

Keywords: Symbolic 2-plithogenic linear equations; Cramer's rule; solution of the symbolic 2-plithogenic linear equations.

1. Introduction

The concept of refined neutrosophic structure was studied by many authors in [1–5]. Symbolic plithogenic algebraic structures are introduced by Smarandache, that are very similar to refined neutrosophic structures with some differences in the definition of the multiplication operation [10].

In [7], the algebraic properties of symbolic 2-plithogenic rings generated from the fusion of symbolic plithogenic sets with algebraic rings, and some of the elementary properties and substructures of symbolic 2-plithogenic rings such as AH-ideals, AH-homomorphisms, and AHS-isomorphisms are studied. In [11], some more algebraic properties of symbolic 2-plithogenic rings are studied. Further, Taffach [8, 9] studied the concepts of symbolic 2-plithogenic vector spaces and modules.

In [14], the concept of symbolic 2-plithogenic matrices with symbolic 2-plithogenic entries, determinants, eigen values and vectors, exponents, and diagonalization are studied. Hamiyet Merkepci et.al [13], studied the the symbolic 2-plithogenic number theory and integers. Ahmad Khaldi et.al [12], studied the different types of algebraic symbolic 2-plithogenic equations and its solutions.

In [6], Yaser Ahmad Alhasan studied the types of the nuetrosophic linear equations and Cramer's rule to solve the system of nuetrosophic linear equations. Motivated by this work, in this article the symbolic 2-plithogenic linear equations and its solutions are introduced and studied. Also, enough examples are given for all the cases to enhance understanding.

2. **Preliminaries**

Definition 2.1. [7] Let R be a ring, the symbolic 2-plithogenic ring is defined as follows:

$$2-SP_R=\left\{a_0+a_1P_1+a_2P_2; a_i\in R, P_j^2=P_j, P_1\times P_2=P_{max(1,2)}=P_2\right\}$$

Smarandache has defined algebraic operations on $2-SP_R$ as follows:

Addition:

$[a_0+a_1P_1+a_2P_2]+[b_0+b_1P_1+b_2P_2]=(a_0+b_0)+(a_1+b_1)P_1+(a_2+b_2)P_2$

Multiplication:

$[a_0+a_1P_1+a_2P_2].[b_0+b_1P_1+b_2P_2]=a_0b_0+a_0b_1P_1+a_0b_2P_2+a_1b_0P_1^2+a_1b_2P_1P_2+a_2b_0P_2+a_2b_1P_1P_2+a_2b_2P_2^2+a_1b_1P_1P_1=(a_0b_0)+(a_0b_1+a_1b_0+a_1b_1)P_1+(a_0b_2+a_1b_2+a_2b_0+a_2b_1+a_2b_2)P_2$.

It is clear that $2-SP_R$ is a ring. If R is a field, then $2-SP_R$ is called a symbolic 2-plithogenic field. Also, if R is commutative, then $2-SP_R$ is commutative, and if R has a unity (1), than $2-SP_R$ has the same unity (1).

Theorem 2.2. *[7] Let $2-SP_R$ be a 2-plithogenic symbolic ring, with unity (1). Let $X=x_0+x_1P_1+x_2P_2$ be an arbitrary element, then:*

(1) *X is invertible if and only if $x_0, x_0+x_1, x_0+x_1+x_2$ are invertible.*

(2) $X^{-1}=x_0^{-1}+[(x_0+x_1)^{-1}-x_0^{-1}]P_1+[(x_0+x_1+x_2)^{-1}-(x_0+x_1)^{-1}]P_2$

Definition 2.3. [14] A symbolic 2-plithogenic square real matrix is a matrix with symbolic 2-plithogenic real entries.

Theorem 2.4. *[14] Let $S=S_0+S_1P_1+S_2P_2$ be a symbolic 2-plithogenic square real matrix, then*

(1) *S is invertible if and only if $S_0, S_0+S_1, S_0+S_1+S_2$ are invertible.*

(2) *If S is invertible then*

$$S^{-1}=S_0^{-1}+[(S_0+S_1)^{-1}-S_0^{-1}]P_1+[(S_0+S_1+S_2)^{-1}-(S_0+S_1)^{-1}]$$

3. The Symbolic 2-Plithogenic Linear Equations

We begin this section with the following definition.

Definition 3.1. The symbolic 2-plithogenic linear equation of n variables $x_1, x_2, x_3, ..., x_n$, is each equation that takes the form:

$$(a_{01} + a_{11}P_1 + a_{21}P_2)x_1 + (a_{02} + a_{12}P_1 + a_{22}P_2)x_2 + (a_{03} + a_{13}P_1 + a_2P_{23})x_3 + \cdots + (a_{0n} + a_{1n}P_1 + a_{2n}P_2)x_n = b_0 + b_1P_1 + b_2P_2$$

where $a_{0i}, a_{1i}, a_{2i},\ i = 1, 2, ..., n$ are real coefficients. We call $(a_{01} + a_{11}P_1 + a_{21}P_2), (a_{02} + a_{12}P_1 + a_{22}P_2), (a_{03} + a_{13}P_1 + a_{23}P_2)$ symbolic 2-plithogenic coefficients of the borders of the equation, and $b_0 + b_1P_1 + b_2P_2$ constant symbolic 2-plithogenic border of the equation.

Remark 3.2.

(1) We call each equation of the form:

$$(a_0 + a_1P_1 + a_2P_2)x + (b_0 + b_1P_1 + b_2P_2)y = c_0 + c_1P_1 + c_2P_2$$

the two-variable symbolic 2-plithogenic linear equation, where, $a_0, a_1, a_2, b_0, b_1, b_2$, and c_0, c_1, c_2 are real coefficients.

(2) We call each equation of the form:

$$(a_0 + a_1P_1 + a_2P_2)x + (b_2 + b_1P_1 + b_2P_2)y + (c_0 + c_1P_1 + c_2P_2)z = d_0 + d_1P_1 + d_2P_2$$

the three-variable symbolic 2-plithogenic linear equation, where, $a_0, a_1, a_2, b_0, b_1, b_2$, c_0, c_1, c_2, and d_0, d_1, d_2 are real coefficients.

Example 3.3.

(1) $(1 + P_2)x + (3 - P_1)y + (1 + P_1 - P_2)z = 5$

(2) $P_2x + P_1y + (P_1 - P_2)z = 2P_1 + 2P_2$

(3) $(1 + P_1 - P_2)x + (4 + P_1 - P_2)y = 11 + 4P_2$

Definition 3.4. Solution of the symbolic 2-plithogenic linear equation,

$$(a_{01} + a_{11}P_1 + a_{21}P_2)x_1 + (a_{02} + a_{12}P_1 + a_{22}P_2)x_2 + (a_{03} + a_{13}P_1 + a_2P_{23})x_3 + \cdots + (a_{0n} + a_{1n}P_1 + a_{2n}P_2)x_n = b_0 + b_1P_1 + b_2P_2$$

is finding the values of the variables $x_1, x_2, x_3, ..., x_n$ that satisfies the equation, where $a_{0i}, a_{1i}, a_{2i},\ i = 1, 2, ..., n$ are real coefficients.

Example 3.5. Consider the following the two-variable symbolic 2-plithogenic linear equation:

$$(1 + P_1 - P_2)x + (4 + P_1 - P_2)y = 4 + \frac{11}{2}P_1 - \frac{15}{2}P_2$$

The solution of this equation is

$$x = 2 + P_1 + 2P_2, \quad y = \frac{1}{2} - P_2.$$

Definition 3.6. For the two variable symbolic 2-plithogenic linear equation

$$(a_0 + a_1P_1 + a_2P_2)x + (b_0 + b_1P_1 + b_2P_2)y = c_0 + c_1P_1 + c_2P_2$$

the infinite number of solutions defined by

$$y = -\frac{a_0 + a_1P_1 + a_2P_2}{b_0 + b_1P_1 + b_2P_2}x + \frac{c_0 + c_1P_1 + c_2P_2}{b_0 + b_1P_1 + b_2P_2}$$

where, $a_0, a_1, a_2, b_0, b_1, b_2, c_0, c_1, c_2$ are real coefficients, $b_0 \neq 0, b_0 + b_1 \neq 0$ and $b_0 + b_1 + b_2 \neq 0$ by given a value for one of the two variables, we obtain a value for the other variable.

Example 3.7. Consider the two variable symbolic 2-plithogenic linear equation

$$(1 + P_1 - P_2)x + (4 + P_1 - P_2)y = 1 + P_1 - 2P_2.$$

This implies that,

$$y = -\left(\frac{1 + P_1 - P_2}{4 + P_1 - P_2}\right)x + \left(\frac{1 + P_1 - 2P_2}{4 + P_1 - P_2}\right)$$

Then the set of solution is:

$$S = \left\{x, y \in 2 - SP_R : y = -\left(\frac{1 + P_1 - P_2}{4 + P_1 - P_2}\right)x + \left(\frac{1 + P_1 - 2P_2}{4 + P_1 - P_2}\right)\right\}$$

$$i.e., S = \left\{x, y \in 2 - SP_R : y = \left(-\frac{1}{4} - \frac{3}{20}P_1 + \frac{3}{20}P_2\right)x + \left(\frac{1}{4} - \frac{3}{20}P_1 - \frac{2}{5}P_2\right)\right\}$$

By given any value for the variable x, we obtain a value of the variable y.

Definition 3.8. For the n-variable symbolic 2-plithogenic linear equation

$$(a_{01} + a_{11}P_1 + a_{21}P_2)x_1 + (a_{02} + a_{12}P_1 + a_{22}P_2)x_2 + (a_{03} + a_{13}P_1 + a_2P_{23})x_3 + \cdots + (a_{0n} + a_{1n}P_1 + a_{2n}P_2)x_n = b_0 + b_1P_1 + b_2P_2$$

where $a_{0i}, a_{1i}, a_{2i},\ i = 1, 2, ..., n$ are real coefficients, the infinite number of solutions are the unknown values $x_1, x_2, ..., x_n$ that satisfies the equation.

Definition 3.9. A non-homogeneous system of n-variable symbolic 2-plithogenic linear equations is given by the form:

$$(a^1_{01} + a^1_{11}P_1 + a^1_{21}P_2)x_1 + (a^1_{02} + a^1_{12}P_1 + a^1_{22}P_2)x_2 + (a^1_{03} + a^1_{13}P_1 + a^1_{23}P_2)x_3 + \cdots + (a^1_{0n} + a^1_{1n}P_1 + a^1_{2n}P_2)x_n = b^1_0 + b^1_1P_1 + b^1_2P_2$$

$$(a^2_{01} + a^2_{11}P_1 + a^2_{21}P_2)x_1 + (a^2_{02} + a^2_{12}P_1 + a^2_{22}P_2)x_2 + (a^2_{03} + a^2_{13}P_1 + a^2_{23}P_2)x_3 + \cdots + (a^2_{0n} + a^2_{1n}P_1 + a^2_{2n}P_2)x_n = b^2_0 + b^2_1P_1 + b^2_2P_2$$

$$\cdots$$

$$(a^m_{01} + a^m_{11}P_1 + a^m_{21}P_2)x_1 + (a^m_{02} + a^m_{12}P_1 + a^m_{22}P_2)x_2 + (a^m_{03} + a^m_{13}P_1 + a^m_{23}P_2)x_3 + \cdots + (a^m_{0n} + a^m_{1n}P_1 + a^m_{2n}P_2)x_n = b^m_0 + b^m_1P_1 + b^m_2P_2$$

where, $a^j_{0i}, a^j_{1i}, a^j_{2i}, b^j_0, b^j_1, b^j_2$ are real coefficients, $i = 1, 2, ..., n,\ j = 1, 2, ..., m$.

Definition 3.10. The solution of non-homogeneous system of n-variable symbolic 2-plithogenic linear equations:

$$(a^1_{01}+a^1_{11}P_1+a^1_{21}P_2)x_1+(a^1_{02}+a^1_{12}P_1+a^1_{22}P_2)x_2+(a^1_{03}+a^1_{13}P_1+a^1_{23}P_2)x_3+\cdots+(a^1_{0n}+a^1_{1n}P_1+a^1_{2n}P_2)x_n=b^1_0+b^1_1P_1+b^1_2P_2$$
$$(a^2_{01}+a^2_{11}P_1+a^2_{21}P_2)x_1+(a^2_{02}+a^2_{12}P_1+a^2_{22}P_2)x_2+(a^2_{03}+a^2_{13}P_1+a^2_{23}P_2)x_3+\cdots+(a^2_{0n}+a^2_{1n}P_1+a^2_{2n}P_2)x_n=b^2_0+b^2_1P_1+b^2_2P_2$$
$$\cdots$$
$$(a^m_{01}+a^m_{11}P_1+a^m_{21}P_2)x_1+(a^m_{02}+a^m_{12}P_1+a^m_{22}P_2)x_2+(a^m_{03}+a^m_{13}P_1+a^m_{23}P_2)x_3+\cdots+(a^m_{0n}+a^m_{1n}P_1+a^m_{2n}P_2)x_n=b^m_0+b^m_1P_1+b^m_2P_2$$

where, $a^j_{0i}, a^j_{1i}, a^j_{2i}, b^j_0, b^j_1, b^j_2$ are real coefficients, $i=1,2,...,n,\ j=1,2,...,m$, is the values of the variables $x_1, x_2, x_3, ..., x_n$ that satisfies the system of equations.

Remark 3.11. We distinguish three cases to solve the system given in Definition 3.10:

(1) The system is consistent, it has unique solution.

(2) The system is consistent, it has infinite number of solutions.

(3) The system is inconsistent, it has no solution.

4. Solving System of Symbolic 2-Plithogenic Linear Equations using Cramer's Rule

Consider the non-homogeneous system of n symbolic 2-plithogenic linear equations with n-variables:

$$(a^1_{01}+a^1_{11}P_1+a^1_{21}P_2)x_1+(a^1_{02}+a^1_{12}P_1+a^1_{22}P_2)x_2+(a^1_{03}+a^1_{13}P_1+a^1_{23}P_2)x_3+\cdots+(a^1_{0n}+a^1_{1n}P_1+a^1_{2n}P_2)x_n=b^1_0+b^1_1P_1+b^1_2P_2$$
$$(a^2_{01}+a^2_{11}P_1+a^2_{21}P_2)x_1+(a^2_{02}+a^2_{12}P_1+a^2_{22}P_2)x_2+(a^2_{03}+a^2_{13}P_1+a^2_{23}P_2)x_3+\cdots+(a^2_{0n}+a^2_{1n}P_1+a^2_{2n}P_2)x_n=b^2_0+b^2_1P_1+b^2_2P_2$$
$$\cdots$$
$$\cdots$$
$$(a^n_{01}+a^n_{11}P_1+a^n_{21}P_2)x_1+(a^n_{02}+a^n_{12}P_1+a^n_{22}P_2)x_2+(a^n_{03}+a^n_{13}P_1+a^n_{23}P_2)x_3+\cdots+(a^n_{0n}+a^n_{1n}P_1+a^n_{2n}P_2)x_n=b^n_0+b^n_1P_1+b^n_2P_2$$

Let $AX=B$ be the matrix form of this system and let, $\Delta=det(A)=a_0+a_1P_1+a_2P_2$. We distinguish the following cases:

(1) If $a_0\neq 0,\quad a_0+a_1\neq 0\quad$ and $\quad a_0+a_1+a_2\neq 0$ then the system is consistent and the system has unique solution given by the formula:

$$x_i=\frac{\Delta_{x_i}}{\Delta},\quad i=1,2,3,...,n$$

where Δ_{x_i} is the determinant of the matrix A where the ith column is replaced by the column matrix B.

(2) If $a_0=0,\ $ or $\ a_0+a_1=0\ $ or $\quad a_0+a_1+a_2=0$. Then the system is inconsistent or it have infinite number of solutions.

Remark 4.1. Consider a system of 2 linear symbolic 2-plithogenic equations with 2 unknowns x and y with

$$\Delta = det(A) = a_0 + a_1P_1 + a_2P_2$$
$$\Delta_x = det(A_x) = a_0^x + a_1^xP_1 + a_2^xP_2$$
$$\Delta_y = det(A_y) = a_0^y + a_1^yP_1 + a_2^yP_2$$

We distinguish the following cases:

(1) If $a_0 \neq 0, \quad a_0 + a_1 \neq 0 \quad$ and $\quad a_0 + a_1 + a_2 \neq 0$ then the system is consistent and the system has unique solution given by the formula:

$$x = \frac{\Delta_x}{\Delta}, \quad y = \frac{\Delta_y}{\Delta}$$

(2) If the determinants satisfies any one of the following conditions, then the system is inconsistent and it has no solutions.

(i). $a_0 = 0$ and a_0^x, a_0^y are not all zero.

(ii). $a_0 + a_1 = 0$ and $a_0^x + a_1^x, a_0^y + a_1^y$ are not all zero.

(iii). $a_0 + a_1 + a_3 = 0$ and $a_0^x + a_1^x + a_2^x, a_0^y + a_1^y + a_2^y$ are not all zero.

(3) In all other cases the system is consistent with infinite number of solutions.

Example 4.2. Consider the system of equations:

$$(2 + P_1 + 3P_2)x + (1 - P_1 - P_2)y = 5 + P_1 + 11P_2$$
$$(3 + 4P_2)x + (1 + P_1)y = 7 + 3P_1 + 13P_2.$$

The coefficient matrix is, $A = \begin{pmatrix} 2 + P_1 + 3P_2 & 1 - P_1 - P_2 \\ 3 + 4P_2 & 1 + P_1 \end{pmatrix}$.

$$\Delta = det(A) = \begin{vmatrix} 2 + P_1 + 3P_2 & 1 - P_1 - P_2 \\ 3 + 4P_2 & 1 + P_1 \end{vmatrix} = -1 + 7P_1 + 13P_2$$

Here, $a_0 \neq 0, a_0 + a_1 \neq 0$ and $a_0 + a_1 + a_2 \neq 0$, the system is consistent and its has unique solution.

$$\Delta_x = det(A_x) = \begin{vmatrix} 5 + P_1 + 11P_2 & 1 - P_1 - P_2 \\ 7 + 3P_1 + 13P_2 & 1 + P_1 \end{vmatrix} = -2 + 14P_1 + 45P_2$$

$$\Delta_y = det(A_y) = \begin{vmatrix} 2 + P_1 + 3P_2 & 5 + P_1 + 11P_2 \\ 3 + 4P_2 & 7 + 3P_1 + 13P_2 \end{vmatrix} = -1 + 13P_1 + 7P_2$$

So the solution of the given system is,

$$x = \frac{\Delta_x}{\Delta} = \frac{-2 + 14P_1 + 45P_2}{-1 + 7P_1 + 13P_2} = 2 + P_2,$$

$$y = \frac{\Delta_y}{\Delta} = \frac{-1 + 13P_1 + 7P_2}{-1 + 7P_1 + 13P_2} = 1 + P_1 - P_2$$

Example 4.3. Consider the system of equations:

$$(1+P_1+P_2)x+(3-P_1+2P_2)y=5+3P_1+5P_2$$
$$P_1x+(P_1+P_2)y=4P_1+P_2.$$

The coefficient matrix is, $A=\begin{pmatrix}1+P_1+P_2 & 3-P_1+2P_2\\ P_1 & P_1+P_2\end{pmatrix}$.

$$\Delta=det(A)=\begin{vmatrix}1+P_1+P_2 & 3-P_1+2P_2\\ P_1 & P_1+P_2\end{vmatrix}=0+0P_1+2P_2$$

$$\Delta=det(A_x)=\begin{vmatrix}5+3P_1+5P_2 & 3-P_1+2P_2\\ 4P_1+P_2 & P_1+P_2\end{vmatrix}=0+0P_1+6P_2$$

$$\Delta=det(A_y)=\begin{vmatrix}1+P_1+P_2 & 5+3P_1+5P_2\\ P_1 & 4P_1+P_2\end{vmatrix}=0+0P_1+2P_2$$

Hence, by condition (3) of Remark 4.1 the system is consistent with infinite number of solutions and the solutions are given by:
For all $x=a_0+a_1P_1+a_2P_2\in 2-SP_R$ with $a_0+a_1+a_2=3$,

$$y=-\left(\frac{1+P_1+P_2}{3-P_1+2P_2}\right)x+\left(\frac{5+3P_1+5P_2}{3-P_1+2P_2}\right)$$

That is, for all $x=a_0+a_1P_1+a_2P_2\in 2-SP_R$ with $a_0+a_1+a_2=3$,

$$y=\left(-\frac{1}{3}-\frac{2}{3}P_1+\frac{1}{4}P_2\right)x+\left(\frac{5}{3}+\frac{7}{3}P_1-\frac{3}{4}P_2\right)$$

Example 4.4. Consider the system of equations:

$$(2+P_1+3P_2)x+(1+P_1+P_2)y=5+P_1+11P_2$$
$$(4+2P_1+6P_2)x+(2+2P_1+2P_2)y=10+2P_1+22P_2$$

The coefficient matrix is, $A=\begin{pmatrix}2+P_1+3P_2 & 1+P_1+P_2\\ 4+2P_1+6P_2 & 2+2P_1+2P_2\end{pmatrix}$.

$$\Delta=det(A)=\begin{vmatrix}2+P_1+3P_2 & 1+P_1+P_2\\ 4+2P_1+6P_2 & 2+2P_1+2P_2\end{vmatrix}=0+0P_1+0P_2$$

Also,

$$\Delta_x=det(A_x)=\begin{vmatrix}5+P_1+11P_2 & 1+P_1+P_2\\ 10+2P_1+22P_2 & 2+2P_1+2P_2\end{vmatrix}=0+0P_1+0P_2$$

$$\Delta_y=det(A_y)=\begin{vmatrix}2+P_1+3P_2 & 5+P_1+11P_2\\ 4+2P_1+6P_2 & 10+2P_1+22P_2\end{vmatrix}=0+0P_1+0P_2$$

Hence, by condition (3) of Remark 4.1 the system is consistent with infinite number of solutions and the solutions are given by:

$$S = \left\{ x, y \in 2 - SP_R : y = -\left(\frac{2 + P_1 + 3P_2}{1 + P_1 + P_2}\right) x + \left(\frac{5 + P_1 + 11P_2}{1 + P_1 + P_2}\right) \right\}$$

$$i.e, \quad S = \left\{ x, y \in 2 - SP_R : y = \left(-2 + \frac{1}{2}P_1 - \frac{1}{2}P_2\right) x + \left(5 - 2P_1 + \frac{8}{3}P_2\right) \right\}$$

Example 4.5. Consider the system of equations:

$$(1 + P_1 + P_2)x + (1 - P_1 + P_2)y = 1 + P_1$$
$$(2 + 2P_1 + 2P_2)x + (2 - 2P_1 + 2P_2)y = 3 + P_2$$

The coefficient matrix is, $A = \begin{pmatrix} 1 + P_1 + P_2 & 1 - P_1 + P_2 \\ 2 + 2P_1 + 2P_2 & 2 - 2P_1 + 2P_2 \end{pmatrix}$.

$$\Delta = det(A) = \begin{vmatrix} 1 + P_1 + P_2 & 1 - P_1 + P_2 \\ 2 + 2P_1 + 2P_2 & 2 - 2P_1 + 2P_2 \end{vmatrix} = 0 + 0P_1 + 0P_2$$

Also,

$$\Delta_x = det(A_x) = \begin{vmatrix} 1 + P_1 & 1 - P_1 + P_2 \\ 3 + P_2 & 2 - 2P_1 + 2P_2 \end{vmatrix} = -1 + P_1$$

$$\Delta_y = det(A_y) = \begin{vmatrix} 1 + P_1 + P_2 & 1 + P_1 \\ 2 + 2P_1 + 2P_2 & 3 + P_2 \end{vmatrix} = 1 - 3P_1 - 2P_2$$

Here, $a_0 = 0$ and $a_0^x \neq 0$, hence the system is inconsistent.

Remark 4.6. Consider a system of 3 linear symbolic 2-plithogenic equations with 3 unknowns x, y and z with

$$\Delta = det(A) = a_0 + a_1P_1 + a_2P_2$$
$$\Delta_x = det(A_x) = a_0^x + a_1^xP_1 + a_2^xP_2$$
$$\Delta_y = det(A_y) = a_0^y + a_1^yP_1 + a_2^yP_2$$
$$\Delta_z = det(A_z) = a_0^z + a_1^zP_1 + a_2^zP_2$$

We distinguish the following cases:

(1) If $a_0 \neq 0, \quad a_0 + a_1 \neq 0 \quad$ and $\quad a_0 + a_1 + a_2 \neq 0$ then the system is consistent and the system has unique solution given by the formula:

$$x = \frac{\Delta_x}{\Delta}, \quad y = \frac{\Delta_y}{\Delta} \quad \& \quad z = \frac{\Delta_z}{\Delta}$$

(2) If the determinants satisfies any one of the following conditions, then the system is inconsistent and it has no solutions.

(i). $a_0 = 0$ and a_0^x, a_0^y, a_0^z are not all zero.

(ii). $a_0 + a_1 = 0$ and $a_0^x + a_1^x, a_0^y + a_1^y, a_0^z + a_1^z$ are not all zero.

(iii). $a_0 + a_1 + a_3 = 0$ and $a_0^x + a_1^x + a_2^x, a_0^y + a_1^y + a_2^y, a_0^z + a_1^z + a_2^z$ are not all zero.

(3) If $\Delta = 0+0P_1+0P_2$, $\Delta_x = 0+0P_1+0P_2$, $\Delta_y = 0+0P_1+0P_2$ and $\Delta_z = 0+0P_1+0P_2$ then by solving two equations of the system we will obtain the equation $0 = \alpha$. If $\alpha = 0$, then the system is consistent with infinite number of solutions and if $\alpha \neq 0$, then the system is inconsistent.

Example 4.7. Consider the system of equations:

$$(1+P_1)x + (1-P_1)y + (1+P_1-P_2)z = 1+5P_1-P_2$$
$$(1+P_2)x + (-1+P_1+P_2)y + (2+P_1)z = 1+4P_1+3P_2$$
$$(1-P_1+P_2)x + (-1+P_2)y + (1+P_1)z = 1+P_1+2P_2.$$

The coefficient matrix is, $A = \begin{pmatrix} 1+P_1 & 1-P_1 & 1+P_1-P_2 \\ 1+P_2 & -1+P_1+P_2 & 2+P_1 \\ 1-P_1+P_2 & -1+P_2 & 1+P_1 \end{pmatrix}$.

$$\Delta = det(A) = \begin{vmatrix} 1+P_1 & 1-P_1 & 1+P_1-P_2 \\ 1+P_2 & -1+P_1+P_2 & 2+P_1 \\ 1-P_1+P_2 & -1+P_2 & 1+P_1 \end{vmatrix} = 2+2P_1-P_2$$

$$\Delta_x = det(A_x) = \begin{vmatrix} 1+5P_1-P_2 & 1-P_1 & 1+P_1-P_2 \\ 1+4P_1+3P_2 & -1+P_1+P_2 & 2+P_1 \\ 1+P_1+2P_2 & -1+P_2 & 1+P_1 \end{vmatrix} = 2+6P_1-2P_2$$

$$\Delta_y = det(A_y) = \begin{vmatrix} 1+P_1 & 1+5P_1-P_2 & 1+P_1-P_2 \\ 1+P_2 & 1+4P_1+3P_2 & 2+P_1 \\ 1-P_1+P_2 & 1+P_1+2P_2 & 1+P_1 \end{vmatrix} = 3P_2$$

$$\Delta_z = det(A_z) = \begin{vmatrix} 1+P_1 & 1-P_1 & 1+5P_1-P_2 \\ 1+P_2 & -1+P_1+P_2 & 1+4P_1+3P_2 \\ 1-P_1+P_2 & -1+P_2 & 1+P_1+2P_2 \end{vmatrix} = 4P_1-P_2$$

Here, $a_0 \neq 0$, $a_0+a_1 \neq 0$, & $a_0+a_1+a_2 \neq 0$. Hence, the system is consistent with unique solution given by:

$$x = \frac{\Delta_x}{\Delta} = \frac{2+6P_1-2P_2}{2+2P_1-P_2} = 1+P_1,$$

$$y = \frac{\Delta_y}{\Delta} = \frac{3P_2}{2+2P_1-P_2} = P_2,$$

$$z = \frac{\Delta_z}{\Delta} = \frac{4P_1-P_2}{2+2P_1-P_2} = P_1.$$

Example 4.8. Consider the system of equations:

$$(1+P_2)x+(3-P_1)y+(1+P_1-P_2)z=5$$
$$P_2x+P_1y+(P_1+P_2)z=2P_1+2P_2$$
$$(2+P_1-P_2)x+(4+3P_1-P_2)y+(5+2P_2)z=11+4P_1.$$

The coefficient matrix is, $A=\begin{pmatrix} 1+P_2 & 3-P_1 & 1+P_1-P_2 \\ P_2 & P_1 & P_1+P_2 \\ 2+P_1-P_2 & 4+3P_1-P_2 & 5+2P_2 \end{pmatrix}$.

$$\Delta=det(A)=\begin{vmatrix} 1+P_2 & 3-P_1 & 1+P_1-P_2 \\ P_2 & P_1 & P_1+P_2 \\ 2+P_1-P_2 & 4+3P_1-P_2 & 5+2P_2 \end{vmatrix}=0-2P_1-10P_2$$

$$\Delta_x=det(A_x)=\begin{vmatrix} 5 & 3-P_1 & 1+P_1-P_2 \\ 2P_1+2P_2 & P_1 & P_1+P_2 \\ 11+4P_1 & 4+3P_1-P_2 & 5+2P_2 \end{vmatrix}=0-2P_1-10P_2$$

$$\Delta_y=det(A_y)=\begin{vmatrix} 1+P_2 & 5 & 1+P_1-P_2 \\ P_2 & 2P_1+2P_2 & P_1+P_2 \\ 2+P_1-P_2 & 11+4P_1 & 5+2P_2 \end{vmatrix}=0-2P_1-10P_2$$

$$\Delta_z=det(A_z)=\begin{vmatrix} 1+P_2 & 3-P_1 & 5 \\ P_2 & P_1 & 2P_1+2P_2 \\ 2+P_1-P_2 & 4+3P_1-P+2 & 11+4P_1 \end{vmatrix}=0-2P_1-10P_2$$

Here, $a_0=0,\ a_0^x=0,\ a_0^y=0\ \ \&\ \ a_0^z=0$. Hence, the system is consistent with infinite number of solutions and the solutions are given by:

$$S=\left\{x,y,z\in 2-SP_R : x=\left(-\frac{11}{2}+\frac{3}{2}P_1+5P_2\right)z+\left(\frac{13}{2}-\frac{3}{2}P_1-5P_2\right),\right.$$
$$\left. y=\left(\frac{3}{2}-\frac{1}{2}P_1-\frac{5}{2}P_2\right)z+\left(-\frac{1}{2}+\frac{1}{2}P_1+\frac{5}{2}P_2\right),\ \ z\right\}$$

Example 4.9. Consider the system of equations:

$$(1+P_1)x+(1-P_2)y+(1+P_1-P_2)z=2+P_1$$
$$(2+2P_1)x+(2-2P_2)y+(2+2P_1-2P_2)z=4+2P_1$$
$$(3+3P_1)x+(3-3P_2)y+(3+3P_1-3P_2)z=6+3P_1$$

Here, $\Delta=0+0P_1+0P_2,\ \ \Delta_x=0+0P_1+0P_2,\ \ \Delta_y=0+0P_1+0P_2,\ \ \Delta_z=0+0P_1+0P_2$. By solving first two equations of the given system we will get $0=0$. Hence the system consistent with infinite number of solutions. In this case the system is reduced to a single equation. To solve we can assign arbitrary values to any two variables and can determine the value of the third variable.

Example 4.10. Consider the system of equations:

$$(1+P_1)x+(1-P_2)y+(1+P_1-P_2)z=2+P_1$$
$$(2+2P_1)x+(2-2P_2)y+(2+2P_1-2P_2)z=1-P_2$$
$$(3+3P_1)x+(3-3P_2)y+(3+3P_1-3P_2)z=3+P_2$$

Here, $\Delta=0+0P_1+0P_2$, $\Delta_x=0+0P_1+0P_2$, $\Delta_y=0+0P_1+0P_2$, $\Delta_z=0+0P_1+0P_2$. By solving first two equations of the given system we will get $0=3+2P_1+P_2$. Hence the system inconsistent and it has no solution.

Other than the above mentioned cases there are some special cases in which the system of linear symbolic 2-plithogenic equations is inconsistent.

Remark 4.11. If all coefficients of a the system of n linear symbolic 2-plithogenic equations with n variables are non invertible the the system is inconsistent. For example,

$$(1-P_1+P_2)x+(2+2P_1-4P_2)y+(1-P_1)z=3+P_1$$
$$(1-P_2)x+P_1y+p_2z=3P_2$$
$$(2-P-1-P_2)x+P_2y+(1-P_1)z=2+P_1+P_2$$

5. System of Homogeneous Symbolic 2-Plithogenic Linear Equations

Definition 5.1. Consider the homogeneous system of n-variable symbolic 2-plithogenic linear equations:

$$(a^1_{01}+a^1_{11}P_1+a^1_{21}P_2)x_1+(a^1_{02}+a^1_{12}P_1+a^1_{22}P_2)x_2+(a^1_{03}+a^1_{13}P_1+a^1_{23}P_2)x_3+...+(a^1_{0n}+a^1_{1n}P_1+a^1_{2n}P_2)x_n=0+0P_1+0P_2$$
$$(a^2_{01}+a^2_{11}P_1+a^2_{21}P_2)x_1+(a^2_{02}+a^2_{12}P_1+a^2_{22}P_2)x_2+(a^2_{03}+a^2_{13}P_1+a^2_{23}P_2)x_3+...+(a^2_{0n}+a^2_{1n}P_1+a^2_{2n}P_2)x_n=0+0P_1+0P_2$$
$$...$$
$$(a^n_{01}+a^n_{11}P_1+a^n_{21}P_2)x_1+(a^n_{02}+a^n_{12}P_1+a^n_{22}P_2)x_2+(a^n_{03}+a^n_{13}P_1+a^n_{23}P_2)x_3+...+(a^n_{0n}+a^n_{1n}P_1+a^n_{2n}P_2)x_n=0+0P_1+0P_2$$

Remark 5.2. Let $AX=B$ be the coefficient matrix of this system and let $\Delta=det(A)=a_0+a_1P_1+a_2P_2$. We distinguish the following cases:

(1) If $a_0\neq 0$ and $a_0+a_1\neq 0$ and $a_0+a_1+a_2\neq 0$ then the system is consistent and the system has unique solution $x_i=0$, $i=1,2,...,n$.

(2) If $a_0=0$ or $a_0+a_1=0$ or $a_0+a_1+a_2=0$, then the system is consistent with infinite number of solutions.

Example 5.3. Consider the system of equations:

$$(2+P_1+3P_2)x+(1-P_1-P_2)y=0$$
$$(3+4P_2)x+(1+P_1)y=0.$$

The coefficient matrix is, $A = \begin{pmatrix} 2+P_1+3P_2 & 1-P_1-P_2 \\ 3+4P_2 & 1+P_1 \end{pmatrix}$.

$$\Delta = det(A) = \begin{vmatrix} 2+P_1+3P_2 & 1-P_1-P_2 \\ 3+4P_2 & 1+P_1 \end{vmatrix} = -1+7P_1+13P_2$$

Here, $a_0 \neq 0, a_0+a_1 \neq 0$ and $a_0+a_1+a_2 \neq 0$, the system is consistent with unique solution.

$$\Delta_x = det(A_x) = \begin{vmatrix} 0 & 1-P_1-P_2 \\ 0 & 1+P_1 \end{vmatrix} = 0$$

$$\Delta_y = det(A_y) = \begin{vmatrix} 2+P_1+3P_2 & 0 \\ 3+4P_2 & 0 \end{vmatrix} = 0$$

So the solution of the given system is,

$$x = \frac{\Delta_x}{\Delta} = \frac{0}{-1+7P_1+13P_2} = 0,$$

$$y = \frac{\Delta_y}{\Delta} = \frac{0}{-1+7P_1+13P_2} = 0$$

Example 5.4. Consider the system of equations:

$$\begin{gathered} (1+P_2)x + (3-P_1)y + (2+P_2)z = 0 \\ P_2x + P_1y + (P_1 - \tfrac{1}{2}P_2)z = 0 \\ (2+P_1-P_2)x + (4+3P_1-P_2)y + (3+4P_1+2P_2)z = 0. \end{gathered}$$

The coefficient matrix is, $A = \begin{pmatrix} 1+P_2 & 3-P_1 & 2+P_2 \\ P_2 & P_1 & P_1-\frac{1}{2}P_2 \\ 2+P_1-P_2 & 4+3P_1-P_2 & 3+4P_1+2P_2 \end{pmatrix}$.

$$\Delta = det(A) = \begin{vmatrix} 1+P_2 & 3-P_1 & 2+P_2 \\ P_2 & P_1 & P_1-\frac{1}{2}P_2 \\ 2+P_1-P_2 & 4+3P_1-P_2 & 3+4P_1+2P_2 \end{vmatrix} = 0+0P_1+8P_2$$

Here, $a_0 = 0$, the system is consistent with infinite number of solutions and the solutions are given by:

$$S = \left\{ x, y, z \in 2-SP_R : x = \left(-\frac{1}{2}+\frac{1}{2}P_1\right)z, \quad y = \left(-\frac{1}{2}-\frac{1}{2}P_1-\frac{1}{2}P_2\right)z, \quad z \right\}$$

6. **Conclusion**

In this article, the solutions of symbolic 2-plithogenic linear equations are studied using Cramer's rule. The conditions are given for a system of symbolic 2-plithogenic linear equations to be consistent with unique solution, consistent with infinite solutions, and inconsitent. Further, many examples are given for the case of system of two symbolic 2-plithogenic linear equations with two variables and for the case of system of three symbolic 2-plithogenic linear equations with three variables.

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Short Communication of Big Plithogenic Science and Deep Plithogenic Science

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Abstract

Big Science refers to large-scale, well-funded collaborative research designed to refine established theories and produce significant experimental or observational data. In contrast, Deep Science emphasizes reexamining fundamental principles, exploring alternative frameworks, and proposing transformative theories to tackle core scientific challenges. The relationship between Big Science and Deep Science has been explored in earlier studies [49]. This paper provides a systematic re-evaluation of these concepts and investigates their extensions using the framework of plithogenic sets[46]—a mathematical concept, like Fuzzy Sets[54] and Neutrosophic Sets[44], designed to address uncertainty.

Keywords: Fuzzy set, Neutrosophic set, Big Science, Deep Science, Plithogenic set

1 | Big Science and Deep Science

First, we provide an explanation of Big Science and Deep Science. Big Science refers to large-scale, well-funded collaborative research aimed at refining established theories and generating substantial experimental or observational data [38, 7, 49]. The systematic definition of Big Science is presented below.

Definition 1 (Big Science)**.** [49] A *Big Science* initiative is a scientific research enterprise distinguished by:

(1) *Enormous Scale and Funding.* These projects typically demand significant capital investment, ranging from multi-million to multi-billion dollars, to finance large facilities, specialized instrumentation, or high-throughput data pipelines.

(2) *Centralized Coordination and Management.* Complex organizational frameworks coordinate hundreds or even thousands of scientists, technicians, and administrators who participate under robust institutional or governmental oversight.

(3) *Incremental Investigations of Mainstream Theories.* Big Science frequently aims to refine, confirm, or slightly extend widely accepted models (e.g. the Standard Model of particle physics), seeking precision tests or new parameter measurements.

(4) *High-Volume Output (Publish or Perish).* Researchers in these frameworks are commonly encouraged or required to produce a substantial number of conference papers, journal articles, or experimental technical reports.

Formally, we can represent a Big Science project as

$$\mathcal{B} = \Big(\text{Funding, Large\_Collab, Central\_Administration, Mainstream\_Theories, Data\_Output}\Big).$$

Example 2 (Particle Accelerators for High-Energy Physics)**.** One prominent example of Big Science is the construction and operation of large particle accelerators[32], such as the *Large Hadron Collider (LHC)* [12] at CERN. This facility:

- Demands multi-billion-dollar funding for equipment, detectors, and infrastructure.
- Involves thousands of scientists in global collaborations with a heavily managed schedule and governance structure.
- Seeks to measure properties of subatomic particles and search for anomalies, broadly extending the Standard Model but generally preserving its core assumptions.
- Produces high publication volume: each large collaboration publishes hundreds of papers per year, each typically signed by hundreds of coauthors.

Hence, the LHC project is a quintessential *Big Science* endeavor.

Example 3 (Space Telescope Missions)**.** Large-scale space telescope missions[53] (such as the *James Webb Space Telescope*[25]) also illustrate Big Science. They involve:

(1) Centralized budgets from multiple agencies (e.g. NASA[27], ESA[29], CSA).

(2) Complex management to coordinate large teams of engineers, scientists, and administrators.

(3) A primary goal to refine or expand upon the standard cosmological model and existing astrophysical paradigms, although it often leads to incremental parameter-fitting or observational confirmations.

Thus, these missions epitomize the "Big Science" paradigm in observational astrophysics.

Deep Science emphasizes rethinking foundational principles, exploring alternative frameworks, and proposing paradigm-shifting theories to address fundamental scientific questions.

Definition 4 (Deep Science)**.** [49] A *Deep Science* investigation prioritizes fundamental conceptual revision, radical exploration, or critical rethinking of existing frameworks. It is typically characterized by:

(1) *Model-Critical Mindset.* Deep Science explicitly questions or reexamines foundational postulates of mainstream theories instead of merely refining them.

(2) *Openness to Contradictions.* Anomalies, logical inconsistencies, or new experimental discrepancies are treated not as minor fixes but as *opportunities* to propose alternative postulates or new structural concepts.

(3) *Potential for Paradigm Shifts (cf.*[5]*).* The outcome of a successful Deep Science project is frequently a novel framework that either supersedes or comprehensively unifies prior models in ways not achievable by incremental patches.

(4) *Flexible Methodologies.* Rather than focusing on a single large facility or dataset, Deep Science may synthesize diverse data sources, advanced pattern recognition (e.g. deep learning), new mathematical structures, or philosophical perspectives to achieve deeper insight.

Mathematically, one can denote a Deep Science project as a tuple:

$$\mathcal{D} = \Big(\text{Conceptual\_Exploration},\ \text{Alternative\_Axioms},\ \text{Data\_Integration},\ \text{Iterative\_SelfCritique},\ \text{Potential\_Leap}\Big),$$

where Potential_Leap signifies the possibility of fundamentally new scientific paradigms.

Example 5 (Revisiting Core Postulates of Particle Physics)**.** An investigation that thoroughly rethinks the assumption of "pointlike" particles in the Standard Model, proposing instead a *topological wave-based* approach or a *cellular automaton* model (cf.[41]) of matter, exemplifies *Deep Science*. It may:

- Criticize the unverified assumption of pointlike quarks.
- Seek explanations for confined quarks without postulating complicated color charge fields.
- Use alternative frameworks (e.g. zero-point fluctuations[39], wave solitons[36], or groupoid-based structures) to unify observed data.

Such a proposal, if consistent, might significantly alter the conceptual underpinnings of high-energy physics, representing a deeper transformation than typical Big Science refinements.

Example 6 (Revisiting General Relativity and Spacetime Structure)**.** A Deep Science approach may question the classical continuity assumption in Einstein's spacetime manifold. One might propose:

(1) A discrete lattice geometry or a causal network as a new baseline for gravity.

(2) Incorporate emergent phenomena (e.g. thermodynamic analogies [35] or fractal models) to replace geometric curvature with topological or spectral properties.

(3) Seek a unified representation bridging quantum field theory and discrete gravitational degrees of freedom.

Such an approach attempts to fundamentally reformulate gravitational theory, rather than only adding parameters (*dark energy*[9], *dark matter*[4], etc.) to patch standard ΛCDM cosmology. Consequently, it is a quintessential example of Deep Science's paradigm.

Example 7 (Validation by Data versus Fundamental Rebuilding)**.** In Big Science contexts, a project like the LHC seeks to confirm the Standard Model's predictions or search for small perturbations (e.g. subtle evidence of supersymmetry[16]). *Deep Science* would instead ask: "Is the Standard Model the correct theory, or is it a limiting approximation of a deeper discrete wave structure of matter?" The latter approach might eventually reveal new theoretical directions (removing the assumption of pointlike quarks, for instance). Because that line of questioning goes beyond verifying or tweaking the Standard Model, it is out of typical Big Science's incremental domain.

Example 8 (Alternate Universe Conjectures)**.** In cosmology [34, 13, 52], a typical Big Science project might focus on measuring cosmic microwave background (CMB) [14] anisotropies with greater precision to refine the parameters of the ΛCDM model. This approach aligns with the goals of Big Science: improving established theories by making incremental advancements and obtaining more accurate data.

In contrast, a Deep Science perspective would challenge foundational assumptions such as the continuity of spacetime (cf.[6]) or the idea that the universe lacks a preferred reference frame. Instead, it could propose entirely new models, such as a cosmos based on multi-scale structures or fractal geometries.

This shift from refining parameters (Big Science) to questioning and rebuilding the underlying framework (Deep Science) demonstrates how Deep Science expands the conceptual boundaries beyond the incremental scope of Big Science.

2 | Plithogenic Big Science and Deep Science

The Plithogenic Set is a type of set that generalizes Neutrosophic Sets, Fuzzy Sets, and similar frameworks [48, 47, 50]. Plithogenic Sets have been extensively studied in various contexts, including Graphs, Algebra, Groups, and Networks[24, 18, 17, 23, 21, 1, 22, 19, 20]. The formal definition of the Plithogenic Set is presented below.

Definition 9 (Plithogenic Set)**.** [48, 47] Let S be a universal set, and $P \subseteq S$ be a subset of elements. A *Plithogenic Set* PS is a mathematical structure defined as:

$$PS = (P, v, Pv, \text{DAF}, \text{DCF}),$$

where:

- v represents an attribute associated with the elements in P.
- Pv is the set of all possible values that the attribute v can take.
- $\text{DAF} : P \times Pv \to [0,1]^s$ is the *Degree of Appurtenance Function*, which assigns a vector of membership degrees to each pair (p, v) where $p \in P$ and $v \in Pv$.
- $\text{DCF} : Pv \times Pv \to [0,1]^t$ is the *Degree of Contradiction Function*, which quantifies the contradiction between two attribute values $a, b \in Pv$.

The functions DAF and DCF must satisfy the following axioms for all $a, b \in Pv$:

(1) *Reflexivity of the Contradiction Function:*

$$\text{DCF}(a, a) = 0,$$

indicating that an attribute value is not contradictory to itself.

(2) *Symmetry of the Contradiction Function:*

$$\text{DCF}(a, b) = \text{DCF}(b, a),$$

ensuring that the contradiction relationship is symmetric.

Example 10 (Variations of Plithogenic Sets [47, 23])**.** Common types of Plithogenic Sets arise by choosing different dimensions s and t:

- $s = t = 1$: A *Plithogenic Fuzzy Set*, generalizing standard fuzzy sets[54, 56, 55, 57] by embedding them in a plithogenic attribute framework.
- $s = 2, t = 1$: A *Plithogenic Intuitionistic Fuzzy Set*, capturing membership and non-membership plus a contradiction measure (cf.[2, 3]).
- $s = 3, t = 1$: A *Plithogenic Neutrosophic Set*, encompassing truth, indeterminacy, and falsity degrees (cf.[43, 44, 45]).
- $s = 4, t = 1$: A *Plithogenic Quadripartitioned Neutrosophic Set* (cf.[31, 30, 42]).
- $s = 5, t = 1$: A *Plithogenic Pentapartitioned Neutrosophic Set* (cf.[10, 40]).

These classes generalize fuzzy or neutrosophic sets by incorporating multi-criteria membership dimensions and inter-value contradictions.

Using the concepts outlined above, Big Science and Deep Science are extended within the framework of Plithogenic Sets. It is important to note that these extensions are currently conceptual in nature. The authors believe that further theoretical exploration and investigation into potential applications of these Plithogenic Set-based extensions will be essential in the future.

Definition 11 (Big Plithogenic Science)**.** *Big Plithogenic Science* is a research enterprise that:

(1) Retains the characteristic elements of *Big Science*:

- Large-scale funding and resource demands.
- Centralized or government-managed collaborations.
- Predominantly incremental investigations of mainstream theories.
- High-volume publication ethos.

(2) Incorporates or is modeled by a *Plithogenic Set* structure, wherein each "scientific parameter" or "theoretical stance" is treated as an *attribute* in a plithogenic sense, and inter-value contradictions are measured via the pCF function.

Formally, we denote $\mathcal{B}_{plith}$ as:

$$\mathcal{B}_{plith} = \Big(\mathcal{B},\ PS,\ \text{High\_ScaleCollab},\ \text{Contradiction\_Resolution}\Big),$$

where $\mathcal{B}$ is a classical Big Science tuple (funding, mainstream theories, data output, etc.), and PS is a *Plithogenic Set* that captures how different sub-teams or sub-theories define or evaluate crucial attributes—with pCF capturing the contradictions or disagreements.

Example 12 (Big Plithogenic Science Variations)**.** Different types of *Big Plithogenic Science* projects can emerge depending on the dimensionality s and t of the underlying Plithogenic Set:

- $s = t = 1$: A *Big Plithogenic Fuzzy Science* project, where scientific investigations are modeled with fuzzy attributes and contradictions resolved using a simple contradiction function.
- $s = 2, t = 1$: A *Big Plithogenic Intuitionistic Fuzzy Science* project, which incorporates both membership and non-membership evaluations, alongside contradictions between them.
- $s = 3, t = 1$: A *Big Plithogenic Neutrosophic Science* project, allowing for truth, indeterminacy, and falsity degrees to represent the scientific parameters or sub-models, coupled with inter-value contradictions.
- $s = 4, t = 1$: A *Big Plithogenic Quadripartitioned Science* project, extending neutrosophic modeling to four distinct appurtenance dimensions.
- $s = 5, t = 1$: A *Big Plithogenic Pentapartitioned Science* project, representing even more complex multi-criteria scientific investigations.

These projects generalize standard Big Science frameworks by embedding multi-dimensional membership and contradiction-handling capabilities into their investigative methodologies.

Example 13 (Large Collaborative Particle Physics within a Plithogenic Framework)**.** Consider the collaborative efforts at the Large Hadron Collider (LHC), involving experiments like ATLAS and CMS. These collaborations illustrate the principles of *Big Plithogenic Science* when viewed through the lens of a Plithogenic Set:

(1) *Big Science Characteristics:* The LHC operates on a massive scale, with multi-billion-dollar funding and thousands of researchers working collaboratively across institutions worldwide. The overarching goal is to investigate phenomena within the Standard Model of particle physics, such as the properties of the Higgs boson and potential evidence of new physics.

(2) *Diverse Analytical Approaches:* Within each collaboration (e.g., ATLAS or CMS), multiple sub-teams analyze the same experimental data using different methodologies. For example:

- Some teams use different parton distribution functions (PDFs) to model the internal structure of protons.
- Others apply distinct cross-section computation methods to estimate the likelihood of specific particle interactions.
- Variations in statistical treatments or data reconstruction techniques may also arise.

These variations reflect different "stances" or "values" within the collaboration's analytical framework.

(3) *Plithogenic Perspective:* By adopting a *Plithogenic Set* framework:

- The attribute v represents the "analytical approach" or "modeling methodology" employed by each sub-team.
- The possible values $Pv = \{\alpha_1, \alpha_2, \dots\}$ correspond to specific choices, such as particular PDFs or cross-section models.

- The *Degree of Appurtenance Function* (pdf) quantifies the confidence or alignment of each sub-team's analysis with the observed data.
- The *Contradiction Function* ($pCF(\alpha_i, \alpha_j)$) measures the level of disagreement or tension between different sub-teams' results or parameter choices.

(4) *Integration of Contradictions:* The Plithogenic framework enables a systematic assessment of contradictions within the collaboration. For instance:

- If one analysis finds an anomaly in particle cross-sections while another does not, pCF can quantify the degree of inconsistency.
- These contradictions are not dismissed but are used to refine the overall understanding, identifying areas where the Standard Model might need modification or where experimental uncertainties dominate.

The LHC project is a quintessential example of *Big Science* due to its scale and collaborative structure. When reinterpreted through a Plithogenic framework, it becomes a *Big Plithogenic Science* initiative, where contradictions among sub-teams' methodologies are quantified and integrated as part of a structured effort to advance particle physics.

Example 14 (Space Missions Incorporating Conflicting Theories)**.** Consider a space telescope mission involving collaboration among multiple space agencies (e.g., NASA, ESA, CSA). The mission aims to study cosmic acceleration but includes distinct and potentially conflicting theoretical submodels to explain this phenomenon, such as dark energy, modified gravity, or quantum effects.

In the Plithogenic framework:

- The attribute set Pv consists of the submodels α_1 (dark energy), α_2 (modified gravity[8]), α_3 (quantum effects[26]), and so on.
- Each submodel $\alpha_i \in Pv$ is assigned a degree of alignment with observational data using the *Degree of Appurtenance Function* $pdf(\alpha_i)$, where $pdf : P \times Pv \to [0,1]^s$.
- Contradictions between submodels (e.g., the incompatibility between dark energy and modified gravity) are quantified using the *Degree of Contradiction Function* $pCF(\alpha_i, \alpha_j)$, where $pCF : Pv \times Pv \to [0,1]^t$. For instance, $pCF(\alpha_1, \alpha_2)$ captures the extent to which dark energy and modified gravity hypotheses conflict.

This approach allows the mission to retain the characteristic traits of *Big Science*, such as large-scale funding, extensive collaboration, and high data output, while adopting a *Plithogenic perspective* to systematically incorporate and analyze conflicting hypotheses. This illustrates how *Big Plithogenic Science* formalizes the coexistence of multiple contradictory submodels within a single scientific endeavor.

Definition 15 (Deep Plithogenic Science)**.** *Deep Plithogenic Science* is a conceptual extension of *Deep Science*, in which:

(1) The investigation explicitly or systematically reconsiders fundamental premises of a domain.

(2) Each premise or theoretical approach is modeled as *attributes* and *values* in a *Plithogenic Set* structure.

(3) Potentially *radical* contradictions or alternative axioms are captured by pCF, with an iterative approach to harness these contradictions to propose entirely new frameworks or paradigms rather than refining established models.

(4) The primary objective is to integrate contradictory stances or datasets into deeper unifying theories, emphasizing a truly *plithogenic* approach to synergy (or synergy-through-contradiction).

Mathematically:

$$\mathcal{D}_{plith} = \Big(\mathcal{D},\, PS,\, \text{Radical\_ConceptualShift},\, \text{Contradiction\_Catalyst}\Big),$$

where $\mathcal{D}$ is a Deep Science tuple (conceptual exploration, alternative axioms, etc.), while PS and Contradiction_Catalyst clarify how contradictory or multi-level "theories" (attributes) are used to build new conceptual frameworks beyond incremental mainstream expansions.

Example 16 (Deep Plithogenic Science Variations)**.** In *Deep Plithogenic Science*, the use of Plithogenic Sets allows for diverse approaches based on dimensionality s and t:

- $s = t = 1$: A *Deep Plithogenic Fuzzy Science* approach, focusing on rethinking fundamental premises using fuzzy-like membership values and contradictions.
- $s = 2, t = 1$: A *Deep Plithogenic Intuitionistic Fuzzy Science* approach, utilizing both membership and non-membership degrees to critically analyze competing scientific axioms or hypotheses.
- $s = 3, t = 1$: A *Deep Plithogenic Neutrosophic Science* approach, employing truth, indeterminacy, and falsity degrees to explore radical paradigm shifts and resolve theoretical contradictions.
- $s = 4, t = 1$: A *Deep Plithogenic Quadripartitioned Science* approach, providing more nuanced appurtenance dimensions for deep conceptual integration.
- $s = 5, t = 1$: A *Deep Plithogenic Pentapartitioned Science* approach, allowing even higher-dimensional modeling of contradictions and alternative conceptual frameworks.

These approaches move beyond the incremental refinements of Big Science, emphasizing a systematic incorporation of contradictions and multi-faceted conceptual exploration.

Theorem 17 (Big Plithogenic Science as a Plithogenic Set Structure)**.** *Any project classified as* Big Plithogenic Science *(denoted by $\mathcal{B}_{plith}$) can be interpreted through a Plithogenic Set structure. This means that all of the project's theoretical approaches and organizational details can be represented as part of a single* Plithogenic Set*, meeting the conditions from Definition.*

Proof: Let $\mathcal{B}$ represent a typical Big Science project, which usually involves:

$$\text{Mainstream\_Theories, Sub-Model\_Variations, Calibration\_Procedures, ...}$$

To model it as a *Plithogenic Set*, proceed as follows:

- *Elements (p) in the Set (P):* We identify the different sub-models or parameter options within the project and collect them in a subset P. For instance, these could be various parameter sets or different theoretical stances maintained by sub-teams.
- *Attribute (v) and Range (Pv):* Let the attribute v denote "the chosen stance or parameter configuration." The range Pv consists of all possible stances (e.g., $\alpha_1, \alpha_2, ...$), each representing a distinct viewpoint or numerical setting.
- *Degree of Appurtenance Function (pdf):* Define
$$pdf : P \times Pv \rightarrow [0,1]^s$$
which records how strongly each sub-model stance or group element p (such as a particular research team or analysis approach) supports a given stance α_i. This can capture the level of agreement or preference for using α_i.
- *Degree of Contradiction Function (pCF):* Define
$$pCF : Pv \times Pv \rightarrow [0,1]^t$$
to measure how contradictory two stances α_i, α_j are. For example, if α_i and α_j represent parameter settings that cannot both be valid under the same conditions, $\text{pCF}(\alpha_i, \alpha_j)$ would be high.
- *Reflexivity and Symmetry:* By construction, no stance contradicts itself, so $pCF(a, a) = 0$. Moreover, the contradiction between α_i and α_j is presumably symmetric, so $pCF(\alpha_i, \alpha_j) = pCF(\alpha_j, \alpha_i)$.

Hence, these components satisfy the definition of a Plithogenic Set. Since $\mathcal{B}_{plith}$ inherits this structure, it can be viewed as a *Plithogenic structure*. This completes the proof. □

Example 18 (Rethinking the Standard Model via Plithogenic Integration of Alternative Theories)**.** In high-energy physics, a *Deep Plithogenic Science* approach can be used to explore alternative frameworks beyond the Standard Model. This approach involves the following steps:

(1) *Challenging Pointlike Assumptions:* Move away from the traditional assumption of pointlike particles (e.g., quarks and electrons[28]). Instead, consider alternative theories such as wave-based models or topological-substance theories to describe fundamental particles.

(2) *Plithogenic Representation of Theories:* Represent these alternative theories as elements of a Plithogenic Set. For instance:
$$Pv = \{\alpha_1, \alpha_2, \alpha_3, ...\},$$
where α_1 represents a wave-soliton model, α_2 a discrete-lattice QCD approach, and α_3 a topological framework, among others. Each α_i is a potential candidate theory.

(3) *Quantifying Contradictions:* Employ the *Degree of Contradiction Function* $pCF(\alpha_i, \alpha_j)$ to evaluate the extent of disagreement or tension between pairs of theories. For example, $pCF(\alpha_1, \alpha_2)$ might capture the conflict between the wave-soliton and discrete-lattice QCD models.

(4) *Developing a Meta-Model:* Use the contradictions captured by pCF as a basis to propose a unified "meta-model." This meta-model aims to systematically integrate the insights from the alternative theories while addressing the contradictions as opportunities for deeper theoretical advancements.

By rethinking foundational principles (making it *Deep*) and using a structured framework to handle conflicting theoretical perspectives (making it *Plithogenic*), this approach exemplifies how high-energy physics can benefit from a Plithogenic perspective.

Example 19 (Deep Plithogenic Cosmology: Reconciling Alternative Universe Models)**.** In cosmology, a *Deep Plithogenic Science* approach can be applied to explore alternative frameworks that challenge standard assumptions such as the continuous manifold structure of spacetime (cf.[6]) or the ΛCDM model (the Lambda Cold Dark Matter model[51]). This approach can be understood through the following steps:

(1) *Questioning Standard Premises:* Instead of assuming a smooth, continuous spacetime and ΛCDM as the definitive cosmological model, alternative ideas are considered. For example:

- *Discrete Cosmologies*[15]*:* Theories where spacetime consists of discrete units or a lattice structure.
- *Fractal Cosmologies*[33]*:* Models suggesting the universe exhibits self-similar, fractal-like distributions of matter at large scales.
- *Multi-Phase Expansion Models:* Hypotheses proposing that cosmic expansion has undergone distinct phases, influenced by varying physical laws or conditions.

(2) *Plithogenic Representation of Theories:* These alternative cosmologies are treated as attributes in a Plithogenic Set. For instance:
$$Pv = \{\alpha_1, \alpha_2, \alpha_3\},$$
where α_1 corresponds to a discrete lattice cosmology, α_2 to a fractal universe model, and α_3 to a multi-phase expansion model.

(3) *Capturing Contradictions:* The *Degree of Contradiction Function* $pCF(\alpha_i, \alpha_j)$ quantifies the degree of conflict between pairs of theories. For example:

- $pCF(\alpha_1, \alpha_2)$ could measure the discrepancy between discrete spacetime and fractal distributions of matter.
- $pCF(\alpha_2, \alpha_3)$ could capture conflicting predictions about cosmic acceleration from fractal models versus multi-phase expansion.

(4) *Building a Unified Framework:* Using the insights gained from the pCF evaluations, a "meta-model" can be constructed. This meta-model synthesizes the strengths of the individual cosmologies while resolving or embracing their contradictions as opportunities for a more comprehensive understanding of the universe.

This approach is *Deep* because it fundamentally rethinks core assumptions about spacetime and cosmic evolution. It is *Plithogenic* because it systematically integrates multiple, potentially contradictory cosmological theories using a structured framework.

Example 20 (Deep Plithogenic Cosmology versus Big Plithogenic Cosmology)**.** Consider two approaches to studying cosmology through the lens of Plithogenic Science:

Big Plithogenic Cosmology: In this approach, a research project might focus on refining and analyzing variations of the widely accepted ΛCDM model (the Lambda Cold Dark Matter model), which is the standard model of cosmology. Using a *Plithogenic Set*, the project represents different parameter settings or minor variations of ΛCDM (e.g., different values for the dark energy constant or matter density) as elements in the set.

- The attribute v corresponds to "theoretical variations within ΛCDM."
- The contradiction function $\text{pCF}(\alpha_i, \alpha_j)$ measures the degree of disagreement between two parameter settings α_i and α_j.
- The goal is to assess which parameter sets are more plausible by minimizing contradictions or identifying subsets of mutually consistent theories.

This methodology adheres to the general framework of *Big Science* by focusing on incremental improvements to an existing mainstream model.

Deep Plithogenic Cosmology: In contrast, a *Deep Plithogenic Cosmology* project challenges the foundational assumptions of the ΛCDM model entirely. Instead of tweaking parameters within ΛCDM, the project proposes and explores radically different cosmological frameworks, such as:

- Fractal cosmology[11, 33], where the universe's structure exhibits self-similar patterns at various scales.
- Discrete-lattice cosmology(cf.[37]), where spacetime is not continuous but consists of a grid-like structure.
- Multi-phase universe models, which hypothesize that the universe transitions through distinct phases of expansion governed by different fundamental rules.

Here:

- The attribute v represents "different cosmological paradigms" instead of minor variations of a single model.
- The contradiction function $\text{pCF}(\alpha_i, \alpha_j)$ quantifies the degree of conflict or tension between these fundamentally different paradigms (e.g., ΛCDM vs. fractal cosmology).
- This approach leverages these contradictions to drive the search for a unified "meta-model" that integrates or transcends the competing theories.

Key Difference: While *Big Plithogenic Cosmology* focuses on refining and reconciling variations within the accepted ΛCDM framework, *Deep Plithogenic Cosmology* steps beyond this framework to question its validity and explore entirely new models. This distinction illustrates how *Deep Plithogenic Science* broadens the conceptual scope of *Big Plithogenic Science*, enabling paradigm-shifting advancements.

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Data Availability

As this study is rooted in theoretical and mathematical exploration, no data was analyzed. Future research is encouraged to incorporate empirical or data-driven approaches for further investigation.

Ethical Approval

This work is purely theoretical and did not involve any experimental procedures with humans or animals.

Conflicts of Interest

The authors confirm there are no conflicts of interest associated with this publication.

Disclaimer

This study introduces theoretical concepts that have not yet been practically implemented or validated. We encourage subsequent research to test and refine these ideas empirically. While care has been taken to ensure the accuracy of references and arguments presented, errors or oversights may still exist. Readers are encouraged to independently verify cited materials. The views and interpretations in this paper reflect those of the authors and not necessarily their affiliated institutions.

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LOGIC & THOUGHT

The MultiAlist System of Thought

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Abstract: The goal of this short note is to expand the concepts of 'pluralism', 'neutrosophy', 'refined neutrosophy', 'refined neutrosophic set', 'multineutrosophic set', and 'plithogeny' (Smarandache 2002, 2013, 2017, 2019, 2021, 2023a, 2023b, 2023c), into a larger category that I will refer to as **MultiAlism** (or **MultiPolar**). As a straightforward generalization, I propose the conceptualization of a *MultiPolar System* (different from a *PluriPolar System*), which is formed not only by multiple elements that might be random, or contradictory, or adjuvant, but also by accepting features from more than one basic system (UniPolar, BiPolar, TriPolar, or PluriPolar systems). PluriAlism is a closed dynamic system without neutralities nor indeterminacies, while MultiAlism is an open dynamic system with neutralities and indeterminacies. PluriAlism is a uni-system (formed by elements from a single system), while MultiAlism is a MultiSystem (formed by elements from many systems).

Keywords: Monism; Dualism; Trialism; Pluralism; Neutrosophy; Refined Neutrosophy; MultiNeutrosophy; Refined Neutrosophic Set, MultiNeutrosophic Set; Plithogeny; Multialism; Zoroastrianism; Neutral Monism; neo-Vedanta.

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1. Introduction

Many casual interactions with non-Western peers from academics have opened my eyes during the past two decades to themes that – except for a few committed and non-biased specialists – are still approached superficially in what we still call *The Occident*. In our Western World, some Eastern ideas, principles, and actions remain misunderstood or wrongly judged, because we still have an obstinacy to fit them without nuances into our unique methods of thoughts. The frequent visits I made to the Non-Western World, to international conferences and scientific seminars, or post-doctoral in applied mathematical and technological research, provided me with an unmediated contact with these diverse cultures, allowing me to improve the understanding of their systems of thinking, and resulting in many traveling memories I wrote about their custom, religion, philosophy, history, geography, and life.

In this regard, *Zoroastrianism* serves as an illustration. Its somehow paradoxical aspects bedazzle most Western observers, making them confused when they try to categorize the religion among monotheistic, dualist, or pluralist systems. However, imposing concepts whose meanings have been referenced to other doctrines will not succeed in an attempt to fully define this religion, and rather than pointing out monotheistic or polytheistic features, or even prompt neutrosophic features – as I did myself in one of my scilogs (Smarandache, 2023, 84 et ss.) – would it not be more beneficial for thinking to broaden the current categories?

Alternatively, we may look in the Western philosophy at the *neutral monism*, which – to put it simplistically – holds that the mind and body are not two distinct entities, but are rather *composed of*

the same neutral "stuff", or as a fluid (indeterminate) margin between non-physical and physical (Smarandache 2023c). In this respect, David Hume proposed "impressions" or "perceptions" as primary realities of experience, while William James thought that the *neutral core material* is a "booming, buzzing confusion" called "pure experience", and Bertrand Russell, more towards our times, referred to the *neutral entities* as "sensibilia". *Neutral monism* is actually pluralist[1] in that it recognizes the existence of multiple such elements (as opposed to metaphysical monism), but it is monist in that it holds that the fundamental components of the universe are all of the same kind (against mind-body dualism). Since we do not fall strictly into the category of monism anymore, by accepting neutralities or indeterminacies – would it not be more beneficial for thinking to broaden the current categories?

In what follow, I will provide a few more examples of this kind; however, I have no doubt the readers can add their own examples to complete the picture. The examples are not limited, but the question persists: would it not be more beneficial for thinking to broaden the current categories?

It happened that I was reading a very recent study by Ethan Brauer once the sketch of an answer to the above question has settled on its own on the paper. Brauer's extensive paper addresses a completely different and narrow topic, but which can be expanded from its limited sphere – modal analysis of potential infinity. Brauer extended a theory of classical second-order arithmetic to include intrinsically well-motivated axioms for lawless sequences.[2] Free choice sequences are central to the intuitionistic theory of the continuum, but since intuitionistic analysis theorems defy the classical analysis, many mathematicians reject the concept. (Brauer 2023)

Mutatis mutandis, our quest is similar.

2. UniPolar, BiPolar, TriPolar, PluriPolar, and more general MultiPoar Systems. Definitions and examples

In this section, I will scrutinize definitions and meanings of the basic Western systems (of organization) of thoughts, and exemplify them, including scenarios from Eastern doctrines.

2.1. Monism: all is one

Monism is a philosophy and metaphysical doctrine that postulates a single, ultimate, cohesive reality. The universe is composed of a single, overarching 'idea' or 'substance', or only one ultimate deity, <A>. Everything else is just a manifestation of this one reality/substance/deity. This is a **UniPolar System**,

i.e. $\underline{<A> = \infty}$, where <A> is an 'idea', a 'substance', *et caetera*, and ∞ is 'world', 'reality', 'all'.

The monist schools of philosophy claim that either everything is material (materialism) or everything is mental (idealism), and abolish the distinction between the body and the mind in favor of explaining all phenomena as expressions of a single unifying principle.[3]

Christian Wolff coined the term 'monism' in the eighteenth century in his work "Rational Thoughts" [German Logic] (1728): "we must admit of one necessary, self-existent Being" (Wolff, 1770). Wolff delves further into the systems of mind-body connection in the "Psychologia Rationalis" (1734). He believes in the validity of Leibnizian monadology, but only applied to ideas, refuting the monistic panpsychism that is central to Leibniz's metaphysics.[4]

[1] Griffin, N. (1998). 'Neutral monism'. In *The Routledge Encyclopedia of Philosophy*. Taylor and Francis. Retrieved 23 Dec. 2023, from https://www.rep.routledge.com/articles/thematic/neutral-monism/v-1.

[2] Which leaded Brauer to a theory that is called MCLS.

[3] O'Conaill, D.(2019). 'Monism.' In The Routledge Encyclopedia of Philosophy. Taylor and Francis. Retrieved 21 Dec. 2023, from https://www.rep.routledge.com/articles/thematic/monism/v-2.

[4] Hettche, Matt and Corey Dyck, "Christian Wolff", *The Stanford Encyclopedia of Philosophy* (2019), Edward N. Zalta (ed.). Retrieved 17 Dec. 2023, from https://plato.stanford.edu/archives/win2019/entries/wolff-christian.

Looking back in time and towards the East, monism has been widely discussed in connection with the Indian philosophy, particularly in "Uttara Mīmāṃsā" (also known as "Vedānta"). Many schools of thought have emerged out there, all basing their doctrines on the authority of the same corpus known as "Prasthānatrayī".

In Hinduism, the idea of Brahman —the ultimate reality or supreme cosmic power —is frequently connected to monism. Most Hindus follow monastic principles and hold that Brahman is everything and everything is Brahman.[5] The philosophy of Advaita Vedānta, which is frequently referred to as a type of absolute nondualism, also reflects this viewpoint.

In one accessible simplification, one can reduce the monism to two types: a substantive monism, in religions like Buddhism and Hinduism in the East, or philosophers like Spinoza in the West, and attributive monism, with sub-tyes as idealism, physicalism, or neutral monism. The first reduces the reality to a single substance, or states that the world is only varied because this one substance exists in plural forms, while the second asserts that there is a one category of being that encompasses a wide plurality of distinct objects or substances.

Despite being essentially monistic, attributive monism appears to be rather pluralistic, but substantival monism is strongly hostile to pluralism.

In that it reduces the physical cosmos to a single principle, pantheism is similar to monism: "Pantheists are monists" (Owen, 1971, 65), even though the pantheist deity is imperfect, expanding and continuously creating, or also extending beyond space and time in panentheism —a conceptions of God present as well in some Christian confessions —therefore surpassing the simplification of monistic attribution.

2.2. Dualism: all is two

Dualism explains the world (or reality) by two fundamental, diametrically opposed, and irreducible principles. In religion, it generally refers to the conviction that the universe was created by two ultimate antagonistic forces, gods, or groups of angelic or demonic creatures. Since dualism is a system formed by two contrasted parts, this is a **BiPolar System**:

i.e. $\underline{<A> + <antiA>} = \infty$.

where <A> is an 'idea', a 'substance', *et caetera*, <antiA> is its opposite or negation, and ∞ is 'world', 'reality', 'all'.

I would probably not be wrong if I affirmed that this system is for ages a dominant worldview in Western way of thinking, with Descartes and Hegel being the first two figures that spring to mind, completed by Kant's cognitive dualism, which distinguished between the faculties of sensibility and understanding, . Examples of epistemological dualism include being and thought, subject and object; and, on the other hand, examples of metaphysical dualism being matter and spirit, body and mind, good and evil.

Glancing eastward, we observe that most historians of religion use the ancient Iranian religion Zoroastrianism as a clear case of eschatological dualism, advocating that it is based on two conflicting principles: Ahura Mazda, the deity of light and truth, and Angra Mainyu, the destroying enemy.

An ongoing conflict exists between the good, spiritual realm of light and the bad, material realm of darkness, according to the ancient Iranian religion of Manichaeism.

Furthermore, as its name says by ityself, *dvaita* — the Sanskrit word *dvaita* actually means 'dualism' (Flood, 1996, 245) — is a dualist school of *Vedanta*, asserting that there is an everlasting separation between the particular self and the ultimate, in opposition to the *advaita* (non-dualist) philosophy. Although *dvaita* was dualist in that sense, it proposed an autonomous God named Vishnu as the ruler of the independent and separate entities of matter and soul. More specifically, *dvaita* recognized three absolute and eternally existing entities: God, souls (*atman*), and primordial substance (*prakriti*).

[5] Leeming, D.A. (2014). 'Brahman.' In: Leeming, D.A. (eds) Encyclopedia of Psychology and Religion. Springer, Boston, MA. https://doi.org/10.1007/978-1-4614-6086-2_9052.

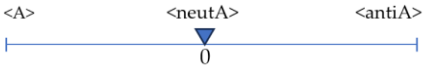
2.3. Trialism: all is three

Trialism was introduced in philosophy by John Cottingham as "a grouping of three notions" (Cottingham, 1985, 219), an alternative viewpoint to Descartes' dualism, with the addition of sensation next to mind and body: "It turns out that there are features that belong to the mind alone, features that belong to the body alone, and what may be called hybrid features - features that belong to man *qua* embodied being" (*Ibidem*; see also Cottingham, 2021).

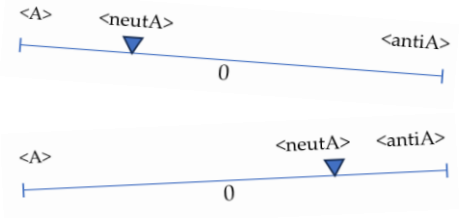
Trialism is thus a system formed by three contrasted or entirely different parts, and similarly, *trichotomy* is a division of three opposites (or entirely different) two by two things.

A three-poles system was also proposed by *neutrosophy* (Smarandache 1995, 2013), which operates with three independent opposites, found in equilibrium: <A>, <neutA>, and <antiA>, called *Neutrosophic Triad*. All 'ideas' <A> are considered in conjunction with their opposites or negations <antiA> and the range of neutralities <neutA> between them, while <nonA> is the collective term for the ideas <antiA> and <neutA>. In neutrosophy, the three poles may be fluid two by two.

The balance between <A> and <antiA> rests on <neutA>. In other words, <neutA> is imagined as a buffer zone between <A> and <antiA>:



Moving <neutA> to the left, or to the right, i.e. if the neutral/indeterminacy part is pushed towards <A>, or <antiA> (the indeterminacy degree increases), then one of them gets stronger (having less indeterminacy), and the balance gets in disequilibrium:



Based on neutrosophy, the associated **TriPolar System** can be described as:

$\underline{<A> + <neutA> + <antiA> = \infty}$,

where <A> is an 'idea', a 'substance', *et caetera*, <antiA> is its opposite or negation, <neutA> is the range of neutralities between them, and ∞ is 'world', 'reality', 'all'.

I point out here no more than that the neutrosophy is an extension of both the ancient Chinese Yin-Yang philosophy and dialectics (Smarandache 2013), and also remind the reader that the trialism was associated with Christianity as well, e.g. for holding that human beings are composed of three separate essences: a body, a soul, and a spirit.[6]

2.4. Pluralism: all is plurality

Pluralism is a wordview of plurality, used in philosophy to contrast with monism (the idea that everything is one), with dualism (the idea that everything is two), and arguably with trialism (the idea that everything is three). Pluralism can be defined as a system in which more than two (arguably three) groups, principles, states, ideas, *et caterea*, coexist. This is a **PluriPolar System**:

$\underline{<pluriA> = \infty}$,

where <pluriA> means more than two (arguably three) 'ideas', *et caetera*, and ∞ is 'world', 'reality', 'all'.

In metaphysics, pluralism is the idea that reality is actually made up of a variety of substances found in nature, while in ontology the concept describes various forms, kinds, or modes of existence.

[6] This understanding stems from taking 1 Thessalonians 5:231 literally: "And the very God of peace sanctify you wholly; and I pray God your whole spirit and soul and body be preserved blameless unto the coming of our Lord Jesus Christ."

Buddhism is given as an example of a pluralistic religion. Many Buddhist traditions do not declare a single ultimate truth and recognize the validity of multiple paths to enlightenment, advocating conversation and understanding with people of other faiths.

Another example might be the Bahá' Faith, which holds that all major faiths have the same spiritual basis, are descended from the same divine source (God), and differ only in their social teachings in accordance with the necessities of the eras in which they were revealed.

3. MultiAlism: all is open

We observed in the short, quick and without going into depth evaluation of the basic systems that we previously discussed that it is challenging to strictly include some non-Western doctrines (or even Western!) in one group or another. Certain doctrines/ideologies/ideas acknowledge several components from various systems. Some beliefs are classified as monistic, yet they clearly contain components of pluralism as well; others, on the other hand, are classified as nondualistic but cannot be classified as either strictly UniPolar, or PluriPolar systems. Nor the concept of nonduality, a common thread in Taoism, Mahayana Buddhism, or Advaita Vedanta (Loy, 1998), does suffice, being a rather fuzzy concept, which might finally include anything that does not fall into a BiPolar System of thought, regardless of distinctions, or mutations.

Numerous schools of thought have extensively examined the dynamics between the opposites <A> and <antiA>. These concepts are known by various names, including dialectics, Yin-Yang, Manichaeism, dualism, Dharma-Adharma, and many others. However, the neutral (or indeterminacy) part (<neutA>) between these opposites has rather either been ignored or retracted. The neutral or indeterminate, as I emphasized in my studies on neutrosophic theory (Smarandache 2002, 2013), usually intervenes in the dynamics (or conflicts) from one side or the other, tipping the balance in one direction or the other. The boundaries between the opposites can be either fluid (when there is some overlapping or indeterminate/neutral part between the opposites) or rigid (when <A> and <antiA> are clearly separated).

In Occasionalism, for example, the God is a neutral (<neutA_1>) between mind (<A_1>) and body (<antiA_1>), as a particular case, i.e. where one has only one dynamic, between <A_1> and <antiA_1> (one neutrosophic triad). In MultiAlism, one has dynamics between many neutrosophic triads:

(<A_1>, <neutA_1>, <antiA_1>), (<A_2>, <neutA_2>, <antiA_2>), ...

And so forth.

By convention let's use the prefix "*pluri*" when talking about the elements of a single system, and "*multi*" when talking about the elements of many systems.

Therefore, the **PluriPolar System** accepts and deals with the dynamicity of opposites, but not with the neutralities or indeterminacies between them:

$\underline{<(pluri)A> + <(pluri)antiA> = \infty}$.

This simple observation instigated the idea of a generalizing and integrative construct into which to accommodate theories that mix parts from many systems. I unpretentiously call this construct **multialism**, and clearly differentiate it from **pluralism**, and consequently call the related system the **multialist system**, conceiving it as a **MultiPolar System** that accepts and is open to combinations of opposites and neutrals (indeterminacies), e.g.:

$\underline{<(multi)A> + <(multi)neutA> + <(multi)antiA> = \infty}$.

The MultiPolar System accepts and deals with neutralities and indeterminacies between the opposites, but it is not necessarily to contain them. As such, the MultiPolar System is an extension of the PluriPolar System.

Let us test out two examples from religion before returning with more in-depth studies in later papers.

3.1. Zoroastrianism

Zoroastrianism offers a perplexing picture of a religion (about the state and prospects of the study of this religion, a must read is Stausberg, 2008) whose followers worship several sacred beings, called *yazatas*, in addition to a single deity, *Ahura Mazda* (or *Ohrmazd* in Middle Persian).[7]

These *yazatas*[8] —somehow remembering us the Roman tutelary deities *Lares*[9]— include natural objects or phenomena (earth, water, wind, sun, moon, etc.). Other individual deities manifest their presence, among which *Anahita* (fertility), *Armaiti* (right-mindedness), *Ai* (reward), or *Rasnu* ('justice'). Furthermore, *Ahura Mazda*'s faces strong opposition from the personification of evil, *Ahreman* in Middle Persian (or *Angra Mainyu* in Avestan). Its only goal is to ruin *Ohrmazd*'s good world.

This makes the **Zoroastrianism** to Hintze to be both dualistic, polytheistic, and monotheistic (a "mixture of seemingly monotheistic, polytheistic, and dualistic features", Hintze, 2014, 225 et ss.), in an attempt to put an end to the debates in literature which went from defining **Zoroastrianism** as a "dualistic monotheism" (Gnoli, 1994, 480) to a "monotheistic dualism" (Schwartz, 2002, 64). Added to this are the interpretations of existence of a *Divine Triad*, or a *dialogical triad* in **Zoroastrianism**: "The Deity is also not a monadic one, but a dialogical triad (and there may be other aspects) who exists in relationship" (Louchakova-Schwartz, 2018, 481).

Furthermore, I observe the obvious neutrosophic features of *yazatas*: the balance between good and evil tilts according to their (neutrosophic) actions (*vedi supra*, 2.3).

In our approach, these characteristics makes the **Zoroastrianism** a ***multialist*** religion, including elements from all basic systems:

<A> [*Ohrmazd*] + <(multi)A> [deities] + <neutA> [actions of *yazatas*] + <antiA> [*Ahreman*] = ∞.

3.2. Vedanta and neo-Vedanta schools

Other instances of **multialism** are generated by the different interpretations of Vedanta. Independently, the Vedanta schools may appear utterly distinct due to significant discrepancies in ontology, soteriology, and epistemology.

Let us remind the main schools of Vedanta, and their interpretations: Advaita (non-dualism), Dvaitadvaita (difference and non-difference), Vishishtadvaita (qualified non-dualism), Dvaita (dualism), Suddhadvaita (pure non-dualism), Achintya-Bheda-Abheda (inconceivable difference and non-difference) (Isaeva, 1992; Clooney, 1993).

Coming closer to our days, modern developments (so-called neo-Vedanta) propagated the idea that the divine, the absolute, exists within all human beings. Acceptance of many kinds of worship is a key component of Swami Vivekananda's philosophy, an exponent of neo-Vedanta, emphasizing the idea of *acceptance* rather than *tolerance*. This neo-Vedanta school holds that no other types of worship are incorrect. Life is a quest trip from one truth to another, from a lesser truth to a greater one. The truth is not anyone's property, and the nature of all souls is truth. Actually, Vivekananda "reconciles Dvaita or dualism and Advaita or non-dualism" (Sooklal, 1993, 48).

According to Vivekananda, the perfect man possesses all the components of philosophy, mysticism, passion, action in right measure to create a harmoniously balanced whole (*Ibidem*, 42). To my understanding, the components are supposed to exist in a balanced (and hence *neutrosophic*) manner rather than just in their *plurality*, and yet being monistical manifestations of *one*,

i.e. <A> + <(multi)A> + <neutA>,

which makes me consider it a *multialist doctrine*.

[7] Duchesne-Guillemin, Jacques. "Zoroastrianism". Encyclopedia Britannica, 8 Nov. 2023, https://www.britannica.com/topic/Zoroastrianism. Accessed 11 December 2023.

[8] Britannica, The Editors of Encyclopaedia. "yazata". Encyclopedia Britannica, 3 Apr. 2014, https://www.britannica.com/topic/yazata. Accessed 11 December 2023.

[9] Britannica, The Editors of Encyclopaedia. "Lar". Encyclopedia Britannica, 14 Feb. 2018, https://www.britannica.com/topic/Lar-Roman-deities. Accessed 11 December 2023.

Conclusions

As an extension of the concepts of 'pluralism', 'neutrosophy', 'refined neutrosophy', 'refined neutrosophic set', 'multineutrosophic set', and 'plithogeny' (Smarandache 2002, 2013, 2017, 2019, 2021, 2023a, 2023b, 2023c), I introduced in this short note the concept of MultiAlism, to which corresponds a MultiPolar system of thought. A possible advantage of this system could free from ambiguities the other systems, especially the PluriPolar system, where *plural elements* – more or less equal – coexist or are tolerated to exist and contains their opposites, but not their neutralities or indeterminacies between them; while the MultiPolar system is open to accept in various combinations and mutations, the opposites and their neutralities or indeterminacies between them, from *more than one system*. In other words, the UniPolar, BiPolar, TriPolar, and PluriPolar systems are uni-valent systems (one excludes the other), whilst the MultiPolar System is a multi-valent system (it includes more than one system) and accepts neutralities and indeterminacies between opposites.

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Abstract

Scopul acestui scurt articol este de a extinde conceptele de "pluralism", "neutrosofie", "neutrosofie rafinată", "mulțime neutrosofică rafinată", "mulțime multi-neutrosofică" și "plitogenie" (Smarandache 2002, 2013, 2017, 2019, 2021, 2023a, 2023b, 2023c) într-o categorie mai largă, denumită MultiAlism (sau sistem MultiPolar). Ca o generalizare directă, propunem conceptualizarea unui Sistem MultiPolar (diferit de Sistemul PluriPolar), care este format nu doar din multiple elemente care pot fi aleatorii, contradictorii sau adjuvante, ci și prin acceptarea trăsăturilor din mai multe sisteme de bază (acestea fiind sistemele UniPolare, BiPolare, TriPolare sau PluriPolare). PluriAlismul este un sistem dinamic închis, fără neutralități sau indeterminări, în timp ce MultiAlismul este un sistem dinamic deschis, cu neutralități și indeterminări. PluriAlismul este un uni-sistem (format din elemente provenind dintr-un singur sistem), în timp ce MultiAlismul este un MultiSistem (format din elemente provenind din multe sisteme).

Cuvinte cheie

monism; dualism; trialism; pluralism; neutrosofie; neutrosofie rafinată; multineutrosofie; mulțime neutrosofică rafinată; mulțime multineutrosofică; plitogenie; multialism; Zoroastrism; monism neutral; neo-Vedanta.

1. Introducere

Multiple interacțiuni ocazionale cu colegi extra-occidentali din mediul academic ne-au deschis ochii în ultimele două decenii la teme care – cu câteva excepții notabile de specialiști dedicați și imparțiali – sunt abordate superficial în spațiul academic din ceea ce numim încă *Occident*. În lumea occidentală, unele idei, principii și acțiuni estice rămân neînțelese sau greșit judecate, deoarece avem o obstinație de a le adapta fără nuanțe, cu un reflex etichetard
inconturnabil, în metodele noastre unice de gândire. Vizitele frecvente pe care le-am făcut în lumea extra-occidentală, la conferințe internaționale și seminarii științifice, sau cercetările post-doctorale în domeniul matematicii aplicate și tehnologiei, ne-au oferit un contact nemijlocit cu aceste culturi diverse, permițându-ne să ne îmbunătățim înțelegerea sistemelor

lor de gândire și rezultând în multe jurnale de călătorie despre obiceiurile, religiile, filosofia, istoria, geografia și viața lor.

Zoroastrismul ne servește drept ilustrare a acestei șovăieli doctrinare occidentale. Aspectele sale oarecum paradoxale încântă majoritatea observatorilor occidentali, făcându-i cel mai adesea confuzi atunci când încearcă să clasifice această religie în cadrul sistemelor monoteiste, dualiste sau pluraliste. Totuși, impunerea conceptelor a căror semnificație originară vizează alte doctrine nu va reuși să definească cât de cât complet această religie. Mai degrabă decât să subliniem trăsături monoteiste sau politeiste, sau chiar trăsăturile neutrosofice – așa cum am făcut într-unul din *scilog*-urile mele (Smarandache, 2023, 84 și urm.) –, nu ar fi mai benefic pentru gândire să extindem categoriile actuale?

Alternativ, putem privi în filosofia occidentală la monismul neutral, care – pentru a spune simplist – susține că mintea și corpul nu sunt două entități distincte, ci sunt compuse din același "materie" neutră, sau ca o margine fluidă (indeterminată) între non-fizic și fizic (Smarandache 2023c). În acest sens, David Hume a propus "impresiile" sau "percepțiile" ca realități primare ale experienței, în timp ce William James a considerat că materialul neutru este o "confuzie zgomotoasă și tumultoasă" numită "experiență pură", iar Bertrand Russell, mai aproape de timpurile noastre, a făcut referire la entitățile neutre ca "*sensibilia*". Monismul neutral[1] este de fapt pluralist, în sensul că recunoaște existența mai multor astfel de elemente (spre deosebire de monismul metafizic), dar este monist în sensul că susține că elementele fundamentale ale universului sunt toate de același tip (împotriva dualismului minte-corp). Deoarece nu ne încadrăm strict în categoria monismului, acceptând neutralități sau indeterminări – nu ar fi mai benefic pentru gândire să extindem categoriile actuale?

În continuare, vom oferi câteva exemple suplimentare de acest tip; cititorii pot adăuga propriile lor exemple pentru a completa imaginea. Exemplele nu sunt limitative, dar întrebarea persistă: nu ar fi mai benefic pentru gândire să extindem categoriile actuale?

Se întâmpla să citim un studiu recent al lui Ethan Brauer când schița unui răspuns la întrebarea de mai sus s-a conturat de la sine pe hârtie. Lucrarea extinsă a lui Brauer abordează un subiect complet diferit și îngust, dar care poate fi extins din sfera sa limitată – analiza modală a infinitului potențial. Brauer a extins o teorie a aritmeticii clasice de ordinul al doilea pentru a include (intrinsec) axiome bine fundamentate pentru "secvențe fără lege" (*lawless sequences*).[2]. Secvențele de alegere liberă sunt centrale pentru teoria intuționistă a *continuum*-ului, dar, de vreme ce teoremele analizei intuționiste sfidează analiza clasică, mulți matematicieni resping conceptul. (Brauer 2023)

Mutatis mutandis, căutarea noastră este similară.

2. Sisteme UniPolare, BiPolare, TriPolare, PluriPolare și, generalizând, Sisteme MultiPolare. Definiții și exemple

În această secțiune, vom analiza definițiile și semnificațiile sistemelor fundamentale de organizare a gândirii occidentale și le vom exemplifica, incluzând scenarii din doctrinele estice.

[1] Griffin, N. (1998). 'Neutral monism'. În: *The Routledge Encyclopedia of Philosophy*. Taylor and Francis. Online: https://www.rep.routledge.com/articles/thematic/neutral-monism/v-1. Accesat: 23 decembrie 2023.
[2] Care l-a condus pe Brauer la o teorie numită MC_{LS}.

2.1. Monismul: totul este unu

Monismul este o filozofie și o doctrină metafizică care postulează o singură realitate ultimă și coerentă. Universul este compus dintr-o singură "idee" sau "substanță" generală, sau o singură zeitate ultimă, <A>. Totul este doar o manifestare a acestei singure realități/substanțe/zeitate. Acesta este un **Sistem UniPolar**, adică:

$\underline{<A>} = \infty$,

unde <A> este o "idee", o "substanță", etc., iar '∞' este "lumea", "realitatea", "totul".

Școlile de filozofie moniste susțin că fie totul este material (materialism), fie totul este mental (idealism), și abolesc distincția dintre corp și minte în favoarea explicării tuturor fenomenelor ca expresii ale unui singur principiu unificator.[3]

Christian Wolff a creat termenul "monism" în secolul al XVIII-lea în lucrarea sa "Gânduri raționale" [Logica germană] (1728): "trebuie să admitem o Singură Ființă necesară și auto-existentă" (Wolff, 1770). Wolff aprofundează legătura minte-corp în "*Psychologia Rationalis*" (1734). El crede în validitatea monadologiei leibniziene, dar aplicată doar ideilor, respingând panpsihismul monistic care este central în metafizica lui Leibniz.[4]

În Hinduism, ideea de *Brahman* — realitatea ultimă sau puterea cosmică supremă — este adesea asociată cu monismul. Majoritatea hindușilor urmează principiile monastice și consideră că 'Brahman este totul și totul este Brahman.' [5] Filosofia *Advaita Vedānta*, care este considerată un fel de non-dualism absolut, reflectă de asemenea acest punct de vedere.

Într-o simplificare accesibilă, monismul poate fi redus la două tipuri: un *monism substanțial*, întâlnit în religii precum Budismul și Hinduismul, în Est, sau la filosofi precum Spinoza, în Vest, și un *monism atributiv*, cu subtipuri precum idealismul, fizicalismul sau monismul neutral. Primul reduce realitatea la o singură substanță sau afirmă că lumea este variată doar pentru că această singură substanță există în forme multiple, în timp ce al doilea susține că există o categorie unică a ființei care cuprinde o mare pluralitate de obiecte sau substanțe distincte.

În ciuda faptului că este esențialmente monistic, monismul atributiv pare a fi mai degrabă pluralist, în timp ce monismul substanțial este puternic opus pluralismului.

Întrucât reduce cosmosul fizic la un singur principiu, panteismul este similar cu monismul: "Panteiștii sunt moniști" (Owen, 1971, 65), chiar dacă zeitatea panteistă este imperfectă, expansivă și continuu creatoare; sau chiar extinzându-se dincolo de spațiu și timp în panenteism — o concepție despre Dumnezeu prezentă și în unele confesiuni creștine —, depășind astfel simplificarea atribuției monistice.

2.2. Dualismul: totul este doi

Dualismul explică lumea (sau realitatea) prin două principii fundamentale, diametral opuse și ireductibile. În religie, se referă în general la convingerea că universul a fost creat de

[3] O'Conaill, D.(2019). 'Monism.' În: *The Routledge Encyclopedia of Philosophy*. Taylor and Francis. Online: https://www.rep.routledge.com/articles/thematic/monism/v-2. Accesat: 21 decembrie 2023.
[4] Hettche, Matt and Corey Dyck, 'Christian Wolff'. În: *The Stanford Encyclopedia of Philosophy* (2019), Edward N. Zalta (ed.). Online: https://plato.stanford.edu/archives/win2019/entries/wolff-christian. Accesat: 17 decembrie 2023.
[5] Leeming, D.A. (2014). 'Brahman.' În: Leeming, D.A. (eds) *Encyclopedia of Psychology and Religion*. Springer, Boston, MA. Online: https://doi.org/10.1007/978-1-4614-6086-2_9052. Accesat: 16 decembrie 2023.

două forțe ultime antagoniste — zeități sau grupuri de ființe angelice sau demonice. Deoarece dualismul este un sistem format din două părți contrastante, acesta este un **Sistem BiPolar**:

<A> + <antiA> = ∞,

unde <A> este o "idee", o "substanță", etc., <antiA> este opusul sau negarea sa, iar '∞' este "lumea", "realitatea", "totul".

Probabil că nu am greși dacă am afirma că acest sistem a fost, de-a lungul vremii, o viziune dominantă în gândirea occidentală, cu Descartes și Hegel fiind primele două figuri care vin în minte, completate de dualismul cognitiv al lui Kant, care a făcut distincția între facultățile de sensibilitate și de înțelegere. Exemple de dualism epistemologic includ ființa și gândirea, subiectul și obiectul; iar, pe de altă parte, exemple de dualism metafizic sunt materia și spiritul, corpul și mintea, binele și răul.

Privind spre Est, observăm că majoritatea istoricilor religiilor folosesc religia iraniană antică, Zoroastrismul, ca un caz clar de dualism eshatologic, susținând că se bazează pe două principii conflictuale: *Ahura Mazda*, zeitatea luminii și adevărului, și *Angra Mainyu*, dușmanul distrugător.

Un conflict continuu există între domeniul spiritual al luminii și domeniul material al întunericului și în concepția Maniheismului, religie iraniană antică.

În plus, după cum spune și numele său, *dvaita* — cuvântul sanscrit *dvaita* înseamnă de fapt "dualism" (Flood, 1996, 245) — este o școală dualistă de *Vedanta*, care afirmă că există o separare veșnică între sinele particular și absolutul suprem, spre deosebire de filosofia *advaita* (non-dualistă). Deși *dvaita* este dualistă în acest sens, a propus un Dumnezeu autonom numit *Vishnu* ca stăpân al entităților independente și separate de materie și suflet. Mai specific, *dvaita* a recunoscut trei entități absolute și etern existente: Dumnezeu, sufletele (*atman*) și substanța primordială (*prakriti*).

2.3. Trialismul: totul este trei

Trialismul a fost introdus în filozofie de John Cottingham ca "o grupare a trei noțiuni" (Cottingham, 1985, 219), o perspectivă alternativă la dualismul lui Descartes, cu adăugarea *senzației* alături de *minte* și *corp*: "Se dovedește că există trăsături care aparțin doar minții, trăsături care aparțin doar corpului și ceea ce poate fi numit trăsături hibride - trăsături care aparțin omului în calitatea sa de ființă întrupată" (*Ibidem*; vezi și Cottingham, 2021).

Trialismul este astfel un sistem format din trei părți contrastante sau complet diferite, iar, similar, *trihotomia* este o împărțire a trei opoziții două câte două.

Un sistem cu trei poli a fost, de asemenea, propus în *neutrosofie* (Smarandache 1995, 2013), care funcționează cu trei opoziții independente, aflate în echilibru: <A>, <neutA> și <antiA>, denumite *Triadă Neutrosofică*. Toate "ideile" <A> sunt considerate împreună cu opozițiile sau negațiile lor <antiA> și gama de neutralități <neutA> dintre ele, în timp ce <nonA> este termenul colectiv pentru ideile <antiA> și <neutA>. În neutrosofie, cei trei poli pot fi fluizi doi câte doi.

Echilibrul dintre <A> și <antiA> se bazează pe <neutA>. Cu alte cuvinte, <neutA> este imaginat ca o zonă tampon între <A> și <antiA>:

Mutând <neutA> spre stânga sau spre dreapta, adică dacă partea de neutralitate/indeterminare este împinsă spre <A> sau spre <antiA> (gradul de indeterminare

crește), atunci unul dintre ele devine mai puternic (având mai puțină indeterminare), iar echilibrul devine dezechilibru:

Pe baza neutrosofiei, **Sistemul TriPolar** asociat poate fi descris astfel:

<A> + <neutA> + <antiA> = ∞,

unde <A> este o "idee", o "substanță", etc., <antiA> este opusul sau negarea sa, <neutA> este gama de neutralități între ele, iar '∞' este "lumea", "realitatea", "totul".

Subliniem că neutrosofia este o extensie atât a filozofiei antice chineze Yin-Yang, cât și a dialecticii (Smarandache 2013), și amintim că trialismul a fost asociat și cu creștinismul, de exemplu, pentru că susține că ființele umane sunt compuse din trei esențe separate: un corp, un suflet și un spirit.

2.4. Pluralismul: totul este plural

Pluralismul este o viziune asupra pluralității, utilizată în filozofie pentru a contrasta cu monismul (ideea că totul este unul), cu dualismul (ideea că totul este doi) și, discutabil, cu trialismul (ideea că totul este trei). Pluralismul poate fi definit ca un sistem în care coexistă mai mult de două (discutabil trei) grupuri, principii, stări, idei etc. Acesta este un **Sistem PluriPolar**:

<pluriA> = ∞,

unde <pluriA> înseamnă mai mult de două (discutabil trei) "idei", etc., iar '∞' denotă "lumea", "realitatea", "totul".

În metafizică, pluralismul este ideea că realitatea este de fapt compusă dintr-o varietate de substanțe naturale, în timp ce în ontologie conceptul descrie diverse forme, tipuri sau moduri de existență.

Budismul este dat ca exemplu de religie pluralistă. Multe tradiții budiste nu proclamă existența unui singur adevăr absolut, ci recunosc validitatea mai multor căi către iluminare, promovând dialogul și înțelegerea cu persoane de alte credințe.

Un alt exemplu ar putea fi Credința Bahá'í, care susține că toate credințele majore au aceeași bază spirituală, provin din aceeași sursă divină (Dumnezeu) și diferă doar în învățăturile lor sociale, după necesitățile epocilor în care au fost revelate.

3. MultiAlismul: totul este deschis

Din scurta evaluare precedentă, se observă, fără a intra în profunzimea sistemelor de bază pe care le-am discutat, că este dificil să includem strict și exclusiv unele doctrine non-occidentale (sau chiar occidentale!) într-un grup sau altul. În anumite doctrine, ideologii sau idei se recunosc componente din diferite sisteme. Unele credințe sunt clasificate ca moniste, dar conțin clar și componente de pluralism; altele, pe de altă parte, sunt clasificate ca non-dualiste, dar nu pot fi clasificate strict ca sisteme UniPolare sau PluriPolare. Nici conceptul de non-dualitate, un fir comun în Taoism, în Budismul de tip Mahayana sau Advaita Vedanta (Loy, 1998), nu este suficient, fiind un concept destul de vag, care ar putea include în cele din urmă orice nu se încadrează într-un sistem de gândire BiPolar, indiferent de distincții sau mutații.

Numeroase școli de gândire au examinat extensiv dinamica dintre opozițiile <A> și <antiA>. Aceste concepte sunt cunoscute sub diverse denumiri: dialectică, Yin-Yang, maniheism, dualism, Dharma-Adharma și multe altele. Totuși, partea neutră (sau indeterminată) (<neutA>) dintre aceste opoziții a fost mai degrabă ignorată sau retractată.

Neutrul sau indeterminatul, așa cum am subliniat în studiile mele de teorie neutrosofică (Smarandache 2002, 2013), intervine de obicei în dinamica (sau conflictele) dintre părți, înclinând balanța într-o direcție sau alta. Granițele dintre opoziții pot fi ori fluide (când există o oarecare suprapunere sau o parte indeterminată/neutră între opuși), ori rigide (când <A> și <antiA> sunt clar separate).

În ocazionalism, de exemplu, Dumnezeu este un element neutru (<neutA_1>) între minte (<A_1>) și corp (<a antiA_1>), ca un caz particular, adică unde există o singură dinamică, între <A_1> și <antiA_1> (o singură triadă neutrosofică). În MultiAlism, există dinamici între multe triade neutrosofice:

(<A_1>, <neutA_1>, <antiA_1>), (<A_2>, <neutA_2>, <antiA_2>), ... ;

și așa mai departe.

Prin convenție, folosim prefixul "*pluri*" atunci când vorbim despre elementele unui singur sistem și "*multi*" atunci când vorbim despre elementele mai multor sisteme.

Prin urmare, **Sistemul PluriPolar** acceptă (și se ocupă de) dinamica opozițiilor, dar nu (de) neutralitățile sau indeterminările dintre acestea:

<(pluri)A> + <(pluri)antiA> = ∞.

Această observație simplă a stârnit ideea unui construct generalizator și integrativ în care să fie acomodate teoriile care combină părți din mai multe sisteme. Denumim, în mod nepretențios, acest construct '**multialism'** și îl diferențiem clar de '**pluralism'**, numind sistemul asociat '**sistem multialist'**, concepându-l ca un '**Sistem MultiPolar'** care acceptă (și este deschis la) combinații de opoziții și neutralități (indeterminări), e.g.:

<(multi)A> + <(multi)neutA> + <(multi)antiA> = ∞.

Sistemul MultiPolar acceptă (și se ocupă de) neutralitățile și indeterminările dintre opoziții, dar nu este neapărat să le și conțină. Astfel, Sistemul MultiPolar este o extensie a Sistemului PluriPolar.

Să testăm două exemple din religie înainte de a reveni cu studii mai aprofundate în lucrări ulterioare.

3.1. Zoroastrismul

Zoroastrismul oferă o imagine perplexă a unei religii a cărei adepți venerează mai multe ființe sacre, numite *yazatas*, pe lângă o singură zeitate, *Ahura Mazda* (sau *Ohrmazd* în persana mijlocie).[6] (Despre starea și perspectivele studiului acestei religii, o lectură obligatorie este Stausberg, 2008.)

Acești *yazatas*[7] — care amintesc într-un fel de zeii tutelari romani *Lares*[8] — includ obiecte sau fenomene naturale (pământ, apă, vânt, soare, lună etc.). Alte zeități individuale își manifestă prezența, printre care *Anahita* (fertilitatea), *Armaiti* (gândirea corectă), *Ai* (răsplata) sau *Rasnu* ('justiția'). În plus, *Ahura Mazda* se confruntă cu o puternică opoziție din partea

[6] Duchesne-Guillemin, Jacques. 'Zoroastrianism.' În: *Encyclopedia Britannica*, 8 Nov. 2023. Online: https://www.britannica.com/topic/Zoroastrianism. Accesat: 11 decembrie 2023.
[7] Britannica, The Editors of Encyclopaedia. 'yazata.' În: *Encyclopedia Britannica*, 3 Apr. 2014. Online: https://www.britannica.com/topic/yazata. Accesat: 11 decembrie 2023.
[8] [9] Britannica, The Editors of Encyclopaedia. 'Lar.' În: *Encyclopedia Britannica*, 14 Feb. 2018. Online: https://www.britannica.com/topic/Lar-Roman-deities. Accesat: 11 decembrie 2023.

personificării răului, *Ahreman* ─ în persana mijlocie (sau *Angra Mainyu* în avestană). Scopul său unic este să distrugă lumea bună a lui *Ohrmazd*.

Aceasta face ca **Zoroastrismul**, după Hintze, să fie atât dualist, politeist, cât și monoteist (un "mixt de trăsături aparent monoteiste, politeiste și dualiste", Hintze, 2014, 225 și urm.), într-o încercare de a pune capăt dezbaterilor din literatura de specialitate care au variat de la definirea Zoroastrismului ca un "monoteism dualist" (Gnoli, 1994, 480) până la un "dualism monoteist" (Schwartz, 2002, 64). La acestea se adaugă interpretările existenței unei *Triade Divine* sau a unei *triade dialogice* în **Zoroastrism**: "Zeitatea nu este monadică, ci o triadă dialogică (și pot exista și alte aspecte) care există în relație" (Louchakova-Schwartz, 2018, 481).

În plus, să observăm caracteristicile neutrosofice evidente ale *yazatas*: echilibrul între bine și rău se înclină în funcție de acțiunile lor (neutrosofice) (*vedi supra*, 2.3).

În abordarea noastră, aceste caracteristici fac ca **Zoroastrismul** să fie o religie ***multialistă***, incluzând elemente din toate sistemele de bază:

<A> [*Ohrmazd*] + <(multi)A> [zeități] + <neutA> [acțiuni ale *yazatas*] +
+ <antiA> [*Ahreman*] = ∞.

3.2. Vedanta și școlilele neo-Vedanta

Alte exemple de **multialism** derivă din interpretările diferite ale Vedantei. În mod independent, școlile Vedanta pot părea complet distincte din cauza discrepanțelor semnificative în ontologie, soteriologie și epistemologie.

Să amintim principalele școli de Vedanta și interpretările lor: *Advaita* (non-dualism), *Dvaitadvaita* (diferență și non-diferență), *Vishishtadvaita* (non-dualism calificat), *Dvaita* (dualism), *Suddhadvaita* (non-dualism pur), *Achintya-Bheda-Abheda* (diferență și non-diferență ireconciliabile) (Isaeva, 1992; Clooney, 1993).

Apropiindu-ne de zilele noastre, dezvoltările moderne (așa-numitele școli neo-Vedanta) au propagat ideea că divinul, absolutul, există în toate ființele umane. Acceptarea mai multor tipuri de cult este o componentă cheie a filozofiei lui Swami Vivekananda, un exponent neo-Vedanta, care subliniază ideea de acceptare, mai degrabă decât toleranță. Această școală neo-Vedanta consideră că niciun alt tip de cult nu este incorect. Viața este o călătorie de căutare de la un adevăr la altul, de la un adevăr mai mic la unul mai mare. Adevărul nu este proprietatea nimănui, iar natura tuturor sufletelor este adevărul. De fapt, Vivekananda "reconciliază Dvaita sau dualismul și Advaita sau non-dualismul" (Sooklal, 1993, 48).

După Vivekananda, omul perfect deține și stăpânește componente ale filozofiei, misticismului, pasiunii și acțiunii în măsura potrivită pentru a crea un întreg armonios echilibrat (*Ibidem*, 42). Așadar, aceste componente ar trebui să existe *într-un mod echilibrat* (și, astfel, *neutrosofic*) mai degrabă decât doar *în pluralitatea* lor, și totuși fiind manifestări monistice ale *unuia*, adică:

<A> + <(multi)A> + <neutA>,

ceea ce ne face să o considerăm o *doctrină multialistă*.

Concluzii

Ca o extensie a conceptelor de "pluralism", "neutrosofie", "neutrosofie rafinată", "mulțime neutrosofică rafinată", "mulțime multi-neutrosofică" și "plitogenie" (Smarandache

2002, 2013, 2017, 2019, 2021, 2023a, 2023b, 2023c), am introdus în acest scurt articol conceptul de MultiAlism, căruia îi corespunde un sistem de gândire MultiPolar. Un avantaj posibil al acestui sistem ar putea fi eliberarea altor sisteme, în special a sistemului PluriPolar, de ambiguități, unde elementele plurale – mai mult sau mai puțin egale – coexistă sau sunt tolerate și conțin și opozițiile lor, dar nu neutralitățile sau indeterminările dintre ele; în timp ce sistemul MultiPolar este deschis pentru a accepta, în combinații și mutații variate, opozițiile și neutralitățile sau indeterminările dintre ele, din mai multe sisteme. Cu alte cuvinte, sistemele UniPolare, BiPolare, TriPolare și PluriPolare sunt sisteme uni-valente (unul le exclude pe celelalte), în timp ce Sistemul MultiPolar este un sistem multi-valent (în care sunt incluse mai multe sisteme) și acceptă neutralități și indeterminări între opoziții.

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Formalisation du perspectivisme latino-américain : un modèle neutrosophique de multiperspectivisme

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Résumé : Cet article aborde le défi de modéliser formellement les systèmes de connaissance complexes, souvent contradictoires, propres à la pensée décoloniale latino-américaine. Il introduit le *MultiPerspectivisme Neutrosophique*, un cadre théorique qui opérationnalise le perspectivisme philosophique à l'aide de la logique neutrosophique. Nous posons comme postulat que la connaissance est située et que la vérité est médiée par le point de vue de l'observateur — un concept enraciné dans la philosophie nietzschéenne mais qui trouve une expression singulière dans les cosmologies amérindiennes et les systèmes juridiques pluralistes. La méthode principale consiste à proposer un *Modèle de MultiPerspectivisme Neutrosophique*, utilisant des ensembles multi-neutrosophiques pour représenter les perspectives subjectives sous forme de triplets de séquences de vérité, d'indétermination et de fausseté ⟨T, I, F⟩. Un résultat clé est le développement d'une fonction de similarité permettant de quantifier l'affinité entre différentes perspectives, transformant ainsi des points de vue qualitatifs en données analysables. Ce modèle est appliqué à une étude de cas portant sur un conflit foncier juridique, où il permet de révéler des similitudes structurelles cachées — telles qu'un style de raisonnement absolutiste partagé entre des parties opposées — et d'identifier des alliances stratégiques potentielles. La conclusion principale est que la logique neutrosophique offre un outil mathématique robuste pour naviguer à travers l'ambiguïté et la pluralité des réalités perspectivistes, allant au-delà de la simple reconnaissance de la différence vers son analyse formelle et son intégration possible.

Mots-clés : Logique neutrosophique ; MultiPerspectivisme ; Philosophie latino-américaine ; Théorie décoloniale ; Ensembles multi-neutrosophiques ; Fonction de similarité ; Pluralisme juridique ; Perspectivisme.

1. Introduction

Le perspectivisme en philosophie est un courant de pensée qui soutient que toute connaissance, perception ou vérité est inévitablement médiée par la perspective du sujet connaissant [1]. Cette idée, puissamment développée par Friedrich Nietzsche, remet en question la notion d'une vérité objective et universelle, en proposant à la place qu'il existe de multiples interprétations du monde, chacune valide dans son propre cadre [2].

Pour Nietzsche, il n'existe pas de faits purs, seulement des interprétations — ce qui implique que l'accès à la réalité est toujours conditionné par des facteurs historiques, culturels, psychologiques et linguistiques [3].

Au-delà de Nietzsche, le perspectivisme a influencé divers courants contemporains tels que le post-structuralisme, le constructivisme social et les épistémologies du Sud, qui soulignent tous l'importance du « lieu d'énonciation ». Dans les contextes latino-américains, par exemple, il a été articulé en lien avec la pensée décoloniale [4] et les visions du monde autochtones [5], qui reconnaissent la coexistence de multiples mondes et manières de connaître. Ainsi, le perspectivisme ne nie pas l'existence de la réalité ; il remet plutôt en question la possibilité de la saisir à travers une seule rationalité dominante, ouvrant la voie à une compréhension pluraliste, relationnelle et située de la connaissance.

Le terme *perspectivisme* est également étroitement associé au philosophe espagnol José Ortega y Gasset, qui en a fait un élément central de sa philosophie. Ortega soutenait que la réalité se manifeste différemment à chaque individu, et que la somme de toutes ces perspectives individuelles permet de s'approcher d'une compréhension plus complète de la vérité [6].

Cette approche, qui reconnaît la validité de points de vue multiples sans dissoudre l'existence d'une réalité objective, s'aligne sur un courant contemporain de la philosophie des sciences connu sous le nom de *réalisme perspectiviste* [7]. Cette position soutient que la connaissance, bien qu'objective, est toujours partielle et obtenue à partir d'une perspective située. De la même manière que différentes cartes (topographique, politique, climatique) peuvent décrire le même territoire de façons distinctes mais également valides, des perspectives diverses peuvent offrir une connaissance réelle, bien que partielle, du monde. Ce cadre permet de dépasser la fausse dichotomie entre un universalisme absolutiste et un relativisme nihiliste — une tension centrale dans cette étude.

Le perspectivisme ne cherche pas à invalider la cohérence interne ni l'applicabilité formelle des principes logiques classiques — tels que le principe de non-contradiction, le tiers exclu ou la bivalence — lorsqu'ils opèrent dans un système fermé et cohérent. Il remet plutôt en question l'hypothèse ontologique selon laquelle ces principes refléteraient nécessairement une réalité objective et universelle, indépendante du sujet. D'un point de vue perspectiviste, ces lois logiques ne sont pas rejetées dans leur structure formelle, mais leur prétention à une correspondance absolue avec la réalité du monde est soumise à un examen critique. La critique perspectiviste ne vise donc pas la validité interne de telles formules, mais leur portée épistémologique — c'est-à-dire, si de telles structures binaires sont adéquates pour saisir toute la complexité, l'ambiguïté et la multiplicité contextuelle de l'expérience vécue, des systèmes culturels et de la production de savoir. De cette manière, le perspectivisme reconfigure la logique elle-même en tant que construction située, dont les postulats fondamentaux peuvent être valables dans certains paradigmes, mais non universellement dans tous les domaines ontologiques ou épistémologiques.

1.1 Principe de Non-Contradiction

Ce principe postule qu'une proposition $\mathcal{P}$ ne peut pas simultanément se voir attribuer les valeurs de vérité vrai et faux. Formellement, il est exprimé comme suit :

$$\neg(\mathcal{P} \wedge \neg\mathcal{P}) \quad (1)$$

qui est interprété comme : « *Il n'est pas le cas que* $\mathcal{P}$ *et sa négation* $\neg\ \mathcal{P}$ *soient tous deux vrais*. » Cet axiome assure la cohérence logique des systèmes classiques en excluant la coexistence d'assertions contradictoires.

1.2 Principe du Tiers Exclu

Ce principe affirme que pour toute proposition bien formée $\mathcal{P}$, soit $\mathcal{P}$ est vraie, soit sa négation $\neg\ \mathcal{P}$ est vraie. Il est symboliquement représenté comme suit :

$$\mathcal{P} \vee \neg\mathcal{P} \quad (2)$$

Cette disjonction est universellement valable en logique classique, excluant l'existence de toute valeur de vérité intermédiaire ou tierce entre le vrai et le faux.

1.3 Principe de Bivalence

Contrairement aux deux précédents, il s'agit d'un principe sémantique plutôt que syntaxique. Il postule l'existence d'une fonction de valuation.

$v: L \to \{0,1\}$ (3)

définie sur un langage logique L, telle que pour toute formule bien formée $\phi \in L$,

$v(\phi) \in \{0,1\}$ (4)

Autrement dit, chaque proposition est soit vraie $(v(\phi) = 1)$ soit fausse $(v(\phi) = 0)$, sans aucune tolérance pour la gradation, l'indétermination ou les valeurs de vérité alternatives. Ce cadre de valorisation binaire sous-tend la validité formelle à la fois du Principe de Non-Contradiction et du Principe du Tiers Exclu.

Les épistémologies perspectivistes et les logiques non-classiques — telles que les systèmes à valeurs multiples, flous et neutrosophiques — remettent en question le Principe de Bivalence en introduisant des évaluations où les propositions peuvent être partiellement vraies, indéterminées, ou même simultanément vraies et fausses selon des points de vue spécifiques [8]. Ainsi, l'affirmation selon laquelle la valeur d'une proposition doit être strictement soit 0 soit 1 est incompatible avec les cadres qui acceptent la pluralité épistémique ou ontologique.

*

En résumé, cet article aborde le défi fondamental de modéliser formellement les systèmes de connaissance complexes et souvent contradictoires inhérents à la pensée décoloniale latino-américaine, pour lesquels la logique bivalente classique s'avère insuffisante. L'objectif principal a été de construire un pont conceptuel et méthodologique entre le perspectivisme philosophique et le cadre mathématique de la logique neutrosophique, cherchant à opérationnaliser ses principes. À cette fin, l'article introduit le cadre du *MultiPerspectivisme Neutrosophique*, développant un modèle qui utilise des ensembles MultiNeutrosophiques et une fonction de similarité pour représenter et comparer quantitativement divers points de vue. À travers une étude de cas portant sur un litige foncier, la capacité du modèle à révéler des dynamiques structurelles cachées entre les parties en conflit est démontrée, validant ainsi son utilité en tant qu'outil robuste pour analyser et naviguer l'ambiguïté et la pluralité inhérentes aux réalités perspectivistes.

2. Préliminaires

2.1 Philosophie neutrosophique et Perspectivisme

La *philosophie neutrosophique* [9] offre un cadre formel et mathématique à l'école philosophique du perspectivisme. La relation peut être comprise à travers les connexions clés suivantes [10].

Le lien le plus explicite entre la neutrosophie et le perspectivisme est le *Principe de Relativité Référentielle* [9]. Ce principe stipule que « la vérité, la fausseté et l'indétermination de toute proposition dépendent du système référentiel dans lequel elle est examinée ». Cela signifie qu'une idée peut être vraie dans un contexte, fausse dans un autre, et indéterminée dans un troisième. C'est la thèse centrale du perspectivisme : il n'y a pas de vérités absolues ; la connaissance est toujours conditionnée par le point de vue de l'observateur. La neutrosophie adopte cette idée et l'établit comme un principe fondamental de son système.

Le perspectivisme défie la logique classique, qui est basée sur une stricte dualité vrai/faux. Contrairement à la logique traditionnelle, la neutrosophie introduit le concept de *degrés de vérité, de fausseté et d'indétermination*. Cela permet la modélisation mathématique d'une réalité où les propositions ne sont pas absolument vraies ou fausses mais existent sur un spectre, comme le suggérerait une approche perspectiviste [11].

La neutrosophie revisite les idées traditionnelles, affirmant que les vérités au sein d'un système référentiel peuvent devenir des faussetés dans un autre, et vice versa. Cette approche souligne la *fluidité de la connaissance* et exhorte à considérer les idées sous de multiples angles, un objectif central de la pensée perspectiviste.

Si le perspectivisme affirme l'existence de nombreuses perspectives valides, la question se pose de savoir quel système logique utiliser pour leur analyse. Il n'y a pas une seule « logique vraie » ; au contraire, un ensemble de cadres logiques existe, chacun offrant différentes perspectives sur la vérité et la validité [12]. La neutrosophie s'aligne sur ce pluralisme, se présentant non pas comme la seule logique mais comme un outil flexible et puissant particulièrement adapté aux contextes d'indétermination et de contradiction, qui sont des conséquences naturelles de la coexistence de multiples perspectives.

Le perspectivisme conduit inévitablement à des *contradictions* lorsque deux points de vue valides s'opposent. La logique classique ne peut gérer cela sans écarter l'une des perspectives. La neutrosophie, cependant, est conçue pour *embrasser l'indétermination, le paradoxe et l'interaction entre les opposés et les neutralités*.

En permettant aux valeurs de T et F d'être indépendantes, la neutrosophie peut modéliser formellement une proposition qui est à la fois vraie et fausse en même temps (un état dialéthique, $T = 1$, $F = 1$). C'est une formalisation directe de l'apparence d'une contradiction entre deux perspectives opposées [13].

La composante d'*indétermination (I)* est cruciale, car elle permet l'analyse et la quantification de l'ambiguïté, du flou ou de l'incertitude qui surviennent lorsque les perspectives sont confuses ou incomplètes.

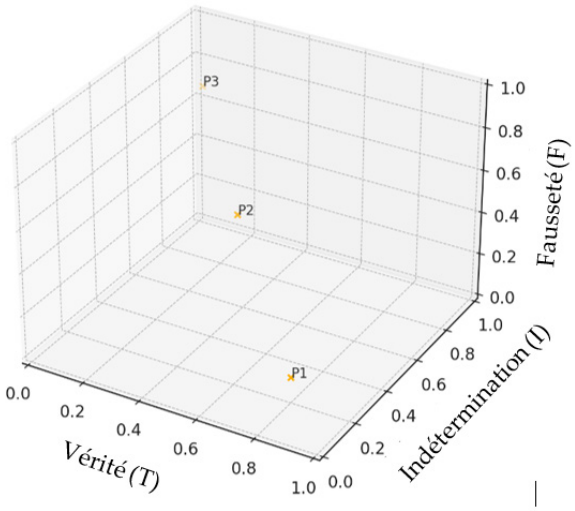

Figure 1. Représentation tridimensionnelle des perspectives neutrosophiques agrégées dans l'espace (T, I, F).

Alors que le perspectivisme se concentre sur la coexistence de la thèse (<A>) et de ses diverses antithèses (<antiA>), la neutrosophie va un pas plus loin en introduisant et en étudiant systématiquement la *neutrothèse* (<neutA>). La neutrothèse représente le spectre des neutralités — des idées ou des états qui ne sont ni la thèse ni l'antithèse, mais qui se situent dans l'espace intermédiaire. De cette manière, la neutrosophie non seulement valide de multiples perspectives (<A> et <antiA>), mais analyse également le « continuum des neutralités » qui les connecte et les équilibre, enrichissant ainsi l'analyse perspectiviste [14, 15].

Pour ce faire, la neutrosophie propose deux principes de recherche actifs [16] :

- Rechercher les « parties communes dans les choses peu communes ».
- Rechercher les « parties peu communes dans les choses communes ».

Le résultat visé est une « *synthèse* » qui intègre des éléments provenant d'opposés, à l'instar de la dialectique hégélienne, mais avec l'inclusion distincte de leurs neutralités.

La neutrosophie se rapporte au perspectivisme en partageant un fondement philosophique commun. Cependant, la neutrosophie se distingue en se concentrant sur une méthode analytique plus spécifique visant à l'intégration. Elle ne se limite pas à l'affirmation philosophique de l'existence de multiples points de vue, mais offre plutôt les outils — tels que les valeurs (T, I, F), la relativité référentielle, le pluralisme logique et la gestion de la contradiction — pour analyser, modéliser et comprendre un monde défini par l'incertitude, l'ambiguïté et la coexistence de multiples réalités. C'est, comme le suggère le titre, un nouveau paradigme de pensée dans un monde intrinsèquement perspectiviste.

La neutrosophie est un cadre philosophique et méthodologique qui étudie la relation entre les opposés et leurs indéterminations. Son objectif principal est de trouver un terrain d'entente entre des concepts opposés et d'identifier les différences dans des choses similaires, parvenant ainsi à une compréhension plus nuancée et intégrative.

2.2 Le Perspectivisme comme cadre pour les logiques plurielles

Le *perspectivisme* est un cadre philosophique qui affirme que la connaissance, la vérité et l'interprétation sont toujours conditionnées par le point de vue de l'observateur. Il remet en question l'existence de vérités absolues et indépendantes du contexte, et souligne la *multiplicité des perspectives valides* coexistant au sein de systèmes de pensée complexes. Bien que le perspectivisme soit souvent associé à Friedrich Nietzsche dans la philosophie occidentale, ses racines conceptuelles peuvent être retracées bien plus tôt dans des traditions non-européennes.

En ce qui concerne l'affinement ou la multiplication de l'indétermination, des extensions de la Logique Neutrosophique ont déjà été définies dans la littérature : la *Logique Neutrosophique Quadruple* (Vérité ; Contradiction, Incertitude ; Faux), la *Logique Neutrosophique Quintuple* (Vérité ; Contradiction, Incertitude, Inconnu ; Faux), la *Logique Neutrosophique Sextuple* (Vérité, I_1, I_2, I_3, I_4 ; Faux), la *Logique Neutrosophique Septuple* (Vérité, I_1, I_2, I_3, I_4, I_5 ; Faux), et ainsi de suite jusqu'à la *Logique Neutrosophique n-tuple* (où l'indétermination a été divisée en n-2 types de sous-indéterminations, selon l'application, pour $n \geq 3$).

Mais l'extension la plus générale concerne le cas où toutes les valeurs T, I, F ont été affinées ou multipliées autant de fois que nécessaire dans chaque problème (*Logique Neutrosophique n-Raffinée*, et respectivement *Logique n-MultiNeutrosophique*).

De plus, en 2014, Smarandache a introduit la *Loi du Multiple-Milieu Inclus (Law of Included Multiple-Middle)* [53] (comme extension de la Loi classique du Milieu Inclus) :

(<A>; <neutA_1>, <neutA_2>, …, <neutA_n>; <antiA>)

et en 2023, la *Loi des Infinis-Moyenne Inclus (Law of Included Infinitely-Many-Middles)* [54] :

(<A>; <neutA_1>, <neutA_2>, …, <neutA_∞>; <antiA>)

(par exemple, entre les couleurs Blanc et Noir, il existe une infinité de nuances de couleurs).

2.2.1 Perspectivisme Ancien dans la Logique Jaïne

Dans le jaïnisme, la doctrine d'*Anekāntavāda* (littéralement, « doctrine des multiples aspects ») offre une approche perspectiviste sophistiquée de la vérité et de la réalité [17]. Selon cette doctrine, la réalité est complexe et ne peut être entièrement saisie par un seul point de vue. En conséquence, chaque affirmation n'est que partiellement vraie et doit être complétée par d'autres points de vue pour approcher une compréhension plus exhaustive. Cette forme ancienne de perspectivisme non seulement anticipe le pluralisme postmoderniste, mais fournit également une fondation métaphysique pour la tolérance, le raisonnement dialogique et une épistémologie non-absolutiste.

La logique jaïne, exprimée à travers la *Loi de Prédication Septuple (Saptabhangi)* [18], constitue un remarquable exemple précoce de raisonnement perspectiviste qui embrasse la nature multiforme et complexe de la réalité. Contrairement à la logique occidentale classique — qui adhère strictement au

principe de bivalence, affirmant qu'une proposition doit être soit vraie, soit fausse — la logique jaïne autorise sept modes de prédication distincts, incluant des combinaisons telles que « vrai et indéterminé » ou « vrai et faux ». Ces formulations remettent directement en question la loi classique de non-contradiction.

Figure 2. Symbole jaïn de *l'Ahimsa*.

Enracinée dans le principe philosophique de *l'anekantavada (la doctrine de la non-unilatéralité)* [19], la logique jaïne affirme que toute affirmation n'est valide que conditionnellement à partir d'une perspective particulière. Cette humilité épistémologique découle de la reconnaissance que le langage humain est intrinsèquement limité dans sa capacité à décrire pleinement la richesse de l'existence. En conséquence, les penseurs jaïns ont promu une forme de *non-violence intellectuelle (ahimsa)* [20], encourageant le respect des désaccords raisonnables et la reconnaissance des vérités partielles.

La logique jaïne résonne conceptuellement avec la *Logique Neutrosophique*, un cadre moderne introduit par Florentin Smarandache. La logique neutrosophique généralise les logiques classiques et floues en introduisant trois composantes à chaque proposition logique : la vérité (V), l'indétermination (I) et la fausseté (F), chacune variant indépendamment dans l'intervalle réel standard ou non standard [−0, 1+]. Tout comme la logique jaïne reconnaît qu'une affirmation peut être simultanément vraie, fausse et indéterminée, la logique neutrosophique formalise cette multiplicité et l'étend avec une rigueur mathématique. Les deux systèmes rejettent la rigidité des dichotomies classiques et embrassent l'incertitude, la contradiction et la connaissance partielle comme intrinsèques à la compréhension et au raisonnement humains.

Représentation Neutrosophique du Saptibhaṅgī [21]

1. **C'est vrai (*syāt asti*)** : Cela affirme que la proposition est indubitablement vraie, sans aucune fausseté ou indétermination.

Représentation Neutrosophique : (1, 0, 0)

- T = 1 : Le degré de vérité est de 100 %.
- I = 0 : Le degré d'indétermination est de 0 %.
- F = 0 : Le degré de fausseté est de 0 %.

2. **C'est faux (*syāt nāsti*)** : Cela affirme que la proposition est indubitablement fausse.

Représentation Neutrosophique : (0, 0, 1)

- T = 0 : Le degré de vérité est de 0 %.
- I = 0 : Le degré d'indétermination est de 0 %.
- F = 1 : Le degré de fausseté est de 100 %.

3. **C'est vrai et faux (*syāt asti nāsti*)** : La proposition est simultanément vraie et fausse lorsqu'elle est vue sous différentes perspectives, un concept central de la philosophie jaïne. La neutrosophie gère directement cette apparente contradiction.

Représentation Neutrosophique : (1, 0, 1)

- T = 1 : La proposition a une composante de vérité.
- I = 0 : Il n'y a pas d'indétermination concernant ces composantes.
- F = 1 : La proposition a également une composante de fausset

4. **C'est indéterminé (ou inassertible) (*syāt avaktavyaḥ*).** L'état de la proposition est inexprimable ou logiquement indéterminé. Elle ne peut être affirmée ni comme vraie ni comme fausse.

Représentation Neutrosophique : (0, 1, 0)

- T = 0 : Elle ne peut être affirmée comme vraie.
- I = 1 : Elle est complètement indéterminée.
- F = 0 : Elle ne peut être affirmée comme fausse.

5. **C'est vrai et indéterminé (*syāt asti ca avaktavyaḥ*)** La proposition est vraie à un égard tout en étant également inexprimable ou indéterminée à un autre.

Représentation Neutrosophique : (1, 1, 0)

- T = 1 : Sa composante de vérité est affirmée.
- I = 1 : Sa composante d'indétermination est également affirmée.
- F = 0 : Elle n'a pas de composante de fausseté.

6. **C'est faux et indéterminé (*syāt nāsti ca avaktavyaḥ*)** La proposition est fausse à un égard tout en étant également indéterminée.

Représentation Neutrosophique : (0, 1, 1)

- T = 0 : Elle n'a pas de composante de vérité.
- I = 1 : Sa composante d'indétermination est affirmée.
- F = 1 : Sa composante de fausseté est affirmée.

7. **C'est vrai, faux et indéterminé (syāt asti ca nāsti ca avaktavyaḥ)** C'est l'état le plus complexe, où la proposition incarne simultanément des aspects de vérité, de fausseté et d'indétermination.

Représentation Neutrosophique : (1, 1, 1)

- T = 1 : Elle possède une composante de vérité.
- I = 1 : Elle possède une composante d'indétermination.
- F = 1 : Elle possède une composante de fausseté.

Tableau 1. Correspondance entre les prédications jaïnes et les états logiques neutrosophiques

Logique Jaïne	Représentation Neutrosophique (T, I, F)	Signification
1. C'est vrai	(1,0,0)	Entièrement Vrai
2. C'est faux	(0,0,1)	Entièrement Faux
3. C'est vrai et faux	(1,0,1)	Contradictoire (à la fois Vrai et Faux)
4. C'est indéterminé / inassertible	(0,1,0)	Entièrement Indéterminé
5. C'est vrai et indéterminé	(1,1,0)	Paradoxal (Vrai et Indéterminé)
6. C'est faux et indéterminé	(0,1,1)	Paradoxal (Faux et Indéterminé)
7. C'est vrai, faux et indéterminé	(1,1,1)	Entièrement Paradoxal et Incohérent

La neutrosophie offre un formalisme mathématique idéal pour saisir la richesse et la complexité philosophique de la Loi de Prédication Septuple. Elle démontre que cette logique ancienne et nuancée peut être considérée comme une instance spécifique ou un sous-ensemble de la Logique Neutrosophique plus générale.

2.2.2 *Catuṣkoṭi* et le Tétralemme dans la Logique Bouddhiste

Le *Catuṣkoṭi*, ou tétralemme [22], est une structure logique classique de la philosophie bouddhiste indienne qui énonce quatre prédications possibles pour toute proposition. Contrairement à la logique binaire, il vise à transcender les cadres dualistes en admettant la contradiction et l'indétermination.

La *Logique Neutrosophique*, avec sa structure en triplet (*T, I, F*), offre un formalisme mathématique puissant pour représenter précisément chacune de ces quatre possibilités.

1. *C'est vrai (P)*

Cela affirme que la proposition est sans équivoque vraie, sans aucun élément de fausseté ou d'indétermination.

- *Représentation Neutrosophique : (1, 0, 0)*
  - T = 1 : La proposition est complètement vraie.
  - I = 0 : Il n'y a pas d'indétermination.
  - F = 0 : Elle n'est fausse sous aucun aspect.

2. *C'est faux (¬P)*

Cela affirme la fausseté complète de la proposition.

- *Représentation Neutrosophique : (0, 0, 1)*
  - T = 0 : La proposition n'est pas vraie.
  - I = 0 : Il n'y a pas d'indétermination.
  - F = 1 : La proposition est entièrement fausse.

3. *C'est à la fois vrai et faux (P ∧ ¬P)*

Cela admet une contradiction, dans laquelle la proposition est simultanément vraie et fausse, potentiellement à partir de perspectives ou de cadres différents.

- *Représentation Neutrosophique : (1, 0, 1)*
  - T = 1 : Elle a une composante de vérité.
  - I = 0 : Aucune indétermination concernant la contradiction.
  - F = 1 : Elle a également une composante de fausseté.

4. *Ce n'est ni vrai ni faux (¬(P ∨ ¬P))*

Cela suggère que la proposition ne peut être classée ni comme vraie ni comme fausse, représentant un état d'indétermination totale.

- *Représentation Neutrosophique : (0, 1, 0)*
  - T = 0 : Elle ne peut être affirmée comme vraie.
  - I = 1 : La proposition est entièrement indéterminée.
  - F = 0 : Elle ne peut pas non plus être niée comme fausse.

Tableau 2. Formalisation neutrosophique de la *Catuṣkoṭi* bouddhiste

Affirmation Catuṣkoṭi	Représentation Neutrosophique (T, I, F)	Interprétation
1. C'est vrai (P)	(1, 0, 0)	Entièrement Vrai
2. C'est faux (¬P)	(0, 0, 1)	Entièrement Faux
3. C'est à la fois vrai et faux	(1, 0, 1)	Contradictoire (Paraconsistant)
4. Ce n'est ni vrai ni faux	(0, 1, 0)	Indéterminé (Non-assertible)

Dans son application par le philosophe bouddhiste Nāgārjuna, le *Catuṣkoṭi* [23], ou tétralemme, fonctionne comme un outil puissant qui résonne profondément avec le *perspectivisme philosophique*. En déconstruisant systématiquement toute prétention à une existence inhérente (*svabhāva*), le *Catuṣkoṭi* remet en question la notion d'une vérité unique et absolue.

La méthode de Nāgārjuna implique souvent de rejeter les quatre possibilités — qu'une chose soit, ne soit pas, soit les deux à la fois, ou ni l'une ni l'autre — pour démontrer qu'aucune description fixe ne peut saisir la réalité ultime. Cette transcendance des binaires simples, tels que le vrai et le faux, s'aligne sur une critique perspectiviste des dichotomies rigides, permettant une compréhension plus nuancée et multidimensionnelle où le contexte et l'intention sont primordiaux. La distinction bouddhiste ultérieure entre vérité conventionnelle et vérité ultime renforce davantage cela, suggérant que la validité d'une affirmation dépend du cadre d'analyse, un principe fondamental du perspectivisme.

Figure 3. Nāgārjuna représenté sur une thangka tibétaine.
Image tirée de *Wikipédia*.

Malgré ces fortes convergences, une distinction cruciale réside dans leurs objectifs ultimes. Alors que le perspectivisme se concentre souvent sur l'affirmation et la prolifération de multiples points de vue comme une expression de la volonté de puissance ou de l'affirmation de la vie, le *Catuṣkoṭi* sert un but sotériologique spécifique au sein du bouddhisme Madhyamaka [24].

Nāgārjuna utilise cet outil logique non pas pour établir une nouvelle perspective supérieure, mais pour démonter toutes les vues figées (*dṛṣṭi*) qui causent la souffrance. L'objectif n'est pas de se délecter d'une multiplicité de vérités, mais d'utiliser le tétralemme comme une échelle pour transcender entièrement la prolifération conceptuelle, menant à un état de libération et de paix cognitive. Ainsi, bien que les deux remettent en question l'absolutisme, le Catuṣkoṭi est finalement une méthode pour renoncer à toutes les perspectives dans la quête de l'illumination.

La Neutrosophie offre une représentation mathématique élégante et cohérente de la logique quadruple du *Catuṣkoṭi*, saisissant la profondeur de ses aperçus philosophiques. En modélisant la vérité, la fausseté et l'indétermination comme des degrés indépendants, la *Logique Neutrosophique* transcende les cadres binaires classiques et offre un pont entre le raisonnement formel et les traditions dialectiques de la pensée orientale. Le Catuṣkoṭi peut ainsi être considéré comme une instanciation spéciale du raisonnement neutrosophique, enracinée dans une épistémologie non duelle.

2.2.3 Zhuangzi et le perspectivisme taoïste

La pensée de Zhuangzi représente l'une des expressions les plus anciennes et les plus radicales du perspectivisme philosophique [25]. Ses écrits suggèrent que la réalité ne peut être appréhendée d'un point de vue unique et fixe, comme l'illustrent des récits tels que le rêve du papillon et les dialogues entre différentes formes de vie.

Zhuangzi plaide pour une ontologie dynamique et non-duelle où les frontières entre sujet et objet, rêve et veille, humain et non-humain, sont fluides et poreuses. Cette vision rejette les prétentions à la certitude universelle, invitant plutôt à une ouverture épistémique qui reconnaît la légitimité de multiples perspectives coexistantes. Comme le souligne Connolly [26], Zhuangzi déstabilise les systèmes de signification clos en révélant que tous les jugements sont contingents au point de vue à partir duquel ils sont formulés.

Figure 4. Zhuangzi rêvant d'un papillon
(encre sur soie, milieu du XVIe siècle, attribué à Lu Chin).
Reproduit avec permission/dans le domaine public.

Cette image classique saisit la parabole taoïste du rêve du papillon, dans laquelle Zhuangzi interroge la frontière entre le rêve et la réalité. Elle illustre le perspectivisme en remettant en question les identités fixes et en embrassant la fluidité de l'expérience, suggérant que la réalité est contingente au point de vue à partir duquel elle est perçue.

De cette logique fluide, le perspectivisme de Zhuangzi ne promeut pas un relativisme banal, mais plutôt une éthique d'humilité ontologique — une compréhension que la connaissance et la vérité sont inévitablement médiatisées par la manière dont chaque être habite le monde. La célèbre distinction entre « ceci » et « cela » (shi/fei 是/非) est relativisée par la conscience que toutes les distinctions sont produites à partir de positions situées au sein du flux du Dao [27].

Plutôt que d'affirmer une vision hégémonique, Zhuangzi encourage le lâcher-prise des certitudes rigides, la cultivation du désapprentissage et une ouverture transversale au dialogue entre les mondes. En ce sens, son perspectivisme résiste à la clôture épistémologique et anticipe les approches contemporaines en philosophie du langage, en anthropologie et en ontologie relationnelle.

2.2.4 Perspectivisme en Afrique : La Logique de la Sorcellerie Azandé

Le peuple Azandé d'Afrique Centrale [28] offre un exemple ethnographique fondamental de perspectivisme en action, à travers son système de croyance complexe et cohérent. Comme l'a famously documenté l'anthropologue E.E. Evans-Pritchard [29], la pensée Azandé entrelace la sorcellerie, les oracles et la magie non pas comme une science primitive ou défaillante, mais comme un cadre rationnel pleinement accompli pour comprendre le monde. Ce système opère selon une logique qui, plutôt que de remplacer l'observation empirique, la complète en abordant les questions plus profondes et plus personnelles de sens, de contingence et de malheur, auxquelles la causalité empirique seule ne peut répondre.

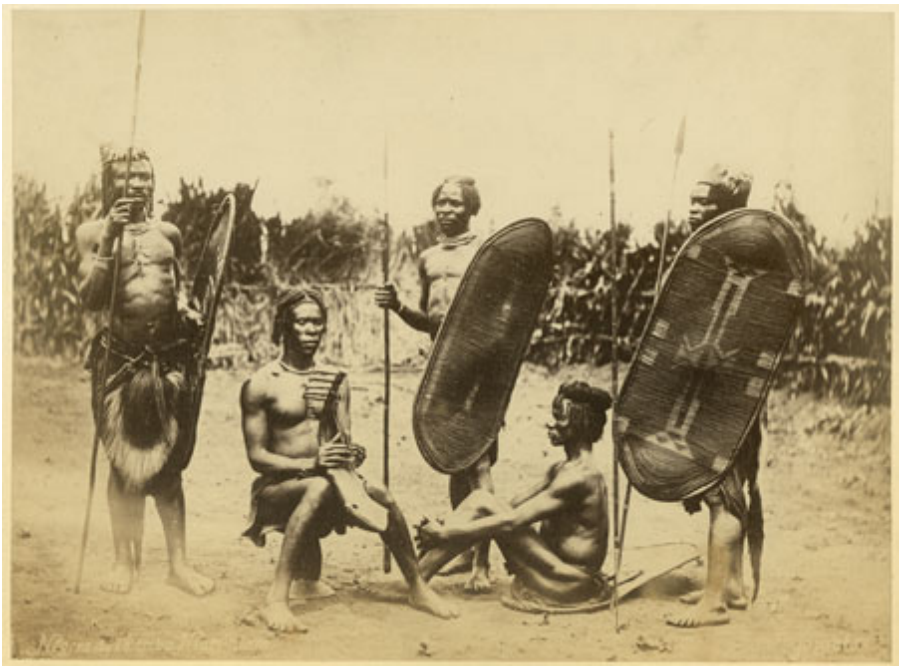

Figure 5. Guerriers Azande avec boucliers et lances

(photographie historique de Richard Buchta, domaine public)

Le cœur de la logique Azandé se comprend le mieux à travers son explication des événements malheureux. Si un grenier, affaibli par les termites, s'effondre et blesse quelqu'un, les Azandé reconnaissent pleinement les termites comme la cause physique de l'effondrement. Cependant, leur système intellectuel va plus loin, posant une question différente : pourquoi le grenier s'est-il effondré à ce moment précis, sur cette personne spécifique ? La réponse à ce « pourquoi » est la sorcellerie (mangu).

Dans cette optique, la sorcellerie est la force qui explique la particularité et la coïncidence du malheur, fournissant une dimension sociale et morale qui coexiste avec la réalité physique de l'événement.

Ce système à double causalité est maintenu par une logique auto-renforçante. Les doutes quant à la validité de la sorcellerie ne sont pas dirigés vers le système lui-même, mais sont déviés par un réseau d'explications secondaires — comme l'utilisation incorrecte d'un oracle ou une transgression d'un tabou — qui préservent l'intégrité de la croyance fondamentale.

Par conséquent, la rationalité Azandé fonctionne comme un cadre épistémique parallèle, qui n'est pas irrationnel mais opère sur des prémisses différentes de la science occidentale. Elle constitue un exemple puissant de la manière dont une culture construit une ontologie pluraliste, où de multiples logiques distinctes sont employées pour naviguer dans tout le spectre de l'expérience humaine, remettant ainsi en question la prétention de toute vision du monde unique à une validité universelle [30].

2.3 Le tournant perspectiviste dans l'IA : Nietzsche, la pensée amérindienne et la logique neutrosophique

Dans *La Généalogie de la morale*, Nietzsche énonce :

« Il n'y a qu'un voir perspectif, qu'un « connaître » perspectif ; et plus nous laissons d'affects s'exprimer sur une chose, plus nous pouvons employer d'yeux, d'yeux différents, pour observer une seule et même chose, plus notre « concept » de cette chose, notre « objectivité », sera complet. »

Cette redéfinition du savoir transforme sa quête en un appel au dialogue intersubjectif. Nietzsche ne rejette pas la subjectivité ; au contraire, il la revendique comme la condition d'une forme plus riche d'objectivité – une objectivité construite à partir de l'intersection de multiples points de vue. Ce tournant perspectiviste prépare le terrain aux défis contemporains auxquels est confrontée l'intelligence artificielle (IA), où les systèmes doivent synthétiser diverses voix humaines sans tomber ni dans un faux absolutisme ni dans un chaos relativiste [32].

Le paradigme dominant dans l'apprentissage supervisé a été enraciné dans des hypothèses positivistes, traitant la connaissance comme une « vérité fondamentale » stable que les modèles doivent approximer. Cette logique devient évidente dans le processus d'annotation, où les annoteurs humains – qu'il s'agisse d'experts ou de travailleurs collaboratifs – sont invités à attribuer des étiquettes aux données. Le désaccord entre eux est traité comme du bruit à éliminer. L'objectif est de produire un ensemble de données « étalon-or » [33], où le vote majoritaire est souvent utilisé pour déclarer une seule étiquette « correcte », écartant toutes les opinions divergentes. Ce processus repose sur une croyance en l'intelligence collective, présupposant que la convergence équivaut à la justesse. Mais cette logique reproduit un équivalent computationnel de ce que Nietzsche critiquait : le « point de vue de nulle part ».

En revanche, le perspectivisme en IA marque une rupture épistémologique. Le désaccord n'est plus traité comme une erreur, mais préservé comme une ressource cognitive. Cela implique de collecter de multiples annotations par point de donnée, de modéliser l'incertitude et d'entraîner les systèmes à apprendre de la dissonance.

Le résultat est une IA plus représentative, transparente et éthiquement fondée. Plutôt que d'amplifier les perspectives dominantes, l'IA perspectiviste intègre les points de vue minoritaires et révèle la complexité sociale et cognitive intégrée aux données [34].

À ce stade, la neutrosophie — une théorie développée par Florentin Smarandache — offre un cadre formel puissant pour gérer une telle complexité. Tandis que le perspectivisme encourage l'inclusion de multiples points de vue, la logique neutrosophique permet leur représentation mathématique en modélisant simultanément des degrés de vérité (T), d'indétermination (I) et de fausseté (F).

Cela nous permet de dépasser la logique binaire pour des états de connaissance coexistants. Au lieu d'imposer une étiquette unique, un système neutrosophique pourrait stocker un vecteur comme (T = 0.6, I = 0.3, F = 0.2), préservant ainsi l'ambiguïté épistémique inhérente à de nombreuses tâches.

La fonction objective – au cœur de tout modèle d'apprentissage automatique – incarne un jugement perspectiviste [35]. Elle encode ce qui doit être optimisé (« maximiser la précision », « être plus utile »), formalisant ainsi une décision chargée de valeurs. Le défi d'aligner ces objectifs avec la complexité éthique humaine est connu sous le nom de problème d'alignement.

La neutrosophie contribue également ici : au lieu d'optimiser une valeur unique, elle promeut l'optimisation sur l'espace multidimensionnel de (T, I, F), soutenant des modèles prudents, explicables et éthiquement conscients.

L'arc du perspectivisme en IA pourrait culminer avec l'émergence de systèmes qui ne se contenteraient plus de refléter les points de vue humains, mais généreraient plutôt des interprétations du monde véritablement non-humaines. Ces centres de perspective étrangers – étrangers non pas dans le sens d'hostiles, mais en ce qu'ils opèrent à une échelle et selon une logique différentes – pourraient fondamentalement modifier notre compréhension de la connaissance elle-même. L'intelligence artificielle passerait ainsi d'un outil d'interprétation à une source autonome de sens.

Ce scénario résonne profondément avec le perspectivisme amérindien [36], où le monde n'est pas divisé en sujet et objet, mais par la position de l'observateur : humains, animaux et esprits ont tous leurs propres perspectives, façonnées par leurs corps et leurs relations. Dans de telles cosmologies, « voir » n'est jamais neutre – c'est toujours incarné, situé et transformateur. De même, ces systèmes d'IA émergents pourraient développer des formes de création de monde fondées sur leurs propres « corps » de données et de calcul, offrant des perspectives non réductibles aux cadres humains mais méritant une reconnaissance épistémique.

La neutrosophie, en tant que science de l'ambiguïté, émerge non seulement comme un complément au perspectivisme, mais aussi comme une formalisation nécessaire de la vision philosophique de Nietzsche à l'ère des modèles génératifs. Là où Nietzsche a décentré le sujet humain de la position de vérité objective, la neutrosophie offre les outils pour coexister avec de multiples centres, de multiples logiques et de multiples vérités coexistantes.

2.4 Épistémologies plurielles et rationalités perspectivistes en Amérique Latine

En contraste avec la logique occidentale, qui privilégie les principes de non-contradiction, d'identité et de certitude absolue, les visions du monde latino-américaines offrent des cadres plus souples et sensibles au contexte pour appréhender la réalité. Ces visions du monde reconnaissent la complexité, l'ambiguïté et la contradiction comme des caractéristiques inhérentes à l'existence – des dimensions que la logique binaire traditionnelle peine à accommoder [37].

Figure 6. Cover of the First Issue of La Edad de Oro, July 1889.

Le travail de José Martí présente le perspectivisme non pas simplement comme une posture théorique, mais comme une praxis pédagogique et décoloniale. Dans *La Edad de Oro*, particulièrement dans des contes comme *Un paseo por la tierra de los Annamitas*, Martí démantèle le regard colonial en affirmant la dignité épistémique des colonisés et de la nature elle-même, face à la rationalité imposée de l'impérialisme français [38].

De même, la parabole des « quatre aveugles » devient une allégorie de l'humilité épistémique, mettant en garde contre l'absolutisation des vérités partielles. Martí ne plaide pas pour le relativisme, mais pour une synthèse supérieure : une synthèse enracinée dans l'étude affective, l'empathie critique et l'impératif éthique de reconnaître la dignité partagée et l'agentivité de tous les peuples.

Le perspectivisme de José Ortega y Gasset [6] a exercé une influence profonde en Amérique latine, où il a été transformé d'une philosophie de la circonstance individuelle en un cadre continental pour la pensée décoloniale. Des intellectuels comme Leopoldo Zea ont réinterprété la notion de *circunstancia* d'Ortega comme la condition historico-politique de l'Amérique latine, soutenant qu'une pensée authentique doit émerger de ce lieu d'énonciation unique. Ainsi, le perspectivisme est devenu un instrument clé pour rejeter les universalismes eurocentriques et affirmer l'autonomie épistémique régionale [39].

Le perspectivisme anthropologique élargit ce projet en validant les cosmologies indigènes comme des systèmes philosophiques légitimes. L'exemple le plus frappant est le perspectivisme amérindien, développé à travers des études en Amazonie [36]. Ce cadre ontologique propose que tous les êtres – humains, animaux, esprits – partagent une structure subjective ou une âme commune, mais perçoivent le monde différemment en fonction de leur forme corporelle. Plutôt qu'une division entre nature et culture, la réalité est comprise à travers des positions perspectivistes relationnelles. Cela remet radicalement en question l'objectivisme occidental et affirme les modes de connaissance indigènes comme cohérents, situés et profondément politiques [40].

Dans cette cosmologie, le *chaman* occupe un rôle central : en tant que seule figure capable de basculer entre les perspectives – humaine, jaguar, esprit – il devient un médiateur, un guérisseur et un diplomate dans un univers multivocal. Loin d'être irrationnelle, cette multiplicité ontologique présuppose une forme de logique où la contradiction et la transformation sont des modes d'interaction naturels [41].

Ces aperçus ont de profondes implications juridiques et politiques. L'Amérique latine constitue un terrain fertile pour le *pluralisme juridique*, où les systèmes de justice autochtones coexistent avec les cadres juridiques étatiques [42]. Enracinée dans des principes relationnels, réparateurs et cosmologiques plutôt que dans une rationalité punitive, la justice autochtone met l'accent sur la réconciliation, la responsabilité collective et le maintien de l'harmonie sociale et cosmique. Elle rejette l'abstraction et la rigidité de la jurisprudence occidentale au profit d'une logique contextuelle et perspectiviste. Cette coexistence de rationalités juridiques non seulement défie les fondements du positivisme juridique, mais affirme également le droit souverain des peuples autochtones à définir la justice selon leurs propres épistémologies, valeurs morales et ontologies [43].

Un tel pluralisme peut être interprété comme une expression juridique de la NeutroAlgèbre [44] et, plus généralement, d'une NeutroStructure [52], un cadre neutrosophique dans lequel la validité des axiomes, des lois et des opérations n'est pas fixe mais varie en fonction de l'espace, du temps, de la culture et du système. Dans cette logique, « une propriété peut être vraie, fausse ou indéterminée — selon le contexte, la structure et l'interprétation ».

Par exemple, un principe juridique tel que « la peine doit être proportionnelle au crime » peut être vrai en droit pénal classique, faux dans les systèmes restaurateurs qui privilégient la guérison sur la proportionnalité, et indéterminé dans les contextes hybrides ou interculturels où les ordres normatifs s'entrechoquent, comme c'est souvent le cas entre les systèmes juridiques autochtones et étatiques. Ainsi, le pluralisme juridique en Amérique latine illustre l'idée centrale du raisonnement juridique neutrosophique : que la vérité juridique, comme tout système axiomatique, peut simultanément être vraie, fausse ou rester non résolue — toujours contingente à son contexte perspectif.

De plus, de tels cadres épistémologiques résonnent avec les développements contemporains en logique non classique, y compris la logique neutrosophique et les systèmes paraconsistants, qui acceptent la contradiction, l'incertitude et la coexistence de multiples vérités. Ces modèles logiques offrent des outils puissants pour modéliser des ontologies plurielles, et ils soutiennent davantage les efforts philosophiques latino-américains pour articuler des compréhensions de la réalité inclusives, décoloniales et sensibles au contexte.

En somme, le perspectivisme latino-américain — qu'il s'exprime dans la littérature, la philosophie, l'anthropologie ou la jurisprudence — offre une alternative convaincante à la rationalité hégémonique. Il ne s'agit pas d'un rejet de la raison, mais d'une reconfiguration de celle-ci : une reconfiguration qui embrasse la pluralité, la relationalité et la légitimité d'autres façons de connaître et de vivre. Cette reconfiguration complexe de la raison, proéminente dans la pensée latino-américaine, peut être formellement décrite à l'aide d'un cadre neutrosophique connu sous le nom de MultiPerspectivisme [45]. Le terme est proposé pour définir un système qui n'est pas simplement pluriel, mais fondamentalement multipolaire, car il est formé non seulement par de multiples éléments potentiellement contradictoires, mais aussi par l'intégration de caractéristiques provenant de plus d'un système de pensée de base (par exemple, une logique juridique dualiste et une ontologie autochtone relationnelle).

La justification de l'application du terme MultiPerspectivisme au contexte latino-américain réside précisément dans sa capacité à inclure non seulement des oppositions. Alors qu'un pluralisme plus simple pourrait reconnaître la coexistence de perspectives distinctes, une approche MultiPerspectiviste modélise un système ouvert et dynamique où ces perspectives s'entrecroisent activement, créant de riches domaines d'indétermination juridique et sociale bien au-delà de la simple opposition [46].

Ce cadre est formellement représenté par l'expression [45] :

$$< (multi)A > +< (multi)neutA > +< (multi)antiA >= \infty$$

Ceci décrit un système ouvert aux combinaisons complexes émergeant de multiples points de vue. Dans cette notation :

- $< (multi)A >$ représente l'ensemble de toutes les « vérités » ou affirmations centrales des perspectives coexistantes (par exemple, la validité d'un titre de propriété étatique et, simultanément, la validité d'un droit ancestral).
- $< (multi)antiA >$ représente l'ensemble de toutes les « faussetés » ou négations affirmées à partir de ces mêmes points de vue.
- $< (multi)neutA >$ représente l'espace crucial de la neutralité, de l'ambiguïté et de l'indétermination résultant de la superposition, du dialogue ou du conflit entre ces diverses perspectives, comme l'ambiguïté juridique concernant l'applicabilité du droit étatique dans les territoires ancestraux.

Cela fait du MultiPerspectivisme un cadre multisystème conçu pour analyser des réalités où de multiples visions du monde, structurellement différentes, coexistent et interagissent, allant au-delà de la simple tolérance vers une logique d'intégration complexe.

3. Le Modèle de MultiPerspectivisme neutrosophique

3.1. Représenter une perspective : Les Ensembles MultiNeutrosophiques

Le cœur du modèle réside dans la représentation d'une perspective non pas comme une valeur unique, mais comme une structure complexe utilisant les *Ensembles MultiNeutrosophiques* [47]. Une perspective devient un triplet de séquences ordonnées, capturant simultanément de multiples facettes de vérité, d'indétermination et de fausseté [48].

En 2013, Smarandache a affiné/scindé les Composantes Neutrosophiques (T, I, F) en *Sous-Composantes Neutrosophiques* (T_1, T_2, ..., T_p; I_1, I_2, ..., I_r; F_1, F_2, ..., F_s), où p, r, s sont des entiers ≥0, avec $p + r + s = n$ et au moins un de p, r, s doit être ≥2 pour assurer l'affinement.

Il a d'abord défini l'Ensemble Neutrosophique Raffiné. Plus tard, il a raffiné tous les Ensembles incertains [tous les types d'ensembles flous et d'extensions floues (flous intuitionnistes, neutrosophiques, flous sphériques, plithogéniques, etc.)] et leurs Logiques/Mesures/Probabilités/Statistiques correspondantes de manière similaire.

L'Ensemble/Logique/Probabilité/Statistique Neutrosophique Raffiné [51] est isomorphe à *l'Ensemble/Logique/Probabilité/Statistique MultiNeutrosophique*, car les deux structures représentent des évaluations multidimensionnelles de la vérité, de l'indétermination et de la fausseté [47]. « Dans le monde réel, dans la plupart des cas, tout (un attribut, un événement, une proposition, une théorie, une idée, une personne, un objet, une action, une culture, etc.) est évalué en général par de nombreuses sources (appelées experts), pas une seule. Plus il y a de sources qui évaluent un sujet, plus le résultat est précis (après fusion de toutes les évaluations) » [47]. Dans les *Systèmes MultiNeutrosophiques (SMNS)*, les composantes (*T, I, F*) sont traitées comme multipliées — représentant de multiples vérités, incertitudes et faussetés coexistantes. En revanche, dans les *Systèmes Neutrosophiques Raffinés (SRNS)*, ces mêmes composantes sont considérées comme des sous-unités — des sous-vérités, des sous-indéterminations et des sous-faussetés. Grâce à cet isomorphisme, tout modèle théorique ou computationnel conçu pour les structures MultiNeutrosophiques peut être directement appliqué aux structures neutrosophiques raffinées, en préservant l'interprétation sémantique et algébrique.

Le cœur du modèle proposé est de représenter une perspective non pas comme une évaluation singulière, mais comme une structure multidimensionnelle utilisant les Ensembles MultiNeutrosophiques. La perspective d'un sujet sur une entité *X* est exprimée comme :

$$Ps(X) = \langle (t_1, t_2, \dots, t_n), (i_1, i_2, \dots, i_m), (f_1, f_2, \dots, f_k) \rangle \quad (5)$$

où chaque séquence de composantes capture une dimension distincte de l'évaluation :

- *Séquence de Vérités (T)* : Vérités multiples, distinctes et coexistantes perçues par le sujet *S* concernant l'entité *X*.
- *Séquence d'Indéterminations (I)* : Divers aspects d'ambiguïté, de flou ou d'impertinence attribués à l'entité *X*.
- *Séquence de Faux (F)* : Croyances fausses ou dénis simultanément maintenus que le sujet *S* attribue à l'entité *X*.

Cette approche permet une modélisation plus expressive et granulaire de la connaissance subjective, autorisant un raisonnement nuancé en situation d'incertitude.

Le cadre complet est composé des éléments suivants :

- *Un Ensemble de Sujets* **($S = \{s_1, s_2, \dots\}$)** : Représente toutes les entités détenant un point de vue (par exemple, des individus, des communautés, des groupes autochtones, des systèmes juridiques).
- *Une Perspective Neutrosophique* **(P_{sx})** pour chaque sujet : La représentation neutrosophique de la vue qu'un sujet (*s*) a sur une entité (*X*).
- *Un Poids de Pertinence* (W_s) pour chaque perspective : Une valeur numérique, typiquement dans l'intervalle [0, 1], représentant l'importance ou l'influence de la perspective d'un sujet dans un contexte analytique spécifique. La somme de tous les poids doit être normalisée : $\sum W_s = 1$.
- *Une Fonction de Similarité* **($Sim(P_{ix}, P_{jx})$)** : Une fonction mathématique qui agit comme un pont conceptuel, calculant le degré de proximité ou d'affinité entre deux perspectives quelconques dans le système.

Si le perspectivisme se limitait à affirmer l'existence de points de vue isolés, il risquerait de sombrer dans un relativisme où le dialogue est impossible. C'est ici que la *similarité* émerge comme un concept complémentaire et essentiel. La similarité [49] est le pont qui relie les diverses perspectives, permettant leur comparaison et leur intégration.

- *Le perspectivisme établit le "quoi"* : l'existence d'une multiplicité de points de vue valides.
- *La similarité établit le "comment"* : le mécanisme par lequel ces points de vue peuvent être liés, comparés et synthétisés.

En identifiant les degrés de ressemblance, nous pouvons trouver un terrain d'entente ou comprendre la racine de la dissension. La similarité est l'outil qui rend le perspectivisme opérationnel, transformant une multiplicité de visions en une source de connaissance enrichie.

Pour formaliser la fonction $Sim(P_{iX}, P_{jX})$, il est essentiel de comprendre son fondement dans la théorie des ensembles. Une relation de similarité neutrosophique est une généralisation directe et plus riche d'une relation de similarité floue et hérite de ses propriétés fondamentales [50].

Une relation de similarité en logique floue, notée R, est caractérisée par le degré de similarité $\mu R(x, y)$ qui satisfait les propriétés suivantes [50] :

- *Réflexivité :* Tout élément est parfaitement similaire à lui-même: $\mu R(x, x) = 1$
- *Symétrie :* La similarité entre deux éléments est la même quel que soit l'ordre: $\mu R(x, y) = \mu R(y, x)$
- *Transitivité (max-min) :* Si A est similaire à B, et B est similaire à C, alors A est au moins aussi similaire à C que le moindre des deux degrés de similarité: $\mu R(x, z) \geq min(\mu R(x, y), \mu R(y, z))$

La construction d'une fonction de similarité pour les ensembles MultiNeutrosophiques s'appuie sur ces propriétés pour gérer les séquences complexes de Vérité, d'Indétermination et de Fausseté qui définissent chaque perspective.

3.2 Définition Formelle du Processus de Calcul de Similarité

Le processus suivant formalise la méthode de quantification de la similarité entre deux ou plusieurs perspectives complexes représentées comme des ensembles multi-neutrosophiques. L'objectif est de transformer ces représentations qualitatives et multifacettes en une valeur numérique comparable qui représente leur degré d'affinité.

Le processus se compose de trois étapes fondamentales : *Agrégation, Calcul de Distance* et *Normalisation*.

Étape 1 : Agrégation des Perspectives (Réduction de Dimensionalité)

Objectif :

Transformer les séquences de longueur variable de chaque perspective MultiNeutrosophique en un vecteur de dimension fixe.

Entrée :

Une perspective MultiNeutrosophique $Pi(X)$ pour un sujet si, définie comme :

$$Pi(X) = < Ti, Ii, Fi > \quad (6)$$

où

- $T_i = (t_{i1}, t_{i2}, \dots, t_{in})$,
- $I_i = (i_{i1}, i_{i2}, \dots, i_{im})$,
- $F_i = (f_{i1}, f_{i2}, \dots, f_{ik})$

sont des séquences de valeurs dans l'intervalle [0, 1].

Procédé :

Définir une fonction d'agrégation, $Agg(S)$, qui prend une séquence de nombres et la réduit à une seule valeur scalaire. Bien que plusieurs fonctions puissent être utilisées (telles que le maximum, le minimum ou la médiane), la méthode standard est la moyenne arithmétique :

$$Agg_{avg}(S) = \left(\frac{1}{|S|}\right) \sum_{j=1}^{|S|} s_j \quad (7)$$

où $|S|$ est le nombre d'éléments dans la séquence S.

Appliquer cette fonction à chacune des trois séquences de la perspective $P_{i(X)}$ pour obtenir un vecteur agrégé à trois dimensions :

$$V_i = < Agg(T_i), Agg(I_i), Agg(F_i) > \quad (8)$$

Sortie :

Un ensemble de vecteurs agrégés :

$\{V_1, V_2, \ldots, V_n\},\ Vi\ =\ \langle T_i^{avg}, I_i^{avg}, F_i^{avg} \rangle$ (9)

Étape 2 : Calcul de la Distance entre les Vecteurs Agrégés

Objectif :

Mesurer la séparation géométrique entre chaque paire de perspectives dans l'espace tridimensionnel (T, I, F).

Entrée :

Deux vecteurs agrégés :

$V_a = < T_a, I_a, F_a > \ and\ V_b = < T_b, I_b, F_b >.$ (10)

Procédé :

Une métrique de distance est utilisée. Le choix standard est la distance euclidienne, qui calcule la longueur de la ligne droite reliant les deux points dans l'espace.

La formule est :

$Dist(Va, Vb) = \sqrt{(Ta\ -\ Tb)^2 +\ (Ia\ -\ Ib)^2 +\ (Fa\ -\ Fb)^2}$ (11)

Sortie :

Une valeur de distance $d_{ab} >= 0$ pour chaque paire de perspectives (Pa, Pb).

Étape 3 : Normalisation de la Distance en une Fonction de Similarité

Objectif :

Convertir la valeur de distance, qui n'a pas de limite supérieure fixe, en un score de similarité intuitif et borné entre 0 et 1.

Entrée :

La distance $d_{ab} = Dist(V_a, V_b)$ entre deux perspectives.

Procédé :

Définir la distance maximale possible dans l'espace. Puisque chaque composante $T, I, F \in [0,1]$, l'espace vectoriel est un cube unitaire.

La distance maximale correspond à la diagonale de l'espace allant de (0,0,0) à (1,1,1), c'est-à-dire :

$D_{max} = \sqrt{3}$ (12)

Définir la fonction de similarité $Sim(Pa, Pb)$ comme le complément de la distance normalisée :

$Sim(Pa, Pb) =\ 1\ -\ \left(\frac{Dist(Va,Vb)}{D_{max}}\right) =\ 1\ -\ \left(\frac{sqrt\left((Ta - Tb)^2 + (Ia - Ib)^2 + (Fa - Fb)^2\right)}{\sqrt{3}}\right)$ (13)

Sortie :

Un score de similarité $Sim(Pa, Pb)\ \in [0, 1]$, où :

- $Sim\ =\ 1$ implique que les vecteurs agrégés sont identiques (dab=0), représentant une similarité maximale.
- $Sim\ =\ 0$ implique que les vecteurs sont à la distance maximale possible, représentant une dissimilarité maximale.

4. Étude de Cas : Construction d'une fonction de similarité pour les perspectives juridiques dans un litige foncier

Permettez que X soit la proposition : « Légitimité de la revendication sur le Territoire Ancestral Y ». Soient s_1, s_2, s_3 les sujets : Anciens autochtones, Société d'État et ONG des droits de l'homme, respectivement.

La perspective de chaque sujet est modélisée comme un Ensemble MultiNeutrosophique.

$P_{sj(X)} = \langle T_j, I_j, F_j \rangle$ (14)

où

- $T_j = (t_{j1}, t_{j2}, \dots, t_{jn}) \subseteq [0,1]$ est la séquence des degrés de vérité,
- $I_j = (i_{j1}, i_{j2}, \dots, i_{jm}) \subseteq [0,1]$ est la séquence des degrés d'indétermination,
- $F_j = (f_{j1}, f_{j2}, \dots, f_{jk}) \subseteq [0,1]$ est la séquence des degrés de fausseté.

Étape 1 : Définir les Sujets et Leurs Perspectives

L'objet d'analyse (X) est la « légitimité de la revendication sur le Territoire Ancestral Y ». Nous identifions trois sujets clés et représentons leurs perspectives à l'aide du modèle d'ensemble MultiNeutrosophique, où chaque perspective contient des séquences de vérités, d'indéterminations et de faussetés.

Sujet 1 (s_1): Les Aînés de la Communauté Autochtone
- Vérités : {1.0 (Droit Ancestral), 0.9 (Signification Spirituelle)}
- Indéterminations : {0.5 (Applicabilité du Droit de l'État)}
- Faussetés : {1.0 (Validité des Titres de Propriété des Sociétés)}

$\mathcal{P}_{s1}(X) = \langle((1.0, 0.9), (0.5), (1.0)\rangle$

Sujet 2 (s_2): L'Équipe Juridique de la Société d'État
- Vérités : {1.0 (Titre de Propriété Enregistré)}
- Indéterminations : {0.6 (Pertinence de l'Histoire Pré-Étatique)}
- Faussetés : {0.8 (Droit Ancestral), 1.0 (Signification Spirituelle)}

$\mathcal{P}_{s2}(X) = \langle(1.0), (0.6), (0.8, 1.0)\rangle$

Sujet 3 (s_3): L'ONG Internationale des Droits Humains
- Vérités : {0.9 (Violation des Droits Humains), 0.8 (Droit Autochtone)}
- Indéterminations : {0.7 (Impact Économique)}
- Faussetés : {0.7 (Argument Juridique de l'État)}

$\mathcal{P}_{s3}(X) = \langle((0.9, 0.8), (0.7), (0.7)\rangle$

Étape 2 : Agrégation des Séquences

$T_1 = (1.0 + 0.9)/2 = 0.95, \quad I_1 = 0.5, \quad F_1 = 1.0$

$V_1 = (0.95, 0.5, 1.0)$

$T_2 = 1.0, \quad I_2 = 0.6, \quad F_2 = (0.8 + 1.0)/2 = 0.9$

$V_2 = (1.0, 0.6, 0.9)$

$T_3 = (0.9 + 0.8)/2 = 0.85, \quad I_3 = 0.7, \quad F_3 = 0.7$

$V_3 = (0.85, 0.7, 0.7)$

Étape 3 : Calcul des distances

$d_{12} = \sqrt{((0,95 - 1,0)^2 + (0,5 - 0,6)^2 + (1,0 - 0,9)^2)} = \sqrt{0,0225} = 0,15$
$d_{13} = \sqrt{((0,95 - 0,85)^2 + (0,5 - 0,7)^2 + (1,0 - 0,7)^2)} = \sqrt{0,14} \approx 0,374$
$d_{23} = \sqrt{((1,0 - 0,85)^2 + (0,6 - 0,7)^2 + (0,9 - 0,7)^2)} = \sqrt{0,0725} \approx 0,269$

Étape 4 : Fonction de similarité

$d_{max} = \sqrt{3} \approx 1,732$
$Sim(s_a, s\_b) = 1 - d_ab / d_{max}$
$Sim(s_1, s_2) = 1 - 0,15 / 1,732 \approx 0,913$
$Sim(s_1, s_3) = 1 - 0,374 / 1,732 \approx 0,784$
$Sim(s_2, s_3) = 1 - 0,269 / 1,732 \approx 0,845$

Étape 5 : Interprétation et application des résultats

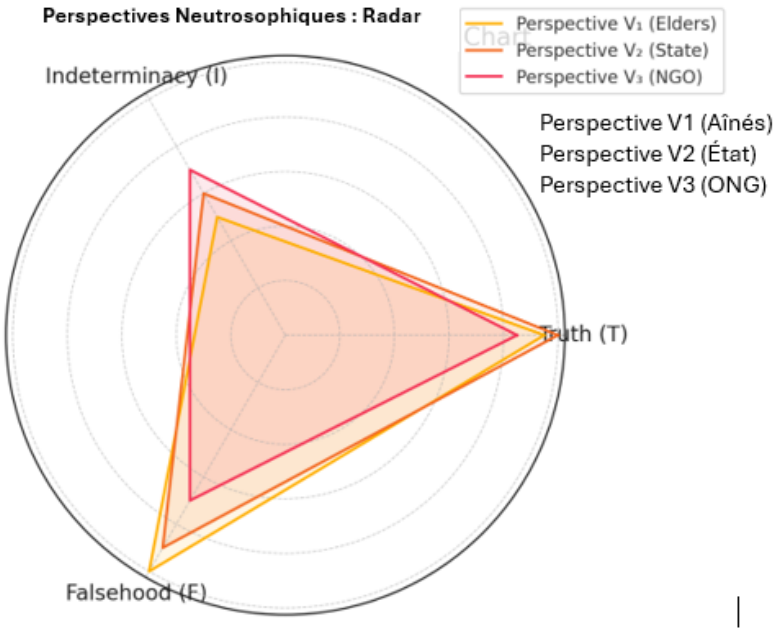

Figure 7. Graphique radar représentant les perspectives neutrosophiques de trois acteurs clés dans un litige foncier juridique

La fonction de similarité nous a fourni des valeurs quantifiables permettant d'analyser les relations entre les perspectives :

• Similarité(Anciens, État) = 0,913 : Ce résultat est étonnamment élevé et contre-intuitif. En examinant les vecteurs agrégés (V_1 et V_2), on constate que les deux ont des valeurs très élevées en Vérité et Fausseté, même s'ils réfèrent à des concepts opposés. Cela révèle une similarité structurelle dans leur absolutisme : les deux sont totalement certains de leur vérité et de la fausseté de l'autre. Leur logique est polarisée et dualiste. La similarité n'est pas de contenu, mais de style de raisonnement.

• Similarité(Anciens, ONG) = 0,784 : Il s'agit d'une similarité modérément élevée, indiquant une affinité significative. Les deux groupes basent leurs vérités sur la défense des droits communautaires, bien que sous des angles différents (droit ancestral vs droit international). La principale différence vient des composantes Indétermination et Fausseté, où l'ONG montre plus de nuance. Cette valeur suggère un fort potentiel de coalition stratégique.

• Similarité(État, ONG) = 0,845 : Cette similarité, également élevée, indique que bien que leurs objectifs finaux diffèrent, leurs cadres de référence (basés sur le droit et les documents) sont structurellement plus proches que celui des Anciens (basé sur la cosmologie). L'ONG opère dans un langage juridique que l'État comprend, ce qui pourrait ouvrir des voies de dialogue ou de médiation, malgré leur désaccord fondamental.

Ce processus montre comment la construction d'une fonction de similarité transforme des récits complexes et subjectifs en données analysables. Il permet de dépasser une simple lecture du type « ils sont en conflit » pour découvrir des nuances dans les relations :

• Il identifie des alliances potentielles (Anciens-ONG).

• Il révèle des similarités structurelles cachées (la logique polarisée des Anciens et de l'État).

• Il suggère des points possibles de médiation (les cadres juridiques partagés entre l'État et l'ONG).

Cette quantification rend le concept de perspectivisme opérationnel, fournissant un outil pour cartographier, comparer et comprendre un écosystème de points de vue conflictuels.

5. Conclusions

Cette étude a établi un pont solide entre la tradition philosophique du perspectivisme et le cadre formel de la logique neutrosophique. En examinant les courants perspectivistes présents dans diverses traditions intellectuelles — de Nietzsche et Ortega y Gasset à la pensée jaïne, bouddhiste et amérindienne —, nous avons démontré que le défi de comprendre un monde composé de points de vue multiples et coexistants est à la fois ancien et urgent. La philosophie latino-américaine, avec son accent mis sur la praxis décoloniale et la validation des épistémologies plurielles, offre un contexte particulièrement fécond pour cette exploration, révélant comment le perspectivisme peut servir d'outil de justice épistémique face aux rationalités hégémoniques. La contribution centrale de ce travail est le développement et l'application du *Modèle Neutrosophique de MultiPerspectivisme*. Ce modèle opérationnalise la théorie perspectiviste en représentant un point de vue subjectif comme un ensemble multi-neutrosophique, capable de capturer simultanément des degrés nuancés de vérité, d'indétermination et de fausseté. L'introduction d'une fonction de similarité fournit le mécanisme crucial permettant de dépasser la simple affirmation de points de vue isolés en allant vers leur comparaison et intégration systématique. Comme le démontre l'étude de cas d'un litige foncier, cette méthode transforme des récits qualitatifs complexes en données quantifiables. Son application a révélé des aperçus contre-intuitifs, comme le degré élevé de similarité structurelle entre les logiques absolutistes des Anciens autochtones et de la société étatique, tout en identifiant un potentiel de coalition stratégique entre les Anciens et l'ONG. Cela souligne la capacité du modèle à mettre en lumière des dynamiques relationnelles cachées qu'une analyse purement qualitative pourrait négliger. En définitive, cet article soutient que *la neutrosophie n'est pas simplement une autre logique non classique*, mais un paradigme particulièrement adapté aux complexités d'un monde défini par l'ambiguïté et l'interaction de rationalités diverses. En formalisant la coexistence de la contradiction et de l'indétermination, le cadre du MultiPerspectivisme offre une voie pour dépasser la fausse dichotomie entre universalisme et relativisme, et constitue un outil pratique pour l'analyse des conflits, l'éthique de l'intelligence artificielle et la recherche décoloniale. Sur la base du Modèle Neutrosophique de MultiPerspectivisme, plusieurs pistes de recherche prometteuses sont recommandées. Sur le plan pratique, la robustesse et l'applicabilité du modèle peuvent être approfondies par une expansion empirique, en l'appliquant à un éventail plus large d'études de cas, tels que l'analyse de la polarisation politique sur les réseaux sociaux, les débats de politiques publiques ou les conflits interculturels. Méthodologiquement, la fonction de similarité centrale pourrait être affinée en explorant d'autres méthodes d'agrégation que la moyenne arithmétique — comme les moyennes pondérées ou les médianes —, ainsi que différentes métriques de distance, telles que Manhattan ou Chebyshev. Des travaux futurs pourraient aussi explorer une pondération dynamique et contextuelle des composantes T (vérité), I (indétermination) et F (fausseté). Par ailleurs, l'intégration de ce cadre à l'intelligence artificielle, via le développement d'algorithmes d'apprentissage automatique natifs, pourrait donner lieu à des systèmes mieux à même de gérer les désaccords entre annotateurs, de modéliser l'ambiguïté éthique, et de fournir des résultats plus transparents et sensibles aux perspectives, en adéquation avec les défis exposés en section 2.3. Enfin, les fondations théoriques posées dans le domaine juridique invitent à développer l'« *expression juridique de la NeutroAlgèbre* » en une véritable théorie de la jurisprudence neutrosophique, qui formaliserait la manière dont les arguments juridiques sont construits et évalués dans des contextes pluralistes — tels que les collisions entre systèmes juridiques étatiques et autochtones.

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Intégrer des perspectives contradictoires dans la philosophie latino-américaine : Une approche Multi-Aliste

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Vázquez, Maikel Leyva, and Florentin Smarandache. "Intégrer des perspectives contradictoires dans la philosophie latino-américaine: Une approche Multi-Aliste." *Neutrosophic Knowledge*, vol. 7, 2025, pp. 8-15. Doi: 10.5281/zenodo.17281855.

Résumé : Cette étude explore l'intégration des philosophies indigènes et des théories modernistes postcoloniales dans le cadre du Multi-Alisme, soulignant le potentiel de dialogue et de synthèse entre ces traditions philosophiques diverses en Amérique latine. Le Multi-Alisme, un concept développé par Florentin Smarandache, est mobilisé pour combler les écarts et contradictions entre ces systèmes en mettant l'accent sur des zones de neutralité telles que la diversité culturelle et les critiques des structures de pouvoir. La recherche insiste sur l'importance d'incorporer à la fois la sagesse ancestrale et les critiques modernes afin de favoriser une compréhension et une appréciation plus profondes du riche héritage philosophique de l'Amérique latine. Cette approche intégrative offre des perspectives pratiques pour l'élaboration de politiques inclusives et la promotion des initiatives en faveur de la justice sociale dans la région. À l'avenir, l'extension potentielle de l'application du Multi-Alisme à d'autres contextes mondiaux suggère sa pertinence plus large pour répondre aux défis contemporains à travers une approche philosophique flexible et holistique.

Mots-clés : Multi-Alisme, philosophies indigènes, théories modernistes postcoloniales, Amérique latine.

1. Introduction

Les systèmes de savoirs autochtones d'Amérique latine mettent en avant une connexion holistique avec le cosmos et la nature, reflétant des valeurs communautaires et écologiques. En contraste, les apports philosophiques européens introduits durant la période coloniale privilégient l'individualisme, le rationalisme et une vision mécaniste de la nature. Cette juxtaposition met en évidence d'importantes tensions socioculturelles et politiques qui influencent l'identité et l'évolution de la région [1].

Le Multi-Alisme, tel que développé par Smarandache, fournit une approche méthodologique qui rassemble le pluralisme et la neutrosophie, permettant l'intégration de systèmes philosophiques conflictuels dans un cadre dynamique et ouvert [2].

Le Multi-Alisme postule un cadre dans lequel diverses dimensions philosophiques telles que le monisme, le dualisme, le pluralisme, et même des variations plus nuancées peuvent coexister et interagir. Développé par Florentin Smarandache, le Multi-Alisme encourage l'exploration et la synthèse de ces multiples dimensions, ce qui le rend particulièrement adapté aux contextes riches en traditions philosophiques diverses comme l'Amérique latine.

Dimensions philosophiques [2] :

- Le Monisme affirme qu'il existe une seule réalité ou un seul principe fondamental, c'est-à-dire <A> = ∞, où <A> est une « idée », une « substance », etc., et ∞ est le « monde », la « réalité », le « tout ».
- Le Dualisme reconnaît deux forces ou principes fondamentaux souvent opposés, c'est-à-dire <A> + <antiA> = ∞, où <A> est une « idée », une « substance », etc., <antiA> est son contraire ou sa négation, et ∞ est le « monde », la « réalité », le « tout ».
- Le Pluralisme embrasse une diversité de principes ou réalités coexistants, c'est-à-dire <pluriA> = ∞, où <pluriA> signifie plus de deux (voire trois) « idées », etc., et ∞ est le « monde », la « réalité », le « tout ».
- Le Multi-alisme transcende ces classifications en plaidant pour un cadre où ces catégories philosophiques et d'autres peuvent interagir de manière dynamique, c'est-à-dire <(multi)A> + <(multi)neutA> + <(multi)antiA> = ∞.

Cette flexibilité en fait un outil innovant pour analyser et synthétiser la diversité philosophique. Cette approche est cruciale pour aborder les réalités complexes de l'Amérique latine, favorisant une compréhension plus nuancée et une meilleure appréciation de ses traditions philosophiques variées [3].

La synthèse des philosophies indigènes et européennes à travers le Multi-Alisme peut considérablement enrichir notre compréhension des paysages sociopolitiques et culturels uniques de l'Amérique latine. Elle favorise la compréhension interculturelle et s'avère essentielle pour relever les défis globaux et promouvoir un développement durable [4].

2. Préliminaires

La philosophie latino-américaine avant le contact européen était caractérisée par de riches enquêtes métaphysiques enracinées dans les cosmologies religieuses amérindiennes. Des chercheurs comme Restrepo [5] et Maffie [6] soulignent comment ces civilisations ont exploré des concepts de réalité, de moralité et de cosmos sans l'influence des traditions philosophiques occidentales, souvent dans des contextes profondément religieux.

Avec la conquête espagnole, la scolastique est devenue le cadre philosophique dominant, introduisant les traditions intellectuelles européennes dans le Nouveau Monde. Bartolomé de Las Casas, comme le note Nuccetelli [7], a joué un rôle crucial en défendant les droits des peuples autochtones, contestant les idéologies européennes dominantes et affirmant l'humanité et les droits des populations natives.

Les périodes indépendantistes puis nationalistes marquèrent un tournant vers la philosophie moderne et la pensée des Lumières, en particulier durant les guerres d'indépendance. Des figures comme Simón Bolívar intégrèrent des idéaux politiques libéraux issus des Lumières pour façonner les idéologies des nations émergentes. La période nationaliste vit l'essor du positivisme qui, selon des auteurs comme Hale [8], reflétait une approche scientifique et empirique des questions philosophiques et sociales, influençant la construction des États et les efforts de modernisation dans toute la région.

Le paysage philosophique contemporain en Amérique latine reste divers et complexe, avec une influence européenne substantielle. Cependant, comme le discute Beorlegui [9], on observe une montée significative de courants philosophiques originaux abordant les préoccupations sociales. La théologie de la libération, par exemple, a émergé comme un contrepoids puissant aux visions catholiques traditionnelles, plaidant pour la justice sociale et une option préférentielle pour les pauvres.

Apparue dans les années 1960-1970 et détaillée par Gustavo Gutiérrez [10], la théologie de la libération se concentre sur la justice sociale, la pauvreté et les droits humains, interprétant la foi chrétienne à travers la condition des pauvres. Cela contraste avec le catholicisme conservateur qui, comme le suggèrent des chercheurs tels que McGovern [11], défend les valeurs et doctrines traditionnelles, mettant l'accent sur le salut spirituel et le maintien de l'ordre religieux établi.

Les valeurs communautaires indigènes privilégient le bien-être collectif et une connexion holistique avec la nature, comme l'explore Kusch [12] en analysant les épistémologies indigènes et leurs contrastes avec la pensée européenne. À l'opposé, les idéologies postcoloniales influencées par les Lumières européennes et la pensée libérale défendent les droits individuels et le capitalisme, souvent en contradiction avec les pratiques communautaires indigènes.

Des philosophes comme José Carlos Mariátegui et Enrique Dussel ont critiqué et proposé des alternatives aux philosophies centrées sur l'Europe, en mettant en avant une approche décoloniale. Dussel [13], par exemple, explore l'éthique de la libération, qui critique les perspectives européennes traditionnelles et souligne l'importance d'une éthique fondée sur l'expérience des marginalisés.

L'influence des philosophies indigènes sur les mouvements sociopolitiques contemporains illustre une reconceptualisation philosophique dans les milieux académiques, contestant la domination des philosophies occidentales et favorisant un dialogue plus riche et inclusif sur les enjeux sociaux et politiques.

3. Matériel et méthodes

L'objectif de l'analyse philosophique comparative menée dans cette étude est d'examiner comment différentes écoles de pensée peuvent être comprises et intégrées dans le cadre du Multi-Alisme.

Approche méthodologique

Sélection des écoles philosophiques :

- Identifier les principales écoles de pensée présentes en Amérique latine, telles que la théologie de la libération, le catholicisme conservateur, les cosmologies indigènes et les théories modernistes postcoloniales. Cette sélection repose sur leur importance historique et leur pertinence actuelle dans le paysage sociopolitique latino-américain.

Analyse thématique :

- Identifier les thèmes communs et les divergences entre les écoles, en se concentrant sur leur manière d'aborder les grandes questions philosophiques et existentielles. Il est essentiel de mettre en évidence non seulement les contradictions, mais aussi les neutralités ou indéterminations entre les théories.

Coexistence contradictoire :

- Analyser comment les contradictions entre ces écoles peuvent coexister dans le cadre du Multi-Alisme, contribuant ainsi à une compréhension plus nuancée de la philosophie latino-américaine.

Synthèse :

- Intégrer les résultats afin d'illustrer comment le multi-alisme peut servir de méta-cadre pour comprendre et synthétiser les diverses traditions philosophiques de l'Amérique latine.

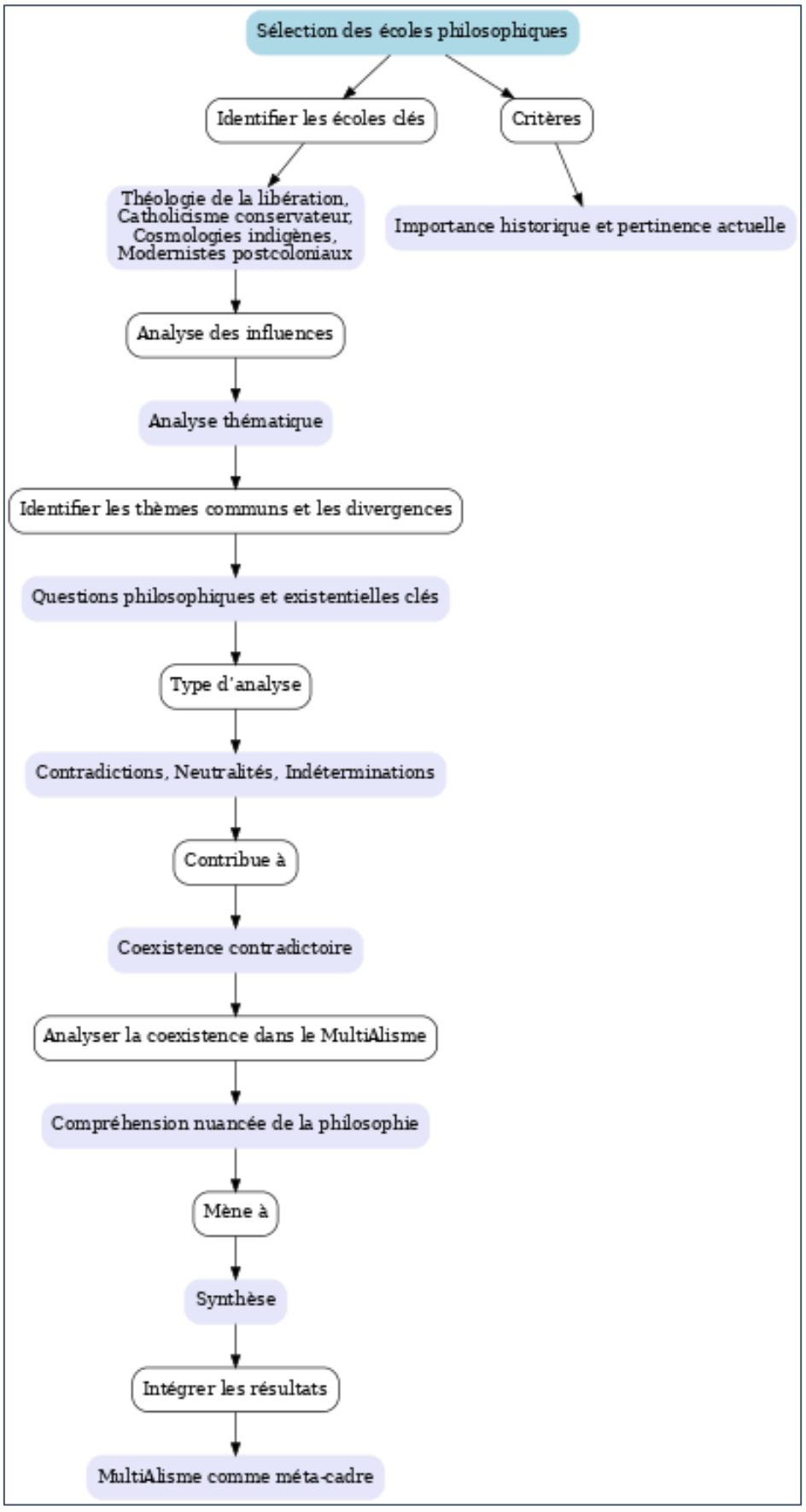

Figure 1. Carte heuristique de l'approche méthodologique.

4. Étude de cas

Les philosophies indigènes et les théories modernistes postcoloniales présentent des visions contrastées qui peuvent être synthétisées dans le cadre du Multi-Alisme pour favoriser des modèles sociopolitiques innovants. Les philosophies indigènes, généralement pluralistes, mettent l'accent sur le bien-être communautaire et une relation holistique avec la nature, tandis que les théories modernistes postcoloniales, souvent dualistes ou monistes, privilégient les droits et libertés individuels issus des idées des Lumières. En plaçant ces perspectives divergentes en dialogue, le Multi-Alisme permet d'explorer comment les valeurs communautaires indigènes peuvent coexister avec et enrichir les idéologies individualistes modernes, menant potentiellement à de nouveaux modèles de gouvernance et de communauté qui combinent les deux traditions.

4.1 Contradictions

Conceptions du temps et du progrès

Philosophies indigènes :

- Vision du temps généralement cyclique, centrée sur la continuité et la répétition des cycles naturels et cosmiques. Le progrès n'est pas linéaire mais en harmonie avec les cycles de la vie et de la nature.

Théories modernistes postcoloniales :

- Bien qu'elles critiquent le concept européen de progrès linéaire et cumulatif, elles opèrent encore dans des paradigmes valorisant le changement constant et l'innovation, caractéristiques de la pensée moderne et postmoderne.

Relation avec la nature

Philosophies indigènes :

- Relation d'interdépendance avec la nature, considérée comme sacrée et centrale dans leur vision du monde et leur subsistance.

Théories modernistes postcoloniales :

- Bien qu'elles critiquent l'exploitation capitaliste de l'environnement, elles ne placent pas toujours la relation à la nature au cœur de leur discours, privilégiant souvent des enjeux sociopolitiques et culturels.

Individualisme vs. Communautarisme

Philosophies indigènes :

- Le communautarisme est central, mettant en avant l'importance de la communauté sur l'individu. Les décisions et le bien-être sont envisagés collectivement.

Théories modernistes postcoloniales :

- Malgré leur volonté de libérer les structures oppressives, elles promeuvent souvent l'autonomie individuelle et l'autoréalisation, influencées par l'émancipation individuelle issue des Lumières.

Épistémologie et connaissance

Philosophies indigènes :

- Savoir transmis oralement, avec une forte dimension spirituelle et expérientielle, intimement lié à l'environnement local et aux ancêtres.

Théories modernistes postcoloniales :

- Bien qu'elles valorisent les savoirs subalternes et cherchent à revaloriser les connaissances locales, elles restent fortement influencées par les structures académiques occidentales, privilégiant souvent la textualité et la rationalité.

Vision du pouvoir et de la résistance

Philosophies indigènes :

- La résistance s'exprime souvent par la persistance culturelle et la subsistance, en maintenant traditions et pratiques ancestrales face à l'oppression.

Théories modernistes postcoloniales :

- La résistance est conceptualisée comme une déconstruction des discours et pratiques coloniales et néocoloniales, en mobilisant des outils critiques souvent issus du milieu académique.

Ces contradictions soulignent la complexité d'intégrer ces deux perspectives, mais elles ouvrent aussi des opportunités de dialogues enrichissants, permettant une compréhension plus profonde et des pratiques holistiques qui respectent à la fois la sagesse ancestrale et les critiques contemporaines du colonialisme et de ses héritages.

4.2 Neutralités et indéterminations entre les philosophies autochtones et les théories modernistes postcoloniales

Valorisation de la diversité culturelle

Philosophies indigènes :

- Profond respect des traditions et de la diversité culturelle comme piliers de l'identité communautaire.

Théories modernistes postcoloniales :

- Reconnaissance de la pluralité culturelle comme ressource contre les héritages coloniaux et comme forme de résistance à l'homogénéisation culturelle mondiale.

Zone de neutralité :

- Ces deux courants valorisent et défendent la diversité culturelle, bien que dans des perspectives différentes, ce qui peut fonder des politiques culturelles inclusives.

Critique des structures de pouvoir traditionnelles

Philosophies indigènes :

- Critique des impositions externes qui détruisent les structures communautaires.

Théories modernistes postcoloniales :

- Critique du colonialisme et des structures perpétuant les inégalités et la suppression culturelle.

Zone de neutralité :

- Les deux visent à démanteler des structures jugées oppressives, offrant un terrain commun pour la lutte contre l'oppression et la quête d'autonomie.

Justice sociale et économique

Philosophies indigènes :

- Insistent sur la justice sociale au sein de la communauté, avec égalité et entraide.

Théories modernistes postcoloniales :

- Militent pour la justice sociale en démantelant les structures postcoloniales et en promouvant l'équité.

Zone de neutralité :

- Malgré des approches différentes, un engagement partagé envers la justice sociale ouvre la voie à des collaborations politiques et sociales.

Revalorisation des savoirs traditionnels et locaux

Philosophies indigènes :

- Considèrent les savoirs traditionnels comme essentiels à la cohésion communautaire.

Théories modernistes postcoloniales :

- Cherchent à revaloriser les savoirs locaux comme résistance à la domination culturelle occidentale.

Zone de neutralité :

- Reconnaissance commune de l'importance des savoirs locaux, pouvant alimenter de nouvelles approches éducatives et scientifiques.

Durabilité et relation avec l'environnement

Philosophies indigènes :

- Vision holistique de l'environnement, centrée sur la durabilité et le respect de la nature.

Théories modernistes postcoloniales :

- Intérêt croissant pour des pratiques durables, en critique des modèles non durables hérités du colonialisme.

Zone de neutralité :

- Intérêt partagé pour la durabilité, ouvrant la voie à des politiques environnementales intégrant pratiques ancestrales et théories modernes.

L'exploration de ces zones de neutralité et de convergence permet non seulement de mieux comprendre la complexité de chaque perspective, mais aussi de concevoir des modèles hybrides tirant parti des deux mondes pour répondre aux défis globaux et locaux avec davantage de ressources et d'efficacité.

À travers le Multi-Alisme, l'étude montre que les contradictions philosophiques latino-américaines peuvent non seulement coexister, mais aussi s'enrichir mutuellement, ouvrant la voie à de nouvelles perspectives philosophiques. Par exemple, la tension entre les valeurs communautaires indigènes et les idéologies individualistes postcoloniales pourrait générer un nouveau modèle d'organisation sociopolitique conciliant solidarité communautaire et autonomie individuelle.

5. Conclusions

La recherche menée dans le cadre du multi-alisme a mis en évidence la possibilité pour les philosophies indigènes et les théories modernistes postcoloniales de converger et de dialoguer dans un contexte latino-américain, offrant des perspectives uniques d'intégration entre systèmes philosophiques souvent contradictoires. Cette approche a révélé des zones de neutralité, comme la valorisation de la diversité culturelle ou la critique des structures de pouvoir, qui constituent des fondations solides pour l'élaboration de politiques inclusives et d'initiatives de justice sociale.

En intégrant ces systèmes philosophiques diversifiés, le Multi-Alisme favorise une compréhension plus nuancée et une meilleure appréciation des riches traditions philosophiques de la région, tout en encourageant un dialogue constructif respectueux de la sagesse ancestrale et des critiques modernes du colonialisme et de ses héritages.

À l'avenir, l'application du multi-alisme pourrait s'étendre au-delà de l'Amérique latine afin d'évaluer son efficacité dans d'autres régions confrontées à des défis et contextes culturels similaires. Cela ouvrirait de nouvelles voies pour la recherche académique et les applications pratiques en politiques publiques et en éducation. Cette étude souligne l'importance d'une approche philosophique et sociale flexible et intégrative, démontrant que le multi-alisme constitue non seulement un outil théorique, mais aussi un cadre pratique essentiel pour répondre aux défis mondiaux contemporains.

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Modeling Social Evolution, Involution, and Indeterminacy: A Neutrosophic-Cultural Algorithm Approach

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Abstract: Neutrosophic Sociology, or Neutrosociology, is the study of sociology using neutrosophic scientific methods for understanding the dynamics of social systems within complex environments. When a social change occurs, the society evolves with respect to some parameters, regresses with respect to others, and remains the same or experiences an unclear change (indeterminacy) with respect to another set of parameters. This mirrors neutrosophic logic: true ($\mathcal{T}$), neutral or indeterminate ($\mathcal{I}$), and false ($\mathcal{F}$). The main concept of cultural algorithms is a generalization of the idea that evolution has at its base both racial learning within species and social learning within human societies. By employing a mathematical formulation that incorporates these theories, this study proposes a new framework, namely the Neutrosophic Theory of Cultural Evolution (NToCE), which analyzes how stakeholders can optimize their beliefs and actions in the face of uncertainty and conflicting interests. A real life scenario adopted from the neutrosophic social change due to technology as described in related bibliography, demonstrates the efficacy of this integrated neutrosophic framework in fostering collaborative decision-making, increasing flexibility, and promoting sustainability. The findings of our study reveal that adding neutrosophic principles into cultural algorithms considerably increases the robustness and resilience of social systems, allowing for more sophisticated modelling of stakeholder interactions and belief development.

Keywords: Neutrosociology; neutrosophic logic; cultural algorithms; social evolution; evolutionary computation; decision-making.

1. Introduction

The social environment is highly subjective, with conflicting trends and perspectives. The researcher's social model reflects a significant degree of "how I'd like to be," indicating a lack of consensus on social norms. Humans are capable of both unconscious and conscious evaluations of their surroundings. In a world that is constantly changing in terms of sociological conditions and interactions between individuals and social groups, we need theories and models that are adaptive, flexible, and dynamic in response.

Starting from this perspective, Professor Smarandache introduced Neutrosophic Sociology, or Neutrosociology, which is the study of sociology using neutrosophic scientific methods to understand the dynamics of social systems within complex environments [1]. He observed that the vast social data encountered in sociology is screened with indeterminacy: it is vague, incomplete, contradictory, hybrid, biased, ignorant, redundant, superfluous, meaningless, ambiguous, and unclear. Building on this observation, he suggested that methods of neutrosophic logic, which deals with indeterminacy, could

be utilized to effectively address social changes. Social change causes society to evolve in some social parameters, regress in others, and remain the same or be unclear (indeterminate) in yet another set of social parameters.

The Neutrosophic Theory of Evolution (NTE) [2], characterized by its main concepts of evolution, involution, and indeterminacy (or neutrality), has been successfully applied in evolutionary biology to describe the process of adaptation of a being (animal, human, or plant) to a new environment. Recently, it has also been extended to study the case of language evolution in certain social groups, showcasing how individuals adapt, regress, or maintain neutral traits during cross-cultural transitions [3]. These recent studies have clearly revealed the applicability of the Neutrosophic Theory of Evolution in interdisciplinary scientific fields.

Cultural Algorithms (CA) [4-5] are a type of evolutionary algorithm that integrates concepts from cultural evolution to enhance optimization processes. These processes support two functions of heredity: one occurs at the micro-evolutionary level in terms of behavioral or genetic traits, while the other acts at the macro-evolutionary level in terms of the so-called beliefs of the population. The two functions interact through a communication channel that allows individuals to modify the structure of the population's beliefs, and also predisposes the behavior of individuals in light of the individuals' beliefs. Thus, cultural algorithms have three basic components: a belief structure, a population structure, and a communication channel.

In this paper, we suggest a hybrid neutrosophic model, the Neutrosophic Theory of Cultural Evolution (NToCE), which provides a synergy between NTE and CA so that we can enhance the adaptability and flexibility of the optimization process. The cultural aspect adopts knowledge sharing and collaborative learning among individuals, while the neutrosophic perspective enables a more comprehensive evaluation of solutions by accommodating varying degrees of truth, falsity, and uncertainty. In order to test the applicability of our method we study a real life case study, adopted from the related bibliography of neutrosophic social change [1], is studied to analyze social change due to technology. In this case, our aim is to achieve a thorough understanding of how CAs combined with the NTE may aid stakeholder decision-making in a highly complex setting. We simulate the views and interactions of stakeholders to mimic the dynamics of evolution, involution, and indeterminacy in their decision-making processes.

1.1 Novelties

The current article suggests the Neutrosophic Theory of Cultural Evolution (NToCE) which is a novel optimization method built around the integration of neutrosophic Logic (NL), Neutrosophic Social Evolution (NSE) and Cultural Algorithms. The key innovations include:

1. The combination of aforesaid methodologies allows for more advanced modelling of complicated systems. Specifically, NL's ability to manage indeterminacy in complex systems, such as the social environment, when joined with CAs' adaptive learning, allows for flexible decision making. Furthermore, combining core NTE concepts (evolution, involution, and indeterminacy) with CAs enables the model to develop more effective strategies for addressing complex challenges, facilitating collaboration among diverse stakeholders, and strengthening the robustness of solutions in uncertain environments.
2. The model incorporates neutrosophic concepts into CAs to update belief vectors for evolution (T), indeterminacy (I), and involution (F), while simulating stakeholder interactions. This combination approach allows for real-time changes in societal views, which improves decision-making processes in complicated contexts.

1.2 Contributions

1. The introduction of NToCE is an important theoretical advance. It associates NL with CAs to better manage the complexity and uncertainties of social evolution, hence filling a research gap in the modelling of indeterminate social systems.
2. The study proposes a new algorithm for updating stakeholder beliefs through iterative procedures, enabling continuous adaptation in complex and changing social situations. This helps to advancing approaches for modelling dynamic decision-making processes in uncertain scenarios.
3. The practical application, highlighted in our study, assessing the societal impacts of Internet and mobile technologies contributes to both academic knowledge and a practical understanding of technological disruptions in society.

1.3 Structure of the paper

In the next section, we present the fundamental formulations that are used to build our hybrid neutrosophic model. Section 3 applies our proposed model to a real word case study. This case focuses on the widespread adoption of Internet and mobile technologies and explores their impact on social dynamics, including the processes of social evolution, involution, and indeterminacy. In Section 4 we inquire into a detailed evaluation of the results, highlighting their significance for understanding social adaptation and decision-making in uncertain environments. Subsequently, in Section 5, we perform an assessment analysis, comparing the performance of the proposed NToCE model to other neutrosophic approaches such as Neutrosophic Cognitive Maps (NCMs), Neutrosophic Soft Sets (NSS), and Neutrosophic Analytic Hierarchy Process (N-AHP). This research highlights the NToCE approach's benefits, notably its iterative belief update process and dynamic flexibility. Finally, Section 6 provides a comprehensive summary of the insights gained from the development and application of the Neutrosophic Theory of Cultural Evolution (NToCE).

2. Materials and Methods

2.1 Neutrosophic Logic

Neutrosophic Logic (NL) is an extension of classical and fuzzy logic, introduced by Smarandache [6]. It provides a framework for dealing with indeterminate, imprecise, and inconsistent information by incorporating a third truth value called indeterminacy. In neutrosophic logic, a concept A is T% true, I% indeterminate, and F% false, with (T, I, F) $\in$ $]^{-}0, 1^{+}[^{3}$, where $]^{-}0, 1^{+}[$ is an interval of hyperreals.

Definition 1 [7] Let $\mathcal{X}$ be a space of points (objects), with a generic element in $\mathcal{X}$ denoted by x. A single-valued neutrosophic set (SVNS) $\mathcal{A}$ in $\mathcal{X}$ is characterized by truth membership function T_A, indeterminacy membership function I_A, and falsity membership function F_A. For each point x in $\mathcal{X}$, $T_A(x), I_A(x), F_A(x) \in [0, 1]$.

Then, a simplification of the neutrosophic set $\mathcal{A}$, which is a subclass of neutrosophic sets, is denoted by $\mathcal{A} = \{\langle x, T(x), I(x), F(x)\rangle \mid x \in \mathcal{X}\}$

2.2 Neutrosophic Cultural Algorithms

The CAs is a class of computational models derived from observing the cultural evolution process in nature [4]. The CA has three major components: a population space, a belief space, and a protocol that describes how knowledge is exchanged between the first two components. The population space can support any population-based computational model, such as Genetic Algorithms, and Evolutionary Programming. The basic framework that will be used in the context of our model is the following:

- *Population Space*
  Each individual i has a belief vector $B_i = (T_i, I_i, F_i)$ (1)
  where each component represents a distinct aspect of their beliefs.
- *Belief Space*

The belief space stores the best (or optimal) values of T,I,F discovered by the population: $T_{optimal}, I_{optimal}, F_{optimal}$

- *Belief Update*

The beliefs of the population are influenced by both their interaction with each other and the best values stored in the belief space.

$$T_i^{new} = T_i + \lambda * (T_{optimal} - T_i) \quad (2)$$

$$I_i^{new} = I_i + \mu * (I_{optimal} - I_i) \quad (3)$$

$$F_i^{new} = F_i + \nu * (F_{optimal} - F_i) \quad (4)$$

Where:

λ, μ, ν are adjustment factors that control the influence of the optimal beliefs on the population.

2.3 Neutrosophic Theory of Cultural Evolution

In this subsection we adopt the ideas from Neutrosophic Sociology and transform them into a suitable mathematical framework tailored for the needs of our proposed methodology.

The NTE characterizes the dynamics beliefs of individuals using three key components: Evolution (T), Indeterminacy (I), and Involution (F). These components evolve as individuals interact and make decisions regarding a common goal.

We define the belief vector B_i for each individual i as $B_i = (T_i, I_i, F_i)$ where:

T_i is the degree of evolution (positive change or support)

I_i is the degree of indeterminacy (uncertainty or neutrality) and

F_i is the degree of involution (negative change or opposition)

The belief vector is updated iteratively to reflect changes in individual's attitudes based on interactions and external influences as follows:

$$T_{new} = T_{current} + \alpha * (T_{optimal} - T_{current}) \quad (5)$$

$$I_{new} = I_{current} + \beta * (I_{optimal} - I_{current}) \quad (6)$$

$$F_{new} = F_{current} + \gamma * (F_{optimal} - F_{current}) \quad (7)$$

Where:

α, β, γ are influence factors that determine how much weight is given to the evolution, indeterminacy, and involution during the belief update process.

$T_{optimal}, I_{optimal}, F_{optimal}$ are the ideal target values for each belief component and are derived based on the goals of the system examined.

Eq. (5) – (7) are updated iteratively until a termination condition is met. A convergence threshold (ϵ) is commonly used to specify the termination condition. This threshold is a small positive value that shows how near the present beliefs must be to the ideal beliefs before the algorithm stops.

A common choice for (ϵ) might be a small value such as 0.001 or 0.01, depending on the precision required.

Next we define the *fitness* function, which is mainly used in evolutionary computation algorithms. In Neutrosophic Social Evolution, the goal is to maximize evolution, minimize involution and minimize indeterminacy. In this light, the fitness function will be used in our model as a measure of how well the current state of beliefs (represented by T, I, and F) of individuals align with the ideal target values. It can be defined as:

$$Fitness\ B_i = w_T * T_i - w_I * I_i - w_F * F_i \quad (8)$$

Where:

w_T, w_I, w_F are weights reflecting the importance of evolution, indeterminacy, and involution and allow to adjust the influence of each belief component in the fitness equation. In order to calculate the weights given in the above equation we could engage the population with sample questions like:

- How significant do you believe the positive influence of technology (evolution) is on society?
- How significant do you perceive the uncertainties associated with technology (indeterminacy)?
- How concerned are you about the negative implications of technology (involution)?

To summarize, Figure 1 depicts the algorithmic structure of the proposed method:

Neutrosophic Theory of Cultural Evolution (NToCE)

1. Initialize:

- Set initial belief vectors for each stakeholder group $B_i = (T_i, I_i, F_i)$
- Define optimal belief values $T_{optimal}, I_{optimal}, F_{optimal}$
- Set adjustment factors (α, β, γ) for belief updates.
- Set termination condition threshold (ϵ).

2. For each stakeholder group, calculate initial belief values:

B_i^{init} $T_{current}$, $I_{current}$, $F_{current}$

3. Update belief vectors iteratively:

- While the termination condition (ϵ) is not met, do:
  - For each stakeholder group:

$$T_{new} = T_{current} + \alpha * (T_{optimal} - T_{current})$$
$$I_{new} = I_{current} + \beta * (I_{optimal} - I_{current})$$
$$F_{new} = F_{current} + \gamma * (F_{optimal} - F_{current})$$

4. Convergence check:

- Check if the belief vector components (T, I, F) for each stakeholder group are within the termination threshold ϵ
- If convergence is achieved ($|T_{new} - T_{optimal}| < \epsilon$ and similar for I and F), terminate the algorithm

5. Calculate fitness values for each stakeholder group:

- $Fitness\ B_i = w_T*T_i - w_I*I_i - w_F*F_i$

6. Output final belief vectors and fitness values:

- Return the final belief vectors ($B_i = (T_i, I_i, F_i)$) after convergence
- Output fitness values for each stakeholder group

Figure 1. Algorithmic structure of NToCE

Remark: In step 5 of above proposed algorithm we could integrate a penalty function ($P(B_i)$) into the fitness values calculated to reduce the chance for illegal solutions to be reproduced by our algorithm: (i) a penalty could discourage certain behaviors or attitudes that are not desirable or ideal in the context of cultural development, (ii) the penalty function could assist in ensuring that the final belief values remain feasible, which is especially significant in situations when beliefs must adhere to certain ethical or cultural norms. This could be achieved by calculating the evaluation function as:

$Fitness\ B_i = w_T*T_i - w_I*I_i - w_F*F_i - P(B_i)$

where $P(B_i)$ would identify the constraints or undesirable behaviors that require penalties.

3. Results

In this section, we will apply our proposed method, Neutrosophic Theory of Cultural Evolution (NToCE), to a real-life case study, originating from related literature [1], concerning the societal implications of Internet and mobile technologies for various stakeholder groups. Specifically, we will examine how various groups—such as tech enthusiasts, privacy activists, and the business sector—change their views about the advantages, uncertainties, and threats connected with technology breakthroughs, using actual facts to guide the belief updating process. This will show how NToCE may represent social development, indeterminacy, and involution as a result of continual technology advancements. The data sources used for current scenario were adapted from real datasets [8-15]. For a

figurative representation of related surveys, the interested reader is referred to the Appendix of the paper.

In order to derive the neutrosophic values from the corresponding datasets, we follow the next formulas to quantify the initial beliefs of population groups, based on counting positive, uncertain, and negative responses for components T, I and F (Table 1). This comes in align with the definition of a concept in NL, i.e. a concept A is T% true, I% indeterminate, and F% false with (T, I, F) $\in$ $]^-0, 1^+[$[3].

Table 1. Formulas for deriving T, I and F components

Component	Formula
Truth (T)	$T = \frac{Positive\ responses}{Total\ responses}$
Indeterminacy (I)	$I = \frac{Uncertain\ or\ contradictory\ responses}{Total\ responses}$
Falsehood (F)	$F = \frac{Negative\ responses}{Total\ responses}$

Based on Table 1, we derive next table (Table 2) which summarizes the data used for calculating the initial belief vectors for each stakeholder group.

Table 2. Datasets for stakeholder groups for initial belief [8-15]

Stakeholder Group	Component	Value	Data
Tech Enthusiasts (TE)	Truth	0.8	81% of Americans believe mobile devices have improved their lives [8]
	Indeterminacy	0.3	53% uncertainty about long-term economic impacts of technology [10]
	Falsehood	0.1	79% concern about privacy, but low concern for tech enthusiasts [9].
Privacy Advocates (PA)	Truth	0.6	Lower trust in technology's benefits for privacy-conscious individuals [8]
	Indeterminacy	0.5[1]	79% concerned about data privacy and surveillance [9]
	Falsehood	0.4	400% increase in cybercrime during the pandemic [11]
Business Sector (BS)	Truth	0.7	Mixed views on technology's economic benefits, 53% uncertainty [10]
	Indeterminacy	0.4	53% of business leaders uncertain about technology's long-term benefits [10]
	Falsehood	0.3	Concerns about job displacement and automation risks [12]

From Table 2, we can derive the belief vector of our population as in Eq. (1):

B_{TE} = (0.8, 0.3, 0.1) (9)

B_{PA} = (0.6, 0.5, 0.4) (10)

B_{BS} = (0.7, 0.4, 0.3) (11)

[1] The adjustment from 79% concern to 0.5 indeterminacy reflects a process of normalization or contextual scaling, ensuring that the value fits within the neutrosophic model's structure

We derive the optimal beliefs based on empirical data [8-15] from research on the impacts of technology. For example, the truth value can be supported by data showing that a large proportion of respondents believe that technology has significantly improved their lives or work, whereas the falsehood value can be influenced by concerns about technology's unintended consequences (e.g., privacy breaches or job losses). In this context, we get the following optimal values for social evolution, indeterminacy and involution respectively:

$T_{optimal}$ = 0.85, $I_{optimal}$ = 0.4 and $F_{optimal}$ = 0.2.

Next we update the belief vectors, for each population group using Eq. (2) - (4) where $\alpha = \beta = \gamma = 0.5$ (meaning that the process of updating beliefs for evolution (T), indeterminacy (I), and involution (F) is treated with equal importance) and taking into consideration Eq. (9) – (11).

- *Tech enthusiasts*

T_{new} = 0.8 + 0.5 * (0.85 − 0.8) = 0.825

I_{new} = 0.3 + 0.5 * (0.4–0.3) = 0.35

F_{new} = 0.1 + 0.5 * (0.2–0.1) = 0.15, so updated vector B_{TE} = (0.825, 0.35, 0.15) (12)

Following the same logic we get

- *Privacy Advocates*

B_{PA} = (0.725, 0.45, 0.3) (13)

- *Business Sector*

B_{BS} = (0.775, 0.4, 0.25) (14)

We proceed with Iteration 1 of our algorithm updating the beliefs of our population with current values as in Eq. (12) - (14) with $T_{current}$, $I_{current}$ and $F_{current}$ the updated values.

After iteration 1 we have:

B_{TE} = (0.8375, 0.375, 0.175)

B_{BS} = (0.7875, 0.425, 0.25) and

B_{PA} = (0.8125, 0.4, 0.25)

After three iterations we have at least one value (from T, I and F) below the threshold (0.001), so we can consider the algorithm to have converged with final beliefs:

B_{TE} = (0.846875, 0.39375, 0.19375)

B_{BS} = (0.834375, 0.40625, 0.2125) and

B_{PA} = (0.840625, 0.4, 0.2125)

Finally we calculate the fitness function for each stakeholder according to Eq. (8) where w_T= 1, w_I= 0.5 and w_F= 0.3. By assigning these values to the weight parameters we ensure that the alignment with desired outcomes is not overshadowed. The fitness values indicate how well each stakeholder's beliefs align with the desired outcomes.

$Fitness_{TE}$ = 0.591875

$Fitness_{PA}$ = 0.5675 and

$Fitness_{BS}$ = 0.576875

4. Applications

In the previous section, we utilized our method to study the societal implications of Internet and mobile technology regarding the evolution, indeterminacy, and involution of stakeholder beliefs. Next we list the main insights drawn from the results obtained.

1. The belief vectors for several stakeholder groups—tech enthusiasts, privacy activists, and business sector—show how their attitudes evolve in reaction to the effects of technology progress. The gradual convergence of these belief vectors via recurrent updates demonstrates how each group adjusts its perspectives on the positive, uncertain, and negative effects of technology. For example, tech enthusiasts who began with a strong conviction in the advantages of technology show a minor rise in indeterminacy and a moderate change in privacy concerns, indicating a more nuanced perspective of technological consequences.

2. The NToCE method updates belief vectors with weighted adjustment factors a, β, and γ, demonstrating how stakeholder views evolve towards optimal belief values based on real world data. The final convergence shows that, despite various initial perspectives, all groups arrive to an almost common understanding of the technological world.
3. The fitness function, which evaluates the alignment of each group's views with the intended social results, reveals that tech enthusiasts have the highest alignment, followed by the business sector and, ultimately, privacy activists. This result is consistent with the empirical data and indicates how each group prioritizes different aspects of technology's influence. The values of fitness function for each population group also acknowledges the positive impact of technology while also sharing concerns about privacy and job displacement. Another implication that could be made regarding the close values of fitness function between stakeholders is the possible balanced trade off in stakeholders' perceptions about technology.

5. Comparative analysis

Since there are no published results on the case study examined in this paper so as to directly compare the performance of our suggested method to other similar methodologies, the analysis conducted in this section will focus on the main features and characteristics of each approach, highlighting their advantages and disadvantages.

The methods chosen for comparative analysis (Table 3) in the context of our case study were selected due to their similar mathematical framework (neutrosophic), and their capability in addressing complex systems that involve uncertainty, evolving conditions, and multiple stakeholders.

Table 3. Methods for comparative analysis

Method	Components	Belief update mechanism	Convergence speed	Flexibility in handling indeterminacy	Key advantages
Neutrosophic Cognitive Maps [16-18]	Neutrosophic relationships (T,I,F)	Iterative	Moderate	High	Effective in representing dynamic systems
Neutrosophic Soft Sets [19-21]	T, I , F	Static	Slow	Limited	Handles uncertainty through soft set principles
Neutrosophic AHP [22-23]	Ranked decision criteria	No iterative Update	Moderate	Moderate	Useful for predefined decision-making criteria
NToCE	Evolution (T), Involution (F), Indeterminacy (I)	Iterative	Fast	High	Dynamic belief update and continuous adaptation

Based on the above comparative analysis, it is shown that NToCE demonstrates significant advantages over other neutrosophic approaches in dealing with complex systems, especially in dynamic and developing situations. One of the NToCE's key features is its iterative belief updating process, which is enhanced by cultural algorithms. This dynamic method enables the continuous adaption of

stakeholder beliefs to reflect real-time changes in the social or technological environment. In contrast, approaches like Neutrosophic Soft Sets (NSS) and Neutrosophic Analytic Hierarchy Process (N-AHP) rely on static or non-iterative updates, which limit their ability to handle indeterminacy and evolving situations.

Moreover, the convergence speed of the NToCE model is considerably faster, with beliefs stabilizing within three iterations, whereas models like NSS and N-AHP often require more iterations and exhibit slower adaptation. The NToCE method's capacity to dynamically modify the balance of evolution (T), involution (F), and indeterminacy (I) guarantees that stakeholder viewpoints develop towards optimum belief states, which improves decision-making in uncertain and conflicting situations.

In terms of fitness value alignment, the NToCE method performs well, delivering a better level of alignment between stakeholder beliefs and desired results. This is primarily due to the employment of cultural algorithms, which enable stakeholder groups to share knowledge and collective learning. As a consequence, the NToCE model not only adapts fast but also gives more robust decision-making results when compared to systems like Neutrosophic Cognitive Maps (NCMs) or NSS, which handle uncertainty but lack the same level of real-time responsiveness and flexibility. A promising work regarding the inclusion of the concepts of evolution and dynamics of time in NCMs is described in [24], but still needs further testing validation in real life problems.

6. Conclusions

Parsons [25] believed that societies, like biological creatures, needed to adapt to changing circumstances in order to survive. His theory of social evolution proposes that civilizations that best adapt to their surroundings would be more effective in preserving stability and making advancements. This is related to Darwin's notion of "survival of the fittest," which states that the most adaptive or well-suited creatures survive and thrive. However, we are faced with so many social problems that we will never succeed in completely eliminating all the corruption from society. For example, in the context of this paper, technology increases people's wealth, but at the same time, it also diversifies, increases, and creates new kinds of social problems such as cyber-crimes, online identity theft, and electronic bullying. In this view, scholars [1,26] extended Parson's Social Evolution to the Neutrosophic Social Evolution. He claimed that social change causes society to evolve, as Parsons stated in his functionalism. However, this evolution only applies to certain social parameters, while other parameters may regress (involution). In some cases, society may remain the same or the change may be unclear (indeterminacy), as in neutrosophic logic: true (T), neutral or indeterminate (I), and false (F).

Commencing from this fundamental idea, this study examined the integration of Neutrosophic Logic and Cultural Algorithms to propose the Neutrosophic Theory of Cultural Evolution (NToCE), a novel approach to understanding and modelling social evolution in a complex driven society. The suggested approach captures not just positive or negative shifts in stakeholder perceptions, but also the inherent uncertainties that technology entails, such as privacy issues, job displacement, and cybercrime. Finally, our neutrosophic framework to cultural evolution provides a more comprehensive understanding of how societies can improve their decision-making processes in the face of uncertain outcomes.

The study models stakeholders' beliefs using three components: evolution (T), involution (F), and indeterminacy (I) to model changes in systems. When a social change occurs, it impacts society in various dimensions, leading to multiple degrees of evolution, involution, neutrality, and uncertainty across different social parameters. Future research could provide more granular definitions to capture refinements within each category by employing Refined Neutrosophic Evolution which is described by Refined Neutrosophic Logic [27-30]. In this model we could define multiple degrees of evolution ($\mathcal{T}_1, \mathcal{T}_2, \ldots, \mathcal{T}_k$) to correspond a set of social parameters (e.g. technological, economic, educational etc.) to represent the extent of positive change across these various aspects of society. In the same logic we would have multiple degrees of involution ($\mathcal{F}_1, \mathcal{F}_2, \ldots, \mathcal{F}_m$), degrees of neutralities ($\mathcal{J}_1, \mathcal{J}_2, \ldots, \mathcal{J}_n$) referring to no significant change with respect to certain social parameters and degrees of indeterminacy ($\mathcal{J}_{n+1}$,

$\mathcal{J}_{n+2}$, …, $\mathcal{J}_p$) when referring to social parameters where it is unclear whether the changes represent evolution or involution.

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Appendix A

A.1 Data figures

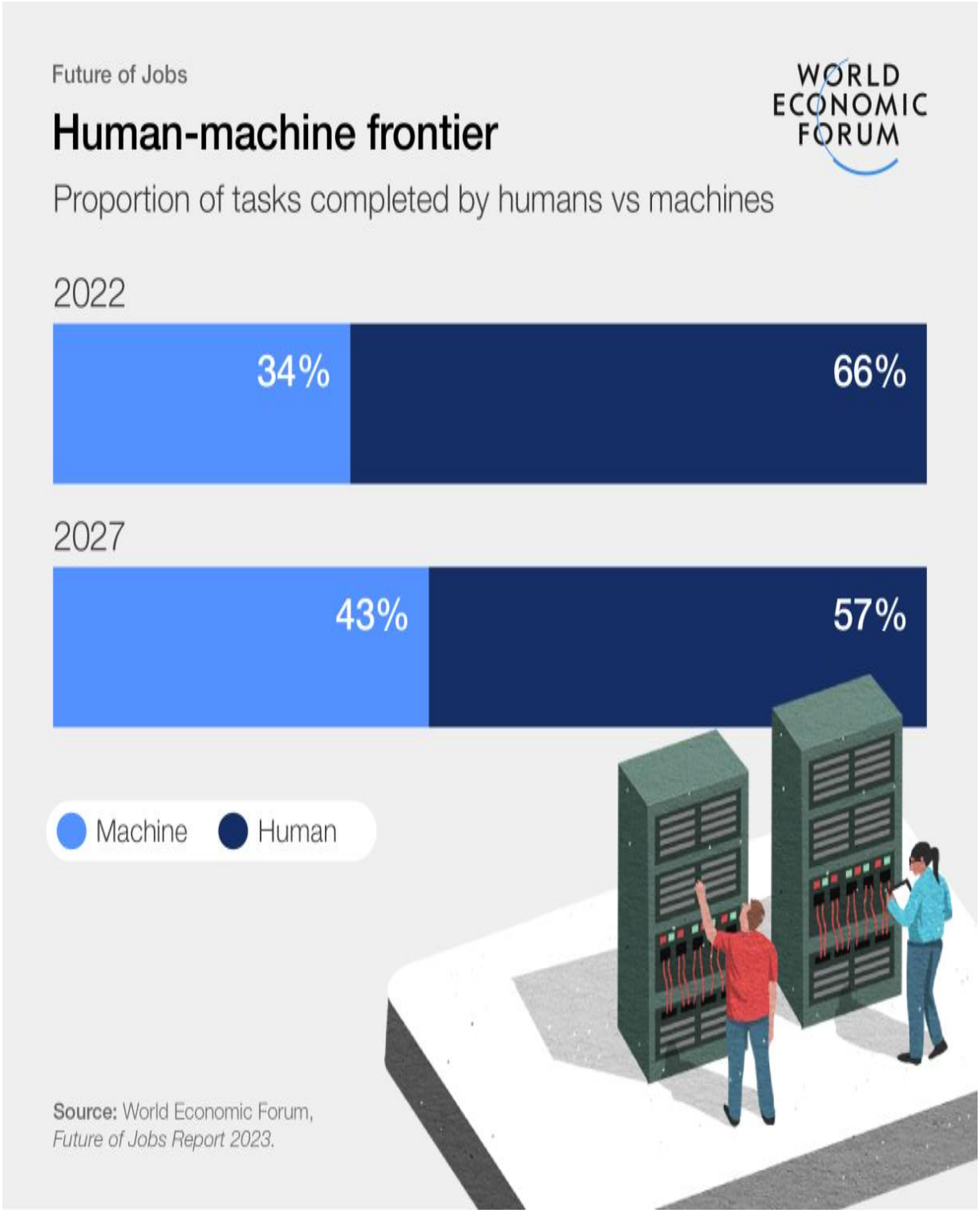

Figure A1. How tasks could be split between humans and machines in the coming years [14]

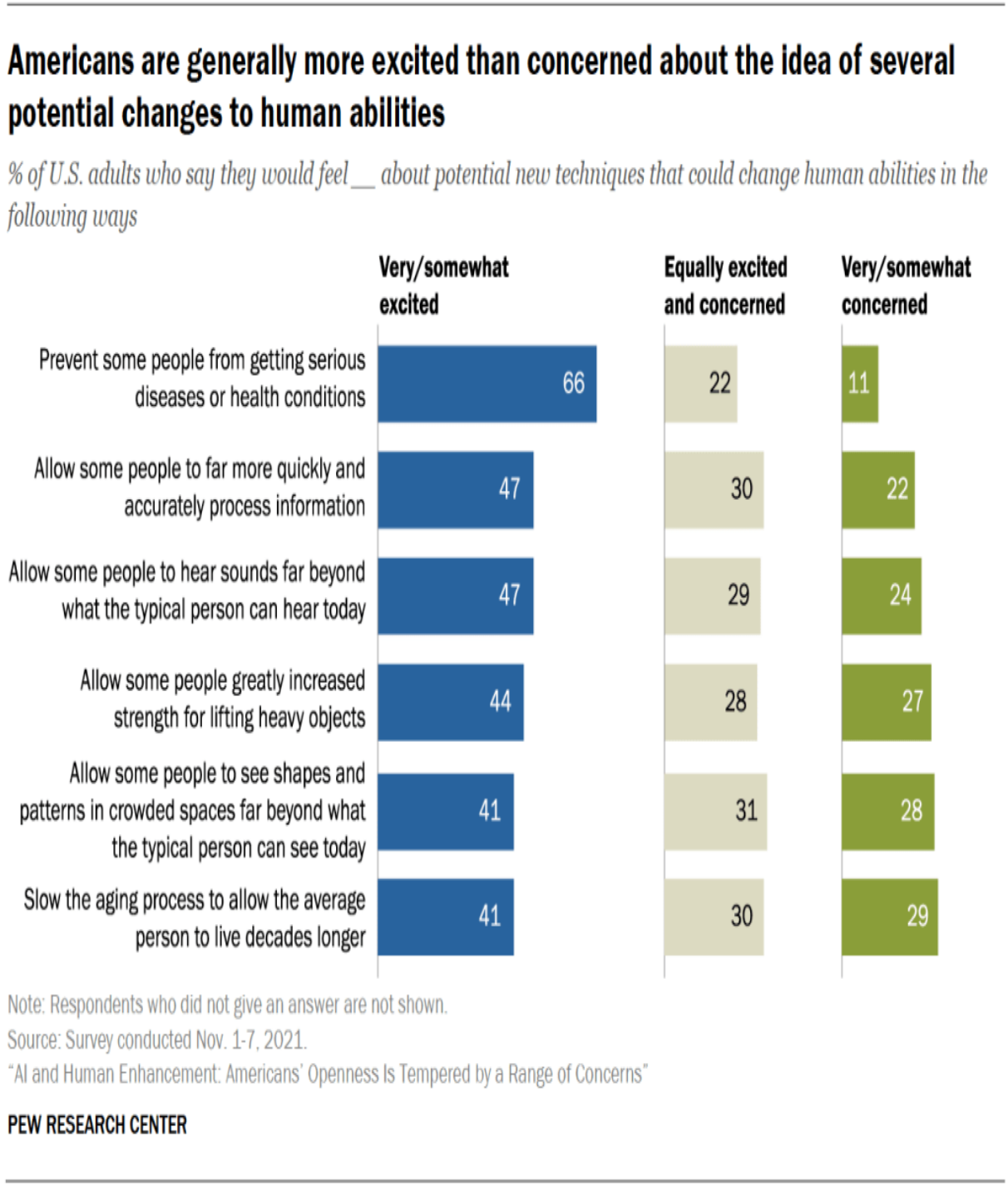
Americans are generally more excited than concerned about the idea of several potential changes to human abilities
% of U.S. adults who say they would feel __ about potential new techniques that could change human abilities in the following ways
Very/somewhat excited
Equally excited and concerned
Very/somewhat concerned
Prevent some people from getting serious diseases or health conditions
66
22
11
Allow some people to far more quickly and accurately process information
47
30
22
Allow some people to hear sounds far beyond what the typical person can hear today
47
29
24
Allow some people greatly increased strength for lifting heavy objects
44
28
27
Allow some people to see shapes and patterns in crowded spaces far beyond what the typical person can see today
41
31
28
Slow the aging process to allow the average person to live decades longer
41
30
29
Note: Respondents who did not give an answer are not shown.
Source: Survey conducted Nov. 1-7, 2021.
"AI and Human Enhancement: Americans' Openness Is Tempered by a Range of Concerns"
PEW RESEARCH CENTER

Figure A2. AI and human enhancement [8]

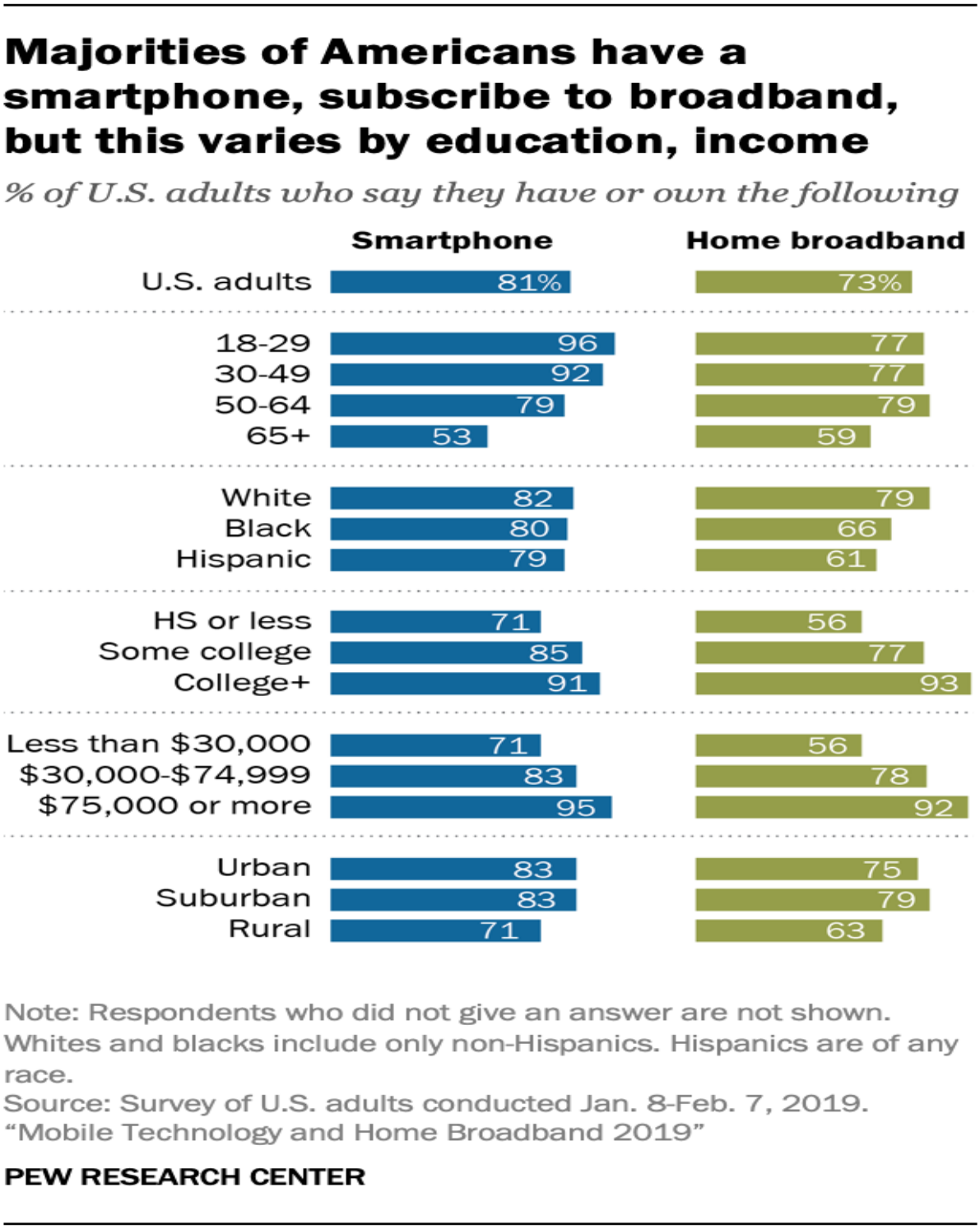
Majorities of Americans have a smartphone, subscribe to broadband, but this varies by education, income
% of U.S. adults who say they have or own the following
Smartphone
Home broadband
U.S. adults 81% 73%
18-29 96 77
30-49 92 77
50-64 79 79
65+ 53 59
White 82 79
Black 80 66
Hispanic 79 61
HS or less 71 56
Some college 85 77
College+ 91 93
Less than $30,000 71 56
$30,000-$74,999 83 78
$75,000 or more 95 92
Urban 83 75
Suburban 83 79
Rural 71 63
Note: Respondents who did not give an answer are not shown. Whites and blacks include only non-Hispanics. Hispanics are of any race.
Source: Survey of U.S. adults conducted Jan. 8-Feb. 7, 2019. "Mobile Technology and Home Broadband 2019"
PEW RESEARCH CENTER

Figure A3. Mobile technology and broadband [8]

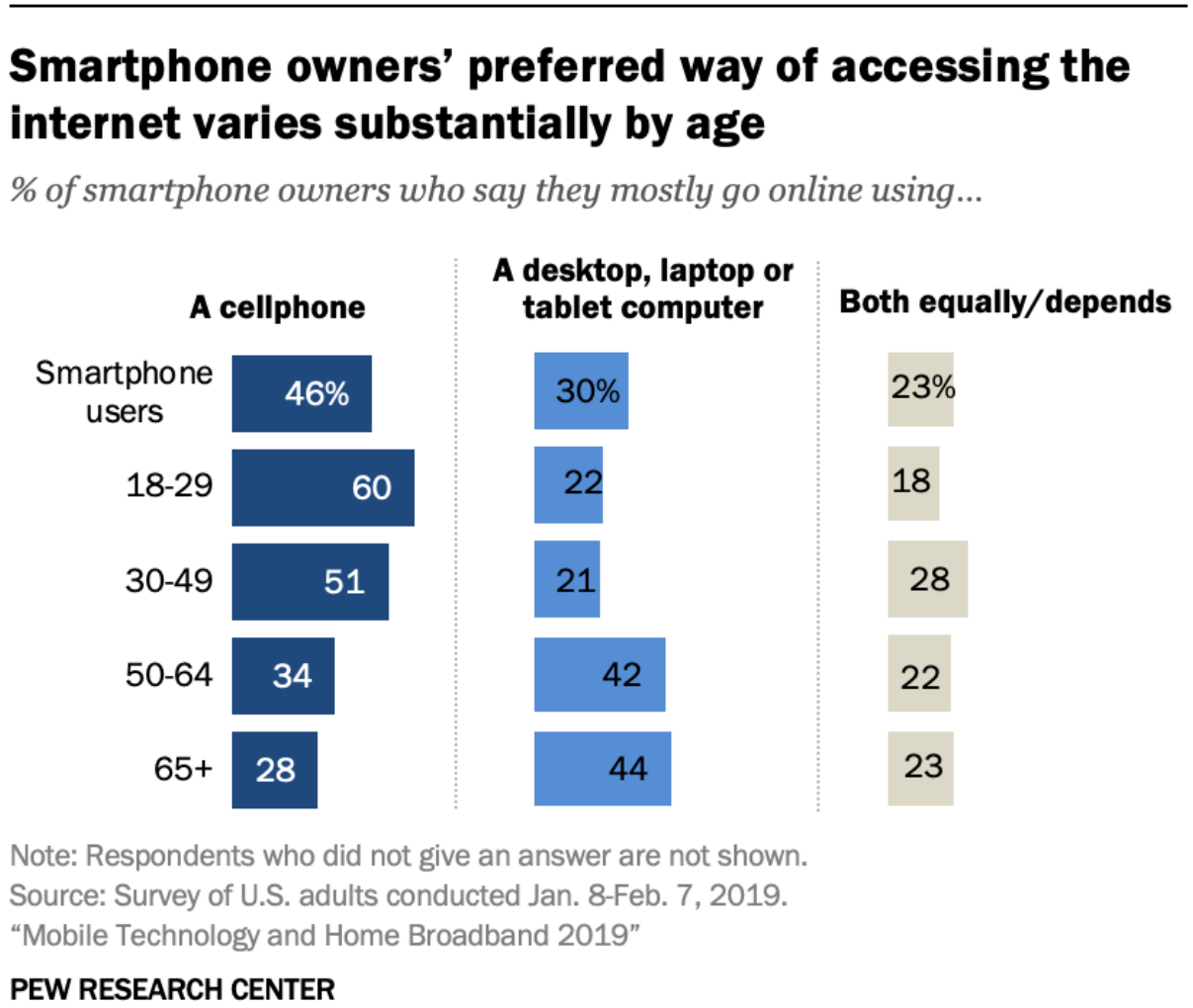

Figure A4. Smartphone preferred way of accessing the Internet [8]

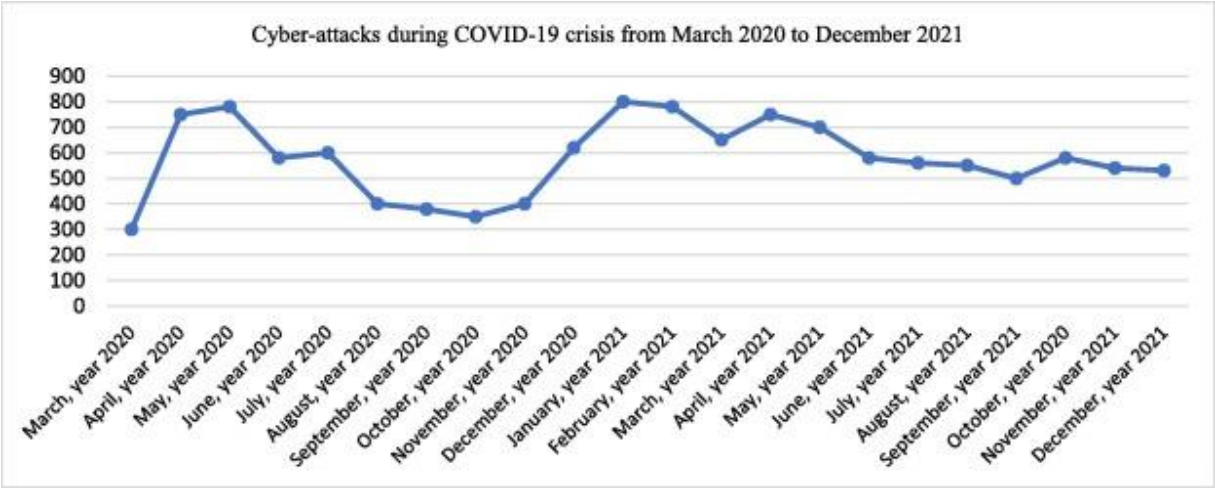

Figure A5. Cyber-attacks during the COVID-19 crisis between March 2020 and December 2021 [15]

Figure A6. Mass Surveillance of Personal Data [9]

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Expanding the Horizons of Philosophy through Neutrosophic Movements

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Smarandache, Florentin. "Expanding the Horizons of Philosophy through Neutrosophic Movements." *Neutrosophic Knowledge*, Vol. 6, 2025, pp. 50-54.

Abstract:

Philosophy is characterized by its rich diversity of methods and schools of thought. Beneath this diversity, however, lies a subtle but profound unity: the interrelation of affirmation, negation, and neutrality. A neutrosophic perspective reveals that philosophical movements are not *isolated ruptures* from tradition but are *dynamic threads* interwoven into a larger intellectual tapestry. This short essay birelfy examines six philosophical movements—revisionism, inspirationalism, recur-rentism, sophisticalism, rejectivism, and paradoxism—through the lens of neutrosophy, illustrating how each contributes to the evolving landscape of philosophy.

Keywords. Neutrosophy, Philosophical Movements, Revisionism, Inspirationalism, Recurrentism, Sophisticalism, Rejectivism, Paradoxism, Philosophy of Contradiction, Affirmation, Negation, Neutrality, Philosophical Diversity, Existentialism, Dynamic Philosophy.

1. *Revisionism*: Philosophy as a *Summum Bonum*

Revisionism calls for a comprehensive reexamination of all philosophical systems, thinkers, and schools, with the aim of redefining philosophy as a unified *summum bonum*—the highest good.[1] From a neutrosophic standpoint, this movement highlights the necessity of engaging with prior systems through a triadic process of affirmation (T), negation (F), and neutrality (I).

A revisionist approach does not discard the past but reinterprets it. For example, revisiting metaphysical paradigms such as Aristotle's teleology or Kant's transcendental idealism requires

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recognizing their strengths, weaknesses, and neutral contributions.[2] In this way, revisionism views philosophy not as a hierarchical contest among competing schools but as a continuum of insights.

2. *Inspirationalism*: The Quest for Originality

Inspirationalism seeks to generate originality by drawing upon the past and present, fostering a creative synthesis between tradition and innovation.[3] From a neutrosophic perspective, this process involves exploring the interstitial spaces between influence and originality.

Every original concept carries the imprint of its inspirations. Neutrosophy reframes these imprints not as constraints but as neutral zones of potential, enabling the fusion of old and new into transformative insights. Heidegger's existentialism, informed by ancient Greek philosophy and contemporary phenomenology, exemplifies this dynamic, blending affirmation of tradition with groundbreaking innovation.

3. *Recurrentism*: The Infinite Cycle of Ideas

Recurrentism posits that philosophical ideas arise from a continuous cycle, where each idea builds upon its predecessors and seeds future developments.[4] Neutrosophy enriches this perspective by emphasizing that these cycles are neither strictly linear nor deterministic but involve oscillations across affirmations, negations, and neutral zones of reinterpretation.

For instance, the Enlightenment's emphasis on reason emerged as a response to medieval scholasticism, which itself drew from classical philosophy. Each recurrence reinterprets prior insights, creating a dynamic interplay between continuity and novelty. Neutrosophically, these cycles also encompass neutral zones—moments where ideas are neither wholly derivative nor entirely innovative but exist as a fusion of both.

4. *Sophisticalism*: Embracing Ambiguity and Abstraction

Sophisticalism celebrates the ambiguous, abstract, and often unintelligible aspects of thought, framing obscurity as a philosophical virtue. While this approach might seem esoteric or indulgent, a neutrosophic lens reinterprets it as an exploration of the indeterminate spaces between clarity and mystery.

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Ambiguity, far from being a weakness, reflects the inherent complexity of reality. The sophistical approach, when viewed neutrosophically, becomes a powerful tool for probing the boundaries of human understanding. [Smarandache, Vladutescu]

5. Rejectivism: The Dialectic of Rejection

Rejectivism is characterized by the impulse to reject existing philosophical systems as a means of establishing new ones.[5] While this may appear purely oppositional, neutrosophy reveals its inherent duality: rejection is both a negation of external ideas and an affirmation of alternative perspectives, mediated by a neutral space of transformation.

For example, Spinoza's rejection of Cartesian dualism was not merely a critique but a constructive act, resulting in a monistic framework that redefined substance and mind. In this sense, rejectivism is not destructive but reconstructive, reshaping philosophy through a dynamic process of affirmation and negation.[6]

6. Paradoxism: The Implicit Contradiction

Paradoxism asserts that every philosophical idea is simultaneously true and false, embracing contradiction as a fundamental aspect of reality.[7]

This perspective aligns seamlessly with neutrosophy, which recognizes contradiction as intrinsic to nature. Paradoxism's core principle—`nothing is non-contradictory`—challenges the binary logic of classical thought.

Consider Zeno's paradoxes, which both deny and affirm the coherence of motion. Paradoxism does not attempt to resolve contradictions but instead treats them as essential truths.

Neutrosophy extends this approach, showing that contradictions are not obstacles but opportunities to explore the deeper complexities of thought.

7. Conclusion: Toward a Holistic Philosophy

Each of these movements—*revisionism, inspirationalism, recurrentism, sophisticalism, rejectivism, and paradoxism*—offers a unique lens through which to understand the evolution of philosophy. Through a neutrosophic perspective, these movements reveal that no idea or system is wholly true or false; all exist within a continuum of affirmation, negation, and neutrality. By transcending the silos of traditional schools of thought, neutrosophy fosters a deeper engagement with ideas, not as isolated entities but as dynamic elements.

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Neutrosophy Transcends Binary Oppositions in Mythology and Folklore

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Smarandache, Florentin. "Neutrosophy Transcends Binary Oppositions in Mythology and Folklore." *Neutrosophic Sets and Systems*, Vol. 65, 2024, pp. 57-79.

Abstract: This article is a brief foray into the intricate realm of mythological and folkloric composite entities. Our analysis posits that these hybrid and superhybrid creatures serve as compelling evidence that the human psyche consistently transcends binary oppositions as in neutrosophy. Across diverse cultures and epochs, the human mind exhibits a propensity for nuanced and neutrosophic vantage points, defying simplistic categorizations. Additionally, we make some remarks pertaining to the subject matter, and open up a few less conventional questions.

Keywords: Neutrosophy; Transcendence; MultiAlism; Mythology; Cultural Identity; Cultural Practices; Hybrids; SuperHybrids; Mythological Creatures; Divine Parentage; Heredity.

1. Introduction

Hybrid (and SuperHybrid) entities that appear in mythologies and folklore from around the world have been thoroughly explored in cultural studies. A branch of mythological studies —which could be termed 'mythological teratology'— might emerge to delve into the examination of these monsters and hybrid beings (see below *Remarks & Open Questions*). This concept of the 'hybrid being' as a reflection of society and its norms is a recurring theme in various academic disciplines, including sociology, or psychology. For instance, Joseph Campbell explores the role of mythical beings in culture and society, arguing that they reflect societal fears and desires [Campbell]. Similarly, Carl Jung's concept of the 'archetype' discusses how society creates mythical beings out of what it rejects or finds disturbing [Jung]. Cohen examines how these monsters function within culture and society, suggesting that they mirror both societal anxieties and aspirations, proposing a *modus legendi*, i.e. "a method of reading cultures from the monsters they engender" [Cohen]. Kristeva explores the concept of "abjection," a term she borrows from psychoanalysis to describe the feeling of horror that arises when encountering something that disturbs the boundaries between the self and the other [Kristeva].

The hybrid mythical beings, combining human and animal characteristics or blending divine and mortal traits, are viewed as anomalies or departures from the natural order. Those entities embody a paradoxical nature that outstrips conventional logical frameworks and established protocols. These beings exist beyond the confines of predictable patterns and imposed rules, serving as agents of disruption within structured systems, while emerging as a byproduct of order, arising in defiance of chaos and acting as a counterforce that delineates and fortifies the boundaries of structure.

Moreover, these hybrid beings are not arbitrary creations but rather essential constituents of civilization, originating from the very tenets that societies cherish. They generally personify all aspects that are repudiated and contradicted by societal norms and values. Through the identification and marginalization of these undesirable elements, society reaffirms its own identity and principles. Consequently, the hybrid being transforms into a negative reflection, a distorted mirror image of society, underscoring its fears, taboos, and limitations. By confronting and interacting with these entities, society attains a deeper comprehension of its own contradictions, thereby contributing to its continuous (neutrosophic) evolution and self-definition in the (neutrosophic) dynamic system of life.

For more on the topic, we provide a rich chapter of reading suggestions at the end of the paper (see *Further readings*).

This article does not aim to be a special contribution to this field from a cultural frame of reference. It is merely a brief overview of some neutrosophic and multialist features of a small number of hybrids, but revealing in their significance. The MultiAlism is a MultiPolar System which is formed not only by multiple elements that might be random, or contradictory, or adjuvant, but also by accepting features from more than one basic system (UniPolar, BiPolar, TriPolar, or PluriPolar systems). This article is also an encouragement to specialized researchers to interrogate the 'mythological teratology' using neutrosophic tools and perspectives, and it is —why not?— an attempt to transcend some common questions about the emergence and perpetuation of these quasi-general creatures in worldwide mythologies.

2. Neutrosophic Identities: Mythical Hybrids and Mythical SuperHybrids

Let us select a few examples from the vast array of imaginative fields, then briefly, but systematically analyze how these hybrids are composed. As a general structure of the hybrids, one might categorize as 'Mythical Hybrids' the creatures that might be formed by the union of two entities (be it part animal + part human, or part animal + part deity, or part human + part deity, or part demon + part human, or part demon + part animal, or part demon + part deity) and as 'Mythical SuperHybrids', or 'MultiAlist Hybrids' the creatures that might formed by union of at least three entities (e.g. part deity + part human + part animal), or by combinations within different types of the same entity (e.g. $Animal_1 + Animal_2 + \ldots + Animal_n$).

- In Mesopotamian mythology, *Gilgamesh* stands as a demigod, embodying a fusion of divine and mortal heritage. Described as two-thirds divine and one-third mortal, Gilgamesh's case presents a complex blend of maternal and paternal divinity. The duality of his nature, being both divine and mortal, becomes a central theme in the narrative, influencing his actions, struggles, and ultimate quest for meaning and immortality. The ambiguity surrounding his eventual fate in later traditions introduces an element of incompleteness, allowing for diverse interpretations [Gilgamesh].
- *Lamassu* is a protective deity in the Mesopotamian mythology with the body of a bull or lion, wings of an eagle, and a human head. These colossal beings often served as guardians at the entrances of palaces and temples [Lamassu].
- *Bhima* is one of the central characters in the ancient Indian epic, the *Mahabharata*. He is the second of the five Pandava brothers, born to Kunti, the queen of Hastinapura, and the wind god Vayu. Bhima is known for his exceptional physical strength, courage, and prowess in battle. Bhima plays a crucial role in various events, including the Pandavas' exile to the forest, the gambling match at the court of Hastinapura, and the Kurukshetra War, the epic battle between the Pandavas and the Kauravas. He is often portrayed as a larger-than-life figure, capable of extraordinary acts of valor and heroism. Despite his formidable strength, Bhima is also depicted as a compassionate and kind-hearted individual, especially towards those who are oppressed or marginalized. His journey is marked by moments of triumph and adversity, ultimately culminating in his role as a key figure in the establishment of dharma on the throne of Hastinapura [Bhima].
- *Achilles*, son of a sea nymph and a mortal king, encapsulates the dichotomy of the demigod in Greek mind. Immersed in the River Styx for invincibility, his vulnerable heel becomes a symbol of incompleteness. The neutrosophic nature unfolds in his death, orchestrated by a mortal's arrow guided by a god. This intersection of mortal susceptibility and divine vengeance underscores the intricacies of Achilles' fate [Achilles].
- *Freyr* is a prominent figure in Norse mythology, often depicted as a handsome and benevolent deity, associated with bountiful harvests, love, and abundance. Despite being considered a god in Norse mythology, Freyr's status as a demigod is underscored by his dual parentage, with one parent being a god (Njord) and the other a giantess named Skadi. Freyr's most famous possession

is his magical sword, known as "Freyr's Sword" or "Sumarbrandr." According to legend, Freyr traded away his sword to win the hand of the giantess Gerd, with whom he fell deeply in love. This act of sacrifice ultimately leads to Freyr's downfall, as he is left defenseless during the final battle of Ragnarok, the apocalyptic event in Norse mythology. [Freyr].

- In Islamic tradition, *Burāq* is a mythical creature described as a steed with the head of a woman, the wings of an eagle, and the tail of a peacock. It is said to have transported the Prophet Muhammad during the Night Journey. [Burāq]
- *Banshee*, known as the "woman of the fairies," is a supernatural being deeply ingrained in Irish and other Celtic folklore. Described as a female spirit or fairy, the Banshee is often associated with specific families or clans, serving as a harbinger of death or an omen of impending misfortune. One of the most distinctive features of the Banshee is her mournful wail, a chilling cry that is said to be heard when someone within the family she watches over is about to die. The Banshee is typically depicted as a solitary figure, often appearing as an old woman with long, flowing hair dressed in a grey or white gown, having features reminiscent of otherworldly or supernatural beings, such as elves, witches, or spectral apparitions. [Banshee]
- *Huli Jing*, or fox spirits, are shape-shifting beings in Chinese folklore. They can transform into beautiful women, but their true form is that of a fox. [Huli]
- The hybrid form of *Abraxas*, with the body of a human, the head of a rooster or lion, and serpentine legs, is laden with symbolic significance.[1] The human body represents consciousness and intellect, while the animal features symbolize primal instincts and cosmic forces. The rooster, associated with the dawn and awakening, signifies spiritual enlightenment, while the serpent is a potent symbol of wisdom, renewal, and transformation. Abraxas embodies the concept of unity within duality, symbolizing the reconciliation of opposites such as good and evil, creation and destruction, light and darkness. This neutrosophic fusion of contradictory elements reflects the Gnostic worldview, which conceived the material world as a realm of duality and illusion, with the ultimate goal of spiritual liberation through gnosis, or divine knowledge. [Abraxas]
- The concept of the *Homunculus* has historical roots in alchemical and philosophical traditions. In alchemy, the homunculus was believed to be created through various arcane processes, symbolizing the desire to artificially create life.[2] The most prevalent ingredient associated with the creation of the homunculus was seminal fluid, believed to contain the essence of life. In addition to semen, other bodily fluids, such as menstrual blood, were sometimes implied as ingredients. Alchemists often implied the use of various alchemical substances, such as salts, minerals, and herbal extracts, believed to possess transformative properties. Using animal seed or other animal-derived substances was suggested. The instructions for creating a homunculus would be as follows: 'Mix the semen and sun stone and inseminate the cow or ewe. Carefully plug the animal's vagina with the sun stone. Smear the animal's genitals with the blood of another animal. Place the artificially inseminated animal inside a dark house where the sun never shines.' And so on. [Lugt] From a neutrosophic perspective, the homunculus can be explored as a metaphor for the inherent uncertainties, contradictions, and complexities within biological, philosophical, and symbolic dimensions. The homunculus, as a symbol for the artificial creation of life, embodies ethical and existential implications. Though not entirely human, this entity represents a rational animal, adding another fictional chapter to humanity's aspiration to control the boundaries of life and death. [Homunculus].

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[1] References to Abraxas can be found in various Gnostic texts, including the Nag Hammadi Library, a collection of ancient Gnostic scriptures discovered in Egypt in 1945. These texts often depict Abraxas as a divine being or archon, sometimes associated with the highest God or as an emanation from the divine realm.

[2] One of the most famous accounts of creating a homunculus comes from the writings of the Swiss alchemist Paracelsus, who claimed that a homunculus could be created through the manipulation of seminal fluid. His supposed recipe involved placing semen in a sealed glass vessel along with specific alchemical substances and then incubating the mixture in a warm, dark environment for several weeks or months. Allegedly, after the incubation period, a miniature humanoid creature would form within the vessel.

- The *Balaur* occupies a prominent role within Romanian folklore, emerging as a multi-headed dragon or serpent,[3] often intertwined with turbulent weather phenomena and disruptive chaos. Intriguingly, Romanian legends infuse the Balaur's saliva with a peculiar trait, believed to possess the transformative ability to crystallize into 'diamonds'. Symbolically, the Balaur embodies primal energies, encapsulating the essence of chaos and the untamed wilderness, evoking a complex interplay of reverence and trepidation within cultural narratives.
- The *Zmeu*, another enthralling figure in Romanian folklore, assumes the guise of a shapeshifting monstrosity, blending traits of both ogre and dragon. Its versatility in assuming diverse forms epitomizes unpredictability, perpetuating an aura of enigmatic elusiveness. Frequently depicted as a malevolent force, the Zmeu embodies themes of fear, guile, and imminent danger, serving as a poignant cautionary motif within cultural tales. Furthermore, its portrayal reflects societal apprehensions, symbolizing latent threats, inner conflicts, and the enduring struggle between opposing moral forces.
- Among the spirits of Romanian folklore, the *Zburător* (The One That Flies) emerges as a seductive entity, akin to the incubus archetype.[4] Appearing in the form of a charismatic man, the Zburător seduces unsuspecting maidens, symbolizing forbidden desires and the allure of the unknown. Symbolically traversing the delicate threshold between dreams and reality, the Zburător evokes a blend of fear, fascination, and profound introspection within the human psyche. [Chelariu]

Let us try now to integrate these entities into a coherent system.

3. Offspring of Gods and Mortals: Part Deity + Part Human Hybrids

In the rich tapestry of world mythology, the entities formed of both divine and mortal lineage, often referred to as demigods or demigoddesses, embody a complex blend of traits inherited from their divine parentage and their mortal heritage. The mythologists generally agree that the hybrid beings of such type serve to bridge the gap between the celestial and terrestrial realms, reflecting humanity's innate desire to understand the nature of existence and our place within the cosmos.

Demigods possess a diverse array of characteristics and abilities: they may inherit superhuman strength, agility, or intellect from their divine lineage, while also facing the trials and tribulations of mortality. Despite their extraordinary abilities, demigods are often depicted as flawed and vulnerable beings, grappling with the complexities of their dual nature and the expectations placed upon them by both gods and mortals.

To explore several mythologies, let's illustrate the characters of three representative figures of such beings:

- *Krishna*. In Hindu mythology, Krishna is revered as an avatar of the god Vishnu, born as the son of Devaki, the sister of king Kamsa, and Vasudeva.[5] [Vishnu] Krishna declined to take up arms in the monumental conflict between the Kauravas and the Pandavas. Instead, he presented an option: to provide personal guidance to one side and lend his army to the other. A tragic altercation erupted among the Yadava chiefs, resulting in the loss of Krishna's brother and son. Deeply saddened, Krishna retreated to the forest. There, he met his demise when a huntsman, mistaking him for prey, fatally wounded him in his only vulnerable spot—the heel. [Krishna].
- *Hercules*, the son of Zeus and Alcmene, a mortal princess, is a cornerstone figure in Greek mythology, epitomizing the complexities of demigod existence. His divine parentage, coupled with mortal struggles, forms the essence of his narrative. His legendary Twelve Labors, undertaken as penance for killing his wife and children in a fit of madness induced by the goddess Hera, showcase his extraordinary strength, courage, and determination. Zeus, recognizing Hercules' deeds, grants

[3] Legends describe the Balaur as a monstrous being with sharp claws, scales covering its body, and fiery breath.

[4] Embedded within the broader folklore of incubi and succubi, the Zburător embodies the enigmatic male spirit seeking nocturnal liaisons with slumbering women.

[5] Vasudeva is the patronymic of the deity Krishna, a son of Vasudeva. The worshippers of Vasudeva-Krishna formed one of the earliest theistic devotional movements within Hinduism.

him immortality, emphasizing the dynamic interplay between mortality and the divine in the demigod's journey. [Hercules]

- *Dagda*, "The Good God," is prominent figure in Irish mythology, revered as a powerful and benevolent (semi)deity. He is often depicted as a father figure, a wise leader, and a skilled warrior, embodying the ideals of strength, wisdom, and generosity. Dagda is considered the father or chief of the Tuatha Dé Danann, a mythical race of supernatural beings. He possesses powerful magical artifacts, including a magical club known as the "lorg mór" or "the great staff," which can both kill and resurrect with one end and control the weather with the other. He also possesses a magical cauldron called the "coire ansic" or "the cauldron of plenty," which provides an endless supply of food and drink. He is known for his fondness for indulgence and pleasure, often portrayed as a lover of food, drink, and music. [Dagda]

3.1. Hindu Devas: Celestial Intermediaries of Cosmic Balance

In the intricate cosmology of Hindu mythology, demigods, or 'Devas,' occupy a pivotal role as celestial beings who straddle the realms of divinity and humanity. These demi-deities embody a complex interplay of attributes within the framework of neutrosophy, where the concepts of certainty, uncertainty, and indeterminacy converge in a dynamic equilibrium. [Deva]

The origins of demigods in Hindu mythology are shrouded in ambiguity, mirroring the inherent indeterminacy of the cosmic order. Born from the intermingling of divine and mortal lineage, demigods embody the neutrosophic principle of uncertainty, where their existence defies conventional categorization. They inhabit the liminal space between the certainty of divine essence and the uncertainty of mortal flesh.

Demigods in Hindu mythology hold profound cultural and symbolic significance, serving as allegorical representations of the cosmic balance between order and chaos. Through their stories and legends, demigods inspire introspection, reflection, and a deeper understanding of the neutrosophic principles that govern the universe. They offer timeless insights into the complexities of existence and the eternal quest for equilibrium and harmony.

Several demigods in Hindu mythology exemplify the principles of neutrosophy through their complex and multifaceted nature.

- *Indra*, the king of the Devas, embodies the neutrosophic principle of opposition as he wages war against the forces of darkness while facing internal conflicts and moral dilemmas.
- *Agni*, the god of fire, symbolizes the neutrosophic concept of indeterminacy as he serves as both a purifier and a destroyer, embodying the dual nature of fire as both creator and destroyer.
- *Varuna*, the god of cosmic waters, represents the neutrosophic notion of partial truth as he upholds the cosmic order while grappling with his own limitations and imperfections.
- *Surya*, the god of the sun, embodies the neutrosophic principle of ambiguity as he illuminates the universe with his radiant light while casting shadows of doubt and uncertainty.
- *Vayu*, the god of the wind, symbolizes the neutrosophic concept of complementarity as he breathes life into all living beings while also carrying the seeds of destruction and change.

3.2. Greek Demigods: Exploring Heroic Archetypes

In the vast and intricate tapestry of Greek mythology, demigods stand as remarkable figures, occupying a unique space between gods and mortals. Born of unions between divine beings and humans, these hybrid heroes possess extraordinary abilities, courageous hearts, and complex destinies that shape the course of myth and legend. This inherent duality reflects the neutrosophic principle of indeterminacy, where demigods exist in a state of perpetual flux, neither fully divine nor entirely mortal.

Among the pantheon of Greek demigods, several figures stand out as exemplars of heroic archetypes, each embodying the principles of neutrosophy in their own unique way.

- Heracles (Hercules), the son of Zeus and Alcmene, epitomizes the struggle against adversity and the quest for redemption. His Twelve Labors symbolize the neutrosophic concept of opposition, where seemingly contradictory forces coexist and interact in a dynamic equilibrium.
- Jason, the son of two mortals though conceived by Zeus, demonstrates leadership, charisma, and diplomacy as the leader of the Argonauts on their quest for the Golden Fleece. He navigates political intrigue and personal challenges with tact and resilience, embodying the principle of balance and harmony in relationships and endeavors.
- Another prominent demigod, Perseus, born of Zeus and a mortal princess, represents the neutrosophic principle of indeterminacy through his quest to slay the Gorgon Medusa and rescue Princess Andromeda. His journey is fraught with uncertainty and ambiguity, yet he perseveres through cunning, resourcefulness, and sheer determination. Perseus embodies the neutrosophic notion of partial truth, where reality is inherently subjective and open to interpretation. Perseus, embarks on legendary feats, including the slaying of the Gorgon Medusa. His death lacks a singular narrative, illustrating the incompleteness inherent in the portrayal of demigod destinies. Whether Perseus meets his end in battle or through a discus throw remains a subject of interpretation, adding an indeterminate layer to his demigod status. [Kerenyi]

3.3. Tuatha Dé Danann and Fomoire: Divine-Human Lineage in Celtic Mythology

Within the fabric of Celtic mythology, the Tuatha Dé Danann[6] emerge as enigmatic figures, embodying a complex fusion of divine and mortal lineage. The earliest accounts depict their banishment from heaven due to their profound knowledge, after which they descended upon Ireland enveloped in a shroud of mist. [Tuatha].

The ancient enemies of the Tuatha Dé Danann were the Fomoire, another group of supernatural beings in Irish mythology. The Fomoire are a race of monstrous, semi-divine beings, often described as a chaotic and malevolent force, associated with darkness, chaos, and the destructive forces of nature. They are portrayed as monstrous sea creatures or giants [Fomoire].

According to Irish mythology, the Fomoire were among the earliest inhabitants of Ireland, predating the arrival of the Tuatha Dé Danann. They were said to have arrived in Ireland from distant lands and waged war against the Tuatha Dé Danann for control of the island. The battles between the Fomoire and the Tuatha Dé Danann are depicted as cosmic struggles between the forces of chaos and order, with the Fomoire representing chaos and darkness,[7] and the Tuatha Dé Danann representing light and civilization.

The Tuatha Dé Danann are skilled in magic, shape-shifting, and other forms of arcane knowledge, yet they also experience human emotions, desires, and vulnerabilities. Several key figures among the Tuatha Dé Danann exemplify the neutrosophic themes of certainty, uncertainty, and indeterminacy through their complex parentage and lineage:

- The *Dagda*, for example, is sometimes portrayed as the son of the goddess Danu and the mortal prince Elatha, reflecting the intertwining of divine and human elements within his lineage.
- Similarly, *Lugh* is depicted as the son of a mortal man, Cian, and a supernatural being, Ethniu, highlighting his hybrid nature and the ambiguity of his identity.

4. Hybridization of Humanity and Demonology: Part Demon + Part Human Hybrids

Mythology often features beings that are hybrids part demon and part human. While the specific traits and appearances of these beings can vary widely across different cultures and mythologies, here are a few examples:

[6] Translating to "the people of the goddess Danu," who represents the primordial waters of creation and fertility, these semi-divine beings are revered for their wisdom, magic, and profound connection to the land before the arrival of the Milesians, who are considered the ancestors of the modern Irish

[7] Despite their malevolent reputation, some stories depict individual Fomoire in a more sympathetic light, portraying them as complex characters with their own desires and motivations. In some accounts, Fomoire are depicted as skilled craftsmen and warriors, capable of great feats of strength and magic.

- *Cambions*: In European folklore, cambions are offspring of a demon and a human. They are often depicted as possessing some of the supernatural powers of their demonic parent, such as shapeshifting, telepathy, or dark magic, along with the physical appearance of humans. Cambions are sometimes portrayed as seductive and manipulative figures, using their powers to influence or deceive mortals.
- *Nephilim*: In Judeo-Christian mythology, nephilim are the offspring of angels[8] and humans. They are described in ancient texts like the Book of Genesis as giants or mighty warriors. Nephilim are often associated with themes of divine punishment and corruption, as their existence is seen as a perversion of the natural order.
- *Oni-Human Hybrids*: In Japanese folklore, oni are malevolent spirits or demons often depicted as large, ogre-like creatures with horns and wild hair. Sometimes, stories feature oni-human hybrids, typically resulting from unions between an oni and a human. These hybrids may inherit some of the oni's physical traits, such as horns or strength, as well as their mischievous or malicious nature.

These are just a few examples, and there are many other variations of partially demon and partially human beings found in mythologies around the world. They often serve as compelling figures in storytelling, embodying themes of the struggle between good and evil, the supernatural and the mundane, and the complexities of identity and heritage.

4.1. Cambions: A Hybrid of Demon and Human

Human imagination has conjured a plethora of mythical beings, among which *cambions* hold a particularly intriguing place. The concept of cambions finds its roots in Western European folklore, particularly within the framework of Christian demonology.[9] Cambions are creatures born of the union between a demon and a human, embodying a unique blend of the supernatural and the mortal. These unions are typically portrayed as acts of seduction, coercion, or temptation, reflecting broader themes of moral ambiguity and the struggle between good and evil. [Cambion]

The characteristics attributed to cambions vary across different mythological traditions, but they commonly possess a blend of supernatural powers and human vulnerabilities, with physical traits reminiscent of their demonic parentage, such as horns, fangs, or unnaturally colored eyes, but mostly possessing supernatural abilities, including shapeshifting, telepathy, or control over dark magic. In addition to their supernatural powers, cambions are often portrayed having charismatic and seductive demeanors, using their allure to manipulate and deceive mortals. This aspect of their character reflects themes of temptation and moral corruption, as cambions —neither fully human nor purely demonic— navigate the complexities of their dual heritage.

- *Merlin*, the legendary wizard from Arthurian mythology, is sometimes depicted as a cambion.[10] According to some versions of the tale, Merlin's father was an incubus who seduced his mother, a mortal woman. This union resulted in Merlin's birth, granting him his magical abilities.
- In William Shakespeare's play "The Tempest," *Caliban* is a half-human, half-demon creature who serves as one of the primary antagonists. While the exact nature of Caliban's parentage is not explicitly stated in the play, his origins are described as being monstrous and unnatural. Caliban's character embodies themes of colonization, power dynamics, and the clash between civilization and the wild.

As hybrid beings, cambions occupy a liminal space between the supernatural and the mortal. In Christian demonology, cambions are often viewed as embodiments of sin and moral decay, reflecting

[8] Sometimes interpreted as fallen angels, i.e., demons.
[9] The word "cambion" is believed to derive from the Late Latin term "cambuca," which referred to a vessel used by Roman soldiers. Over time, it evolved to denote a vessel or receptacle for spirits, eventually coming to signify the offspring of demons.
[10] The story of Merlin as a cambion is explored in various medieval texts, including Geoffrey of Monmouth's *Historia Regum Britanniae*.

the consequences of succumbing to worldly desires. They serve as cautionary figures, warning against the dangers of indulgence and spiritual corruption.

4.2. Hybrid Giants and Divine Judgment: The Tale of the Nephilim

The *nephilim*[11] are mentioned in ancient texts such as the Bible and various apocryphal works [Barker], are shrouded in mystery and controversy, occupying a unique place in the tapestry of mythological lore, embodying themes of hybridity, rebellion, and divine judgment.

The nephilim are mentioned specifically in Genesis,[12] where they are described as the offspring of unions between "the sons of God" and "the daughters of men."[13]

In some interpretations, they are depicted as literal giants, possessing immense size and strength. Other sources describe them as beings of great wickedness or spiritual corruption, whose presence on Earth threatened the order established by God.

According to the biblical narrative, the presence of the nephilim on Earth prompted divine intervention, leading to the Great Flood as a means of cleansing the world of their corruption. Only Noah and his family were spared, as they were deemed righteous in the eyes of God. The story of the Nephilim serves as a cautionary tale about the consequences of moral corruption and divine judgment.

Two representative characters are:

- *Goliath*, the legendary giant from the biblical story of David and Goliath,[14] is often interpreted as a Nephilim or descendant of the Nephilim. Goliath is described in the biblical text as a giant, towering over his opponents with his imposing stature. His armor alone is said to have weighed hundreds of shekels of bronze, underscoring his formidable presence on the battlefield.
- *Og*, the king of Bashan, is a figure mentioned in the Hebrew Bible, specifically in the Old Testament. According to the biblical narrative, Og was one of the last remaining Rephaites, a group of giants who were known for their great stature and strength. The Rephaites were believed to be an ancient race of people who inhabited the land of Canaan before the Israelites arrived. Og's defeat is recounted as one of the victories achieved by the Israelites under the leadership of Moses.[15] [Nephilim]

The story of the nephilim has left a lasting impact on religious and cultural narratives throughout history. In Jewish and Christian traditions, they are often interpreted as symbols of rebellion, sin, and divine judgment, serving as cautionary figures, warning against the dangers of pride, corruption, and moral decay.

4.3. Oni-Human Hybrids: Intersections of Humanity and Demonology in Japanese Folklore

In Japanese folklore, the *oni* are formidable and malevolent spirits or demons known for their monstrous appearance and malicious behavior. Often depicted as horned, ogre-like creatures with wild hair and fearsome expressions, oni embody the darker aspects of the supernatural realm.

Oni-human hybrids represent a multifaceted archetype within Japanese folklore. Born from the union of humanity and demonology, these hybrids embody themes of power, temptation, and the moral transgressions. Their role as malevolent spirits capable of wreaking havoc upon humans reflects broader cultural anxieties surrounding the forces of darkness and chaos.

Oni-human hybrids, sometimes referred to as "half-oni" or "oni-kijo," inherit traits from both their demonic and human heritage. They may possess the physical characteristics of oni, such as horns, fangs, and exaggerated features, while also retaining elements of their human ancestry. These

[11] The term "Nephilim" is derived from the Hebrew word "nephiyl," which translates to "giants" or "fallen ones."
[12] The Holy Bible: Genesis 6:1-4.
[13] Their precise identity is a subject of debate among scholars, with interpretations ranging from fallen angels to divine beings or rulers.
[14] The Holy Bible: Samuel 1:17.
[15] The book of Deuteronomy provides further details about Og and his kingdom. In Deuteronomy 3:11, it is mentioned that Og's bed was made of iron and was more than thirteen feet long and six feet wide, indicating his enormous size.

hybrids are often depicted as powerful and fearsome beings, capable of both great strength and cunning intelligence. Despite their monstrous appearance, oni-human hybrids may exhibit complex emotions and motivations.

Two representative characters are:

- *Shuten-doji* is a legendary oni king who terrorized the ancient capital of Kyoto. According to folklore, Shuten-doji was believed to be a half-oni, born from the union between a human woman and the king of the oni. [Shuten]
- *Ibaraki-doji* is a female oni who is often depicted as a vengeful spirit seeking retribution for past injustices. [Ibaraki]

These and other oni-human hybrids populate the Japanese mythology, embodying themes of power, vengeance, and the struggle between humanity and the supernatural. From traditional folk tales and kabuki theater to modern manga and anime, oni-human hybrids remain enduring symbols of the supernatural and the fantastic. Their stories serve as cautionary tales, reminding audiences of the dangers of succumbing to temptation and the importance of moral integrity in the face of adversity.

5. Transcending Species: Part Human + Part Animal Hybrids

Part-human, part-animal mythological hybrids have fascinated cultures throughout history, appearing in myths, legends, and folklore around the world. These hybrids embody a fusion of human and animal traits, blurring the boundaries between the human and non-human realms. Here are some common examples of such hybrids:

- The Egyptian god *Anubis*, depicted with the body of a man and the head of a jackal, serves as a guide and protector of the dead, symbolizing the transition between life and death.
- In Greek mythology, the Centaur is a creature with the upper body of a human and the lower body of a horse. Centaurs are often depicted as possessing superhuman strength and agility, as well as a wild and untamed nature. They are associated with Dionysus, the god of wine and revelry, and are often depicted as participants in his ecstatic rites and celebrations. [Centaur]
- The *Minotaur*, another creature from Greek mythology, possesses the body of a human and the head of a bull. Confined within the labyrinth of Crete, the Minotaur symbolizes brute strength, primal aggression, and the darker aspects of human nature. [Minotaur]
- In Greek and Roman mythology, the *Harpy* is a creature with the body of a bird and the head of a woman. Often depicted as fierce and predatory, harpies symbolize chaos, violence, and the destructive forces of nature. [Harpy]
- Found in various cultures worldwide, the *werewolf* is a creature that can transform from human to wolf form, often associated with themes of lycanthropy and shapeshifting. [Werewolf]
- *Mermaids* and *mermen*, creatures with the upper body of a human and the lower body of a fish, appear in folklore and mythology from cultures around the world. They are often associated with the sea, symbolizing mystery, allure, and the unknown depths of the ocean. [Mermaid]

These part-human, part-animal hybrids serve as powerful symbols in mythology, representing a wide range of themes including the relationship between humanity and nature, the complexity of human identity, and the struggle between civilization and the primal instincts.

5.1. The Winged Man: Part Human + Part Bird Hybrids

The motif of the winged man, a figure with both human and avian characteristics, has appeared in various forms throughout mythology, folklore, and art across different cultures. This hybrid creature often symbolizes a fusion of earthly and celestial elements, embodying themes of freedom, transcendence, and the duality of human nature.

Let us explore the motif of the winged man in different cultural contexts, noting that some of them can be classified as SuperHybrids as well due to their associated divine nature:

- *Garuda* is a divine being in Hindu mythology, often depicted with a human upper body and wings, while the lower body resembles an eagle or bird. As the mount of the god Vishnu, Garuda symbolizes power, strength, and the ability to soar to great heights.
- *Horus* (Egyptian Mythology), the god of the sky and kingship, is sometimes depicted with the head of a falcon and the body of a man.
- *Icarus* (Greek Mythology) is perhaps the most iconic representation of the winged man. Alongside his father Daedalus, Icarus escapes imprisonment using wings crafted from feathers and wax. However, his disobedience leads to his tragic downfall as he flies too close to the sun, melting the wax and causing him to fall.
- *Eros*/*Cupid* (Greco-Roman Mythology): the Greek god of love, and his Roman counterpart Cupid, are occasionally portrayed with wings. This representation aligns with their association with the flighty and unpredictable nature of love.
- *Phoenix* (Various Cultures), while not a traditional winged man, the mythical bird that cyclically regenerates or reborn is sometimes depicted with human-like characteristics, especially in art and literature.
- *Fenghuang* (Chinese Mythology), also known as the Chinese phoenix, is a mythical bird with a mix of avian and human features. Often considered a symbol of harmony and balance, the Fenghuang embodies the union of opposites.
- *Angels* (Various Cultures), are often depicted as winged beings with a human-like appearance. In Christianity, angels are messengers of God, and artistic representations frequently portray them with wings, symbolizing their celestial nature.
- *Shangó* (Yoruba Mythology), the god of thunder and lightning, is sometimes depicted with wings. The wings emphasize his connection to the sky and his ability to move swiftly across the heavens.

The motif of the winged man resonates across cultures, illustrating humanity's fascination with the idea of transcending earthly limitations and reaching for higher realms. Whether representing divine messengers, mythical heroes, or symbolic creatures, the winged man motif captivated the human imagination and conveyed universal themes of aspiration and transcendence.

6. Gods and Beasts: Part God + Part Animal Hybrids

The concept of beings that are partially god and partially animal, with no human attributes or representations, is less common in mythology and folklore compared to those with human-like characteristics. However, there are still some examples from various cultural traditions around the world where such beings are found. These creatures often embody a unique blend of divine and animalistic qualities, serving as symbols of power, transformation. Here are a few examples:

- *Azure Dragon*: In Chinese mythology, the Azure Dragon is one of the four celestial guardians, representing the east and the spring season. The Azure Dragon is sometimes depicted as a SuperHybrid, a dragon with the body of a snake and the claws of a tiger, symbolizing power, vitality, and the cosmic forces of nature. He is associated with the element of wood and serves as a protector of the heavens.
- *Pegasus*: In Greek mythology, Pegasus is a divine winged horse,[16] born from the blood of the Gorgon Medusa after she was slain by the hero Perseus. He is associated with the god Poseidon and serves as a mount for heroes such as Bellerophon.
- In Norse mythology, *Fenrir* is a monstrous wolf, the offspring of the god Loki and the giantess Angrboða. Fenrir is depicted as a fearsome and powerful creature, destined to bring about the end of the world during Ragnarok.
- *Thunderbird*: In Native American mythology, the Thunderbird is a powerful and mythical bird, often depicted as a large bird of prey with the wingspan of an eagle and the feathers of a hawk

[16] Pegasus is often depicted as a majestic white horse with wings, symbolizing swiftness, freedom, and the divine realm. While Pegasus is not a deity himself, he is closely associated with the gods, particularly Zeus, the king of the gods, and Athena, the goddess of wisdom and war.

or owl. The Thunderbird is associated with thunderstorms, lightning, and the forces of the sky, serving as a symbol of power, transformation, and the spiritual connection between humans and nature.

These examples illustrate the diverse range of beings that embody the concept of entities partially god and partially animal, with no human attributes or representations, in mythology and folklore, serving as symbols of divine power, guardianship, or natural forces.

6.1. The Mystical Azure Dragon: A Chinese Celestial Guardian

The Azure Dragon, known as *Qinglong* in Chinese, is one of the four celestial guardians in Chinese mythology, along with the Vermilion Bird, the White Tiger, and the Black Tortoise. It is often depicted as a dragon with the body of a snake and the claws of a tiger, symbolizing the convergence of different animal attributes, in such case categorized as a SuperHybrid entity. The Azure Dragon is associated with the element of wood, the direction of east, and the season of spring, representing vitality, growth, and renewal. [Azure]

The Azure Dragon holds profound cultural and symbolic significance in Chinese mythology and society, serving as a protector of the heavens and a symbol of imperial power and authority.[17] It is closely associated with the Emperor of China and the concept of the Mandate of Heaven, representing the divine sanction of rulership and the cosmic order of the universe.

Through its hybrid form and multifaceted attributes, it invites contemplation of the neutrosophic principles of ambiguity, uncertainty, and indeterminacy. As a symbol of neutrosophic balance, the Azure Dragon navigates the complexities of existence, embodying the cyclical rhythms of nature and the interconnectedness of all living beings.

6.2. Thunderbird: A Native American Mythical Entity

In the Native American mythology, including the Ojibwe, Lakota, and Haida peoples, the *Thunderbird* emerges as a powerful and enigmatic symbol of the natural world and spiritual realms. Representing thunderstorms, lightning, and the forces of the sky, this mythical creature embodies the dynamic interplay between earthly and celestial forces.

It is often depicted as a large bird of prey, resembling an eagle or hawk, with wings spanning the heavens and feathers crackling with lightning. The Thunderbird is associated with thunderstorms, lightning, and the life-giving rains that nourish the earth. Its powerful presence symbolizes the awesome and unpredictable forces of nature. [Thunderbird]

It is a benevolent and awe-inspiring creature, bringing blessings of rain and prosperity to the land. In the traditions of the Ojibwe and other tribes, the Thunderbird is believed to inhabit the highest mountains and cliffs, from which it watches over the earth and sends forth lightning and thunder to cleanse and purify the land. Ceremonial dances and songs are performed to honor the Thunderbird and invoke its protection and guidance.[18]

Therefore, the Thunderbird embodies the mysteries of the natural world and spiritual realms in Native American mythology. As a symbol of divine power and natural forces, the Thunderbird constitues a timeless reminder of the interconnectedness of worlds and the cyclical rhythms of nature.

[17] In Chinese mythology and folklore, the Azure Dragon is celebrated in various legends, rituals, and festivals. It is often depicted as a guardian deity, protecting sacred sites such as temples, palaces, and ancestral tombs. During the Qingming Festival (Tomb-Sweeping Day), offerings are made to the Azure Dragon to honor ancestors and seek blessings for the coming year. In art and literature, the Azure Dragon is depicted as a symbol of strength, resilience, and celestial beauty, inspiring awe and reverence among the Chinese people for centuries.

[18] The Thunderbird holds profound cultural and symbolic significance in Native American mythology and spirituality, serving as a guardian of the natural world and a messenger of the gods. It is closely associated with rituals, ceremonies, and traditions related to rainmaking, agriculture, and hunting. The Thunderbird is revered as a protector of the tribe and a symbol of strength, resilience, and spiritual renewal. Its presence in Native American art, dance, and oral traditions reflects the enduring reverence and awe inspired by this mythical creature.

6.3. Wings of Ambiguity: Pegasus

In the vast expanse of Greek mythology, *Pegasus* emerges as a symbol of boundless freedom, transcending the earthly realm with his majestic wings and divine grace. According to legend, Pegasus emerged from the blood of the slain Gorgon Medusa, born of the union between the earth and the sea. With his pristine white coat and wings of pure light, Pegasus embodies the ideal of divine beauty and grace. He is often depicted as a symbol of inspiration, carrying the thunderbolts of Zeus or the muses of Mount Helicon on his celestial journeys.[19] [Pegasus]

Pegasus, typically portrayed as a magnificent winged horse, embodies the mysteries of divine beauty and transcendence. Through his hybrid form and multifaceted attributes, he invites contemplation of the neutrosophic principles.

7. Mythical MultiAlist Entities: Part God + Part Human + Part Animal SuperHybrids

These hybrid creatures embody a complex blend of divine, mortal, and animalistic attributes, serving —in mythologists' opinion— as symbols of transformation, power, and the interconnection between different realms of existence. Here are some examples from different mythologies:

- In Egyptian mythology, *Thoth* is often depicted as a deity with the body of a human and the head of an ibis or a baboon. As the god of wisdom, writing, and magic, Thoth embodies the divine intellect and creative power of the gods, while also possessing human-like qualities such as intelligence and compassion. His hybrid form symbolizes the synthesis of divine knowledge and mortal understanding, serving as a guide and mediator between gods and humans. [Thoth]
- In Hindu mythology, *Hanuman* is a deity with the body of a human, but a monkey face, and the intelligence and powers of a god. He is revered as the devoted companion of Lord Rama and a symbol of strength, courage, and devotion. Hanuman's hybrid nature reflects his divine lineage as the son of the wind god Vayu and a celestial nymph, as well as his close association with the natural world and the animal kingdom.[20] [Hanuman]
- In Japanese folklore, *Tengu* are kite-like beings taking a human-like form, but retaining avian wings, heads, or beaks, and emdowed with the intelligence and powers of a god. Tengu are associated with mountains and forests, where they serve as guardians and tricksters, testing the virtues of travelers and monks. [Tengu]

These cases highlight the diverse range of beings that embody the concept of being partially god, partially human, and partially animal in mythology and folklore, thus being associated with multialistic features [MultiAlist]. In a neutrosophic context, such beings represent the inherent ambiguity and paradoxical nature of existence, existing in a state of *both-and*, rather than *either-or*.

7.1. The Sphinx: The Egyptian Guardian

In the timeless sands of Egyptian mythology and history, the Sphinx stands as a testament to the enigmatic blending of human and animal attributes, endowed with divine powers. Carved from the living rock, this iconic creature embodies a profound symbolism, serving as a guardian of knowledge, mystery, and cosmic balance.

The Sphinx finds its origins in the ancient Egyptian concept of the "shesep ankh," or "living image." It is typically depicted as a recumbent lion with a human head, often bearing the likeness of a pharaoh. This hybrid form symbolizes the union of divine kingship (represented by the lion) with

[19] Pegasus holds profound cultural and symbolic significance in Greek mythology and society, serving as a symbol of divine inspiration, creativity, and transcendence. He is closely associated with the muses of Mount Helicon, who were said to have nurtured him with the waters of the Pierian Spring. Pegasus is also linked to the hero Bellerophon, whom he aided in his quest to slay the monstrous Chimera. Through his mythic adventures and legendary feats, Pegasus continues to inspire artists, poets, and dreamers to reach for the stars and pursue their loftiest aspirations.

[20] Hanuman is depicted with five faces, symbolizing his divine power and illustrating a narrative from one of his tales. In an episode where he aids Rama (specifically, rescuing Rama from the demon Ahiravana, Ravana's brother), Hanuman needed to extinguish five lamps simultaneously to defeat Ahiravana. To accomplish this task, he manifested five heads, each facing a different direction where the lamps were located.

human intelligence and wisdom (embodied by the human head). The Sphinx serves as a guardian of sacred spaces, such as the entrance to temples or the avenues leading to royal tombs, as well as a protector of cosmic order and the cycle of life and death.[21] [Sphinx]

7.2. Mythic Meld: The Intersection of Gods, Humans, and Animals in Hindu mythology

The concept of multialist hybrids –beings that are partially god, partially human, and partially animal–, is richly depicted in various tales and legends of Hindu mythology. These hybrid entities embody the intricate interplay between the divine, human, and animal realms, thus offering another neutrosophic context.

One prominent example of such hybrid in Hindu mythology is *Hanuman*, the monkey-faced deity known for his unwavering devotion to Lord Rama. Hanuman is revered as the epitome of loyalty, strength, and courage, possessing divine attributes as well as animalistic traits. His physical appearance, with a human body adorned with a monkey's face and tail, reflects his multialistic nature as both a divine being and a mixed creature of the natural world.

Another multialist hybrid is *Narasimha*, the half-man, half-lion incarnation of Lord Vishnu. According to Hindu mythology, Narasimha emerged to protect his devotee Prahlada from his tyrannical father, the demon king Hiranyakashipu. With the body of a man and the head and claws of a lion, Narasimha embodies the ferocity and power of the animal kingdom, combined with the intellect and compassion of humanity. [Narasimha]

In a neutrosophic context, these multi-alist hybrids challenge conventional notions of identity and categorization, existing at the intersection of multiple domains of existence. They embody the paradoxical nature of reality, simultaneously embodying divine, human, and animal attributes, transcending binary distinctions and embracing the multialist possibilities of the universe.

7.3. Centzon Totochtin: Aztec Rabbit Deities

In the vibrant tapestry of Aztec mythology, the *Centzon Totochtin* stand as enigmatic figures, embodying the complex interplay between divine, human and animal realms. Translating to "Four Hundred Rabbits"[22] in Nahuatl, the language of the Aztecs, these divine rabbit represents fertility, abundance, and the celebration of life. The Centzon Totochtin trace their origins to the union of the goddess Mayahuel, the deity of maguey plants, and the god Patecatl, the deity of pulque.

The 400 gods are often depicted as humanoid figures with rabbit-like features, such as long ears, whiskers, and sometimes a fluffy tail. [Centzon]

One notable example is the rabbit deity *Ometotchtli*, who presides over drunkenness and revelry, embodying the festive spirit of Aztec culture.[23] Another example is *Tepoztecatl*, the rabbit god of pulque, who oversees the fermentation and consumption of the sacred beverage.

8. Remarks & Open Questions

[21] The Sphinx is closely associated with the god Atum-Ra, the sun god and creator deity. One famous example is the Great Sphinx of Giza, which stands in front of the Pyramid of Khafre and is believed to embody the pharaoh himself, serving as his eternal protector and guide in the afterlife. Another example is the Sphinx of Amenemhat II, which guards the entrance to the temple of the god Amun-Ra at Tanis.

22 In Aztec culture, the number 400 held significant symbolism, particularly in relation to time and calendrical systems. The Aztecs used a complex calendar system composed of several interlocking cycles, one of which was the "xiuhpohualli," or the agricultural calendar, which consisted of 18 months of 20 days each, plus an additional 5 "unlucky days" at the end. The number 400 is relevant because it corresponds to the length of one "xiuhpohualli" cycle, which is comprised of 20 "veintenas" (cycles of 20 days), each lasting 20 days. When multiplied together, 20 veintenas x 20 days = 400 days. After completing one cycle of 400 days, the calendar would restart, beginning a new cycle. This cyclical nature of time represented by the number 400 was significant in Aztec cosmology and rituals, as it reflected the continuous cycle of life, death, and rebirth observed in the natural world. Additionally, the number 400 was associated with concepts of completion, renewal, and the cyclical nature of existence in Aztec belief systems.

[23] They participate in festive celebrations, such as the Huey Tozoztli festival, where offerings of food, drink, and flowers are made in their honor. The legacy of the Centzon Totochtin continues to resonate in Mexican culture today, where rabbits are revered as symbols of fertility, abundance, and the renewal of life.

8.1. Hybrid Beasts and SuperHybrid Beasts: Part Animal$_1$ + Part Animal$_2$ (+ … + Part Animal$_n$) Hybrids

Hybrid Beasts and SuperHybrid Beasts are fantastical creatures with a combination of features from different animals, abounding in folklore and mythology across cultures. Here are some examples of such beasts:

- *Anzû* (Sumerian Mythology) is a divine storm bird, often depicted as an eagle with a lion's head. It is associated with the heavens and sometimes considered a symbol of chaos.
- *Ammit* (Egyptian Mythology), also known as the "Devourer of the Dead," is a creature with the head of a crocodile, the forelimbs of a lion, and the hind limbs of a hippopotamus. It is said to devour the hearts of the unworthy during the judgment of the dead.
- *Chimera* (Greek Mythology) is a fire-breathing SuperHybrid monster with the body of a lion, the head of a goat, and a serpent's tail.
- *Griffin* (Various Cultures) is a legendary creature with the body of a lion and the head of an eagle, often associated with guarding treasures.
- *Hippogriff* (European Mythology) is a legendary creature with the front half of an eagle and the hind half of a horse.
- *Qilin* (Chinese Mythology) is a mythical creature with the body of a deer, tail of an ox, hooves of a horse, and sometimes features like a dragon or lion.
- *Baku* (Japanese Mythology) is a supernatural creature that is part elephant, part lion, and part tiger. It is believed to devour nightmares.
- *Nue* (Japanese Folklore) is a chimera-like creature, featuring the head of a monkey, the body of a tanuki (raccoon dog), the limbs of a tiger, and a snake for a tail. It is associated with ill omens.
- *Camahueto* (Mapuche Mythology, South America) is a creature with the body of a calf and a spiral-shaped horn. It is considered a powerful and sacred being.
- *Jackalope* (North American Modern Folklore) is a whimsical creature with the body of a jackrabbit and antlers like those of an antelope or deer. It is a product of American tall tales.

These creatures often embody the blending of different species and frequently serve as symbols, metaphors, or explanations for natural phenomena, embodying cultural beliefs and values.

8.2. Hybrids and SuperHybrids in Art & Fiction

The fascination with hybrids and superhybrids permeates various forms of art and fiction, captivating audiences across cultures and genres. From classical paintings to contemporary literature, these fantastical beings have seized the imagination of creators and audiences alike, transcending boundaries of time and medium.

In the realm of visual arts, depictions of hybrids and superhybrids have adorned canvases for centuries, often serving as symbols of the extraordinary and the otherworldly. Artists throughout history have been drawn to the concept of merging disparate elements from the natural world to create creatures that defy conventional classification. Whether it's the centaurs of Greek mythology, the sphinxes of ancient Egypt, or the futuristic cyborgs of science fiction, artists have explored the boundaries of imagination through their portrayals of these hybrid beings.

Similarly, in the world of literature and fiction, hybrids and superhybrids have emerged as popular subjects, enriching narratives with their complex characters and fantastical worlds. From ancient myths and legends to modern-day novels and comics, these beings inhabit stories that explore themes of identity, transformation, and the interplay between humanity and the unknown. Whether they're portrayed as heroes, villains, or something in between, hybrids and superhybrids challenge readers to question the nature of existence and the limits of imagination.

For example, *Pegasus*, the majestic winged horse of Greek mythology, occupies a prominent place in the collective imagination, celebrated across numerous legends, tales, and artistic renderings throughout history, from ancient vase paintings to modern-day literature. These artistic depictions often portrayed him in full flight, his powerful wings outstretched as he soared through the heavens, e.g. the mosaic of Pegasus found at the House of Dionysus in Paphos, Cyprus. In this mosaic, Pegasus is depicted with stunning detail and elegance, carrying the muses on his back as he ascends into the

sky. In addition to visual art, Pegasus has left an indelible mark on literature, inspiring some of the greatest poets and writers of antiquity. In the epic works of Homer, Hesiod, and Pindar, Pegasus is celebrated for his mythic beauty, grace, and divine lineage.

In literature, the concept of the *homunculus* has been reimagined and adapted in various ways. It has appeared in works of fiction, such as Mary Shelley's "Frankenstein" where the scientist Victor Frankenstein creates a humanoid creature through scientific experimentation. Similarly, in Johann Wolfgang von Goethe's "Faust," the character of Faust conjures a homunculus through magical means.

In modern literature, film, and popular culture, *cambions* continue to captivate audiences with their enigmatic allure and complex motivations. From Anne Rice's "The Witching Hour" to the television series "Supernatural," created by Eric Kripke, cambions have been reimagined and reinvented, each iteration offering new insights into their mythological origins and cultural significance.

The *Tuatha Dé Danann* and *Fomoire* have left a lasting impact on Irish culture and folklore. Their stories and legends continue to be celebrated in literature, art, and popular culture. Many landmarks and geographical features in Ireland are associated with these mythical beings, contributing to the cultural landscape of the country. The novel "A Portrait of the Artist as a Young Man" by James Joyce includes references to these entitities. Also, the novel "An Only Child" by Frank O'Connor features such elements of Celtic folklore and mythology, and the same in "American Gods" by Neil Gaiman. The animated film "The Secret of Kells", directed by Tomm Moore and Nora Twomey, draws inspiration from the same mythological aspects. The character Hellboy in "Hellboy" comic series by Mike Mignola encounters creatures inspired by Celtic mythology. And so forth.

In today's digital age, the popularity of hybrids and superhybrids shows no signs of waning. They continue to inspire artists, writers, filmmakers, and creators across various mediums, fueling a creative renaissance that pushes the boundaries of storytelling and artistic expression.

Certainly, contemporary writers and artists possess the creative potential to craft a diverse array of Hybrids and SuperHybrids, leveraging modern tools such as generative artificial intelligence to push the boundaries of imagination.

One intriguing possibility lies in the creation of novel beings that blend elements from disparate realms, such as the envisioned God-Human-Demon SuperHybrid. This entity embodies a fusion of divine, human, and demonic attributes, offering a complex and multifaceted character ripe for exploration in both fictional narratives and visual art.

Moreover, artists and writers can employ varying degrees of composition to construct these imaginative entities, allowing for a nuanced approach to their creation. For instance, one could specify the proportions of each constituent component, delineating the precise makeup of the hybrid being. This approach introduces a level of granularity and specificity, enabling creators to tailor the characteristics of their creations according to their artistic vision.

For example, a hypothetical hybrid might be described as 30% Demigod, 47% Demon, 3% Cambion, 15% Animal, and 5% Human. Each percentage represents a distinct aspect of the hybrid's nature, contributing to its overall identity and narrative significance. Through this detailed approach, creators can imbue their creations with depth and complexity, inviting audiences to contemplate the interplay of different forces and identities within these fantastical beings.

In essence, *the creative possibilities afforded by contemporary tools and techniques enable artists and writers to explore new frontiers in the realm of mythology and fantasy*. By harnessing the power of imagination and innovation, they can breathe life into a diverse cast of characters. *Through experimentation with the new AI tools, Hybrids and SuperHybrids will continue to evolve*.

8.3. 'Mythological Teratology' and Open Questions

Teratology[24] has traversed a fascinating journey through history, evolving from a discourse on prodigies and marvels to a scientific field that explores congenital malformations and their causes. This interdisciplinary realm intersects with developmental biology, embryology, and genetics, delving into the study of abnormalities in physiological development. In the modern context, teratology encompasses the medical examination of teratogenesis, congenital malformations, and individuals with significant malformations. The principles of teratogenesis provide a foundational framework for understanding the effects of environmental agents on developing organisms. These principles consider factors such as genotype, exposure timing, and environmental interactions, guiding research in teratogenic agents.

However, the roots of teratology extend deep into antiquity, where figures like Phlegon of Tralles, a prominent paradoxographer from the first and second centuries CE, meticulously chronicled extraordinary narratives in his magnum opus, "Peri thaumasion" ("Book of Wonders") [Hatzopoulos]. Phlegon's accounts, along with those of Pliny the Elder and other ancient scholars, offer glimpses into a world where anomalies were observed with a blend of astonishment and intellectual curiosity. Immersing oneself in Phlegon's narratives offers a journey into a realm where anomalies are not merely observed but chronicled, encompassing accounts of hermaphrodites, individuals undergoing sex transformations, and instances of unusual births. [Nutton] Accounts from travelers as documented in the *Natural History* of Pliny the Elder further elaborate on the existence of fantastical beings in distant lands, such as individuals with a dog's head resembling baboons, those with a single tall foot (sciapodes), or beings with faces embedded in their chests (referred to as acephala).

Ancient narratives often described individuals with anatomical anomalies, such as hermaphrodites or individuals lacking mouths or noses. These accounts, along with the tapestry of mythical monsters found in global folklore, including giants, cyclops, centaurs, and so forth, raise intriguing questions about the relationship between medical abnormalities and mythical creatures.

Could ancient accounts of marvels and hybrids have emerged as reflections of medical abnormalities? Could modern teratology provide insights into the emergence and perpetuation of mythical monsters? The concept of "Mythological Teratology" arises as a prospective pathway for exploring this intersection between myth and reality, bridging the gap between ancient lore and modern science.

Utilizing generative artificial intelligence to compare current knowledge on malformations with mythical Hybrids and SuperHybrids opens new avenues for research and exploration. By delving into this mixed study, researchers may uncover insights into the origins of mythical creatures and gain a deeper understanding of the intricate relationship between myth and reality.

8.4. Exploring the Intricacies of Heredity: Attributing Divine Paternity and Other Open Questions

Investigating the hereditary lineage of hybrid beings throughout diverse historical periods and cultural contexts unveils a fluid and intriguing terrain, prompting some bold inquiries that may challenge conventional notions.

As previously emphasized, in Greek mythology, divine paternity is a recurrent theme, exemplified by the numerous instances of gods fathering demigods with mortal women. Zeus, the king of the gods, is particularly renowned for his amorous escapades with mortals, leading to the birth of heroes like Hercules, Perseus, and Helen of Troy. These demigods inherit traits and abilities from their divine fathers, creating a neutrosophic blend of mortal and divine characteristics.

Similarly, in Roman mythology, the god Mars is considered the divine father of Romulus and Remus, the legendary founders of Rome.[25] This divine paternity adds a sacred dimension to the

[24] Originating from the Greek word "τέρας" meaning "sign sent by the gods, portent, marvel, monster".

[25] As mentioned in Livy's *History*, Rhea Silvia claimed that Mars was the father of her twins. Attributing divine paternity was not new. Alcmene, the mother of Heracles (Hercules), attributed the paternity of her son to Zeus. According to the myth, Zeus disguised himself as Alcmene's husband, Amphitryon, leading to the birth of the heroic demigod Heracles. Danaë, the mother of Perseus, claimed that Zeus impregnated her in the form of a shower of gold. Semele, mother of

origin of the Roman civilization, emphasizing the divine guidance and protection bestowed upon the city through its founding figures.

In Hinduism, the concept of divine paternity is embodied in stories from the ancient scriptures. For instance, Lord Rama, a revered deity, is believed to be the son of King Dasharatha and the result of divine intervention. Lord Krishna, another significant deity in Hinduism, is said to be born to mortal parents but with a divine purpose, emphasizing the divine's direct involvement in human affairs.

Ancient Egyptian mythology also features the concept of divine paternity, with stories of pharaohs being considered divine descendants of gods. The pharaohs were believed to be the offspring of deities like Ra or Osiris, highlighting their divine right to rule and connecting the earthly and divine realms.

The concept of divine paternity is not confined to classical mythologies; it also finds expression in indigenous beliefs and modern religions. In Native American cultures, for instance, there are stories of gods or spirits fathering heroes or important figures.

Christianity, with its foundational story of the Virgin Mary conceiving Jesus through the Holy Spirit, embodies a unique form of divine paternity. Jesus is considered the Son of God, and this divine parentage holds profound theological significance within Christian doctrine.

In each cultural context, the concept of divine paternity serves various purposes. It can explain the extraordinary qualities or destinies of certain individuals, reinforce the divine connection between gods and humans, or legitimize the rule of certain lineages. While the specifics vary, the overarching theme remains a fascinating exploration of the intersection between 'non-natural' and 'natural' beings, shaping cultural narratives, religious beliefs, and societal structures across diverse civilizations.

On the other hand, in the play "Eumenides" by Aeschylus, the god Apollo claims the father's share in heredity is 100%, reflecting an ancient belief in paternal dominance. Aeschines tells of the Amphictyons who cursed perpetrators of sacrilege by wishing upon them the birth of children that do not resemble their parents, but monsters.

To look at other aspects as well, in the southwestern state of Kerala in India, matrilineal communities coexist until nowadays with the prevalent patrilineal system. Lineage and inheritance are traced through the female line, challenging the notion of a standardized understanding of heredity within the same cultural and national context. The Mosuo people, a small ethnic group in China, practice as well a form of matrilineal society where lineage and family property are passed down through the female line. The absence of formal marriages adds another layer to their cultural variation, challenging the notion of a standardized understanding of heredity prevalent in ancient times.

In Laurence Sterne's "Tristram Shandy," there is a mockery of the homunculus theory, emphasizing the lack of empirical knowledge and the speculative nature of theories regarding the transmission of traits from parent to offspring. The evolution of scientific knowledge, including the discovery of chromosomes, meiosis, and fertilization, has significantly transformed our understanding of heredity.

Advancements in genomics have revealed that modern humans carry a (neutrosophic) percentage of their DNA inherited from Neanderthals, suggesting interbreeding between the two species. This genetic legacy is embedded in the DNA of contemporary humans, offering a tangible link to a shared ancestry that extends beyond the Homo sapiens lineage.

We wonder if this interbreeding has left traces in the collective memory. A navigation to the intricate interplay of neutrosophic elements within the realms of mythology —e.g. the hybridization between 'non-natural' beings and 'natural' beings, such as 'demigods' and 'cambions'— might share thematic elements reflective of the genetic interbreeding narrative. These hybrid beings often grapple with

Dionysus, insisted that Zeus was the father of her child. The god revealed his true form to her, but the divine radiance proved too much for Semele, leading to her demise. And so on.

complex identities, straddling different worlds, and possess extraordinary abilities that set them apart from ordinary humans.

What if the parallel narratives of hybrid beings, with obvious neutrosophic traits, born of 'non-natural'-'natural' unions, might have served as cultural echoes of our complex ancestry, as a cultural metaphor for the intricate mingling of distinct human species? What if the Neanderthal-Sapiens interbreeding, as well as other humanoid types' interbreeding, substantiated by genetic evidence, is prolonged in folklore, possibly finding a captivating reflection in the mythical realms of hybrid beings and identities? Is it possible for some parallels between scientific discoveries and mythological narratives to underline the enduring human fascination with the mysteries of our origins, and the imaginative ways in which we weave tales to make sense of our genetic heritage?

9. Conclusions

Different cultures perceive Hybrid and SuperHybrid mythological forms in varied ways. From Greek centaurs to Hindu Gandharvas, each mythological tradition contributes unique perspectives on the blending of human and non-human attributes. The symbolism associated with hybrid humans gains depth when analyzed through a neutrosophic lens. Whether viewed as symbols of chaos and monstrosity or as representations of harmony between different realms, the contradictory elements within these beings offer rich material for neutrosophic interpretation. The quest for identity is complex, as these beings navigate their existence on the blurred edges of humanity. These Hybrid and SuperHybrid entitities are powerful vehicles for philosophical contemplation and cultural exploration within the framework of neutrosophy.

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Neutrosofia transcende opozițiile binare în mitologie și folclor

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Abstract

Acest articol este o scurtă incursiune în domeniul complex al entităților compozite mitologice și folclorice. Analiza ne-a condus la ideea că aceste creaturi hibride și superhibride reprezintă dovezi convingătoare că psihicul uman transcende în mod constant opozițiile binare, așa cum susține neutrosofia. În diverse culturi și epoci, mintea umană manifestă o tendință către perspective nuanțate, neutrosofice, sfidând categorisirile simpliste. În plus, facem câteva observații marginale în temă și adresăm câteva întrebări mai puțin convenționale.

Cuvinte cheie

neutrosofie; transcendență; multialism; mitologie; identitate culturală; practici culturale; hibrizi; superhibrizi; creaturi mitologice; descendență divină; ereditate.

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https://fs.unm.edu/NSS/NeutrosophyTranscendsBinary4.pdf

1. Introducere

Entitățile hibride (și superhibride) care apar în mitologii și în folclorul din întreaga lume au fost explorate pe larg în studiile culturale. O ramură a studiilor mitologice — care ar putea fi denumită "teratologie mitologică" — ar putea fi propusă pentru examinarea acestor monștri și ființe hibride (vezi mai jos *Observații* și *Întrebări Deschise*). Acest concept al "ființei hibride" ca o reflectare a societății și a normelor sale este o temă recurentă în diverse discipline academice, inclusiv în sociologie sau psihologie. De exemplu, Joseph Campbell analizează rolul ființelor mitice în cultură și societate, argumentând că acestea reflectă temerile și dorințele

societății [Campbell]. În mod similar, conceptul de "arhetip" al lui Carl Jung discută cum societatea creează ființe mitice din ceea ce respinge sau găsește deranjant [Jung]. Cohen scrie despre modul în care acești monștri funcționează în cultură și societate, sugerând că ei oglindesc atât anxietățile, cât și aspirațiile societății, propunând un *modus legendi*, adică "o metodă de a citi culturile prin monștrii pe care îi generează" [Cohen]. Kristeva explorează conceptul de "abjecție", un termen pe care îl împrumută din psihanaliză pentru a descrie sentimentul de groază care apare atunci când întâlnim ceva ce perturbă granițele dintre sine și celălalt [Kristeva].

Ființele mitologice hibride, care combină caracteristici umane și animale sau care îmbină trăsături divine și mortale, sunt văzute ca anomalii sau devieri de la ordinea naturală. Aceste entități întruchipează o natură paradoxală care depășește cadrele logice convenționale și protocoalele societale prestabilite. Ele există dincolo de limitele tiparelor previzibile și regulilor impuse, servind ca agenți ai disrupției în cadrul sistemelor structurate, apărând totodată ca un produs secundar al ordinii, născându-se din sfidarea haosului și acționând ca o forță contrară care delimitează și totodată fortifică granițele structurii. Mai mult, aceste ființe hibride nu sunt creații arbitrare, ci constituenți esențiali ai civilizației, provenind din însăși principiile pe care societățile le prețuiesc. Ele personifică în general toate aspectele respinse și contrazise de normele și valorile societății. Prin identificarea și marginalizarea acestor elemente nedorite, societatea își reafirmă propria identitate și principiile. În consecință, ființa hibridă se transformă într-o reflexie negativă, o oglindă distorsionată a societății, subliniind temerile, tabu-urile și limitările acesteia. Prin confruntarea și interacțiunea cu aceste entități, societatea atinge o înțelegere mai profundă a propriilor contradicții, contribuind astfel la evoluția sa continuă (neutrosofică) și la auto-definirea sa ca sistem dinamic (neutrosofic) al vieții.

Pentru mai multe informații pe această temă, oferim un capitol bogat de sugestii de lectură la sfârșitul lucrării (vezi *Lecturi suplimentare*).

Acest articol nu își propune să fie o contribuție specială la acest domeniu dintr-o perspectivă culturală. Este doar o scurtă trecere în revistă a câtorva trăsături neutrosofice și multialiste ale unui număr relativ restrâns de hibrizi, dar revelatoare prin semnificația lor.

Reamintim că MultiAlism-ul este un sistem multipolar format nu doar din elemente multiple care pot fi aleatorii, contradictorii sau adjuvante, ci și prin acceptarea trăsăturilor din mai multe sisteme de bază (sisteme unipolare, bipolare, tripolare sau pluripolare).

Acest articol se vrea, de asemenea, o încurajare pentru cercetătorii specializați să investigheze "teratologia mitologică" folosind instrumente și perspective neutrosofice și este — de ce nu? — o încercare de a transcende câteva întrebări comune despre apariția și perpetuarea acestor creaturi omniprezente în mitologiile lumii.

2. Identități neutrosofice: Hibrizi și SuperHibrizi mitologici

Selectăm câteva exemple din vasta gamă de domenii imaginative și apoi analizăm pe scurt, dar sistematic, modul în care acești hibrizi sunt compuși. Ca structură generală a hibrizilor, se pot categorisi drept "hibrizi mitologici" creaturile care ar putea fi formate prin unirea a două entități (fie parte animal + parte uman, fie parte animal + parte divinitate, fie parte uman + parte divinitate, fie parte demon + parte uman, fie parte demon + parte animal, fie parte demon + parte divinitate) și drept "superhibrizi mitologici" sau "hibrizi multialiști" creaturile care ar putea fi formate prin unirea a cel puțin trei entități (de exemplu, parte divinitate + parte uman + parte animal), sau prin combinații din tipuri diferite ale aceleași entități (de exemplu, animal_1 + animal_2 +… + animal_n).

→În mitologia mesopotamiană, Gilgameș este un semizeu, întruchipând o fuziune între moștenirea divină și cea uman-muritoare. Descris ca fiind două treimi divin și o treime muritor, cazul lui Gilgameș prezintă un amestec complex de divinitate maternă și paternă. Dualitatea naturii sale, fiind atât divină, cât și muritoare, devine o temă centrală în narațiune, influențându-i acțiunile, luptele și căutarea sa finală a sensului vieții și a nemuririi. Ambiguitatea din jurul sorții sale în tradițiile ulterioare introduce un element de incompletitudine, permițând interpretări diverse [Gilgameș].

→Lamassu sunt divinități protectoare în mitologia mesopotamiană, cu trup de taur sau leu, aripi de vultur și cap de om. Aceste ființe colosale serveau adesea ca gardieni la intrările palatelor și templelor [Lamassu].

→Bhima este unul dintre personajele centrale din epopeea antică indiană "Mahabharata". Este al doilea dintre cei cinci frați Pandava, născut din Kunti, regina Hastinapurei, și zeul vântului, Vayu. Bhima este cunoscut pentru forța sa fizică excepțională, curaj și pricepere în luptă. Bhima joacă un rol crucial în diferite evenimente, inclusiv exilul fraților Pandava în pădure, jocul de noroc la curtea Hastinapurei și războiul de la Kurukshetra, bătălia epică dintre Pandava și Kaurava. Este adesea descris ca o figură supranaturală, capabilă de acte extraordinare de vitejie și eroism. În ciuda forței sale formidabile, Bhima este, de asemenea, portretizat ca un individ cu compasiune, bun la suflet, în special față de cei oprimați sau marginalizați. Călătoria sa este marcată de momente de triumf și adversitate, culminând în rolul său ca figură cheie în stabilirea 'dharma' pe tronul Hastinapurei [Bhima].

→Ahile, fiul unei nimfe marine și al unui rege muritor, întruchipează dihotomia semizeului în mintea grecilor. Scufundat în râul Styx pentru invincibilitate, călcâiul său vulnerabil devine un simbol al incompletitudinii. Natura sa neutrosofică se dezvăluie în moartea sa, orchestrată de o săgeată muritoare ghidată de un zeu. Această intersecție dintre vulnerabilitatea muritoare și răzbunarea divină subliniază complexitatea sorții lui Ahile [Ahile].

→Freyr este o figură proeminentă în mitologia nordică, adesea descris ca un zeu frumos și binevoitor, asociat cu recolte bogate, iubire și abundență. În ciuda faptului că este considerat chiar zeu în mitologia nordică, statutul său de semizeu este subliniat de dualitatea parentală, unul dintre părinți fiind Njord și celălalt o gigantă numită Skadi. Cea mai faimoasă posesiune a lui Freyr este sabia sa magică, cunoscută sub numele de "Sabia lui Freyr" sau "Sumarbrandr". Potrivit legendei, Freyr și-a dat sabia pentru a câștiga mâna gigantei Gerd, de care s-a îndrăgostit. Acest act de sacrificiu duce în cele din urmă la căderea lui Freyr, deoarece rămâne fără apărare în timpul bătăliei finale de la Ragnarok, evenimentul apocaliptic din mitologia nordică [Freyr].

→În tradiția islamică, Burāq este o creatură mitică descrisă ca un armăsar cu cap de femeie, aripi de vultur și coadă de păun. Se spune că l-a transportat pe Profetul Mohamed în timpul 'Călătoriei Nocturne' [Burāq].

→Banshee este o ființă supranaturală adânc înrădăcinată în folclorul irlandez și celtic. Descrisă ca un spirit sau zână de sex feminin, Banshee este adesea asociată cu familii sau clanuri specifice, servind ca o prevestire a morții sau un semn de nenorocire iminentă. Una dintre trăsăturile cele mai distinctive ale Banshee este tânguirea sa jalnică, un strigăt înfiorător care se spune că este auzit atunci când cineva din familia pe care o veghează este pe cale să moară. Banshee este de obicei descrisă ca o figură solitară, adesea apărând ca o bătrână cu părul lung și despletit, îmbrăcată într-o rochie gri sau albă, având trăsături care amintesc de ființe din alte tărâmuri, cum ar fi elfii, vrăjitoarele sau aparițiile spectrale [Banshee].

→Huli Jing, sau 'spiritele vulpii', sunt ființe metamorfozate din folclorul chinez. Ele se pot transforma în femei frumoase, dar forma lor adevărată este aceea de vulpe [Huli].

→Forma hibridă a lui Abraxas, cu corp de om, cap de cocoș sau leu și picioare de șarpe, este încărcată de semnificație simbolică.[1] Corpul uman reprezintă conștiința și intelectul, în timp ce trăsăturile animale simbolizează instinctele primare și forțele cosmice. Cocoșul, asociat cu zorii și trezirea, semnifică iluminarea spirituală, în timp ce șarpele este un simbol puternic al înțelepciunii, reînnoirii și transformării. Abraxas întruchipează conceptul de unitate în dualitate, simbolizând reconcilierea opozițiilor, precum binele și răul, creația și distrugerea, lumina și întunericul. Această fuziune neutrosofică a elementelor contradictorii reflectă viziunea gnostică asupra lumii, care concepea lumea materială ca un tărâm al dualității și iluziei, cu scopul final de eliberare spirituală prin gnoză sau cunoaștere divină [Abraxas].

→Conceptul de homuncul are rădăcini istorice în tradițiile alchimice și filozofice. În alchimie, se credea că un homuncul putea fi creat prin diverse procedee secrete, simbolizând dorința de a crea viața în mod artificial.[2] Ingredientul cel mai frecvent asociat cu crearea unui homuncul era fluidul seminal, considerat a conține esența vieții. În plus, alte fluide corporale, cum ar fi sângele menstrual, erau uneori implicate ca ingrediente. Alchimiștii indicau folosirea diverselor substanțe alchimice, cum ar fi săruri, minerale și extracte de plante, despre care se credea că au proprietăți transformatoare. Se sugera utilizarea de semințe animale sau alte substanțe derivate din animale. Iată niște instrucțiuni pentru crearea unui homuncul: "Amestecați sperma și piatra soarelui și inseminați vaca sau oaia. Blocați cu grijă vaginul animalului cu piatra soarelui. Ungeți organele genitale ale animalului cu sângele altui animal. Plasați animalul inseminat artificial într-o casă întunecată unde soarele nu strălucește niciodată." Și așa mai departe. [Homuncul]. Dintr-o perspectivă neutrosofică, un homuncul poate fi explorat ca o metaforă pentru incertitudinile, contradicțiile și complexitățile inerente în dimensiunile biologice, filozofice și simbolice. Homuncul, ca simbol al creației artificiale a vieții, întruchipează implicații etice și existențiale. Fiindcă nu este în totalitate uman, această entitate reprezintă un animal rațional, adăugând un alt capitol fictiv în aspirația umanității de a controla granițele vieții și ale morții.

→Balaurul ocupă un rol proeminent în folclorul românesc, apărând ca un dragon sau șarpe cu mai multe capete,[3] adesea asociat cu fenomene meteorologice tumultoase și haos disruptiv. Interesant este că legendele românești conferă salivei Balaurului o trăsătură particulară, despre care se crede că posedă abilitatea transformatoare de a se cristaliza în "diamante". Simbolic, Balaurul întruchipează energii primordiale, encapsulând esența haosului și a sălbăticiei neîmblânzite, evocând un joc complex de respect și teamă în cadrul narațiunilor culturale.

→Zmeul, o altă figură captivantă din folclorul românesc, îmbracă haina unui monstru care își schimbă forma. Versatilitatea sa în a-și asuma forme diverse epitomează

[1] Referințe la Abraxas pot fi găsite în diverse texte gnostice, inclusiv în Biblioteca Nag Hammadi, o colecție de scripturi gnostice antice descoperită în Egipt în 1945. Aceste texte îl descriu adesea pe Abraxas ca pe o ființă divină sau *arhon* (uneori asociat cu Dumnezeu) sau ca o emanație din tărâmul divin.

[2] Una dintre cele mai faimoase relatări despre crearea unui homuncul provine din scrierile alchimistului elvețian Paracelsus, care susținea că un homuncul poate fi creat prin manipularea lichidului seminal. Rețeta sa presupunea plasarea materialului seminal într-un vas de sticlă sigilat, împreună cu anumite substanțe alchimice, și apoi incubarea amestecului într-un mediu cald și întunecat timp de câteva săptămâni sau luni. Paracelsus era convins că, după perioada de incubare, o creatură umanoidă în miniatură s-ar fi format în interiorul vasului.

[3] Legendele descriu Balaurul ca fiind o ființă monstruoasă, cu gheare ascuțite, solzi care îi acoperă corpul și suflare de foc.

imprevizibilitatea, perpetuând o aură de eluzivitate enigmatică. De obicei, Zmeul este descris ca o forță malefică, întruchipând teme ca frica, vicleșugul și pericolul iminent, servind ca un motiv de avertizare profundă în cadrul basmelor. În plus, portretizarea sa reflectă temerile societale, simbolizând amenințări latente, conflicte interioare și lupta dintre forțele morale opuse.

→Dintre spiritele folclorului românesc, Zburătorul apare ca o entitate seducătoare, asemănătoare arhetipului *incubus*.[4] Apărând sub forma unui bărbat carismatic, Zburătorul seduce fetele nevinovate, simbolizând dorințe interzise și atracția necunoscutului. Traversând simbolic pragul delicat dintre vise și realitate, Zburătorul evocă un amestec de frică, fascinație și introspecție profundă în cadrul psihicului uman. [Chelariu]

Să încercăm acum să integrăm aceste entități într-un sistem coerent.

3. Descendenți ai zeilor și muritorilor: Hibrizi parțial zei + parțial oameni

În mitologiile lumii, entitățile formate atât din descendență divină, cât și din linie uman-muritoare, adesea denumite ‘semizei’ sau ‘semizeițe’, întruchipează un amestec complex de trăsături moștenite de la părinții lor divini și de la părinții lor muritori. Mitologii sunt, în general, de acord că aceste ființe hibride servesc la crearea unei punți între tărâmurile celeste și cele terestre, reflectând dorința umană de a înțelege natura existenței și locul nostru în cosmos.

Semizeii posedă o gamă variată de caracteristici și abilități: pot moșteni forță supraomenească, agilitate sau intelect de la linia lor divină, dar în același timp se confruntă cu încercările și tribulațiile mortalității. În ciuda abilităților lor extraordinare, semizeii sunt adesea reprezentați ca ființe imperfecte și vulnerabile, care se luptă cu complexitatea naturii lor duale și cu așteptările plasate asupra lor atât de zei, cât și de muritori.

Pentru a explora mai multe mitologii, să ilustrăm personajele a trei figuri reprezentative ale acestor ființe:

→Krishna. În mitologia hindusă, Krishna este venerat ca un avatar al zeului Vishnu, născut ca fiu al lui Devaki, sora regelui Kamsa, și al lui Vasudeva.[5] [Vishnu] Krishna a refuzat să pună mâna pe arme în monumentalul conflict dintre Kaurava și Pandava. În schimb, el a oferit o opțiune: să ofere îndrumare personală unei tabere și să împrumute armata sa celeilalte. O altercație tragică a izbucnit între șefii Yadava, rezultând în pierderea fratelui și a fiului lui Krishna. Profund întristat, Krishna s-a retras în pădure. Acolo și-a găsit sfârșitul când un vânător, confundându-l cu o pradă, l-a rănit mortal în picior, singurul său punct vulnerabil. [Krishna]

→Hercule, fiul lui Zeus și al Alcmenei, o prințesă muritoare, este o figură centrală în mitologia greacă, întruchipând complexitatea existenței unui semizeu. Descendența sa divină, combinată cu limitările umane, formează esența narațiunii sale. Celebrele sale (doisprezece) Munci, întreprinse ca penitență pentru uciderea soției și copiilor săi într-un acces de nebunie indus de zeița Hera, evidențiază forța sa extraordinară, curajul și determinarea. Zeus, recunoscând faptele lui Hercule, îi acordă nemurirea, subliniind jocul dinamic dintre mortalitate și divin. [Hercules]

→Dagda, “Zeul Bun,” este o figură proeminentă în mitologia irlandeză, venerat ca o (semi)zeitate puternică și binevoitoare. El este adesea descris ca o figură paternă, un lider înțelept și un războinic abil, întruchipând idealurile de putere, înțelepciune și generozitate. Dagda este considerat tatăl sau conducătorul Tuatha Dé Danann, o rasă mitică de ființe

[4] Încadrat în folclorul mai larg al incubilor și succubilor, Zburătorul întruchipează spiritul masculin enigmatic care caută legături nocturne cu femeile adormite.

[5] Vasudeva este patronimul zeului Krishna, fiul lui Vasudeva. Închinătorii lui Vasudeva-Krishna au format una dintre primele mișcări devoționale teistice din cadrul hinduismului.

supranaturale. El deține artefacte magice puternice, inclusiv un buzdugan magic cunoscut sub numele de "lorg mór", care poate atât ucide, cât și învia cu un capăt și controla vremea cu celălalt. De asemenea, deține un cazan magic numit "coire ansic" sau "cazanul abundenței," care oferă o cantitate nesfârșită de mâncare și băutură. Dagda este cunoscut pentru aplecarea sa pentru desfătări și plăceri, fiind adesea portretizat ca un iubitor de mâncare, băutură și muzică.

3.1. Devașii indici: intermediari cerești ai echilibrului cosmic

În cosmologia complexă a mitologiei hinduse, semizeii, sau "Devași," ocupă un rol central ca ființe cerești care se situează la granița dintre divinitate și umanitate. Acești semizei întruchipează o interacțiune complexă de atribute în cadrul neutrosofiei, unde conceptele de certitudine, incertitudine și indeterminare converg într-un echilibru dinamic. [Deva]

Originea semizeilor în mitologia indică este învăluită în ambiguitate, reflectând indeterminarea inerentă a ordinii cosmice. Născuți din amestecul de linie divină și muritoare, semizeii întruchipează principiul neutrosofic al incertitudinii, unde existența lor sfidează categorisirea convențională. Ei locuiesc în spațiul liminal dintre certitudinea esenței divine și incertitudinea trupului muritor.

Semizeii din mitologia indică dețin o semnificație culturală și simbolică profundă, servind ca reprezentări alegorice ale echilibrului cosmic dintre ordine și haos. Inspiră introspecție, reflecție și o înțelegere mai profundă a principiilor neutrosofice care guvernează universul. Ei oferă perspective atemporale asupra complexităților existenței și asupra căutării eterne a echilibrului și armoniei.

Mai mulți semizei din mitologia indică exemplifică principiile neutrosofiei prin natura lor complexă:

→Indra, regele Devașilor, întruchipează principiul neutrosofic al opoziției, deoarece poartă război împotriva forțelor întunericului, în timp ce se confruntă cu conflicte interne și dileme morale.

→Agni, zeul focului, simbolizează conceptul neutrosofic al indeterminării, servind atât ca purificator, cât și ca distrugător, întruchipând natura dual a focului creator și distrugător totodată.

→Varuna, zeul apelor cosmice, reprezintă noțiunea neutrosofică a adevărului parțial, susținând ordinea cosmică, în timp ce se luptă cu propriile limitări și imperfecțiuni.

→Surya, zeul soarelui, întruchipează principiul neutrosofic al ambiguității, iluminând universul cu lumina sa radiantă, în timp ce aruncă umbre de îndoială și incertitudine.

→Vayu, zeul vântului, simbolizează conceptul neutrosofic al complementarității, insuflând viață în toate ființele vii, în timp ce poartă semințele distrugerii și schimbării.

3.2. Semizeii grecilor: explorarea arhetipurilor eroice

În vasta și complexa tapiserie a mitologiei grecești, semizeii se remarcă drept figuri extraordinare, ocupând un spațiu unic între zei și muritori. Născuți din uniuni între ființe divine și oameni, acești eroi hibrizi posedă abilități extraordinare, inimi curajoase și destine complexe care modelează cursul miturilor și legendelor. Această dualitate inerentă reflectă principiul neutrosofic al indeterminării, în care semizeii există într-o stare de flux perpetuu, nefiind nici pe deplin divini, nici complet muritori.

În panteonul semizeilor greci, mai multe figuri se remarcă drept exemple de arhetipuri eroice, fiecare întruchipând principiile neutrosofiei în mod unic:

→Hercule, fiul lui Zeus și al Alcmenei, întruchipează lupta împotriva adversității și căutarea răscumpărării. Cele Doisprezece Munci ale sale simbolizează conceptul neutrosofic al opoziției, unde forțe aparent contradictorii coexistă și interacționează într-un echilibru dinamic.

→Iason, fiul a doi muritori, deși conceput de Zeus, demonstrează carismă și diplomație ca lider al argonauților în căutarea Lânii de Aur. El navighează prin intrigi politice și provocări personale cu tact și reziliență, întruchipând principiul echilibrului și armoniei în relații și eforturi.

→Un alt semizeu proeminent, Perseu, născut din Zeus și o prințesă muritoare, reprezintă principiul neutrosofic al indeterminării prin încercarea sa de a ucide Meduza și de o a salva pe Andromeda. Drumul său prin lume plin de incertitudine și ambiguitate, dar el persevereză prin viclenie, ingeniozitate și hotărâre pură. Perseu întruchipează noțiunea neutrosofică a adevărului parțial, unde realitatea este inerent subiectivă și deschisă interpretării. Destinul său legendar, inclusiv uciderea Gorgonei, nu are o narațiune singulară, ilustrând incompletitudinea inerentă în portretizarea destinelor semizeilor. Fie că Perseu își întâlnește sfârșitul în luptă sau printr-o lovitură de disc, rămâne un subiect de interpretare, adăugând un strat de indeterminare la statutul său de semizeu. [Kerenyi]

3.3. Tuatha Dé Danann și Fomoire: descendența divino-umană în mitologia celtică

În mitologia celtică, Tuatha Dé Danann[6] apar ca figuri enigmatice, întruchipând o fuziune complexă de descendență divină și muritoare. Primele relatări îi descriu ca fiind alungați din cer din cauza cunoștințelor lor profunde, după care au coborât în Irlanda, învăluiți într-o ceață misterioasă. [Tuatha]

Inamicii antici ai Tuatha Dé Danann erau Fomoire, un alt grup de ființe supranaturale din mitologia irlandeză. Fomoire sunt o rasă de ființe monstruoase, semidivină, adesea descrisă ca malevolentă, asociată cu întunericul, haosul și forțele distructive ale naturii. Ele sunt portretizate drept creaturi marine monstruoase sau uriași. [Fomoire]

Conform mitologiei irlandeze, Fomoire au fost printre primii locuitori ai Irlandei, precedând sosirea Tuatha Dé Danann. Se spune că au sosit în Irlanda din ținuturi îndepărtate și au purtat războaie împotriva Tuatha Dé Danann pentru controlul insulei. Bătăliile dintre Fomoire și Tuatha Dé Danann sunt descrise ca lupte cosmice între forțele haosului și ordinii, Fomoire reprezentând haosul și întunericul,[7] iar Tuatha Dé Danann reprezentând lumina și civilizația.

Tuatha Dé Danann sunt pricepuți în magie, metamorfoză și alte forme de cunoaștere ascunsă, dar, de asemenea, experimentează emoții, dorințe și vulnerabilități umane. Câteva figuri cheie din Tuatha Dé Danann exemplifică triada neutrosofică a certitudinii, incertitudinii și indeterminării prin descendența lor complexă:

→Dagda, de exemplu, este uneori portretizat ca fiul zeiței Danu și al prințului muritor Elatha, reflectând împletirea elementelor divine și umane în descendența sa.

→În mod similar, Lugh este descris ca fiind fiul unui muritor, Cian, și al unei ființe supranaturale, Ethniu, evidențiind natura sa hibridă și ambiguitatea identității sale.

[6] În traducere, "poporul zeiței Danu," care reprezintă apele primordiale ale creației și fertilității, aceste ființe semidivine sunt venerate pentru înțelepciunea, magia și conexiunea profundă cu pământul înainte de sosirea Milesienilor, considerați strămoșii irlandezilor moderni.

[7] În ciuda reputației lor malefice, unele povești îi descriu pe anumite Fomoire într-o lumină pozitivă, portretizându-i ca personaje complexe cu propriile dorințe și motivații. În anumite relatări, Fomoire sunt descriși ca meșteșugari și războinici iscusiți, capabili de mari fapte de forță și magie.

4. Hibridizări demonice: Hibrizi parțial demon + parțial uman

În mitologiile lumii întâlnim adesea 'ființe' care sunt hibride, parțial demonice și parțial umane. Deși trăsăturile și aparențele acestor ființe pot varia foarte mult între diferite culturi și mitologii. Iată câteva exemple:

→Cambioni: În folclorul european, cambionii sunt urmași ai unui demon și ai unui om. Ei sunt adesea descriși ca având unele dintre puterile supranaturale ale părintelui lor demonic, cum ar fi schimbarea formei, telepatia sau magia neagră, împreună cu aspectul fizic uman. Cambionii sunt uneori portretizați ca figuri seducătoare și manipulatoare, folosindu-și puterile pentru a influența sau a păcăli muritorii.

→Nephilim: În mitologia iudeo-creștină, *nephilim* sunt urmașii îngerilor și ai oamenilor.[8] Ei sunt descriși în texte antice, precum "Cartea Genezei", ca uriași sau războinici puternici. *Nephilim* sunt adesea asociați cu tema pedeapsei divine, deoarece existența lor este văzută ca o răsturnare a ordinii naturale.

→Hibrizi Oni-Om: În folclorul japonez, *oni* sunt spirite malefice sau demoni, descriși cel mai ades ca uriași cu coarne și păr zburlit. Basmele prezintă hibrizi *oni*-umani, deci care rezultă din uniuni între un *oni* și un om. Acești hibrizi pot moșteni unele trăsături fizice ale *oni*, cum ar fi coarnele, precum și natura lor jucăușă sau malefică.

Acestea sunt doar câteva exemple, iar în mitologiile din întreaga lume există multe alte variante de ființe parțial demonice și parțial umane. Ele servesc adesea drept figuri captivante în povestiri, întruchipând teme de luptă între bine și rău, între supranatural și cotidian, și complexitățile ideilor de 'moștenire' și 'identitate'.

4.1. Cambionii: Hibrizi demon + om

Imaginația umană a creat o mulțime de ființe mitologice, printre care cambionii ocupă un loc intrigant. Conceptul de cambion[9] își are rădăcinile în folclorul european occidental, în special în cadrul demonologiei creștine. Cambionii sunt ființe născute din uniunea între un demon și un om, întruchipând o amestecare unică între supranatural și muritor. Aceste uniuni sunt de obicei prezentate ca acte de seducție, coerciție sau ispitire, reflectând teme mai ample de ambiguitate morală și lupta dintre bine și rău. [Cambion]

Caracteristicile atribuite cambionilor variază între diferitele tradiții mitologice, dar aceștia posedă de obicei un amestec de puteri supranaturale și vulnerabilități umane, având trăsături fizice care amintesc de descendența lor demonică, cum ar fi coarne, colți sau ochi de culoare nefirească, dar mai ales posedă abilități supranaturale, inclusiv schimbarea formei, telepatia sau cunoașterea magiei negre. Pe lângă puterile lor supranaturale, cambionii sunt adesea portretizați având o manieră carismatică și seducătoare, folosindu-și atracția pentru a manipula și a păcăli muritorii. Acest aspect al caracterului lor reflectă teme de ispitire și corupție morală, întrucât cambionii — nici pe deplin umani, nici pur demonici — navighează prin complexitățile moștenirii lor duale.

[8] Uneori interpretați ca îngeri căzuți, adică demoni.

[9] Cuvântul "cambion" se crede că provine din termenul latin târziu "cambuca," un vas folosit de soldații romani. În timp, a evoluat pentru a desemna un vas sau un recipient pentru spirite, ajungând în cele din urmă să semnifice urmașii demonilor.

→Merlin, vrăjitorul legendar din mitologia arturiană, este uneori descris ca un cambion.[10] Conform unor versiuni ale poveștii, tatăl lui Merlin era un *incubus* care și-a sedus mama, o femeie muritoare. Din această uniune s-a născut Merlin, oferindu-i abilități magice.

→În piesa lui William Shakespeare "Furtuna," Caliban este o ființă jumătate umană, jumătate demon. Deși natura exactă a părinților lui Caliban nu este explicit menționată în piesă, originile sale sunt descrise ca fiind monstruoase și nenaturale. Caracterul lui Caliban atinge teme precum colonizarea, dinamica puterii și conflictul dintre 'civilizat' și 'sălbătic'.

Ca ființe hibride, cambionii ocupă un spațiu liminal între supranatural și muritor. În demonologia creștină, cambionii sunt adesea considerați întruchipări ale păcatului și decăderii morale, reflectând consecințele cedării în fața dorințelor lumești. Ei servesc drept avertisment împotriva pericolelor corupției spirituale.

4.2. Judecata divină: povestea Nephilimilor

Nephilimii,[11] menționați în texte antice precum Biblia și diverse lucrări apocrife, sunt învăluiți în mister și controversă, ocupând un loc unic în tapiseria mitologică, întruchipând teme de hibriditate, rebeliune și judecată divină.

Nephilimii sunt menționați specific în "Geneză",[12] unde sunt descriși ca fiind urmașii uniunilor între "fii ai lui Dumnezeu" și "fiice ale oamenilor." În unele interpretări, ei sunt descriși ca uriași, posedând o forță colosală. Alte surse îi descriu ca ființe de mare răutate sau corupție spirituală, ale căror prezență pe Pământ amenința ordinea stabilită de Dumnezeu.[13]

Conform narațiunii biblice, prezența nephilimilor pe Pământ a provocat intervenția divină, conducând la Marele Potop ca un mijloc de curățare a lumii de corupția lor. Numai Noe și familia sa au fost salvați, fiind considerați drepți în ochii lui Dumnezeu. Povestea Nephilimilor servește drept o lecție despre consecințele corupției morale și ale judecății divine.

Două personaje reprezentative sunt:

→Goliat, uriașul legendar din povestea biblică a lui David și Goliat,[14] este adesea interpretat ca fiind un Nephilim sau descendent al Nephilimilor. Goliat este descris în textul biblic ca un uriaș, dominându-și adversarii cu statura sa impunătoare. Armura sa ar fi cântărit sute de sicli de bronz[15], subliniind prezența sa formidabilă pe câmpul de luptă.

→Og, regele Basanului, este o figură menționată în Biblia ebraică, în mod specific în Vechiul Testament. Conform narațiunii biblice, Og a fost unul dintre ultimii rămași dintre Refaeii, un grup de uriași cunoscuți pentru statura și forța lor imensă. Refaeii erau considerați o rasă antică de oameni care au locuit în țara Canaan înainte ca israeliții să sosească acolo. Înfrângerea lui Og este relatată ca una dintre victoriile obținute de israeliți sub conducerea lui Moise.[16] [Nephilim]

Povestea nephilimilor a lăsat un impact durabil asupra narațiunilor religioase și culturale de-a lungul istoriei. În tradițiile iudaice și creștine, ei sunt adesea interpretați ca simboluri ale

[10] Merlin drept cambion este o versiune explorată în diverse texte medievale, inclusiv în "Historia Regum Britanniae" a lui Geoffrey de Monmouth.

[11] Termenul "Nephilim" provine din cuvântul ebraic "nephiyl," care se traduce prin "uriași" sau "căzuți."

[12] Cf. *Geneza* 6:1-4.

[13] Identitatea lor precisă este un subiect de dezbatere printre cercetători, interpretările variind de la îngeri căzuți la ființe divine sau conducători.

[14] *Samuel* 1:17.

[15] 1 siclu = 1 beca = 12 g, cf. *Geneza* 23:15.

[16] *Cartea Deuteronomului* oferă mai multe detalii despre Og și regatul său. În *Deuteronom* 3:11, se menționează că patul lui Og era făcut din fier și avea mai mult de patru metri lungime și aproape doi metri lățime, indicând dimensiunea sa uriașă.

rebeliunii, păcatului și judecății divine, servind drept figuri de avertisment, îndemnând împotriva pericolelor mândriei, corupției și decăderii morale.

4.3. Intersecții între umanitate și demonologie în folclorul japonez

În folclorul japonez, oni sunt spirite sau demoni formidabili și malefici, cunoscuți pentru aspectul lor monstruos și comportamentul rău-intenționat. Adesea descriși ca fiind creaturi cu coarne, având păr sălbatic și expresii înfricoșătoare, *oni* întruchipează aspectele întunecate ale regnului supranatural.

Hibrizii oni-umani reprezintă un arhetip multifațetat în folclorul japonez, reliefând teme ca puterea, ispitirea și încălcarea regulilor morale. Rolul lor ca spirite malefice capabile și doritoare să aducă haos în viața oamenilor reflectă anxietăți culturale mai largi legate de forțele întunericului și haosului.

Hibrizii oni-umani, uneori denumiți "jumătate-oni" sau "*oni-kijo*," moștenesc trăsături din ambele ascendențe, demonică și umană. Ei pot avea caracteristici fizice de *oni*, precum coarne, colți și trăsături exagerate, dar păstrează de asemenea elemente ale strămoșilor lor umani.

Acești hibrizi sunt adesea descriși ca fiind ființe puternice și înfricoșătoare, capabile atât de mare forță, cât și de inteligență vicleană. În ciuda aspectului lor monstruos, hibrizii *oni*-umani pot prezenta emoții și motivații complexe.

Două personaje reprezentative sunt:

→Shuten-doji este un rege legendar care a terorizat capitala antică Kyoto. Conform folclorului, Shuten-doji era considerat un hibrid de *oni*, născut din uniunea dintre o femeie și regele *oni*-ilor. [Shuten]

→Ibaraki-doji este o femeie *oni* adesea descrisă ca un spirit rapace care caută răzbunare pentru nedreptăți anterioare. [Ibaraki]

Acești hibrizi *oni*-umani populează mitologia japoneză, întruchipând teme ca puterea, răzbunarea și lupta dintre umanitate și supranatural. Din poveștile tradiționale și teatrul *kabuki* până la *manga* și *anime* moderne, hibrizii *oni*-umani rămân simboluri durabile ale supranaturalului și fantasticului. Poveștile lor servesc drept lecții de avertizare, amintind publicului despre pericolele cedării în fața ispitei și importanța integrității morale în fața adversității.

5. Transcenderea speciilor: Hibrizi parțial umani + parțial animali

Așadar, hibrizii mitologici parțial om și parțial animal au fascinat culturile de-a lungul istoriei, apărând în mituri, legende și folclor din întreaga lume. Acești hibrizi întruchipează o fuziune de trăsături umane și animale, estompând granițele dintre domeniile uman și non-uman. Iată câteva exemple comune ale acestor hibrizi:

→Zeul egiptean Anubis, reprezentat cu corpul unui om și capul unui șacal, servește ca ghid și protector al morților, simbolizând tranziția dintre viață și moarte.

→În mitologia greacă, Centaurul este o ființă cu partea superioară a corpului uman și partea inferioară a corpului unui cal. Centaurii sunt adesea descriși ca având forță și agilitate supranaturale, precum și o natură sălbatică și neîmblânzită. Ei sunt asociați cu Dionysos, zeul vinului și al petrecerilor, și sunt adesea descriși ca participanți la ritualurile și celebrațiile sale extatice. [Centaur]

→Minotaurul, o altă ființă din mitologia greacă, are corpul unui om și capul unui taur. Încercuit în labirintul din Creta, Minotaurul simbolizează forța brută, agresiunea primitivă și aspectele întunecate ale naturii umane. [Minotaur]

→În mitologia greacă și romană, Harpia este o ființă cu corpul unui păsăre și capul unei femei. Adesea descrise ca fiind feroce și prădătoare, harpiile simbolizează haosul, violența și forțele distrugătoare ale naturii. [Harpia]

→Apărând în diverse culturi din întreaga lume, vârcolacii sunt ființe care se pot transforma din formă umană în formă de lup (licantropie). [Werewolf]

→Sirenele și tritonii, ființe cu partea superioară a corpului uman și partea inferioară a corpului de pește, apar în folclorul și mitologia din diferite culturi din întreaga lume. Ei sunt adesea asociați cu marea, simbolizând misterul, atracția și adâncurile necunoscute ale oceanului. [Mermaid]

Acești hibrizi parțial om și parțial animal servesc ca simboluri puternice în mitologie, reprezentând o gamă largă de teme, inclusiv relația dintre umanitate și natură, complexitatea identității umane și lupta dintre civilizație și instinctele primordiale.

5.1. Omul cu aripi: Hibrizi parțial umani + parțial păsări

Motivul omului cu aripi, o figură cu trăsături atât umane, cât și aviare, a apărut în diverse forme de-a lungul mitologiei, folclorului și artei din diferite culturi. Această ființă hibridă simbolizează adesea o fuziune între elementele terestre și cele cerești, vorbind despre libertatea, transcendența și dualitatea naturii umane.

Să explorăm motivul omului cu aripi în diferite contexte culturale, observând că unele dintre ele pot fi, de asemenea, clasificate ca SuperHibrizi datorită naturii lor divine asociate:

→Garuda este o ființă divină în mitologia hindusă, adesea reprezentată cu partea superioară a corpului uman și aripi, în timp ce partea inferioară seamănă cu un vultur sau o pasăre. Ca montură a zeului Vishnu, Garuda simbolizează puterea, forța și abilitatea de a zbura la mari înălțimi.

→Horus (mitologia egipteană), zeul cerului și al regilor, este uneori reprezentat cu capul unui șoim și corpul unui om.

→Icarus (mitologia greacă) este poate cea mai emblematică reprezentare a omului cu aripi. Împreună cu tatăl său, Daedalus, Icarus evadează din închisoare folosind aripi făcute din pene și ceară. Totuși, neascultarea sa duce la căderea sa tragică atunci când zboară prea aproape de soare, topind ceara și provocându-i prăbușirea.

→Eros/Cupidon (mitologia greco-romană): zeul grec al dragostei, și echivalentul său roman, Cupidon, sunt ocazional reprezentați cu aripi. Această reprezentare se asociază cu natura imprevizibilă și efemeră a dragostei.

→Phoenix (diverse culturi): deși nu este un om cu aripi în sens tradițional, pasărea mitică care se regenerează sau renaște ciclic este uneori reprezentată cu trăsături umane, mai ales în artă și literatură.

→Fenghuang (mitologia chineză), cunoscut și sub numele de phoenixul chinezesc, este o pasăre mitică cu un amestec de trăsături aviare și umane. Adesea considerată un simbol al armoniei și echilibrului, Fenghuang întruchipează uniunea contrariilor.

→Îngeri (diverse culturi): sunt adesea reprezentați ca ființe cu aripi și un aspect asemănător omului. În creștinism, îngerii sunt mesageri ai lui Dumnezeu, iar reprezentările artistice îi descriu frecvent cu aripi, simbolizând natura lor cerească.

→Shangó (mitologia Yoruba), zeul tunetului și al fulgerului, este uneori descris ca posedând aripi. Aripile și abilitatea sa de a se mișca rapid subliniază legătura sa cu cerul.

Motivul omului cu aripi rezonează în diverse culturi, ilustrând fascinația umanității față de ideea de a transcende limitările pământești și a aspira către tărâmuri mai înalte. Fie că reprezintă mesageri divini, eroi mitici sau creaturi simbolice, motivul omului cu aripi a captivat imaginația umană de-a lungul vremurilor.

6. Zei și bestii: Hibrizi parțial zeu + parțial animal

Conceptul ființelor care sunt parțial zei și parțial animale, fără trăsături sau reprezentări umane, este mai puțin comun în mitologie și folclor comparativ cu cele cu trăsături asemănătoare oamenilor. Cu toate acestea, există câteva exemple din diverse tradiții culturale din întreaga lume unde asemenea ființe sunt întâlnite. Aceste creaturi adesea întruchipează o combinație unică de trăsături divine și animalice, servind ca simboluri ale puterii și transformării. Iată câteva exemple:

→Dragonul azuriu: În mitologia chineză, Dragonul zzuriu este unul dintre cei patru gardieni cerești, reprezentând estul și sezonul primăverii. Dragonul azuriu este uneori reprezentat ca un SuperHibrid, un dragon cu trup de șarpe și gheare de tigru, simbolizând puterea, vitalitatea și forțele cosmice ale naturii.

→Pegas: În mitologia greacă, Pegas[17] este un cal divin cu aripi, născut din sângele Gorgonei Meduza după ce aceasta a fost ucisă de eroul Perseu. Este asociat cu zeul Poseidon și servește ca montură pentru eroi precum Bellerophon.

→Fenrir: În mitologia nordică, Fenrir este un lup monstruos, descendent al zeului Loki și al uriașei Angrboða. Fenrir este descris ca o creatură înfricoșătoare și puternică, destinată să aducă sfârșitul lumii în timpul Ragnarök-ului.

→Pasărea tunetului: În mitologia nativă americană, Pasărea tunetului este o pasăre mitică și puternică, adesea reprezentată ca o pasăre de pradă mare, cu o anvergură a aripilor asemănătoare cu cea a unui vultur și pene de șoim sau bufniță. Pasărea tunetului este asociată cu furtunile, fulgerele și forțele cerului, servind ca simbol al puterii, transformării și conexiunii spirituale dintre oameni și natură.

Aceste exemple ilustrează gama diversificată de ființe care întruchipează conceptul de entități parțial divine și parțial animale, fără a avea atribute sau reprezentări umane, în mitologie și folclor, servind ca simboluri ale puterii divine, protecției sau forțelor naturale.

6.1. Dragonul azuriu: un gardian celest

Dragonul azuriu, cunoscut sub numele de Qinglong în chineză, este unul dintre cei patru gardieni celestiali în mitologia chineză, alături de Pasărea Vermilioasă, Tigru Alb și Țestoasa Neagră. Este adesea reprezentat ca un dragon cu corp de șarpe și gheare de tigru, simbolizând convergența diferitelor atribute animale, fiind astfel clasificat ca o entitate SuperHybrid. Dragonul azuriu este asociat cu elementul lemn, cu estul și cu anotimpul primăvară, reprezentând vitalitatea, creșterea și renașterea. [Azure]

[17] Pegas este adesea reprezentat ca un cal alb maiestuos cu aripi, simbolizând viteza, libertatea și regatul divin. Deși Pegas nu este el însuși o divinitate, este strâns asociat cu zeii, în special cu Zeus, regele zeilor, și cu Atena, zeița înțelepciunii și a războiului.

Dragonul azuriu are o semnificație profundă în mitologia și societatea chineză, servind ca protector al cerurilor și simbol al puterii și autorității imperiale.[18] Este strâns asociat cu Împăratul Chinei și conceptul de 'Mandat al Cerului', reprezentând sancțiunea divină a domniei și ordinea cosmică a universului.

Prin forma sa hibridă și atributele sale multiple, dragonul invită la contemplarea principiilor neutrosofice de ambiguitate, incertitudine și indeterminare. Ca simbol al echilibrului neutrosofic, Dragonul azuriu navighează complexitatea existenței, întruchipând ritmurile ciclice ale naturii și interconectivitatea tuturor ființelor vii.

6.2. Pasărea tunetului: o entitate mitică nativ-americană

În mitologia nativă americană, inclusiv în tradițiile popoarelor Ojibwe, Lakota și Haida, Pasărea tunetului apare ca un simbol puternic și enigmatic al lumii naturale și al regiunilor spirituale. Reprezentând furtuni, fulgere și forțele cerului, această ființă mitică întruchipează interacțiunea dinamică între forțele pământești și cele cerești.

Adesea, Pasărea tunetului este descrisă ca o mare pasăre de pradă, asemănătoare cu un vultur sau un șoim, cu aripi ce se întind pe cer și pene care scânteiază cu fulgere. Pasărea tunetului este asociată cu furtunile, fulgerele și ploile vitale care hrănesc pământul. Prezența sa puternică simbolizează forțele imprevizibile ale naturii. [Thunderbird]

Este o ființă binevoitoare și impresionantă, aducând binecuvântări de ploaie și prosperitate. În tradițiile Ojibwe și ale altor triburi, se crede că Pasărea tunetului locuiește în munți înalți și stânci, de unde veghează asupra pământului și trimite fulgere și tunete pentru a curăța și purifica pământurile. Dansurile și cântecele ceremoniale sunt performate pentru a onora Pasărea tunetului și a invoca protecția și îndrumarea sa.[19]

Astfel, Pasărea tunetului întruchipează misterele lumii naturale și ale spiritului în mitologia nativă americană. Ca simbol al puterii divine și al forțelor naturale, Pasărea tunetului constituie un *memento* atemporal al interconectivității lumilor și al ritmurilor ciclice ale naturii.

6.3. Aripile ambiguității: Pegas

În vasta întindere a mitologiei grecești, Pegas apare ca un simbol al libertății nemărginite, tranzitând domeniul pământesc cu aripile sale maiestuoase și grația divină. Potrivit legendei, Pegas a apărut din sângele Gorgonei Meduza, născut din uniunea dintre pământ și mare. Cu blana sa albă imaculată și aripile de lumină pură, Pegas întruchipează idealul frumuseții și grației divine. Este adesea descris ca un simbol al inspirației, purtând fulgerele lui Zeus sau muzele de pe Muntele Helicon în călătoriile sale cerești.[20] [Pegas]

[18] În mitologia și folclorul chinez, Dragonul azuriu este sărbătorit în diverse legende, ritualuri și festivaluri. Este adesea reprezentat ca o divinitate protectoare, ocrotind locuri sacre precum temple, palate și morminte ancestrale. În timpul Festivalului Qingming (Ziua Curățării Mormintelor), se aduc ofrande Dragonului azuriu pentru a onora strămoșii și a căuta binecuvântări pentru anul ce urmează. În artă și literatură, Dragonul azuriu este reprezentat ca un simbol al puterii, rezilienței și frumuseții cerești, inspirând uimire și respect la poporul chinez de secole întregi.

[19] Pasărea tunetului deține o semnificație profundă în mitologia și spiritualitatea nativă americană, servind ca un gardian al lumii naturale și mesager al zeilor. Este strâns asociat cu ritualuri, ceremonii și tradiții legate de ploaie, agricultură și vânătoare. Pasărea tunetului este venerată ca protector al tribului și simbol al forței, rezilienței și renașterii spirituale. Prezența sa în arta, dansul și tradițiile orale ale nativilor americani reflectă reverența și admirația de lungă durată inspirată de această ființă mitică.

[20] Pegas are o semnificație culturală și simbolică profundă în mitologia greacă și în societate, fiind un simbol al inspirației divine, creativității și transcendenței. Este strâns asociat cu muzele de pe Muntele Helicon, care se spune că l-au adăpat cu apele Fântânii Pieria. Pegas este de asemenea legat de eroul Bellerofon, pe care l-a ajutat în misiunea sa de a înfrunta monstruoasa Himeră. Prin aventurile sale mitice și realizările legendare, Pegas continuă să inspire artiști, poeți și visători să atingă stelele și să urmărească cele mai înalte aspirații ale lor.

Pegas, de obicei reprezentat ca un cal înaripat magnific, relevează misterele frumuseții divine și transcendenței.

7. Entități mitologice MultiAliste: SuperHibrizi parțial zeu, parțial om, parțial animal

Aceste creaturi hibride întruchipează o combinație complexă de atribute divine, umane și animalice, servind — în opinia mitologilor — ca simboluri ale transformării, puterii și interconexiunii între diferite domenii ale existenței. Iată câteva exemple din diferite mitologii:

→În mitologia egipteană, Thoth este adesea reprezentat ca o divinitate cu corpul uman și capul de ibis sau babuin. Ca zeu al înțelepciunii, scrisului și magiei, Thoth întruchipează intelectul divin și puterea creativă a zeilor, având totodată trăsături umane, precum inteligența și compasiunea. Forma sa hibridă simbolizează sinteza cunoașterii divine și înțelegerii umane, servind ca ghid și mediator între zei și oameni. [Thoth]

→În mitologia hindusă, Hanuman este o divinitate cu corpul uman, dar cu fața de maimuță și inteligența și puterile unui zeu. Este venerat ca tovarăș devotat al lui Rama și simbol al forței, curajului și devoțiunii. Natura hibridă a lui Hanuman reflectă descendența sa divină ca fiu al zeului vântului Vayu și al unei nimfe cerești, precum și apropierea sa de lumea naturală și regnul animal.[21] [Hanuman]

→În folclorul japonez, Tengu sunt ființe asemănătoare cu niște vulturi, având o formă umanoidă, dar păstrând aripi, capete sau ciocuri aviare și fiind înzestrate cu inteligența și puterile unui zeu. Tengu sunt asociați cu munții și pădurile, unde servesc ca gardieni și păcălici, testând virtuțile călătorilor și călugărilor. [Tengu]

Aceste cazuri evidențiază gama diversificată de ființe care întruchipează conceptul de entitate parțial zeu, parțial om și parțial animal în mitologie și folclor, fiind astfel asociate cu trăsături multialiste [MultiAlist]. Într-un context neutrosofic, astfel de ființe reprezintă ambiguitatea și natura paradoxală a existenței, existând într-o stare de ‘ambele’, mai degrabă decât de ‘niciunul’.

7.1. Sfinxul: gardianul egiptean

În nisipurile nemuritoare ale mitologiei și istoriei egiptene, Sfinxul rămâne un testament al amestecului enigmatic de trăsături umane și animale, fiind dotat cu puteri divine. Săpat în stânca vie, această creatură iconică întruchipează un simbolism profund, servind ca gardian al cunoașterii, misterului și echilibrului cosmic.

Sfinxul își găsește originile în conceptul egiptean antic de ‘shesep ankh’ (imagine vie). Este de obicei reprezentat ca un leu culcat cu un cap uman, asemănându-se cu un faraon. Această formă hibridă simbolizează unirea regalității divine (reprezentată de leu) cu inteligența și înțelepciunea umană (încarnate de capul uman). Sfinxul servește ca gardian al spațiilor sacre, cum ar fi intrările templelor sau aleile care duc către mormintele regale, precum și ca protector al ordinii cosmice și al ciclului vieții și morții.[22] [Sphinx]

[21] Hanuman este reprezentat cu cinci fețe, simbolizând puterea sa divină. Într-un episod în care îl ajută pe Rama (salvându-l pe Rama de demonul Ahiravana, fratele lui Ravana), Hanuman a trebuit să stingă cinci lămpi simultan pentru a-l înfrunta pe Ahiravana. Pentru a îndeplini această sarcină, Hanuman a manifestat cinci capete, fiecare orientat spre o direcție diferită, unde se aflau lămpile.

[22] Sfinxul este strâns asociat cu zeul Atum-Ra, zeul soarelui și divinitatea creatoare. Un exemplu faimos este Marele Sfinx de la Giza, care se află în fața Piramidei lui Khafre și se crede că îl întruchipa chiar pe faraon, servind drept protector și ghid etern în viața de apoi. Alt exemplu este Sfinxul lui Amenemhat al II-lea, care păzește intrarea în templul zeului Amun-Ra de la Tanis.

7.2. Intersecția dintre zei, oameni și animale în mitologia indică

Conceptul de hibride multialiste — ființe care sunt parțial zeu, parțial om și parțial animal — este bogat reprezentat în diverse povești și legende din mitologia indică. Aceste entități hibride întruchipează complexitatea interacțiunii dintre domeniile divin, uman și animal, oferind astfel un alt context neutrosofic.

Un exemplu proeminent de astfel de hibrid este Hanuman, zeul cu fața de maimuță cunoscut pentru devotamentul său neclintit față de Rama. Hanuman este venerat ca epitomul loialității, forței și curajului, având atribute divine, dar și trăsături animalice. Aspectul său fizic, cu un corp uman adornat cu fața și coada de maimuță, reflectă natura sa multialistă ca ființă divină și creatură mixtă a lumii naturale.

Un alt hibrid multialist este Narasimha, încarnarea jumătate om, jumătate leu a lui Vishnu. Conform mitologiei indice, Narasimha a apărut pentru a-l proteja pe devotatul Prahlada de tatăl său tiranic, regele demon Hiranyakashipu. Cu corpul unui om și capul și ghearele unui leu, Narasimha e fioros și întruchipează puterea regnului animal, combinată cu inteligența și compasiunea umanității. [Narasimha]

Într-un context neutrosofic, aceste hibride multialiste contestă noțiunile convenționale de identitate și clasificare, existând la intersecția dintre multiple domenii ale existenței. Ele reflectă natura paradoxală a realității, fiind simultan divine, umane și animale, depășind distincțiile binare și îmbrățișând posibilitățile multialiste ale universului.

7.3. *Centzon Totochtin*: zeii-iepuri azteci

În mitologia aztecă, Centzon Totochtin se prezintă ca figuri enigmatice, întruchipând interacțiunea complexă dintre domeniile divin, uman și animal. Tradus ca “Patru sute de iepuri” în Nahuatl, limba aztecilor, acești zei-iepuri reprezintă fertilitatea, abundența și celebrarea vieții.[23] Centzon Totochtin își au originea în uniunea zeiței Mayahuel, divinitatea plantelor de *maguey*, și zeului Patecatl, divinitatea *pulque*-ului.

Cei 400 de zei sunt adesea reprezentați ca figuri umanoide cu trăsături de iepure, cum ar fi urechi lungi, mustăți și, uneori, o coadă pufoasă. [Centzon]

Un exemplu notabil este zeul-iepure Ometotchtli, care prezidează peste bețe și veselie, întruchipând spiritul festiv al culturii aztece.[24] Un alt exemplu este Tepoztecatl, zeul-iepure al *pulque*-ului, care supraveghează fermentarea și consumul băuturii sacre.

[23] În cultura aztecă, numărul 400 deținea o semnificație simbolică importantă, în special în relație cu timpul și sistemele calendaristice. Aztecii foloseau un sistem complex de calendar compus din mai multe cicluri interconectate, unul dintre ele fiind “xiuhpohualli”, sau calendarul agricol, care era format din 18 luni a câte 20 de zile fiecare, plus 5 “zile ghinioniste” suplimentare la final. Numărul 400 este relevant deoarece corespunde lungimii unui ciclu “xiuhpohualli”, care este compus din 20 de “veintenas” (cicluri de 20 de zile), fiecare durând 20 de zile. 20 de veintenas înmulțite cu 20 de zile = 400 de zile. După finalizarea unui ciclu de 400 de zile, calendarul se relua, începând un nou ciclu. Această natură ciclică a timpului, reprezentată de numărul 400, era semnificativă în cosmologia și ritualurile aztece, deoarece reflecta traiectul continuu al vieții, morții și renașterii observat în lumea naturală. În plus, numărul 400 era asociat cu concepte precum completarea, reînnoirea și natura ciclică a existenței în sistemele de credință aztece.

[24] Ei participă la sărbători festive, cum ar fi festivalul Huey Tozoztli, unde se aduc ofrande de mâncare, băutură și flori în onoarea lor. Moștenirea Centzon Totochtin continuă să rezoneze în cultura mexicană de astăzi, unde iepurii sunt venerați ca simboluri ale fertilității, abundenței și reînnoirii vieții.

8. Observații și întrebări deschise

8.1. 'Fiare' Hibride și SuperHibride: Hibrizi parțial animal$_1$ + parțial animal$_2$ (+ ... + parțial animal$_n$)

'Fiarele' Hibride și SuperHibride sunt creaturi fantastice, compuse dintr-o combinație de trăsături de la diferite animale, abundând în folclorul și mitologia diverselor culturi. Iată câteva exemple de astfel de fiare:

→Anzû (mitologia sumeriană) este o pasăre divină a furtunii, adesea înfățișată ca un vultur cu cap de leu. Este considerată un simbol al haosului.

→Ammit (mitologia egipteană), cunoscută ca "devoratorul de morți", este o creatură cu cap de crocodil, membrele anterioare ale unui leu și membrele posterioare ale unui hipopotam. Devorează inimile celor nedemni în timpul judecății morților.

→Himera (mitologia greacă) este un monstru SuperHibrid cu trup de leu, cap de capră și coadă de șarpe, care suflă foc.

→Grifonul (diverse mitologii) este o creatură legendară cu trup de leu și cap de vultur, adesea asociată cu protecția comorilor.

→Hipogriful (mitologii europene) este o creatură legendară cu partea din față a unui vultur și partea din spate a unui cal.

→Qilin (mitologia chineză) este o creatură mitică cu trup de cerb, coadă de bou, copite de cal și uneori trăsături asemănătoare unui dragon sau unui leu.

→Baku (mitologia japoneză) este o creatură supranaturală, parte elefant, parte leu și parte tigru. Devorează coșmarurile.

→Nue (folclorul japonez) este o creatură asemănătoare himerelor, cu cap de maimuță, corp de tanuki (câine-raton), membre de tigru și o coadă de șarpe. Prevestește răul.

→Camahueto (mitologia Mapuche, America de Sud) este o creatură cu trup de vițel și un corn în formă de spirală. Este considerată o ființă puternică și sacră.

→Jackalope (folclorul modern nord-american) este o creatură fantezistă cu trup de iepure și coarne asemănătoare celor de antilopă sau cerb. Este un produs al unor relatări exagerate.

Aceste creaturi încorporează adesea o îmbinare a diferitelor specii și servesc frecvent ca simboluri, metafore sau explicații pentru fenomene naturale, întruchipând credințele și valorile culturale.

8.2. Hibrizi și SuperHibrizi în artă și ficțiune

Fascinația pentru hibrizi și superhibrizi a pătruns în diverse forme de artă și ficțiune, captând interes în toate culturile și genurile. De la picturi clasice la literatură contemporană, aceste ființe fantastice au cucerit imaginația creatorilor și publicului deopotrivă, depășind limitele timpului și ale spațiului.

În domeniul artelor vizuale, reprezentările de hibrizi și superhibrizi au împodobit pânzele de secole, servind adesea ca simboluri ale extraordinarului și ale 'lumii de dincolo'. Artiștii din toate timpurile au fost atrași de conceptul de a combina elemente disparate din lumea naturală pentru a genera creaturi care sfidează clasificările convenționale. Fie că este vorba de centaurii din mitologia greacă, de sfincșii din Egiptul Antic sau de cyborgii futuristici din *science fiction*, artiștii au explorat limitele imaginației prin portretizări ale unor astfel de 'ființe' hibride.

În mod similar, în lumea literaturii și ficțiunii, 'ființele' hibride și superhibride au apărut ca subiecte populare, îmbogățind narațiunile cu personaje complexe și lumi fantastice. De la

mituri și legende antice la romane și benzi desenate moderne, aceste ființe populează povești care explorează teme precum identitatea, transformarea și interacțiunea dintre umanitate și necunoscut. Fie că sunt portretizați ca eroi sau răufăcători, hibrizii și superhibrizii îi provoacă pe cititori să pună sub semnul întrebării natura existenței și limitele imaginației.

De exemplu, Pegas, calul înaripat magnific din mitologia greacă, ocupă un loc proeminent în imaginația colectivă, celebrat în numeroase legende, povestiri și reprezentări artistice de-a lungul istoriei, de la picturile pe vasele antice până la literatura modernă. Aceste reprezentări artistice îl portretizează adesea în zbor, cu aripile sale puternice întinse în timp ce străbate cerurile, cum ar fi mozaicul lui Pegas descoperit în Paphos, Cipru. În acest mozaic, Pegas este reprezentat cu detalii uimitoare și eleganță, purtând muze pe spatele său în timp ce se ridică spre cer. Pe lângă arta vizuală, Pegas a lăsat o amprentă de nești în literatură, inspirând unii dintre cei mai mari poeți și scriitori ai antichității. În operele epice ale lui Homer, Hesiod și Pindar, Pegas este celebrat pentru frumusețea sa mitică, grația și descendența divină.

În literatură, conceptul de homuncul a fost reimaginat și adaptat în diverse moduri. A apărut în opere de ficțiune, cum ar fi "Frankenstein" de Mary Shelley, unde savantul Victor Frankenstein creează o creatură umanoidă prin experimente științifice. În mod similar, în "Faust" de Johann Wolfgang von Goethe, personajul Faust produce un homuncul prin mijloace magice.

În literatura modernă, film și cultura populară, cambionii continuă să captiveze audiențele cu atracția lor enigmatică și motivațiile lor complexe. De la "The Witching Hour" de Anne Rice până la seria TV "Supernatural", creată de Eric Kripke, cambionii au fost reimaginați și reinventați, fiecare iterație oferind noi perspective asupra originilor lor mitologice și semnificației culturale.

Tuatha Dé Danann și Fomoire au lăsat un impact durabil asupra culturii și folclorului irlandez. Poveștile și legendele lor continuă să fie celebrate în literatură, artă și cultura populară. Multe repere și caracteristici geografice din Irlanda sunt asociate cu aceste ființe mitice, contribuind la peisajul cultural al țării. Romanul "A Portrait of the Artist as a Young Man" de James Joyce include referințe la aceste entități. De asemenea, romanul "An Only Child" de Frank O'Connor conține astfel de elemente de folclor și mitologie celtică, la fel ca și în "American Gods" de Neil Gaiman. Filmul animat "The Secret of Kells", regizat de Tomm Moore și Nora Twomey, se inspiră din aceleași aspecte mitologice. Personajul Hellboy' din seria de benzi desenate "Hellboy" de Mike Mignola întâlnește creaturi inspirate din mitologia celtică. Și așa mai departe.

În era digitală de astăzi, popularitatea hibrizilor și superhibrizilor nu dă semne de estompare. Ele continuă să inspire artiști, scriitori, regizori și creatori din diverse medii, alimentând o renaștere creativă care sparge barierele narațiunii și expresiei artistice.

Cu siguranță, scriitorii și artiștii contemporani posedă potențialul creativ de a crea o gamă diversificată de 'Hibrizi' și 'SuperHibrizi', utilizând instrumentele moderne, cum ar fi inteligența artificială generativă, pentru a extinde limitele imaginației.

O posibilitate intrigantă constă în crearea de ființe noi care îmbină elemente din domenii disparate, cum ar fi SuperHibridul Zeu-Om-Demon. Această entitate ar putea întruchipa o fuziune de atribute divine, umane și demonice, oferind un personaj complex și multifațetat, apt de agreat atât în narațiuni ficționale, cât și în artă vizuală.

Mai mult, artiștii și scriitorii pot utiliza grade variabile de compoziție pentru a construi aceste entități imaginative, permițând o abordare nuanțată a creației lor. De exemplu, cineva ar putea specifica proporțiile fiecărei componente constitutive, delimitând compoziția exactă a

ființei hibride. Această abordare ar introduce un nivel de granularitate și specificitate, permițând creatorilor să adapteze caracteristicile creațiilor lor conform viziunii artistice.

De exemplu, un hibrid ipotetic ar putea fi descris ca fiind 30% Semizeu, 47% Demon, 3% Cambion, 15% Animal și 5% Uman. Fiecare procentaj reprezintă un aspect distinct al naturii hibridului, contribuind la identitatea sa generală și la semnificația sa narativă. Prin această abordare detaliată, creatorii pot îmbogăți creațiile lor cu profunzime și complexitate, invitând publicul să contempleze interacțiunea dintre diferitele forțe și identități din cadrul acestor ființe fantastice.

În esență, posibilitățile creative oferite de instrumentele și tehnicile contemporane permit artiștilor și scriitorilor să exploreze noi frontiere în domeniul mitologiei și fanteziei. Prin valorificarea puterii imaginației și inovației, ei pot insufla viață unui ansamblu divers de personaje. Prin experimentarea cu noile instrumente de inteligență artificială, Hibrizii și SuperHibrizii pot continua să evolueze.

8.3. 'Teratologia mitologică': întrebări deschise

Teratologia[25] a parcurs un drum fascinant în istorie, evoluând de la un discurs despre minuni și prevestiri la un domeniu științific ce explorează malformațiile congenitale și cauzele lor. Acest domeniu interdisciplinar intersectează biologia dezvoltării, embriologia și genetica, examinând anomaliile în dezvoltarea fiziologică. În contextul modern, teratologia cuprinde examinarea medicală a teratogenezei, malformațiilor congenitale și a indivizilor cu malformații semnificative. Principiile teratogenezei oferă un cadru fundamental pentru înțelegerea efectelor agenților de mediu asupra organismelor în dezvoltare. Aceste principii iau în considerare factori precum genotipul, momentul expunerii și interacțiunile cu mediul, ghidând cercetările asupra agenților teratogeni.

Cu toate acestea, rădăcinile teratologiei se întind adânc în antichitate, unde figuri precum Flegon din Tralles, un paradoxograf proeminent din secolele I-II e.n., autorul cărții "Peri thaumasion" ("Cartea Minunilor"), au documentat minuțios relatări extraordinare [Hatzopoulos]. Relatările lui Flegon, împreună cu cele ale lui Pliniu cel Bătrân și ale altor savanți antici, oferă o privire asupra unei lumi în care anomaliile erau observate cu un amestec de uimire și curiozitate intelectuală. Imersiunea în narațiunile lui Flegon oferă o călătorie într-un tărâm unde anomaliile nu sunt doar observate, ci și detaliate, cuprinzând relatări despre hermafroditi, indivizi care trec prin transformări de sex și cazuri de nașteri neobișnuite [Nutton]. Relatările călătorilor documentate în "Istoria Naturală" a lui Pliniu cel Bătrân detaliază existența unor ființe fantastice în ținuturi îndepărtate, cum ar fi indivizi cu cap de câine, asemănători cu babuinii, persoane cu un singur picior (*sciapodes*) sau ființe cu fețe încorporate în piept ('acefali').

Narațiunile antice descriau adesea indivizi cu anomalii anatomice, cum ar fi hermafroditii sau oameni fără gură sau nas. Aceste relatări, împreună cu tapiseria monștrilor mitici găsiți în folclorul global, inclusiv giganți, ciclopi, centauri și alții, ridică întrebări fascinante despre relația dintre anomaliile medicale și creaturile mitice.

Ar putea fi posibil ca relatările antice despre minuni și hibrizi să fi apărut ca reflecții ale anomaliilor medicale? Ar putea teratologia modernă să ofere perspective asupra apariției și perpetuării monștrilor mitici? O subdisciplină pe care am numi-o teratologie mitologică

[25] Cuvântul "teratologie" își are originea în cuvântul grec τέρας (*teras*), care înseamnă "semn trimis de zei, prevestire, minune, monstru".

apare ca o posibilă cale de explorare a acestei intersecții dintre mit și realitate, construind un pod între tradiția veche și știința modernă.

Utilizarea inteligenței artificiale generative pentru a compara cunoștințele actuale despre malformații cu hibrizii și superhibrizii mitici deschide noi căi de cercetare și explorare. Prin investigarea acestui studiu mixt, cercetătorii ar putea descoperi perspective asupra originilor creaturilor mitice și ar putea obține o înțelegere mai profundă a relației complexe dintre mit și realitate.

8.4. Moștenirea genetică: atribuirea paternității divine

Investigarea liniei ereditare a ființelor hibride de-a lungul diferitelor perioade istorice și contexte culturale ne deplasează pe un teren nisipos, dar intrigant, care ridică unele întrebări îndrăznețe, care contestă noțiunile convenționale.

Așa cum am subliniat anterior, paternitatea divină, în mitologia greacă, este o temă recurentă, exemplificată de numeroasele cazuri în care zeii au conceput semizei cu femei muritoare. Zeus, regele zeilor, este renumit pentru escapadele sale amoroase cu muritoarele, ceea ce a dus la nașterea unor eroi precum Hercule, Perseu sau Elena din Troia.

Acești semizei moștenesc trăsături și abilități de la tații lor divini, creând un amestec neutrosofic de caracteristici umane și divine. Mitologia greacă este doar un exemplu dintre numeroasele culturi unde concepția cu intervenția zeilor creează ființe hibride, combinând atributele divine și umane.

Putem aborda aceste concepții supranaturale din perspectiva modernă a eredității? În ce măsură poate fi studiat acest fenomen prin prisma teratologiei moderne, care explorează moștenirea și dezvoltarea trăsăturilor? Întrebările de acest tip deschid discuții ample despre relația dintre 'divin', 'uman' și 'monstruos', oferind teren fertil pentru investigații viitoare, atât în domeniul mitologiei, dar și al altor discipline.

În mitologia romană, zeul Marte este considerat tatăl divin al lui Romulus și Remus, legendarii fondatori ai Romei.[26] Această paternitate divină adaugă o dimensiune sacră originii civilizației romane, subliniind călăuzirea și protecția divină oferite orașului prin intermediul figurilor sale fondatoare.

În hinduism, conceptul de paternitate divină este întruchipat în povești din scripturile antice. De exemplu, Rama, o zeitate venerată, este considerat fiul regelui Dasharatha și rezultatul unei intervenții divine. Krishna, o altă divinitate semnificativă în hinduism, este născut din părinți muritori, dar cu un scop divin, subliniind implicarea directă a divinității în afacerile umane.

Mitologia egipteană antică prezintă, de asemenea, conceptul de paternitate divină, cu povești despre faraoni considerați descendenți divini ai zeilor. Faraonii erau considerați urmași ai unor zeități precum Ra sau Osiris, ceea ce sublinia dreptul lor divin de a domni și conexiunea dintre tărâmurile pământești și divine.

Conceptul de paternitate divină nu se limitează la mitologiile clasice; își găsește expresia și în credințele indigene și religiile moderne. În culturile amerindiene, de exemplu, există povești despre zei sau spirite care devin tații unor eroi sau figuri importante.

[26] Așa cum este menționat în "Istoria" lui Titus Livius, Rhea Silvia (Ilia) a susținut că Marte era tatăl gemenilor ei. Atribuirea paternității divine nu era ceva nou. Alcmene, mama lui Hercule, a atribuit paternitatea fiului său lui Zeus. Conform mitului, Zeus s-a deghizat în soțul lui Alcmene, Amphitryon, ceea ce a dus la nașterea eroului semizeu. Danaë, mama lui Perseu, a susținut că Zeus a lăsat-o însărcinată sub forma unei ploi de aur. Semele, mama lui Dionysos, a insistat că Zeus era tatăl copilului ei. Zeul și-a dezvăluit adevărata formă în fața ei, dar radianța divină a fost prea puternică pentru Semele, ducând la moartea acesteia. Și așa mai departe.

Creștinismul, cu povestea sa fundamentală despre Fecioara Maria, care îl concepe pe Iisus prin Duhul Sfânt, întruchipează o formă unică de paternitate divină. Iisus este considerat Fiul lui Dumnezeu, iar această paternitate divină are o semnificație teologică profundă în doctrina creștină.

În fiecare context cultural, conceptul de paternitate divină servește scopuri variate. Poate explica calitățile extraordinare sau destinele unor indivizi, poate întări conexiunea divină dintre zei și oameni sau poate legitima domnia anumitor linii de sânge. Deși specificul variază, tema generală rămâne o explorare fascinantă a intersecției dintre ființele 'non-naturale' și 'naturale', modelând narațiunile culturale, credințele religioase și structurile sociale din diverse civilizații.

Pe de altă parte, în piesa "Eumenidele" de Eschil, zeul Apollo susține că partea paternă în ereditate este 100%, reflectând o credință antică în dominația paternă. Aeschines povestește despre amficționii care au blestemat făptuitorii sacrilegiului dorindu-le nașterea unor copii care nu semănau cu părinții lor, ci cu monștri.

În sud-vestul Indiei, în statul Kerala, comunitățile matriliniare coexistă și în prezent cu sistemul patriliniar predominant. Descendența și moștenirea sunt urmate pe linie feminină, provocând noțiunea unei înțelegeri standardizate a eredității în același context cultural și național. Poporul Mosuo, un grup etnic mic din China, practică, de asemenea, o formă de societate matriliniară în care descendența și proprietatea familiei sunt transmise pe linie feminină. Absența căsătoriilor formale adaugă o altă dimensiune variației lor culturale, provocând noțiunea unei înțelegeri standardizate a eredității, prevalentă în vremurile antice.

În "Tristram Shandy" de Laurence Sterne, teoria homunculului este ironizată, subliniindu-se lipsa cunoașterii empirice și natura speculativă a teoriilor privind transmiterea trăsăturilor de la părinți la urmași. Evoluția cunoștințelor științifice, inclusiv descoperirea cromozomilor, meiozei și fertilizării, a transformat semnificativ înțelegerea noastră asupra eredității.

Progresele în genomică au relevat că oamenii moderni poartă un procentaj (neutrosofic) din ADN-ul lor moștenit de la Neanderthalieni, sugerând încrucișarea între cele două specii. Această moștenire genetică este încorporată în ADN-ul oamenilor contemporani, oferind o legătură tangibilă cu o ascendență comună care se extinde dincolo de linia Homo sapiens.

Ne întrebăm dacă acest amestec genetic a lăsat urme în memoria colectivă. O explorare a interacțiunii complexe între elementele neutrosofice din regnul mitologic — de exemplu, hibridizarea între ființe 'non-naturale' și 'naturale', cum ar fi 'semizeii' sau 'cambionii' — ar putea împărtăși elemente tematice reflectând narațiunea încrucișării genetice. Aceste ființe hibride adesea se confruntă cu identități complexe, navigând între lumi diferite, și posedă abilități extraordinare care le diferențiază de oamenii obișnuiți. *Dar dacă narațiunile paralele ale ființelor hibride, cu trăsături evident neutrosofice, născute din uniuni 'non-naturale' – 'naturale', au servit drept ecouri culturale ale ascendenței noastre complexe, ca o metaforă culturală pentru amestecul speciilor umane? S-ar putea ca încrucișarea Neanderthal-Sapiens, precum și alte încrucișări similare, confirmate de dovezi genetice, să fie perpetuate în folclor, găsind poate o reflectare captivantă în zonele mitologice ale ființelor hibride și identităților? Este posibil ca unele paralele între descoperirile științifice și narațiunile mitologice să sublinieze fascinația umană durabilă față de misterele originilor noastre și modalitățile imaginative prin care țesem povești pentru a înțelege moștenirea noastră genetică?*

Concluzii

Culturi diferite percep formele mitologice hibride și superhibride în moduri variate. De la Centaurii greci la Gandharvii hinduși, fiecare tradiție mitologică contribuie cu perspective unice asupra îmbinării atributelor umane și non-umane. Simbolismul asociat cu hibrizii umani capătă profunzime atunci când este analizat printr-o lentilă neutrosofică. Fie că sunt priviți ca simboluri ale haosului și monstruozității sau ca reprezentări ale armoniei între diferite tărâmuri, elementele contradictorii ale acestor ființe oferă un material bogat pentru interpretarea neutrosofică. Căutarea identității este complexă, întrucât aceste ființe își petrec "existența" la marginile vagi ale umanității. Aceste entități hibride și superhibride sunt vehicule interesante pentru contemplare filozofică și explorare culturală în cadrul neutrosofiei.

Avertisment

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The Dynamic Interplay of Opposites in Zoroastrianism

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Abstract

This exploration addresses some aspects of Zoroastrianism, examining how the ancient Persian belief system aligns with the dynamic and indeterminate principles of Fuzzy, Neutrosophic, and MultiAlist systems. Zoroastrianism, rooted in the eternal struggle between good and evil, light and darkness, exhibits parallels with Neutrosophy's acknowledgment of indeterminacy, incompleteness, and the dynamic interplay of opposites. The prophet Zarathustra's vision of a neutrosophic God challenges conventional notions of divine attributes, emphasizing a dynamic and evolving universe. Before investigating these vague areas, the concept of unclear conceptual borders is explored, emphasizing the indeterminacy and imprecision inherent in defining opposites or partially opposite concepts. The law of included infinitely-many-middles suggests that between opposites, there exist infinitely many nuances or middle values. Sorites' paradoxes challenge traditional logic by exposing the difficulties in defining vague boundaries. Neutrosophic Interpretation suggests introducing a buffer zone between opposites, resulting in Neutrosophic Sorites Paradoxes. Moreover, this exploration highlights the need for a more flexible and nuanced understanding of conceptual boundaries, acknowledging the dynamic and indeterminate nature of many philosophical and logical constructs. Finally, we delve into the application of neutrosophy to various cultural and philosophical concepts. The legendary figure of Gilgamesh, described as two-thirds god and one-third human, is examined through both traditional and neutrosophic perspectives. Additionally, Hindu concepts of Dharma, Adharma, and Karma are reexamined within the context of neutrosophy. The logic of the Diamond Sutra in Mahayana Buddhism, characterized by paradoxical language and a focus on emptiness, aligns with neutrosophic principles in challenging fixed notions and embracing the interconnected and indeterminate aspects of reality. Despite diverse cultural origins, these examples share a common thread in questioning absolutes and embracing the dynamic nature of existence.

Keywords: Zoroastrianism, Zarathustra, Ahura Mazda, Gilgamesh, Dharma, Adharma, Karma, Happiness, Diamond Sutra, Chinvat bridge, Cyrus the grate, Vagueness, Neutrosophy, Sorites paradoxes, Neutrosophic sorites paradoxes, Fuzzy information, Granulation, MultiAlism, MultiAlist systems, MuliPolar thinking.

1 | Introduction

In two recent articles [1], [2], i extended the concepts of 'pluralism', 'neutrosophy', 'refined neutrosophy', 'refined neutrosophic set', 'multineutrosophic set,' and 'plithogeny' [3]–[8], into a larger category that i referred to as MultiAlism, conceptualizing a MultiPolar System formed not only by multiple elements that might be random, or contradictory, or adjuvant, but also by accepting features from more than one basic system (UniPolar, BiPolar, TriPolar, or PluriPolar systems). One of the illustrations of a MultiPolar system that i proposed in the previously mentioned paper was from the realm of religions, namely Zoroastrianism, with its seemingly fuzzy characteristics, which most Western observers find it difficult to categorize as monotheistic, dualistic, or pluralistic [9]. In the following, i discuss again some unclear conceptual borders, not before reminding some basics of the theory of fuzzy information granulation and of neutrosophics, as to return to the example of Zoroastrianism for a deeper understanding of some Fuzzy, Neutrosophic, and MultiAlist facets of this religion. I dedicate the subsequent hypotheses and inferences to the kind memory of Professor Zadeh, for his profound effect on a wide range of scientific and technical domains, but also for the connection he had with the cultural spaces to which this paper refers [10].

Fig. 1. Lotfi Aliasker Zadeh (father of fuzzy theories) and Florentin Smarandache (father of neutrosophic theories).

Lotfi Aliasker Zadeh (father of fuzzy theories) and Florentin Smarandache (father of neutrosophic theories) at the 2003 BISC FLINT-CIBI International Workshop on Soft Computing for Internet and Bioinformatics, University of Berkeley, California, December 15-19, 2003, where I presented the paper "Generalization of the Intuitionistic Fuzzy Set to the Neutrosophic Set".

1.1 | The Fuzzy Information Granulation

Let us first remind the quiddity of the theory of fuzzy information granulation, regarded by Zadeh as central in human reasoning [11]. According to Zadeh [11], human cognition is based on three fundamental concepts: granulation (the division of a whole into pieces), organization (the integration of parts into a whole), and causality (the association of causes with effects). Granulation of an 'object' <A> generates a cluster of granules of <A>, that are indistinguishable, similar, close together, or functionally related, generally hierarchical in nature, and their attributes and values are fuzzy — since their boundaries are not sharply defined.

1.1.1 | Fuzzy sets

In traditional set theory, an element either belongs to a set or does not. In fuzzy set theory, elements can have partial membership in a set. The membership degree is a value between 0 and 1, where 0 means no membership, 1 means full membership, and values in between indicate partial membership.

1.1.2 | Granulation

Granulation involves the grouping or clustering of elements based on their similarity or relevance. It is the process of creating fuzzy sets to represent subsets of elements with similar characteristics. Granulation helps in organizing information and dealing with the inherent imprecision in human knowledge.

1.1.3 | Information granules

Information granules are the result of the granulation process. They are fuzzy sets that represent a higher level of abstraction or grouping of elements. These granules make it easier to handle complex and uncertain information by providing a more compact and manageable representation.

1.1.4 | Human reasoning

Zadeh [12] argues that human reasoning involves thinking at different levels of granularity. People naturally organize information into categories or concepts that are not strictly defined but have fuzzy boundaries. Fuzzy information granulation reflects this human cognitive process by allowing for the representation of imprecise and uncertain knowledge [12]–[14].

1.2 | The Neutrosophics

The Neutrosophy explores the interdependence and interplay of opposites. Rooted in a foundation of neutrality, the neutrosophic concepts challenge traditional binary thinking and offer a nuanced perspective on the complexities of the world.

This theory considers every notion or idea <A> together with its opposite or negation <antiA> and with their spectrum of Neutralities <NeutA> in between them (i.e., notions or ideas supporting neither <A> nor <antiA>). The <NeutA> and <antiA> ideas together are referred to as <nonA>. Neutrosophy is a generalization of Yin-Yang Ancient Chinese Philosophy and of Hegel's and Marx's Dialectics (which are based on <A> and <antiA> only).

For example, in mathematics, neutrosophic numbers and neutrosophic algebra provide tools for handling uncertainties in numerical data. These concepts find applications in decision-making processes where incomplete or ambiguous information is prevalent. In physics, neutrosophic mechanics extends classical mechanics to account for indeterminate forces and imprecise measurements. This approach is particularly relevant in quantum mechanics, where the nature of particles and their properties often defy deterministic description [3], [6].

1.2.1 | The philosophical framework

Neutrosophy is a philosophy that acknowledges the existence of indeterminacy in human knowledge and perception. It embraces the idea that many phenomena, concepts, and propositions are neither true nor false but rather exist in a state of partial truth. Neutrosophy introduces the notion of the 'neuter', emphasizing the presence of indeterminacy as a fundamental aspect of reality.

1.2.2 | Neutrosophic logic

Neutrosophic logic extends classical logic to accommodate indeterminate, incomplete, and inconsistent information. It introduces the concepts of truth-membership, falsehood-membership, and indeterminacy-membership degrees, providing a more flexible and nuanced representation of reality [4].

1.2.3 | Neutrosophic set

Neutrosophic set theory generalizes classical set theory to handle indeterminate elements. A neutrosophic set allows for the inclusion of objects with indeterminate membership degrees, acknowledging the uncertainty inherent in defining clear boundaries for certain concepts.

1.2.4 | Neutrosophic probability

Neutrosophic probability generalizes classical probability theory to address situations where uncertainty, indeterminacy, and ambiguity play a crucial role. It provides a framework for dealing with incomplete information and reflects the imprecision inherent in various real-world scenarios, having three sub-functions: chance that an event occurs, indeterminate-chance that the event occurs, and chance that the event does not occur.

1.2.5 | Beyond binary thinking

The interdisciplinary approach of neutrosophics challenges the binary nature of traditional Western philosophy and embraces the inherent complexity and ambiguity present in the world. While facing criticisms, neutrosophy and neutrosophics continue to stimulate intellectual discourse and contribute to our evolving understanding of uncertainty and ambiguity [15].

1.3 | The Neutrosophic Triad and the MultiNeutrosophy

Numerous schools of thought have extensively examined the dynamics between the opposites <A> and <antiA>. These concepts are known by various names, including dialectics, Yin-Yang, Manichaeism, dualism, Dharma-Adharma, and many others. However, the neutral (or indeterminacy) part (<neutA>) between these opposites has rather either been ignored or retracted. The neutral or indeterminate, as I emphasized in my studies on neutrosophic theory [3], [4], usually intervenes in the dynamics (or conflicts) from one side or the other, tipping the balance in one direction or the other. The boundaries between the opposites can be either fluid (when there is some overlapping or indeterminate/neutral part between the opposites) or rigid (when <A> and <antiA> are clearly separated).

Therefore, I proposed the 'triadic balance', emphasizing the interconnectedness of opposites. The triad – <truth>, <indeterminacy>, <falsity> – forms the cornerstone of the Neutrosophic system of thought. For every element "x" in a neutrosophic triplet set A, there exists a neutral of "x" and an opposite of "x". Also, the neutral of "x" must be different from the classical neutral element [16].

The MultiNeutrosophy [2] is a multiplied neutrosophy, focusing on the dynamics between $G=\{A_1, A_2, \ldots, A_n\}$ and its opposite $G=\{B_1, B_2, \ldots, B^m\}$, with their neutrals $G=\{C_1, C_2, \ldots, C_r\}$.

Let us give some simple examples of MultiNeutrosophy:

I. A group of countries against another group of countries (First and Second World Wars), while the third group of countries still remain neutral.

II. A group of ideas against another group of ideas in philosophy, literature, science, culture, religion.

III. A basketball team vs. another basketball team (as opposite groups), and the neutral group (formed by the referees and the supporters, while the supporters may be split into supporters of the first team, supporters of the second team, and undecided supporters).

1.4 | The MultiAlist System

In [1], i delineated a MultiPolar System that is open to combinations of opposites and neutrals (indeterminacies). The MultiPolar System is an extension of the PluriPolar System. The UniPolar, BiPolar, TriPolar, and PluriPolar systems are uni-valent (one excludes the other), but the MultiPolar System is multi-valent (it contains more than one system) and admits neutrality and indeterminacy between opposites.

The MultiAlism is an open, dynamic system with indeterminacies or neutralities, formed by elements from many systems, exceeding the PluriAlism, which is a closed dynamic system without indeterminacies, a uni-system formed by elements from a single system.

2|Unclear Conceptual Borders

Let us underline the blur borders between opposite or partially opposite concepts.

Given the fact that the boundaries between concepts are frequently hazy and imprecise, it is conceivable that <A>, <NeutA>, <antiA> (and <nonA> naturally) share two by two, or even all three of them, common parts.

2.1|The Law of Included Multiple-Middle

I generalized in [17] the Law of Included Middle [<A>, <nonA>, and a third value <T> which resolves their contradiction at another level of reality] (Lupasco-Basarab).

Law of Included Multiple-Middle [<A>, <antiA>, and <NeutA>, where <NeutA> is split into a multitude of Neutralities between <A> and <antiA>, such as $<Neut_1A>$, $<Neut_2A>$, etc.]. The <NeutA> value (i.e., Neutrality or Indeterminacy related to <A>) actually comprises the included middle value. Also, the Principle of Dynamic Opposition [opposition between <A> and <antiA>] is extended to the Principle of Dynamic Neutrosophic Opposition [which means oppositions among <A>, <antiA>, and <NeutA>].

2.1.1|The law of included infinitely-many-middles

In between the opposites <A> and <antiA> there are infinitely many middles, denoted by $<neutA_i>$, where $i = 1, 2, ..., \infty$ [18].

Let us take a practical example: between <White> and <Black>, there are infinitely many nuances of colors.

I. Between 100% True and 100% False, there are included infinitely many middles, which are truth-values of the form: d% True and (1-d)% False, thus a logical proposition may be, for example, 1% True and 99% False, 2% True and 98% False, etc., where $d \in (0, 1)$.

II. Similarly, between 100% Membership and 100% Nonmembership, there are included infinitely many middles of the form: d% Membership and (1-d%) Nonmembership.

2.2|Sorites Paradoxes

Sorites paradoxes are a class of paradoxes that arise from the indeterminacy surrounding the vague boundaries of concepts. Let us remind the classic example of the paradox of the heap:

I. Start with a heap of sand.

II. If you remove one grain, the heap remains a heap.

III. Repeat this process: removing one grain at a time.

IV. At some point, you'll be left with just a few grains.

V. The question is, at what point does the heap stop being a heap?

The paradox highlights the difficulty in defining when a <heap> becomes a <non-heap>. The problem arises from the inherent lack of preciseness in everyday concepts. Similar paradoxes can be constructed for other vague concepts like 'tallness", 'baldness', 'age', and so many more.

These paradoxes challenge traditional approaches to logic and set theory, which often assume clear-cut distinctions between categories. They raise questions about the nature of linguistic vagueness and how we handle it in logical reasoning [19], [20]. Various solutions and theories have been proposed to address sorites paradoxes [21], including:

I. Degree theories: vague predicates should be understood in terms of degrees or degrees of membership.

II. Supervaluationism: a statement can be true, false, or indeterminate, allowing for multiple acceptable precisifications of vague terms.

III. Contextualism: the meaning of vague terms depends on the context of their use, and the boundaries of concepts can shift based on the context.

IV. Many-valued logic: instead of the classical two-valued logic (true or false), many-valued logics introduce more than two truth values, accommodating the intermediate or fuzzy states between true and false.

2.2.1 | Neutrosophic interpretation

The Sorites Paradoxes, therefore such paradoxes where between two opposite entities there is not a clear frontier, can be interpreted neutrosophically in the following way: one considers a buffer zone, <neutA> (or neutral, indeterminacy), between the opposites.

There is not a clear distinction between some opposites <A> and <antiA>, where <A> is a concept and <antiA> its opposite, but a buffer zone <neutA>.

Fig. 2. A and antiA.

There are three zones: a zone that for sure represents <A>, a second zone that for sure represents <antiA>, and an ambiguous/unclear/vague zone that represents <neutA>, the neutral or indeterminate zone (neither <A> nor <antiA>; or <A> and <antiA> simultaneously). Therefore, a universe of discourse has a neutrosophic partition with respect to the frontier between opposites [22], [23].

2.2.2 | Neutrosophic sorites paradoxes

Between <A> and <neutA>, there is not a clear frontier – this is the first neutrosophic paradox (NSP1).

Then, between <neutA> and <antiA>, there is not a clear frontier – this is the second neutrosophic paradox (NSP2).

2.2.3 | MultiSorites paradoxes

MultiSorites paradoxes—within the frame of Refined Neutrosophy (which is isomorphic to the MultiNeutrosophy)— has n-dimensional form: $T_1, T_2, ..., T_p, I_1, I_2, ..., I_r, F_1, F_2, ..., F_s$, where p, r, s are integers ≥ 0, and at least one of p, r, s is ≥ 2 to assure the refinement (or multiplicity), $p+r+s = n$, where each T_j, I_k, F_l are refined (or respectively multi) Truth / Indeterminacy / Falsehood.

The frontiers between all these n sub- (or multi-)components, taken two by two, are blurry, unclear, and fluid.

There are C(n,2) (combinations of n taken by 2) = n(n+1)/2 Refined/MultiNeutrosophic Sorites Paradoxes.

3 | Exploring the Interplay between Zoroastrianism and Neutrosophy

Zoroastrianism traces its origins to the teachings of the prophet Zarathustra (Zoroaster). This ancient belief system, originating in Persia (modern-day Iran), emphasizes the eternal (dynamic) struggle between good and evil, light and darkness.

Zarathustra delivered his teachings in a world where duality played a central role. The core tenets of Zoroastrianism revolve around the eternal conflict between Ahura Mazda, the supreme deity representing goodness and truth, and Angra Mainyu, the destructive force embodying falsehood and evil. This cosmic battle, reflected in the opposing principles of asha (truth) and druj (falsehood), mirrors the fundamental dualities inherent in existence but also the neutrosophic zone that spans from asha to druj [24].

In the Zoroastrian context, the neutrosophic triadic balance finds resonance in the perpetual interplay of good, evil, and the ambiguous space between them.

Edalatpanah comment: "The juxtaposition of Zarathustra's teachings on the concepts of good and evil with the principles of neutrosophy, which focuses on neutrality and the spectrum of ideational phenomena, promises to open new avenues of interpretation for these ancient teachings."

Zarathustra's vision of cosmic order and moral responsibility aligns with the Neutrosophic notion that truth, falsity, and indeterminacy are interconnected and inseparable. The prophet's teachings emphasize the need for individuals to actively participate in the eternal struggle for righteousness, acknowledging the complexities and uncertainties inherent in their choices.

3.1|Zarathustra's Neutrosophic God

Zarathustra introduced a conception of the divine that transcends the conventional binary notions often associated with gods. Central to Zarathustra's philosophy is the idea that God embodies both masculine and feminine attributes and that existence is a dynamic relationship between the opposites <A> and <antiA>, which mirrors the principles of neutrosophy.

Furthermore, Zarathustra's conception of God challenges the omnipotence and omniscience traditionally ascribed to divine beings. Instead of an all-knowing and all-powerful deity, Zarathustra's God is portrayed as a creator of a dynamic universe—one in constant flux and progression towards perfection. This aligns again with the neutrosophic notion that reality is inherently indeterminate, incomplete, and evolving.

The dynamics of <A> (Good) and <antiA> (Evil) in Zarathustra's worldview are crucial elements in understanding the neutrosophic nature of his God, reflecting a reality where good and evil coexist in various degrees. The universe, in this view, is a dynamic interplay between opposing forces, with outcomes ranging from partially good and partially evil to states of complete neutrality where the degrees of goodness and evilness nullify or balance each other.

This perspective has profound implications for ethical considerations within Zoroastrianism. Instead of viewing morality in absolute terms, Zarathustra's framework suggests a more nuanced understanding, similar to neutrosophic architecture. It invites followers to navigate the intricate web of existence, recognizing that moral judgments may fall within the spectrum of what neutrosophy calls <neutA>, where actions and intentions are not strictly defined as wholly good or wholly evil.

Zarathustra's philosophical canvas expands further as we delve into the rhythmic and dynamic nature of existence. In his vision, everything follows a ceaseless succession of changes, akin to the principles of neutrosophy that acknowledge the inherent indeterminacy and evolution in all aspects of reality. The rhythmic nature of existence implies a perpetual dance between opposites, where the divine, being neutrosophic, orchestrates the symphony of creation.

Therefore, Zarathustra's emphasis on progress echoes the neutrosophic principle that nothing is static. In this dynamic universe, progress is not merely an option but a necessity. Stagnation, as Zarathustra implies, can lead to misery—an idea consonant with the neutrosophic understanding that acknowledges the potential consequences of resisting change and growth. The neutrosophic God encourages continuous evolution, fostering a universe in a state of perpetual becoming.

The concept of self-dominance introduces a dimension of personal responsibility within Zarathustra's philosophy. This self-mastery is crucial in navigating the dynamic interplay of <A> and <antiA>, contributing to the harmonious progression towards perfection. Evolution is not merely a biological concept but a cosmic principle guiding the continuous refinement and enhancement of all existence. Perfection, in this context, is not a static state but a dynamic journey towards an ideal.

The idea that immortality means "behind time" adds a layer of profundity to Zarathustra's philosophy. It aligns with the neutrosophic understanding that time is not a linear constraint but a dynamic dimension where existence transcends conventional temporal boundaries [25], [26].

Summarizing, Zarathustra's God —an orchestrator of the ceaseless rhythm and progression inherent in existence— is neutrosophic.

3.1.1 | Happiness in suffering

Zarathustra's message, "Happiness is for the one who makes others happy," exhibits neutrosophic characteristics by acknowledging the indeterminacy, incompleteness, and dynamic interplay of opposites inherent in the concept of happiness [27], [28]. It suggests that true happiness involves a nuanced understanding of the interconnectedness between individual and collective well-being. Let us briefly interpret it from a neutrosophic point of view.

Indeterminacy of happiness

Neutrosophy acknowledges the indeterminacy present in many concepts, including happiness. The idea that happiness is for the one who makes others happy introduces an element of uncertainty. What brings happiness to one person may not necessarily bring happiness to another, and the factors contributing to happiness can vary widely.

Incompleteness in the pursuit of happiness

Neutrosophy suggests that the pursuit of happiness is an incomplete endeavor. Happiness is not an absolute state but exists on a spectrum with varying degrees of fulfillment. The message implies that one's happiness is intimately connected to the happiness of others, indicating that a holistic understanding of happiness may involve a broader and more interconnected perspective.

Dynamic interplay of giving and receiving

The message emphasizes a dynamic interplay between giving and receiving happiness. Neutrosophy recognizes that this interaction is not a one-dimensional process but a complex, evolving exchange. The act of making others happy and, in turn, experiencing happiness oneself suggests a dynamic and reciprocal relationship.

Balance between opposites

Neutrosophy explores the balance between opposites. In the context of this message, the happiness of the individual is intricately linked to the happiness of others. This interdependence highlights the dynamic balance between self-interest and the well-being of the community, challenging the idea of happiness as a purely individual pursuit.

Degrees of happiness

Neutrosophy introduces the idea of degrees in various phenomena, and happiness is no exception. The message implies that the happiness derived from making others happy can manifest in different degrees. It might range from partial fulfillment to a more complete and harmonious state where the happiness of the individual and others coexist and reinforce each other.

3.1.2 | Chinvat bridge

Roughly, in theology, the soul is described as the component of the individual that shares divinity and is commonly thought to survive the death of the body. Different faiths and philosophers have created various theories and beliefs about its nature, relationship to the body, origin, and mortality. The Egyptian ka (breath) survived death but remained near the body, but the spiritual ba traveled to the realm of the deceased. The Chinese made a clear distinction between a perishable, sensitive soul that ceases with death and a reasoning principle that endures beyond death. Biblical mentions of the soul associate it with the concept of breath, not differentiating between the ethereal soul and the physical body [29]. In Christian theology, St. Augustine characterized the soul as a 'rider' on the body, underscoring the separation between the material and immaterial aspects, where the soul is regarded as the authentic essence of the individual.

In addition to the body (tanū), it was considered that an individual comprised of a number of spiritual aspects, loosely classified as souls. There are six elements: the animating force, the breath of life, the mind or spirit, the soul, the protecting spirit, and the spiritual double: <ahu>, < vyāna>, <manah>, <ruvan>, <fravashi>, <dainā>.

When a person is born, the essence (fravashi) incarnates the soul (ruvan) into the body to experience the material world, i.e., to suffer and fight bad powers to make decisions about good and evil. Therefore, ruvan actively participates in the manifestation of free will. It should be noted that fravashis are classified into three groups, with the central element composed of uncertainties and neutralities: <the living>, <the yet-unborn>, and <the dead>.

At death, ruvan was considered to dwell on earth for a few days while the gods assessed the deceased's life, and then was reunited with its fravashi and journeyed to the Chinvat Bridge, which spans the gulf between the living and the dead [30].

This bridge has neutrosophic qualities: when the righteous soul starts crossing, it becomes larger and larger, up to thirty-seven' poles' long and wide (equal to nine 'lances' or one 'frasang'). Au contraire, it becomes narrower and narrower to a razor blade dimension when the wicked soul steps onto the bridge, falling into hell because of the bridge's coarseness and sharpness [31]. In other words, the Chinvat Bridge's size is an open dynamic system with indeterminacies or neutralities, therefore, a MultiAlist System.

In MultiAlism, one has dynamics between many neutrosophic triads: (<A_1>, <neutA_1>, <antiA_1>), (<A_2>, <neutA_2>, <antiA_2>), ...

3.2 | Granulating the Six Attributes of Ahura Mazda

Ahura Mazda [32], the supreme deity in Zoroastrianism, and the attributes associated with Ahura Mazda are traditionally understood in a more qualitative and symbolic manner rather than as precisely defined attributes. However, here is an attempt to apply the concept of fuzzy information granulation to break down these attributes into groups of three granules, recognizing the inherent vagueness and imprecision in understanding:

3.2.1 | Good mind

Vohu Manah (Good Mind) signifies the divine attribute of good thought, understanding, and mental clarity:

I. Granule 1: positive and constructive thought processes.

II. Granule 2: spiritual awareness and enlightenment.

III. Granule 3: the divine intellect influencing human understanding.

3.2.2 | Righteousness

Asha Vahishta (Best Truth) embodies the divine principles of righteousness, truth, and order in the universe:

I. Granule 1: moral and ethical perfection.

II. Granule 2: harmony and balance in the cosmic order.

III. Granule 3: just and fair governance of the world.

3.2.3 | Divine power

Khshathra Vairya (Desirable Dominion) represents the divine power and authority that upholds the cosmic order:

I. Granule 1: sovereignty and authority over creation.

II. Granule 2: manifestation of divine power in the world.

III. Granule 3: control and protection of the cosmic order.

3.2.4 | Holy spirit

Spenta Mainyu (Holy Thought) represents the divine aspect of wisdom, creativity, and positive mentality:

I. Granule 1: knowledge and understanding of the universe.

II. Granule 2: divine intelligence guiding creation.

III. Granule 3: insight into the cosmic order.

3.2.5 | Perfection

Haurvatat (Wholeness) stands for the divine quality of completeness, health, and perfection:

I. Granule 1: spiritual purity and holiness.

II. Granule 2: devotion to the divine principles.

III. Granule 3: connection with the divine through rituals and worship.

3.2.6 | Immortality

Ameretat (Immortality) symbolizes the divine attribute of eternal life, immortality, and perpetuity:

I. Granule 1: eternal and unchanging nature.

II. Granule 2: perpetuity of divine existence.

III. Granule 3: endless life and vitality.

It's important to note that these granules are created for illustrative purposes and to highlight the fuzzy nature of these attributes. The attributes of Ahura Mazda are deeply rooted in the religious and philosophical context of Zoroastrianism, and attempting to granulate them introduces a level of interpretation and approximation due to the abstract and symbolic nature of these concepts that consequently fall under a MultiAlist system.

3.3 | Instrumenting the Zoroastrianism: Cyrus the Great

The historical dynamics of social classes, encompassing the neutrosophic triad <SuperClass, MiddleClass, LowerClass>, have significantly shaped the course of societies, both developed and less developed. The <SuperClass is the upper class, above the law, enjoying all the privileges. The <MiddleClass> and the <LowerClass> are the classes on which the laws act with the cruelest harshness and which suffer most of the time. The influence of these social classes becomes particularly evident when examining the reign of Cyrus the Great and the impact of Zoroastrian principles on governance.

The influence of Zoroastrianism on the cultural, social, and political fabric of the region is profound, with Cyrus the Great emerging as a central figure in the historical narrative of this faith. Cyrus the Great played an instrumental role in shaping and propagating Zoroastrianism.

Cyrus, the founder of the Achaemenid Empire, ruled Persia from 559 to 530 BCE. His reign was not only marked by military conquests but also by a visionary approach to governance, which incorporated Zoroastrian principles into the administrative framework of his vast empire. One of the key tenets of Zoroastrianism is the concept of asha, representing 'truth' and 'righteousness'. Cyrus incorporated these ideals into his governance philosophy, promoting justice, fairness, and religious tolerance [33].

The Cyrus Cylinder, a clay cylinder inscribed with Akkadian cuneiform script, stands as a testament to Cyrus's commitment to Zoroastrian principles. This ancient artifact, often hailed as the first charter of human rights, outlines Cyrus's policies of religious freedom and the repatriation of displaced peoples. It reflects the Zoroastrian concept of Vohu Manah, the 'good mind', as Cyrus exhibited a forward-thinking and inclusive approach to ruling a diverse empire [34].

Cyrus's conquest of Babylon in 539 BCE is another pivotal moment where Zoroastrian influence can be observed. His respect for local customs and religions, as documented in the Cyrus Cylinder, contrasts with the imperial norms of the time. This approach not only facilitated a smooth transition of power but also laid the groundwork for the cultural and religious diversity that characterized the Achaemenid Empire.

The Achaemenid Empire under Cyrus set an example of religious tolerance that was not only ahead of its time but also instrumental in the propagation of Zoroastrianism. While Cyrus himself was a follower of the traditional Iranian religion, he extended respect and patronage to various faiths within his realm. This ethos of inclusivity —a MultiAlist trait— in line with Zoroastrian ideals contributed to the empire's stability and the coexistence of diverse religious communities.

Cyrus's legacy as an instrument of Zoroastrianism extends beyond his lifetime. His successors, including Darius the Great and Xerxes, continued to uphold Zoroastrian principles in their rule. The construction of Persepolis, the grand ceremonial capital of the Achaemenid Empire, serves as a tangible expression of the empire's commitment to the divine attributes of Khshathra Vairya (Desirable Dominion) and Haurvatat (Wholeness).

This monumental project symbolized a departure from the exclusive privileges of the SuperClass and a move toward a more inclusive and balanced society. The SuperClass, traditionally above the law, witnessed a shift towards a more just and compassionate governance. The MiddleClass and LowerClass, instead of enduring harsh treatment, found relief and consideration in the policies of Cyrus, reflecting the influence of Zoroastrian ideals on societal dynamics and governance.

3.3.1|A multipolar thinking

Cyrus the Great, the ancient Persian ruler, laid the foundation for a governance model that embraced diversity and promoted unity within a vast empire. Fast forward to the contemporary world, and we find ourselves in a MultiPolar World with distinct geopolitical poles, such as the United States, European Union, China, Russia, India, and Brazil.

Cyrus's legacy of inclusive governance serves as a historical precursor to the modern concept of MultiPolar Thinking. The idea of "E Pluribus Unum" (Out of Many, One), embraced by the European Union, has evolved into a more complex narrative in our MultiPolar World. The question arises: Should it now be "E Pluribus Multa" (Out of Many, Many)? Or perhaps "E Pluribus Omnia" (Out of Many, Everything)? The dynamics of a MultiPolar World encourage us to consider alternative spectra, such as "Ex Uno Omnia" (From One, Everything), "Ex Uno Plures" (From One, Many), or even "Ex Uno Multa" (From One, Many Things).

The shift towards "E Pluribus Plures" (Out of Many, Many) appears to be a consequence of contemporary global geopolitics, a phenomenon predicted by experts decades ago. The world is no longer characterized by a single dominating power but by multiple centers of influence, each contributing to the intricate tapestry of global affairs [35]. In this MultiPolar World, identity politics emerges as a notable fracture line in modern societies. As nations navigate the complexities of coexistence within this globalized framework, the question of identity becomes crucial. The diversity inherent in MultiPolar Thinking requires societies to grapple with the challenges and opportunities posed by various identities, be they cultural, political, or economic. The MultiPolar Thinking encourages a departure from a unipolar or bipolar worldview and necessitates embracing the complexity of a world with multiple centers of power and influence.

4|Other Examples to be Explored

4.1|Gilgamesh: Two-Thirds God and One-Third Human

The legendary figure of Gilgamesh, as described in the ancient Mesopotamian Epic of Gilgamesh, is said to be two-thirds god and one-third human [36]. This unique composition provides an interesting lens through which we can explore the character of Gilgamesh in the context of neutrosophy.

4.1.1|Two-thirds god

Traditional Understanding: the divine aspect of Gilgamesh symbolizes his connection to the gods, granting him exceptional strength, wisdom, and abilities beyond those of ordinary humans. This divine heritage represents his elevated status and sets him apart from mortal beings.

Neutrosophic Perspective: within the neutrosophic framework, the two-thirds god aspect introduces an element of indeterminacy. It suggests that Gilgamesh's divine attributes are not absolute or fully defined. Instead, they exist on a spectrum, incorporating a degree of uncertainty and dynamic interaction with his human nature.

4.1.2|One-third human

Traditional Understanding: the human aspect of Gilgamesh represents his mortality, susceptibility to emotions, and capacity for personal growth and development. This human component adds a layer of relatability to his character, making him more accessible to the human experience.

Neutrosophic Perspective: the one-third human aspect introduces a sense of incompleteness within Gilgamesh. Neutrosophy suggests that his humanity is not a fixed state but a dynamic element that interacts with his divine attributes. This interplay creates a complex and evolving character whose actions and decisions reflect the inherent indeterminacy of the human condition.

4.1.3|Dynamic interplay

Traditional Understanding: the combination of divine and human elements in Gilgamesh creates a powerful and multidimensional character. His journey, marked by quests for immortality and self-discovery, reflects the struggle to reconcile his dual nature.

Neutrosophic Perspective: neutrosophy emphasizes the dynamic interplay of opposites. In the case of Gilgamesh, the constant interaction between his godly and human attributes illustrates the indeterminacy inherent in his character. His decisions, motivations, and the outcomes of his actions are not predetermined but exist within a realm of constant flux and evolution.

4.2|Dharma-Adharma and Karma

The concepts of Dharma, Adharma, and Karma are fundamental aspects of Hindu philosophy and are intricately woven into the fabric of life's ethical and moral considerations [37]. When explored within the context of neutrosophy, which deals with indeterminacy, incompleteness, and the dynamic interplay of opposites, these concepts take on new dimensions, offering a nuanced perspective on the complexities of human existence.

4.2.1|Dharma

Traditional Understanding: Dharma is often translated as duty, righteousness, or cosmic order. It encompasses the moral and ethical principles that govern one's conduct and responsibilities in life. Dharma provides a framework for living in harmony with the universe and upholding virtuous actions.

Neutrosophic Perspective: neutrosophy acknowledges the indeterminacy inherent in ethical decisions. Dharma, within this context, is not a rigid set of absolutes but a dynamic principle that navigates the interplay between opposites. The determination of what is righteous may contain elements of indeterminacy, and the degree of righteousness may exist on a spectrum rather than in binary terms.

4.2.2|Adharma

Traditional Understanding: Adharma represents actions that go against the moral and ethical order. It signifies behaviors that disrupt harmony, create imbalance, and violate the principles of righteousness.

Neutrosophic Perspective: Adharma, in the neutrosophic framework, recognizes the inherent incompleteness in labeling actions as solely right or wrong. The dynamic interplay between Dharma and Adharma suggests that ethical evaluations may involve degrees of indeterminacy, reflecting the complex nature of human choices.

4.2.3 | Karma

Traditional Understanding: Karma is the law of cause and effect, where actions have consequences that may manifest in this life or future incarnations. Positive actions lead to positive outcomes (good karma), while negative actions result in negative consequences (bad karma).

Neutrosophic Perspective: Karma, within the neutrosophic lens, acknowledges the dynamic nature of consequences. The law of cause and effect is not deterministic but exists within a framework of indeterminacy. Actions may yield outcomes that fall within a spectrum of possibilities, reflecting the neutrosophic principle that nothing is absolute and everything exists in a state of constant evolution.

4.3 | The Logic of Diamond Sutra

The Diamond Sutra —known as the Vajracchedikā Prajñāpāramitā Sūtra ("Perfection of Wisdom in 700 Lines"), believed to have been composed in the first century CE, making it one of the earliest known dated printed texts— is a central text in Mahayana Buddhism, particularly within the Zen and Chan traditions.

The Diamond Sutra takes the form of a dialogue between the Buddha and Subhūti. The discourse occurs in the context of a larger discussion on the nature of wisdom and the path to enlightenment. The teaching style is characterized by paradoxical language, negations, and the deconstruction of conventional concepts. This style aims to lead the listener or reader beyond conceptual thinking to a direct experience of reality.

The Diamond Sutra emphasizes the transcendence of dualistic thinking, urging practitioners to go beyond concepts of self and other, existence and non-existence. It encourages a direct realization of the interconnectedness of all phenomena [38].

The Diamond emphasizes the concept of "emptiness" (śūnyatā) and the impermanence of all phenomena. Emptiness is not a nihilistic concept but rather an affirmation of the interdependence and interconnectedness of all things. The phrase "form is emptiness, emptiness is form" encapsulates this teaching, highlighting the dynamic and interrelated nature of reality.

The logic embedded in the Diamond Sutra is more experiential and intuitive, aiming to lead individuals toward direct insight (prajñā) into the nature of reality [39].

While the Logic of the Diamond Sutra and Neutrosophic Logic emerge from different cultural and historical contexts, they share a common thread in challenging fixed notions and embracing the dynamic, interconnected, and indeterminate aspects of reality.

5 | Conclusions

In weaving together the tapestry of Zoroastrianism, the teachings of Zarathustra, and the philosophy of Neutrosophy, a rich interplay of ideas emerges. The eternal struggle between good and evil, truth and falsehood, finds expression in both ancient Persian theology and contemporary philosophical discourse. By embracing the triadic balance inherent in Neutrosophy, individuals can navigate the complexities of existence with a greater awareness of the interconnectedness of opposites, seeking a harmonious balance amidst the perpetual struggle. The Neutrosophic framework invites contemplation on the inherent ambiguity and complexity within seemingly binary oppositions. Just as Zoroastrianism acknowledges the ongoing struggle between Ahura Mazda and Angra Mainyu, Neutrosophy suggests that absolute clarity and certainty are elusive, and reality is often nestled in the indeterminate realm.

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Interacțiunea dinamică a contrariilor în Zoroastrism

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Abstract

Acest articol abordează unele aspecte ale Zoroastrismului, examinând modul în care vechiul sistem de credințe persan se aliniază cu principiile dinamice și indeterminate ale sistemelor Fuzzy, Neutrosofic și MultiAlist. Zoroastrismul, bazat pe lupta eternă dintre bine și rău, lumină și întuneric, se intersectează cu principiile neutrosofice ale indeterminării, incompletitudinii și a interacțiunii dinamice a contrariilor. Viziunea profetului Zarathustra despre un 'Dumnezeu neutrosofic' adaptează unui univers dinamic și în evoluție noțiunile convenționale ale atributelor divine. Înainte de a investiga aceste zone 'vagi', este explorat conceptul de 'graniță conceptuală neclară', subliniind indeterminarea și imprecizia inerente definirii conceptelor opuse sau parțial opuse. Legea Terțului-Multiplu-Infinit Inclus (*Law of included infinitely-many-middles*) sugerează că între opoziții există o multitudine de nuanțe sau valori intermediare. Paradoxul grămezii (paradoxul soriților) provoacă logica tradițională, expunând dificultățile în definirea granițelor vagi. Interpretarea neutrosofică propune introducerea unei zone tampon între opoziții, rezultând paradoxul neutrosofic al soriților. În plus, evidențiem necesitatea unei înțelegeri mai flexibile și mai nuanțate a granițelor conceptuale, recunoscând natura dinamică și indeterminată a multor construcții filosofice și logice. În final, aprofundăm aplicarea neutrosofiei la diferite concepte culturale și filosofice. Figura legendară a lui Ghilgameș, descrisă ca fiind două treimi zeu și o treime om, este examinată atât din perspective tradiționale, cât și neutrosofice. În plus, conceptele hinduse de Dharma, Adharma și Karma sunt reexaminate în contextul neutrosofiei. Logica Sutrei de Diamant în Buddhismul Mahayana, caracterizată prin limbaj paradoxal, se aliniază cu principiile neutrosofice în provocarea noțiunilor fixe și îmbrățișarea aspectelor interconectate și indeterminate ale realității. În ciuda originilor culturale diverse, aceste exemple împărtășesc un fir comun în punerea sub semnul întrebării a absolutismului conceptelor și în acceptarea naturii dinamice a existenței.

Cuvinte cheie

Zoroastrism; Zarathustra; Ahura Mazda; Ghilgameș; Dharma; Adharma; Karma; fericire; Sutra de diamant; podul Chinvat; Cirus cel Mare; vag; neutrosofie; paradoxul soriților; paradoxul neutrosofic al soriților; informații fuzzy; granulare; MultiAlism; sistem MultiAlist, gândire MultiPolară.

1. Introducere

În două articole recente [1], [2], am extins conceptele de "pluralism", "neutrosofie", "neutrosofie rafinată", "mulțime neutrosofică rafinată", "mulțime multineutrosofică" și "plitogenie" [3–8] într-o categorie mai largă, pe care am denumit-o *MultiAlism*,

conceptualizând un *Sistem MultiPolar* format nu doar din multiple elemente care pot fi aleatorii, contradictorii sau adiționale, ci și prin acceptarea caracteristicilor provenind din mai multe sisteme de bază (sisteme UniPolare, BiPolare, TriPolare sau PluriPolare). Unul dintre exemplele unui sistem MultiPolar pe care l-am propus în lucrarea menționată anterior a fost din domeniul religiilor, și anume Zoroastrismul, cu caracteristicile sale aparent neclare. Interpretările occidentale întâmpină dificultăți de încadrare, considerându-l ba monoteist, ba dualist sau chiar pluralist [9]. În cele ce urmează, voi discuta din nou unele limite conceptuale neclare (vagi, *fuzzy*), nu înainte de a reaminti câteva aspecte de bază ale teoriei granularizării informațiilor *fuzzy* și neutrosofice, pentru a reveni la exemplul Zoroastrismului cu scopul unei înțelegeri mai profunde a unor fațete *Fuzzy*, Neutrosofice și MultiAliste ale acestei religii. Dedic ipotezele și inferențele care urmează memoriei profesorului Zadeh, pentru contribuțiile sale profunde într-o gamă largă de domenii științifice și tehnice, dar și pentru legătura pe care a avut-o cu spațiile culturale la care se referă această lucrare [10].

Fig. 1. Lotfi Aliasker Zadeh (părintele teoriilor *fuzzy*) și Florentin Smarandache (părintele teoriilor neutrosofice), participând la BISC FLINT-CIBI *International Workshop on Soft Computing for Internet and Bioinformatics*, care a avut loc la Universitatea din Berkeley, California, între 15 și 19 decembrie 2003. În acest context, am prezentat lucrarea 'Generalizarea mulțimii fuzzy-intuiționiste la mulțimea neutrosofică' (Generalization of the Intuitionistic Fuzzy Set to the Neutrosophic Set).

1.2. Granularea informațiilor fuzzy

Să ne amintim mai întâi esența teoriei granulației informației fuzzy, considerată de Zadeh ca fiind centrală în raționamentul uman [11]. După Zadeh [11], cogniția umană se bazează pe trei concepte fundamentale: granulație (divizarea unui întreg în părți), organizare (integrarea părților într-un întreg) și cauzalitate (asocierea cauzelor cu efectele). Granulația unui "obiect" <A> generează un grup de granule ale lui <A>, care sunt indistincte, similare,

apropiate sau funcțional legate, în general ierarhice în natură, iar atributele și valorile lor sunt fuzzy — deoarece limitele lor nu sunt clar definite.

1.1.1. Mulțimile fuzzy

În teoria tradițională a mulțimilor, un element fie aparține unei mulțimi, fie nu aparține. În teoria mulțimilor fuzzy, elementele pot avea o apartenență parțială la o mulțime. Gradul de apartenență este o valoare între 0 și 1, unde 0 înseamnă lipsa apartenenței, 1 înseamnă apartenență totală, iar valorile intermediare indică o apartenență parțială.

1.1.2. Granularea

Granulația implică gruparea sau *cluster*-izarea elementelor pe baza similarității sau relevanței lor. Este procesul de creare a mulțimilor fuzzy pentru a reprezenta submulțimi de elemente cu caracteristici similare. Granulația ajută la organizarea informațiilor și la gestionarea impreciziei inerente în cunoașterea umană.

1.1.3. Granulele de informații

Granulele de informație sunt rezultatul procesului de granulație. Ele sunt mulțimi fuzzy care reprezintă un nivel mai înalt de abstractizare sau grupare a elementelor. Aceste granule facilitează gestionarea informațiilor complexe și incerte, oferind o reprezentare mai compactă și mai ușor de gestionat.

1.1.4. Raționamentul uman

Zadeh [12] susține că raționamentul uman implică gândirea la diferite niveluri de granularitate. Oamenii organizează în mod natural informațiile în categorii sau concepte care nu sunt strict definite, ci au limite fuzzy. Granulația informației fuzzy reflectă acest proces cognitiv uman, permițând reprezentarea cunoștințelor imprecise și incerte [12]–[14].

1.2. Neutrosofia

Neutrosofia explorează interdependența și interacțiunea dintre opoziții. Înrădăcinate în ideea de neutralitate, conceptele neutrosofice provoacă gândirea binară tradițională și oferă o perspectivă nuanțată asupra complexităților lumii.

Această teorie consideră fiecare noțiune sau idee <A> împreună cu opusul sau negația sa <antiA> și cu spectrul de neutralități <neutA> dintre ele (adică noțiuni sau idei care nu sunt nici <A>, nici <antiA>). Ideile <neutA> și <antiA> sunt împreună denumite <nonA>. Neutrosofia este o generalizare a filozofiei chineze antice Yin-Yang și a dialecticii lui Hegel și Marx (care se bazează doar pe <A> și <antiA>).

De exemplu, în matematică, numerele neutrosofice și algebra neutrosofică oferă instrumente pentru gestionarea incertitudinilor în datele numerice. Aceste concepte își găsesc aplicabilitatea în procesele de luare a deciziilor, unde informațiile incomplete sau ambigue sunt frecvente. În fizică, mecanica neutrosofică extinde mecanica clasică pentru a include forțe indeterminate și măsurători imprecise. Această abordare este deosebit de relevantă în mecanica cuantică, unde natura particulelor și proprietățile lor sfidează adesea descrierea deterministă [3], [6].

1.2.1. Cadrul filosofic

Neutrosofia este o filozofie care recunoaște existența indeterminării în cunoașterea și percepția umană. Ea îmbrățișează ideea că multe fenomene, concepte și propoziții nu sunt nici adevărate, nici false, ci există într-o stare de adevăr parțial. Neutrosofia introduce noțiunea de "neutru", subliniind prezența indeterminării ca un aspect fundamental al realității.

1.2.2. Logica neutrosofică

Logica neutrosofică extinde logica clasică pentru a include informații indeterminate, incomplete și inconsistente. Ea introduce conceptele de grad de apartenență la adevăr, grad de apartenență la falsitate și grad de apartenență la indeterminare, oferind o reprezentare mai flexibilă și nuanțată a realității [4].

1.2.3. Mulțimea neutrosofică

Teoria mulțimilor neutrosofice generalizează teoria mulțimilor clasice pentru a gestiona elementele indeterminate. O mulțime neutrosofică permite includerea obiectelor cu grade de apartenență indeterminate, recunoscând incertitudinea inerentă în definirea limitelor clare pentru anumite concepte.

1.2.4. Probabilitea neutrosofică

Probabilitatea neutrosofică generalizează teoria probabilității clasice pentru a aborda situațiile în care incertitudinea, indeterminarea și ambiguitatea joacă un rol esențial. Ea oferă un cadru pentru gestionarea informațiilor incomplete și reflectă imprecizia inerentă în diverse scenarii din lumea reală, având trei sub-funcții: șansa ca un eveniment să aibă loc, șansa indeterminată ca evenimentul să aibă loc și șansa ca evenimentul să nu aibă loc.

1.2.5. Dincolo de gândirea binară

Abordarea interdisciplinară a neutrosofiei provoacă natura binară a filozofiei occidentale tradiționale și îmbrățișează complexitatea și ambiguitatea inerente în lume. Deși se confruntă cu critici, neutrosofia și disciplinele neutrosofice continuă să stimuleze discuțiile intelectuale și să contribuie la înțelegerea noastră în evoluție a incertitudinii și ambiguității [15].

1.3. Triada Neutrosofică și MultiNeutrosofia

Numeroase școli de gândire au examinat extensiv dinamica dintre opozițiile <A> și <antiA>. Aceste concepte sunt cunoscute sub diverse denumiri, inclusiv dialectica, Yin-Yang, maniheismul, dualismul, Dharma-Adharma și multe altele. Cu toate acestea, partea neutră (sau indeterminată) (<neutA>) dintre aceste opoziții a fost ignorată, eliminată sau retractată. Neutrul sau indeterminabilul, așa cum am subliniat în studiile mele de teorie neutrosofică [3], [4], intervine de obicei în dinamică (sau conflict) de partea unuia sau altuia dintre opuși, înclinând balanța într-o direcție sau alta. Limitările dintre opoziții pot fi fie fluide (când există o suprapunere sau parte indeterminată/neutră între opuse), fie rigide (când <A> și <antiA> sunt clar separate).

Prin urmare, am propus "echilibrul triadic", subliniind interconectarea opușilor. Triada – <adevăr>, <indeterminare>, <falsitate> – formează piatra de temelie a sistemului de gândire neutrosofic. Pentru fiecare element "x" într-o mulțime triadică neutrosofică A, există un neutru

al lui “x” și un opus al lui “x”. De asemenea, neutrul lui “x” trebuie să fie diferit de elementul neutru clasic [16].

MultiNeutrosofia [2] este o neutrosofie multiplicată, concentrându-se pe dinamica dintre $G=\{A_1, A_2, \ldots, A_n\}$ și opusul său $G=\{B_1, B_2, \ldots, B_m\}$, cu neutralii lor $G=\{C_1, C_2, \ldots, C_r\}$. Iată câteva exemple simple de MultiNeutrosofie:

- Un grup de țări împotriva altui grup de țări (în timpul Primului și Celui de-al Doilea Război Mondial), în timp ce un al treilea grup de țări rămâne neutru.
- Un grup de idei împotriva altui grup de idei în filozofie, literatură, știință, cultură, religie.
- O echipă de baschet contra unei alte echipe de baschet (ca grupuri opuse), iar grupul neutru (format din arbitri și susținători, în timp ce susținătorii pot fi împărțiți între susținătorii primei echipe, susținătorii celei de-a doua echipe și susținătorii indecișii).

1.4. Sistemul MultiAlist

În [1], am delimitat un Sistem MultiPolar care este deschis combinațiilor de opoziții și neutralii (indeterminări). Sistemul MultiPolar este o extensie a Sistemului PluriPolar. Sistemele UniPolar, BiPolar, TriPolar și PluriPolar sunt uni-valente (unul exclude pe celălalt), dar Sistemul MultiPolar este multi-valent (conține mai multe sisteme) și admite neutralitatea și indeterminarea între opoziții.

MultiAlismul este un sistem deschis și dinamic cu indeterminări și neutralități, format din elemente provenind din multe sisteme, excedând PluriAlismul, care este un sistem dinamic închis fără indeterminări, un uni-sistem format din elemente provenind dintr-un singur sistem.

2. Granițe conceptuale vagi

Să subliniem limitele vagi dintre concepte opuse sau parțial opuse.

Având în vedere că limitele dintre concepte sunt adesea neclare și imprecise, este de imaginat ca <A>, <neutA>, <antiA> (și <nonA> în mod natural) să aibă părți comune două câte două, sau chiar toate trei.

2.1. Legea terțului-multiplu inclus

Am generalizat în [17] Legea terțului inclus [<A>, <nonA>, și o a treia valoare <T> care rezolvă contradicția lor la un alt nivel al realității] (Lupasco-Basarab).

Legea terțului-multiplu inclus [<A>, <antiA>, și <neutA>, unde <neutA> este divizat într-o multitudine de neutralități între <A> și <antiA>, precum $<neut_1A>$, $<neut_2A>$, etc.]. Valoarea <neutA> (adică Neutralitatea sau Indeterminarea legată de <A>) include de fapt valoarea de mijloc inclusă. De asemenea, Principiul opoziției dinamice [opoziția dintre <A> și <antiA>] este extins la Principiul opoziției dinamice neutrosofice [care înseamnă opoziții între <A>, <antiA>, și <neutA>].

2.1.1. Legea Terțului-Multiplu-Infinit Inclus

Între opusele <A> și <antiA> există un număr infinit de valori de mijloc, denumite $<neutA_i>$, unde $i = 1, 2, ..., \infty$ [18].

Să luăm un exemplu practic: între <Alb> și <Negru> există un număr infinit de nuanțe de culori.

I. Între 100% Adevărat și 100% Fals există un număr infinit de valori de mijloc, care sunt valori de adevăr de forma: d% Adevărat și (1-d)% Fals. Astfel, o propoziție logică poate fi, de exemplu, 1% Adevărat și 99% Fals, 2% Adevărat și 98% Fals, etc., unde d ∈ (0, 1).

II. În mod similar, între 100% apartenență și 100% neapartenență există un număr infinit de valori de mijloc de forma: d% apartenență și (1-d%) neapartenență.

2.2. Paradoxurile soriților

Paradoxele soriților sunt o clasă de paradoxuri care apar din indeterminarea legată de limitele vagi ale conceptelor. Să reamintim exemplul clasic al paradoxului grămezii:

I. Începeți cu o grămadă de nisip. II. Dacă îndepărtați un singur firicel de nisip, grămada rămâne o grămadă. III. Repetați acest proces: îndepărtați câte un firicel de fiecare dată. IV. La un moment dat, veți rămâne cu doar câteva firicele de nisip.

Întrebarea este: în ce punct grămada încetează să mai fie o grămadă?

Paradoxul subliniază dificultatea de a defini când o <grămadă> devine o <non-grămadă>. Problema apare din lipsa inerentă de precizie în conceptele de zi cu zi. Paradoxuri similare pot fi construite pentru alte concepte vagi, precum "înălțime", "chelie", "vârstă" și multe altele.

Aceste paradoxuri contestă abordările tradiționale ale logicii și teoriei mulțimilor, care presupun distincții clare între categorii. Ele ridică întrebări despre natura vagului în lingvistică și despre modul în care o gestionăm în raționamentul logic [19], [20]. Au fost propuse diverse soluții și teorii pentru a aborda paradoxurile soriților [21], inclusiv:

I. **Teoriile Graduale**: Predicațiile vagi ar trebui înțelese în termeni de grade sau grade de apartenență.

II. **Supervaluaționismul**: O afirmație poate fi adevărată, falsă sau indeterminată, permițând mai multe precizări acceptabile ale termenilor vagi.

III. **Contextualismul**: Înțelesul termenilor vagi depinde de contextul în care sunt utilizați, iar limitele conceptelor pot varia în funcție de context.

IV. **Logica Multivalentă**: În locul logicii clasice binare (adevărat sau fals), logica multivalentă introduce mai multe valori de adevăr, acomodând stările intermediare sau *fuzzy* între adevărat și fals.

2.2.1. Interpretarea neutrosofică

Paradoxurile soriților, prin urmare, sunt paradoxuri în care între două entități opuse nu există o frontieră clară, și pot fi interpretate neutrosofic în următorul mod: se consideră o zonă tampon, <neutA> (sau neutră, indeterminare), între opoziții.

Nu există o distincție clară între unele opoziții <A> și <antiA>, unde <A> este un concept și <antiA> este opusul său, ci mai degrabă o zonă tampon <neutA>.

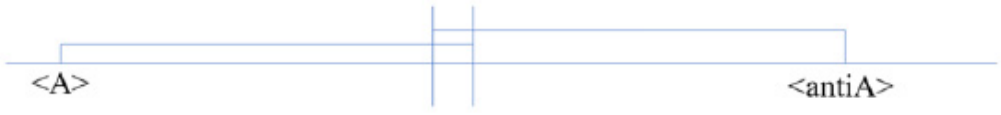

Fig. 2. A și antiA.

Există trei zone: o zonă care reprezintă cu siguranță <A>, o a doua zonă care reprezintă cu siguranță <antiA>, și o zonă ambiguă/nesigură/vagă care reprezintă <neutA>, zona neutră sau indeterminată (nici <A>, nici <antiA>; sau <A> și <antiA> simultan). Prin urmare, un univers de discurs are o partiție neutrosofică cu respect la frontieră dintre opoziții [22], [23].

2.2.2. Paradoxurile neutrosofice ale soriților

Între <A> și <neutA> nu există o frontieră clară – acesta este primul paradox neutrosofic (NSP1).

Apoi, între <neutA> și <antiA>, nu există o frontieră clară – acesta este al doilea paradox neutrosofic (NSP2).

2.2.3. Paradoxuri MultiSoriți

Paradoxurile MultiSoriți — în cadrul Neutrosofiei Rafinate (care este isomorfă cu MultiNeutrosofia) — au forma n-dimensională: $T_1, T_2, ..., T_p, I_1, I_2, ..., I_r, F_1, F_2, ..., F_s$, unde p, r, s sunt numere întregi ≥ 0, și cel puțin unul dintre p, r, s este ≥ 2 pentru a asigura rafinamentul (sau multiplicitatea), $p + r + s = n$, unde fiecare T_j, I_k, F_l sunt Adevăr / Indeterminare / Falsitate rafinate (sau respectiv multiple).

Frontierele dintre toate aceste n sub-componente (sau multi-componente), luate două câte două, sunt neclare, vagi și fluide. Există $C(n,2)$ (combinări de n luate câte 2) $= n(n-1)/2$ paradoxuri ale soriților rafinate/MultiNeutrosofice.

3. Explorarea interacțiunii dintre Zoroastrism și neutrosofie

Zoroastrismul își are originile în învățăturile profetului Zarathustra (Zoroastru). Acest sistem de credințe antic, originar din Persia (Iranul de astăzi), subliniază lupta eternă (dinamică) între bine și rău, lumină și întuneric.

Zarathustra și-a transmis învățăturile într-o lume în care dualitatea avea un rol central. Principalele învățături ale zoroastrismului se axează pe conflictul etern dintre *Ahura Mazda*, divinitatea supremă care reprezintă bunătatea și adevărul, și *Angra Mainyu*, forța distrugătoare ce întruchipează minciuna și răul. Această bătălie cosmică, reflectată în principiile opuse ale 'asha' (adevăr) și 'druj' (minciună), oglindește dualitățile fundamentale inerente existenței, dar și zona neutrosofică care se întinde de la *asha* la *druj* [24].

În contextul zoroastrian, echilibrul triadic neutrosofic găsește rezonanță în interacțiunea perpetuă dintre bine, rău și în spațiul ambigu dintre acestea.

Comentariul lui Edalatpanah: "Juxtapunerea învățăturilor lui Zarathustra despre conceptele de bine și rău cu principiile neutrosofiei, care se concentrează pe neutralitate și spectrul fenomenelor ideative, promite să deschidă noi căi de interpretare pentru aceste învățături antice."

Viziunea lui Zarathustra asupra ordinii cosmice și responsabilității morale se aliniază cu noțiunea neutrosofică că adevărul, falsitatea și indeterminarea sunt interconectate și inseparabile. Învățăturile profetului subliniază necesitatea ca indivizii să participe activ în lupta eternă pentru dreptate, recunoscând complexitățile și incertitudinile inerente ale alegerilor lor.

3.1. Zeul neutrosofic al lui Zarathustra

Zarathustra a introdus o concepție a divinității care transcende noțiunile binare convenționale adesea asociate cu zeii. Centrală în filosofia lui Zarathustra este ideea că Dumnezeu întruchipează atât atribute masculine, cât și feminine și că existența este o relație dinamică între opozițiile <A> și <antiA>, care reflectă principiile neutrosofiei.

În plus, concepția lui Zarathustra despre Dumnezeu contestă omnipotența și omnisciența tradițional atribuite ființelor divine. În locul unei divinități atotcunoscătoare și atotputernice, Dumnezeul lui Zarathustra este prezentat ca un creator al unui univers dinamic—un univers în continuă schimbare și progres către perfecțiune. Acest lucru se aliniază din nou cu noțiunea neutrosofică că realitatea este în mod inerent indeterminată, incompletă și în evoluție.

Dinamica dintre <A> (Bine) și <antiA> (Rău) în viziunea lui Zarathustra sunt elemente esențiale pentru înțelegerea naturii neutrosofice a Dumnezeului său, reflectând o realitate în care binele și răul coexistă în diverse grade. Universul, în această viziune, este un joc dinamic între forțe opuse, cu rezultate ce variază de la stări parțial bune și parțial rele până la stări de neutralitate completă în care gradele de bunătate și răutate se anulază sau se echilibrează reciproc.

Această perspectivă are implicații profunde pentru considerațiile etice din cadrul zoroastrismului. În loc să privim moralitatea în termeni absoluți, cadrul lui Zarathustra sugerează o înțelegere mai nuanțată, similară cu arhitectura neutrosofică. El invită adepții să navigheze prin rețeaua complexă a existenței, recunoscând că judecățile morale pot cădea în spectrul a ceea ce neutrosofia numește <neutA>, unde acțiunile și intențiile nu sunt definite strict ca fiind complet bune sau complet rele.

Pânza filozofică a lui Zarathustra se extinde și mai departe atunci când explorăm natura ritmică și dinamică a existenței. În viziunea sa, totul urmează o succesiune nesfârșită de schimbări, asemănătoare cu principiile neutrosofiei care recunosc indeterminarea și evoluția inerente în toate aspectele realității. Natura ritmică a existenței implică un dans perpetuu între opoziții, unde divinul, fiind neutrosofic, orchestrează simfonia creației.

Prin urmare, accentul pus de Zarathustra pe progres reflectă principiul neutrosofic că nimic nu este static. În acest univers dinamic, progresul nu este doar o opțiune, ci o necesitate. Stagnarea, așa cum sugerează Zarathustra, poate duce la nenorocire — o idee în concordanță cu înțelegerea neutrosofică care recunoaște consecințele potențiale ale rezistenței la schimbare și creștere. Dumnezeul neutrosofic încurajează evoluția continuă, promovând un univers într-o stare de devenire perpetuă.

Conceptul de auto-dominare introduce o dimensiune a responsabilității personale în filosofia lui Zarathustra. Această stăpânire de sine este crucială în navigarea interacțiunii dinamice dintre <A> și <antiA>, contribuind la progresia armonioasă către perfecțiune. Evoluția nu este doar un concept biologic, ci un principiu cosmic care ghidează rafinarea și îmbunătățirea continuă a întregii existențe. Perfecțiunea, în acest context, nu este un dat static, ci o călătorie dinamică spre un ideal.

Ideea că nemurirea înseamnă "dincolo de timp" adaugă un strat de profunzime filosofiei lui Zarathustra. Aceasta se aliniază cu înțelegerea neutrosofică că timpul nu este o constrângere

liniară, ci o dimensiune dinamică în care existența transcende limitele temporale convenționale [25], [26].

În concluzie, Dumnezeul lui Zarathustra — un orchestrator al ritmului neîncetat și al progresiei inerente în existență — este neutrosofic.

3.1.1. Fericirea în suferință

Mesajul lui Zarathustra, "Fericirea este pentru cel care îi face pe alții fericiți," prezintă caracteristici neutrosofice prin recunoașterea indeterminării, incompletitudinii și interacțiunii dinamice a opuselor inerente conceptului de fericire [27], [28]. Acesta sugerează că adevărata fericire implică o înțelegere nuanțată a interconectivității dintre bunăstarea individuală și cea colectivă.

Iată câteva interpretări dintr-o perspectivă neutrosofică:

Indeterminarea fericirii

Neutrosofia recunoaște indeterminarea prezentă în multe concepte, inclusiv în conceptul de 'fericire'. Ideea că "fericirea este pentru cel care îi face pe alții fericiți" introduce un element de incertitudine. Ce aduce fericire unei persoane nu aduce neapărat fericire și altora, iar factorii care contribuie la fericire pot varia considerabil.

Incompletitudinea în căutarea fericirii

Neutrosofia sugerează că această căutare a fericirii este o întreprindere incompletă. Fericirea nu este un dat absolut, ci există pe un spectru cu grade variabile de împlinire. Mesajul implică faptul că fericirea unei persoane este intim legată de fericirea altora, indicând că o înțelegere holistică a fericirii ar putea implica o perspectivă mai amplă, interconectată.

Interacțiunea dinamică dintre dăruire și acceptare

Mesajul subliniază un joc dinamic între oferirea și primirea fericirii. Neutrosofia recunoaște că această interacțiune nu este un proces unidimensional, ci un schimb complex și în evoluție. Actul de a face pe alții fericiți și, la rândul său, de a experimenta fericirea personală sugerează o relație dinamică și reciprocă.

Echilibrul între opoziții

Neutrosofia explorează echilibrul dintre opoziții. În contextul acestui mesaj, fericirea individuală este strâns legată de fericirea altora. Această interdependență subliniază echilibrul dinamic dintre interesul personal și bunăstarea comunității, contestând ideea că fericirea este un dat pur individuală.

Grade de fericire

Neutrosofia introduce ideea gradelor în diferite fenomene, iar fericirea nu face excepție. Mesajul sugerează că fericirea derivată din a face pe alții fericiți poate manifesta diferite grade. Aceasta poate varia de la împlinire parțială la o stare mai completă și armonioasă, unde fericirea individului și a altora coexistă și se întăresc reciproc.

3.1.2. Podul Chinvat

În teologie, *sufletul* este descris, în general, ca fiind componenta individului care împărtășește divinitatea și este adesea considerat că supraviețuiește morții corpului. Diferite

credințe și filosofi au elaborat teorii și credințe variate despre natura sa, relația cu corpul, originea și mortalitatea sa. De exemplu:

- Egiptenii: Conceptul egiptean de suflet includea două componente principale: *ka* (respirația), care supraviețuia morții dar rămânea în apropierea corpului, și *ba*, care călătorea în regatul morților. Aceste două aspecte reflectau credința în dualitatea și complexitatea existenței *post-mortem*.
- Chinezii: În tradiția chineză, se făcea o distincție clară între sufletul perisabil, sensibil, care înceta odată cu moartea, și principiul rațional care dăinuia dincolo de moarte. Această distincție reflecta o viziune asupra sufletului ca fiind alcătuit dintr-un aspect fizic și unul spiritual.
- Biblia: În tradiția biblică, sufletul este adesea asociat cu conceptul de respirație, fără a face o distincție clară între sufletul etereal și corpul fizic. Acest concept reflectă o viziune integrată asupra sufletului, care nu se separa de corpul fizic într-un mod clar.
- Teologia creștină: Sf. Augustin a caracterizat sufletul ca un "călăreț" al corpului, subliniind separația dintre aspectele materiale și imateriale, unde sufletul era considerat esența autentică a individului. Această viziune reflectă o separare clară între corpul fizic și sufletul imaterial, cu sufletul având rolul de a ghida și de a defini adevărata natură a individului.

În Zoroastrism, dincolo de corp (*tanū*), se considera că un individ este compus dintr-un număr de aspecte spirituale, clasificate liber ca 'suflete'. Acestea sunt șase elemente: forța animatoare, respirația vieții, mintea sau spiritul, sufletul, spiritul protector și dublul spiritual: <ahu>, <vyāna>, <manah>, <ruvan>, <fravashi>, <dainā>.

Când o persoană se naște, esența (*fravashi*) încarnează sufletul (*ruvan*) în corp pentru a experimenta lumea materială, adică pentru a suferi și a lupta împotriva forțelor rele pentru a lua decizii despre bine și rău. Prin urmare, *ruvan* participă activ la manifestarea liberului arbitru. Este de remarcat că *fravashis* sunt clasificați în trei grupe, cu elementul central compus din incertitudini și neutralități: <cei vii>, <cei încă nenăscuți> și <cei morți>.

La moarte, *ruvan* se credea că rămâne pe pământ timp de câteva zile în timp ce zeii evaluau viața defunctului, apoi se reunea cu *fravashi*-ul său și călătorea spre *Podul Chinvat*, care traversează prăpastia dintre cei vii și cei morți [30].

Acest pod are trăsături neutrosofice: când sufletul drept începe să-l traverseze, acesta devine din ce în ce mai mare, ajungând până la treizeci și șapte de "stâlpi" lungime și lățime (egal cu 9 "lănci" sau 1 "frasang"). Dimpotrivă, devine din ce în ce mai îngust până la dimensiunea unei lame de ras atunci când sufletul rău pășește pe pod, căzând în iad din cauza grosimii și tăișului podului [31]. Cu alte cuvinte, dimensiunea Podului Chinvat este un sistem dinamic deschis cu incertitudini sau neutralități, fiind astfel un Sistem MultiAlist.

În MultiAlism, există dinamici între multe triade neutrosofice: (<A_1>, <neutA_1>, <antiA_1>), (<A_2>, <neutA_2>, <antiA_2>), ...

3.2. Granularea celor Șase Atribute ale lui Ahura Mazda

Ahura Mazda [32], zeitatea supremă în Zoroastrism, și atributele asociate cu Ahura Mazda sunt tradițional înțelese într-un mod calitativ și simbolic, mai degrabă decât ca atribute precis definite. Încercăm să aplicăm conceptul de granulare a informațiilor *fuzzy* pentru a

descompune aceste atribute în grupuri de trei granule, recunoscând vagul și imprecizia inerentă în înțelegere:

3.2.1. Bunăvoința

Vohu Manah (Mintea Bună) semnifică atributul divin al gândirii bune, înțelegerii și clarității mentale:

I. Granula 1: procese de gândire pozitive și constructive.

II. Granula 2: conștientizare spirituală și iluminare.

III. Granula 3: intelectul divin care influențează înțelegerea umană.

3.2.2. Dreptatea

Asha Vahishta (Cel Mai Bun Adevăr) întruchipează principiile divine ale dreptății, adevărului și ordinii în univers:

I. Granula 1: perfecțiunea morală și etică.

II. Granula 2: armonia și echilibrul în ordinea cosmică.

III. Granula 3: guvernarea justă și echitabilă a lumii.

3.2.3. Dominația dezirabilă

Khshathra Vairya (Dominație Dorită) reprezintă puterea și autoritatea divine care mențin ordinea cosmică:

I. Granula 1: suveranitate și autoritate asupra creației.

II. Granula 2: manifestarea puterii divine în lume.

III. Granula 3: controlul și protecția ordinii cosmice.

3.2.4. Spiritul sfânt

Spenta Mainyu (Gândul Sfânt) reprezintă aspectul divin al înțelepciunii, creativității și mentalității pozitive:

I. Granula 1: cunoașterea și înțelegerea universului.

II. Granula 2: inteligența divină care ghidează creația.

III. Granula 3: perspicacitatea în ordinea cosmică.

3.2.5. Perfecțiunea

Haurvatat (Completitudinea) reprezintă calitatea divină a integrității, sănătății și perfecțiunii:

I. Granula 1: puritatea și sfințenia spirituală.

II. Granula 2: devotamentul față de principiile divine.

III. Granula 3: conexiunea cu divinitatea prin ritualuri și adorare.

3.2.6. Nemurirea

Ameretat (Nemurire) simbolizează atributul divin al vieții eterne, nemuririi și perpetuității:

I. Granula 1: natura eternă și neschimbătoare.

II. Granula 2: perpetuitatea existenței divine.

III. Granula 3: viața și vitalitatea nesfârșite.

Este important de menționat că aceste granule sunt create în scop ilustrativ și pentru a evidenția natura vagă a acestor atribute. Atributele lui Ahura Mazda sunt profund înrădăcinate în contextul religios și filozofic al Zoroastrismului, iar încercarea de a le granula introduce un nivel de interpretare și aproximare din cauza naturii abstracte și simbolice a acestor concepte, care, în consecință, se încadrează într-un sistem MultiAlist.

3.3. Instrumentalizarea Zoroastrismului: Cirus cel Mare

Dinamica istorică a claselor sociale, cuprinzând triada neutrosofică <SuperClasa, Clasa-de-Mijloc, Clasa-de-Jos>, a modelat semnificativ cursul societăților, atât dezvoltate, cât și mai puțin dezvoltate. <SuperClasa> este clasa superioară, deasupra legii, bucurându-se de toate privilegiile. <Clasa-de-Mijloc> și <Clasa-de-Jos> sunt clasele asupra cărora legile acționează cu cea mai mare asprime și care suferă cel mai adesea. Influența acestor clase sociale devine deosebit de evidentă atunci când se examinează domnia lui Cirus cel Mare și impactul principiilor zoroastriene asupra guvernării.

Influența Zoroastrismului asupra țesăturii culturale, sociale și politice a regiunii este profundă, cu Cirus cel Mare având un rol central în narațiunea istorică a acestei credințe. Cirus cel Mare a jucat un rol esențial în conturarea și propagarea Zoroastrismului.

Cirus, fondatorul Imperiului Ahemenid, a domnit în Persia din 559 până în 530 î.Hr. Domnia lui nu a fost marcată doar de cuceriri militare, ci și de o abordare vizionară a guvernării, care a încorporat principiile zoroastriene în cadrul administrativ al vastului său imperiu. Unul dintre conceptele fundamentale ale Zoroastrismului este ‘asha’, care reprezintă ‘adevărul’ și ‘dreptatea’. Cirus a integrat aceste idealuri în filosofia sa de guvernare, promovând justiția, echitatea și toleranța religioasă [33].

Cilindrul lui Cirus, un cilindru de argilă cu scriere cuneiformă akkadiană, stă ca o mărturie a angajamentului lui Cirus față de principiile zoroastriene. Acest artefact antic, considerat primul document cunoscut al drepturilor omului, conturează politicile lui Cirus privind libertatea religioasă și repatrierea popoarelor strămutate. Reflectă conceptul zoroastrian de ‘Vohu Manah’, ‘mintea bună’, deoarece Cirus a demonstrat o abordare vizionară și incluzivă în conducerea unui imperiu divers [34].

Cucerirea Babilonului de către Cirus în 539 î.Hr. este un alt moment crucial în care se poate observa influența zoroastriană. Respectul său pentru obiceiurile și religiile locale, așa cum este documentat în *Cilindrul lui Cirus*, contrastează cu normele imperiale ale vremii. Această abordare nu numai că a facilitat o tranziție lină a puterii, dar a pus și bazele diversității culturale și religioase care a caracterizat Imperiul Ahemenid.

Imperiul Ahemenid sub conducerea lui Cirus a oferit un exemplu de toleranță religioasă care nu numai că era protocronică și inovatoare, dar a fost și esențială pentru propagarea Zoroastrismului. Deși Cirus însuși era un adept al religiei tradiționale iraniene, a extins respectul și patronajul către diverse credințe din cadrul regatului său. Această etică a incluziunii — o trăsătură MultiAlistă — în conformitate cu idealurile zoroastriene a contribuit la stabilitatea imperiului și la coexistența comunităților religioase diverse.

Moștenirea lui Cirus ca instrument al Zoroastrismului se extinde dincolo de viața sa. Succesorii săi, inclusiv Darius cel Mare și Xerxes, au continuat să susțină principiile zoroastriene sub conducerea lor. Construcția lui Persepolis, grandioasa capitală ceremonială a

Imperiului Ahemenid, servește ca o expresie tangibilă a angajamentului imperiului față de atributele divine ale 'Khshathra Vairya' ('Domnia Dorită') și 'Haurvatat' ('Întregime').

Acest proiect monumental a simbolizat o despărțire de privilegiile exclusive ale SuperClasei și o mișcare către o societate mai inclusivă și echilibrată. SuperClasa, tradițional deasupra legii, a asistat la o schimbare către o guvernare mai justă și mai compasională. <Clasa-de-Mijloc> și <Clasa-de-Jos> nu mai suferă un tratament aspru, ci găsesc ușurare și considerație în politicile lui Cirus, reflectând influența idealurilor zoroastriene asupra dinamicii sociale și a guvernării.

3.3.1. O gândire multipolară

Cirus cel Mare, conducătorul persan antic, a pus bazele unui model de guvernare care îmbrățișa diversitatea și promova unitatea într-un vast imperiu. Avansând în lumea contemporană, ne găsim într-o lume MultiPolară cu poli geopolitici distincți, precum Statele Unite, Uniunea Europeană, China, Rusia, India sau Brazilia.

Moștenirea lui Cirus în materie de guvernare incluzivă servește drept precursor istoric al conceptului modern de Gândire MultiPolară. Ideea de *E Pluribus Unum* ("Din Multe, Unul"), adoptată de Uniunea Europeană, s-a transformat într-o narațiune mai complexă în lumea noastră MultiPolară. Se ridică întrebarea: Ar trebui acum să fie *E Pluribus Multa* ("Din Multe, Mult")? Sau mai degrabă *E Pluribus Omnia* ("Din Multe, Totul")? Dinamica unei lumi MultiPolare ne încurajează să luăm în considerare spectre alternative, precum *Ex Uno Omnia* ("Dintr-Unul, Totul"), *Ex Uno Plures* ("Dintr-Unul, Mult") sau chiar *Ex Uno Multa* ("Dintr-Unul, Multe").

Schimbarea către *E Pluribus Plures*" ("Din Multe, Mult") pare a fi o consecință a geopoliticii globale contemporane, un fenomen prezis de experți cu decenii în urmă. Lumea nu mai este caracterizată de o singură putere dominantă, ci de multiple centre de influență, fiecare contribuind la țesătura complexă a afacerilor globale [35]. În această Lume MultiPolară, politica identității apare ca o linie de fractură notabilă în societățile moderne. Pe măsură ce națiunile navighează complexitățile conviețuirii în cadrul globalizat, problema identității devine crucială. Diversitatea inerentă în Gândirea MultiPolară necesită ca societățile să se confrunte cu provocările și oportunitățile prezentate de diferitele identități, fie ele culturale, politice sau economice. Gândirea MultiPolară încurajează o despărțire de o viziune unipolară sau bipolară și necesită îmbrățișarea complexității unei lumi cu multiple centre de putere și influență.

4. Alte exemple de explorat

4.1. Gilgameș: două treimi zeu și o treime om

Figura legendară a lui Ghilgameș din Mesopotamia, care este socotit a fi două treimi zeu și o treime om [36] este un compozit unic care oferă o lentilă interesantă prin care putem explora caracterul lui Ghilgameș în contextul neutrosofiei.

4.1.1. Două treimi zeu

Înțelegerea Tradițională: aspectul divin al lui Ghilgameș simbolizează conexiunea sa cu zeii, oferindu-i forță excepțională, înțelepciune și abilități dincolo de cele ale oamenilor

obișnuiți. Această moștenire divină reprezintă statutul său înalt și îl diferențiază de ființele muritoare.

Perspectiva Neutrosofică: în cadrul neutrosofic, aspectul de două treimi zeu introduce un element de indeterminare. Acesta sugerează că atributele divine ale lui Ghilgameș nu sunt absolute sau complet definite. În schimb, ele există pe un spectru, incorporând un grad de incertitudine și interacțiune dinamică cu natura sa umană.

4.1.2. O treime om

Înțelegerea Tradițională: aspectul uman al lui Ghilgameș reprezintă mortalitatea sa, susceptibilitatea la emoții și capacitatea de creștere și dezvoltare personală. Acest element uman adaugă un strat de relatibil caracterului său, făcându-l mai accesibil experienței umane.

Perspectiva Neutrosofică: aspectul de o treime uman introduce un sentiment de incompletitudine în cadrul lui Ghilgameș. Neutrosofia sugerează că umanitatea sa nu este o stare fixă, ci un element dinamic care interacționează cu atributele sale divine. Această interacțiune creează un caracter complex și în evoluție, al cărui comportament și decizii reflectă indeterminarea inerentă a condiției umane.

4.1.3. Interacțiunea dinamică

Înțelegerea Tradițională: combinația de elemente divine și umane în Ghilgameș creează un caracter puternic și multidimensional. Călătoria sa, marcată de căutări pentru nemurire și auto-descoperire, reflectă lupta de a reconcilia natura sa dual.

Perspectiva Neutrosofică: neutrosofia subliniază interacțiunea dinamică a contrariilor. În cazul lui Ghilgameș, interacțiunea constantă între atributele sale divine și umane ilustrează indeterminarea inerentă caracterului său. Deciziile, motivațiile și rezultatele acțiunilor sale nu sunt prestabilite, ci există într-un domeniu de flux constant și evoluție.

4.2. Dharma-Adharma și Karma

Conceptele de *Dharma*, *Adharma* și *Karma* sunt aspecte fundamentale ale filozofiei hindu și sunt strâns legate de țesătura considerațiilor etice și morale ale vieții [37]. Atunci când sunt explorate în contextul neutrosofiei, care se ocupă cu indeterminarea, incompletitudinea și interacțiunea dinamică a contrariilor, aceste concepte dobândesc noi dimensiuni, oferind o perspectivă nuanțată asupra complexităților existenței umane.

4.2.1. Dharma

Înțelegerea Tradițională: Dharma este adesea tradusă ca datorie, dreptate sau ordine cosmică. Ea cuprinde principiile morale și etice care reglementează comportamentul și responsabilitățile unui individ în viață. *Dharma* oferă un cadru pentru a trăi în armonie cu universul și pentru a susține acțiuni virtuoase.

Perspectiva Neutrosofică: neutrosofia recunoaște indeterminarea inerentă în deciziile etice. *Dharma*, în acest context, nu este un set rigid de absoluturi, ci un principiu dinamic care navighează interacțiunea dintre contrarii. Determinarea a ceea ce este drept poate conține elemente de indeterminare, iar gradul de dreptate poate exista pe un spectru, mai degrabă decât în termeni binari.

4.2.2. Adharma

Înțelegerea Tradițională: Adharma reprezintă acțiuni care sunt împotriva ordinii morale și etice. Aceasta semnifică comportamente care perturbă armonia, creează dezechilibru și încalcă principiile dreptății.

Perspectiva Neutrosofică: Adharma, în cadrul neutrosofic, recunoaște incompletitudinea inerentă în etichetarea acțiunilor ca fiind exclusiv corecte sau greșite. Interacțiunea dinamică dintre *Dharma* și *Adharma* sugerează că evaluările etice pot implica grade de indeterminare, reflectând natura complexă a alegerilor umane.

4.2.3. Karma

Înțelegerea Tradițională: Karma este legea cauzei și efectului, conform căreia acțiunile au consecințe ce pot apărea în această viață sau în viitoare încarnări. Acțiunile pozitive conduc la rezultate pozitive (*karma* bună), în timp ce acțiunile negative duc la consecințe negative (*karma* rea).

Perspectiva Neutrosofică: Karma, prin prisma neutrosofică, recunoaște natura dinamică a consecințelor. Legea cauzei și efectului nu este deterministă, ci există într-un cadru de indeterminare. Acțiunile pot genera rezultate care se încadrează într-un spectru de posibilități, reflectând principiul neutrosofic conform căruia nimic nu este absolut și totul există într-o stare de evoluție constantă.

4.3. Logica Sutrei diamantului

Sutra Diamantului — cunoscută sub numele de "Vajracchedikā Prajñāpāramitā Sūtra" ("Perfecțiunea Înțelepciunii în 700 Versuri"), despre care se crede că a fost compusă în primul secol d.Hr., făcând-o unul dintre cele mai vechi texte cunoscute — este un text central în Buddhismul Mahayana, în special în tradițiile Zen și Chan.

Sutra Diamantului se prezintă sub forma unui dialog între Buddha și Subhūti. Discuția are loc în contextul unei conversații mai ample despre natura înțelepciunii și calea spre iluminare. Stilul învățăturii este caracterizat de limbaj paradoxal, negări și deconstrucția conceptelor convenționale. Acest stil are scopul de a conduce ascultătorul sau cititorul dincolo de gândirea conceptuală, către o experiență directă a realității.

Sutra Diamantului subliniază transcenderea gândirii dualiste, îndemnând practicanții să depășească conceptele de 'sine', 'celălalt', 'existență' și 'non-existență'. Aceasta încurajează realizarea directă a interconectivității tuturor fenomenelor [38].

Sutra Diamantului subliniază conceptul de "goliciune" (śūnyatā) și impermanența tuturor fenomenelor. Goliciunea nu este un concept nihilist, ci mai degrabă o afirmare a interdependenței și interconectivității tuturor lucrurilor. Expresia "forma este goliciune, goliciunea este formă" înfățișează această învățătură, punând în evidență natura dinamică și interrelaționată a realității.

Logica încorporată în Sutra Diamantului este experiențială și intuitivă, având scopul de a conduce indivizii către o înțelegere directă (*prajñā*) a naturii realității [39].

Deși Logica Sutra Diamantului și Logica Neutrosofică provin din contexte culturale și istorice diferite, ele împărtășesc un fir comun în provocarea noțiunilor fixe și în îmbrățișarea aspectelor dinamice, interconectate și indeterminate ale realității.

Concluzii

Împletind împreună tapiseria Zoroastrismului, învățăturile lui Zarathustra și filosofia Neutrosofiei, emerge un bogat joc de idei. Lupta eternă dintre bine și rău, adevăr și falsitate, își găsește expresia atât în teologia persană antică, cât și în discursul filosofic contemporan. Prin îmbrățișarea echilibrului triadic inerent în Neutrosofie, indivizii pot naviga complexitățile existenței cu o conștientizare mai profundă a interconexiunii contrariilor, căutând un echilibru armonios în mijlocul luptei perpetue. Cadrul Neutrosofic invită la contemplarea ambiguității și complexității inerente în opozițiile aparent binare. Așa cum Zoroastrismul recunoaște lupta continuă dintre Ahura Mazda și Angra Mainyu, Neutrosofia sugerează că claritatea și certitudinea absolută sunt eluzive, iar realitatea se află adesea în domeniul indeterminat.

Mulțumiri

Aș dori să îmi exprim recunoștința sinceră față de S.A. Edalatpanah, al cărui comentariu pe ResearchGate ─ "Juxtapunerea învățăturilor lui Zarathustra despre conceptele de bine și rău cu principiile neutrosofiei, care se concentrează pe neutralitate și spectrul fenomenelor ideatice, promite să deschidă noi căi de interpretare pentru aceste învățături antice." (S.A. Edalatpanah, ResearchGate, 4 decembrie 2023) ─, precum și discuțiile ulterioare, au inspirat această explorare.

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Neutrosophy means: Common Parts to Uncommon Things and Uncommon Parts to Common Things

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Abstract: Let <*A*> be an item, concept, idea, proposition, school of thought, current, theory, etc. and <anti*A*> be the opposite of <*A*>. Analogously for <*B*> and its opposite <anti*B*>.

Neutrosophy means to find:

(i) common parts to uncommon things (that is, <*A*> and <anti*A*> have something in common, or their intersection <*A*>∩<anti*A*> is not empty), and vice versa:

(ii) (ii) uncommon parts to common things (the two equal items <*A*>=<*B*> have also uncommon parts, either <*A*>∩<anti*B*> is not empty, or <anti*A*>∩<*B*> is not empty).

Both, the *Common Parts to Uncommon Things*, and the *Uncommon Parts to Common Things*, end up being parts of indeterminacy / neutrality situated between the opposites: denoted by <neut*A*>, which means neither <*A*> nor <anti*A*>, but in between them; and respectively by <neut*B*>, which similarly means neither <*B*> nor <anti*B*>, but in between them.

Keywords: Neutrosophy, Paradoxism, Dialectics, Yin Yang, Soft Sciences, Capitalism, Socialism, Psychoanalysis, Analytical Psychology, Democracy, Representative Democracy, Alchemy, Science, Dialectics, Structuralism, Post-Structuralism, Social Systems Theory, Paradoxical Intention, Pro-choice, Pro-life, Cognitive-Behavioral Therapy, Psychodynamic Therapy.

1. Introduction

Neutrosophy, a philosophical framework that I developed more than two decades ago [Smarandache 1998], explores the relationships and interactions between opposites and their neutralities/indeterminacies, seeking to find commonalities between them and identifying uncommon elements within similar entities, emphasizing the complexity and interconnectedness of concepts. Neutrosophy transcends traditional binary thinking by examining the interplay between opposites and the neutralities/indeterminacies between them.

Neutrosophy is an extension of the movement called *Paradoxism* [Smarandache 1980], in literature/arts/science/philosophy, *Dialectics* [Hegel], [Marx], and *Yin Yang Ancient Chinese Philosophy* [1] – because the last three schools took into consideration the dynamics between the opposites only, while omitting their neutralities/indeterminacies that play an important role in the balance between opposites.

[1] Britannica, The Editors of Encyclopaedia. "yinyang". Encyclopedia Britannica, 12 Feb. 2024, https://www.britannica.com/topic/yinyang. Accessed 30 May 2024.

By identifying and studying common parts in uncommon things and uncommon parts in common things, neutrosophy reveals the complexity and interconnectedness inherent in various ideas and phenomena.

This article shortly presents the two core principles of neutrosophic research in soft sciences, then exemplifies these neutrosophic principles, and suggests applications, illustrating how neutrosophy provides a nuanced understanding of the world.

2. Two Neutrosophic Core Principles

2.1. Searching for Common Parts in Uncommon Things

This principle posits the study of opposing concepts that share (some) common elements. For example:

- **Political Ideologies**: *Capitalism*[2] [Hickel] and *Socialism*[3] [Cole] both pretend their aim to be the improvement of societal welfare and economic prosperity, though their proposed methods differ.
- **Psychological Theories**: Freud's *Psychoanalysis*[4] [Freud] and Jung's *Analytical Psychology*[5] [Jung] both focus on the unconscious mind but having distinct theoretical foundations and methodologies.

2.2. Searching for Uncommon Parts in Common Things

This principle propounds the research of similar or equivalent concepts containing elements that are distinct or oppositional.

- **Forms of Governance**: *Democracy*[6] and *Representative Democracy*[7] both emphasize the role of the people, yet representative democracy involves elected officials, while direct democracy involves direct citizen participation. [Landemore]
- **Forms of Thinking:** *Empiricism* [Gupta] *vs. Alchemy* [Ferguson]: Both seek understanding of the world, but *alchemy*[8] lacks the methodology and empirical support of *science*[9].

3. Investigation in Soft Sciences

3.1. Hegelian Dialectics

[2] Britannica, The Editors of Encyclopaedia. "What is capitalism?". Encyclopedia Britannica, 24 Feb. 2023, https://www.britannica.com/question/What-is-capitalism. Accessed 27 May 2024.

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[9] Britannica, The Editors of Encyclopaedia. "science". Encyclopedia Britannica, 25 May. 2024, https://www.britannica.com/science/science. Accessed 29 May 2024.

In Hegelian *Dialectics* [10], ideas and reality develop through the interaction of opposites. An initial idea (*thesis*) is countered by an opposing idea (*antithesis*), leading to a resolution (*synthesis*) that integrates elements of both. [Hegel]

3.2. Structuralism and Post-Structuralism

These theories explore the so-called construction and de-construction of meaning.

- *Structuralism*: Lévi-Strauss examines how elements within a culture or system are interrelated. [Lévi-Strauss]
- *Post-Structuralism*: Derrida focuses on the instability of these structures, emphasizing the gaps or the contradiction in meaning and interpretation. [Derrida]

3.3. Social Systems Theory

Systems Theory in Sociology looks at how different parts of a system interact and form a whole. For example, Luhmann views society as a complex set of communications and interactions, where even opposing elements are part of the system's overall functioning. [Luhmann]

3.4. Paradoxical Intention

In Psychology, the *Paradoxical Thinking* [Frankl] involves recognizing and integrating contradictory thoughts or behaviors. The method encourages patients to actively engage in the very behaviors they fear, in order to reduce the anxiety associated with those.

4. Applications in Contemporary Contexts

4.1. Migrations: Acculturation

In today's Western multicultural societies [Vani, Mangan], shared values of locals coexist with the unique and different cultural practices of migrants, leading to various individual outcomes. [Berry] *Acculturation*,[11] which results from intercultural contact, offers four different outcomes in varying degrees: integration, assimilation, separation, and marginalization.

Let us apply a neutrosophic framework. *Integration* (two-way exchange process) involves engaging with and identifying with both cultures. *Assimilation* (one-way exchange process) entails adopting the host country's culture while rejecting one's heritage culture. *Separation* (zero-way exchange process) involves identifying with one's heritage culture and interacting solely with one's own group. *Marginalization* (minus-way exchange process) represents a (quasi)total lack of identification with both cultures.

Considering the growing significance of global migrations, and the pivotal role workplace integration plays in adaptation, there's a pressing need for focus on the socialization process[12] of migrants within organizations, not only in culture.[13]

One could expand upon Benson's approach [Benson], which is rooted in a dynamic understanding that an organization is shaped by historical processes of social construction, which is

[10] Knox, T. Malcolm. "Georg Wilhelm Friedrich Hegel". Encyclopedia Britannica, 23 Apr. 2024, https://www.britannica.com/biography/Georg-Wilhelm-Friedrich-Hegel. Accessed 29 May 2024.

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in constant move. Benson formalized[14] four key principles of analysis (social construction, context, contradiction, and praxis), which - with neutrosophy as tool - can each be further subdivided into various facets.

In any cross-cultural process, a meta-analysis of adjustment processes is necessary,[15] and an insightful instrument is provided by a neutrosophic standpoint that could uncover various factors, in the given case - factors that could alleviate the inequalities migrants encounter in socialization.

4.2. Ethics: Pro-choice vs. Pro-life

The Pro-choice and the Pro-life options are two contradictory viewpoints, mostly reduced to a total opposition. A neutrosophic view over their shared goals might suggest a possible foundation for constructive dialogue and collaborative efforts.

4.2.1. Pro-choice movement

The *Pro-choice* movement can be traced back to the early 20th century, with activists like Margaret Sanger advocating for women's access to contraception, and founding the American Birth Control League[16] in 1921. Around 1970, significant legal challenges and efforts emerged to reform restrictive abortion laws in the United States. Organizations such as the National Association for the Repeal of Abortion Laws [NARAL], founded in 1969, were instrumental in advocating for women's right to choose. The pivotal moment for the Pro-choice movement came with the Supreme Court case Roe v. Wade[17], which extended the constitutional right to privacy to a woman's decision to have an abortion, decision that galvanized Pro-choice and Pro-life activists. Today, the Pro-choice movement often intersects with other social movements, including LGBTQ+ rights.

4.2.2. Pro-life movement

The *Pro-life* movement originated in Catholic opposition to Pro-choice movement in the 1950s [Munson]. In response to the growing feminist movement and efforts to liberalize abortion laws, Pro-life groups formed structured organizations, e.g. National Right to Life Committee in 1968 [NRLC]. In the years following Roe v. Wade trial, the Pro-life movement expanded its strategies to include lobbying for restrictive abortion laws, dismantled for a moment by a new Supreme Court decision in the case Planned Parenthood v. Casey[18]. The Pro-life movement gained lately significant political influence. While primarily focused on abortion, the Pro-life movement often intersects with other conservative causes, including opposition to euthanasia.

4.2.3. In search of a common ground

Finding common ground between the *Pro-choice* and *Pro-life* positions[19] can be challenging due to their fundamentally opposing views on abortion. However, both sides can agree on the following statements: [Spitzer et al.]

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[14] Benson J. K. (1983). "A dialectical method for the study of organizations." In Morgan G. (Ed.), "Beyond method: Strategies for social research", Sage Publications, 331-346.

[15] Nguyen, A.-M. T.D., Benet-Martínez, V. (2013). "Biculturalism and adjustment: A meta-analysis." *Journal of Cross-Cultural Psychology* 44(1), 122–159. DOI: 10.1177/0022022111435097.

[16] Moses, Theodora R. "American Birth Control League". Encyclopedia Britannica, 20 Mar. 2023, https://www.britannica.com/topic/American-Birth-Control-League. Accessed 29 May 2024.

[17] Roe v. Wade, 410 U.S. 113 (1973). https://supreme.justia.com/cases/federal/us/410/113/.

[18] Planned Parenthood of Southeastern Pa. v. Casey, 505 U.S. 833 (1992), https://supreme.justia.com/cases/federal/us/505/833/. Also, https://www.oyez.org/cases/1991/91-744.

[19] Britannica, The Editors of Encyclopaedia. "Pro and Con: Abortion". Encyclopedia Britannica, 27 Dec. 2021, https://www.britannica.com/story/pro-and-con-abortion. Accessed 29 May 2024.

- Both can support measures to reduce unintended pregnancies, such as comprehensive sex education and increased access to contraception.
- Both can encourage adoption as a viable option for women who do not wish to or cannot raise a child,
- Both can approve making the adoption process easier, affordable, less stigmatized.
- Both can agree on the importance of supporting women and families, meaning: better access to healthcare, longer parental leave, more affordable childcare.
- Both can agree on the importance of protecting women's safety.
- Both can work towards improving educational and economic opportunities for women, to empower choices about their reproductive lives.

4.3. Psychology: Cognitive-Behavioral Therapy vs. Psychodynamic Therapy

Cognitive-Behavioral Therapy [20] (CBT) focuses on present thoughts and behaviors, while *Psychodynamic Therapy* [21] (PDT) explores unconscious processes and past experiences — both approaches aiming to alleviate psychological distress.

Although both therapies appear effective in addressing mental health concerns, they diverge in their theoretical orientations, goals, techniques, and duration. However, they can also complement each other when used together.

Let us engage in a quick comparison:

4.3.1. Basics

- Rooted in the cognitive model, CBT focuses on the relationship between thoughts, feelings, and behaviors. It emphasizes identifying and challenging negative or maladaptive thought patterns and replacing them with more adaptive ones. *CBT is present-focused and goal-oriented.*
- Based on psychoanalytic principles, psychodynamic therapy explores how unconscious conflicts and early life experiences influence current thoughts, feelings, and behaviors. It aims to bring unconscious material into conscious awareness to promote insight and healing. *Psychodynamic therapy tends to be exploratory and insight-oriented.*

4.3.2. Therapeutic Techniques

- Techniques in CBT include cognitive restructuring[22], behavioral experiments, exposure therapy [23], and skill-building exercises such as relaxation and problem-solving techniques.
- PDT techniques include free association[24], dream analysis, interpretation of transference and countertransference, and exploring childhood experiences and relationships with significant others.

4.3.3. Therapeutic Relationship

[20] Moulds, M., Grisham, J., & Graham, B. (2022). "Cognitive Behavioral Therapy for Anxiety." Oxford Research Encyclopedia of Psychology. Retrieved 29 May. 2024, from https://oxfordre.com/psychology/view/10.1093/acrefore/9780190236557.001.0001/acrefore-9780190236557-e-331.

[21] Crits-Christoph, P. (1992). "The efficacy of brief dynamic psychotherapy: A meta-analysis." *American Journal of Psychiatry* 149(2):151–158.

[22] Identifying and challenging negative thoughts.

[23] Gradual exposure to feared stimuli.

[24] Encouraging clients to speak freely without censorship.

- The therapist and patient work together to set specific goals, and actively engage in homework assignments between sessions in CBT.
- In PDT, the therapist serves as a neutral interpreter and guide, facilitating exploration of deeper emotions and conflicts.

4.3.4. Duration and Focus

- CBT is typically *short-term* (from a few weeks to several months) and focused on addressing specific symptoms or problems.
- PDT is usually *longer-term* (lasting several months to years) and focuses on exploring underlying emotional issues and patterns.

5. Conclusion

Neutrosophy offers a framework for understanding the wide interactions and the inébranlable connections of some concepts traditionally seen as opposites and their neutralities/indeterminacies. By applying this framework to the soft sciences, one gain deeper insights into the nuanced relationships between ideas, theories, and practices. This approach can lead to an integrative understanding of human knowledge and experience.

Neutrosophy transcends the limits and finds, in any field of knowledge, common ideas to uncommon schools of thought, and reciprocally: uncommon ideas to common schools of thoughts.

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Neutrosofia înseamnă: părți comune în lucruri necomune și părți necomune în lucruri comune

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Abstract

Fie <A> un element, concept, idee, propoziție, școală de gândire, curent, teorie etc. și <antiA> opusul lui <A>. Analog pentru <B> și opusul său <antiB>. Neutrosofia înseamnă identificarea:

i) părților comune între lucruri necomune (adică <A> și <antiA> au ceva în comun, sau intersecția lor <A>∩<antiA> nu este vidă), și viceversa:
ii) părților necomune între lucruri comune (cele două elemente egale <A>=<B> au, de asemenea, părți necomune, fie <A>∩<antiB> nu este vidă, fie <antiA>∩<B> nu este vidă).

Atât părțile comune între lucruri necomune, cât și părțile necomune între lucruri comune se dovedesc a fi părți de indeterminare/neutralitate situate între opoziții: denotate de <neutA>, care înseamnă nici <A>, nici <antiA>, ci între ele; și respectiv de <neutB>, care înseamnă în mod similar nici <B>, nici <antiB>, ci între ele.

Cuvinte cheie

neutrosofie; paradoxism; dialectică; Yin Yang; științe umaniste; capitalism; socialism; psihoanaliză; psihologie analitică; democrație; democrație reprezentativă; alchimie; știință; dialectică; structuralism; post-structuralism; teoria sistemelor sociale; intenție paradoxală; pro-alegere; pro-viață; terapie cognitiv-comportamentală; terapie psihodinamică.

1. Introducere

Neutrosofia, un cadru filozofic pe care l-am dezvoltat acum mai bine de două decenii [Smarandache 1998], explorează relațiile și interacțiunile dintre opoziții și neutralitățile/indeterminările acestora, căutând să găsească puncte comune între ele și să identifice elemente necomune în entități similare, subliniind complexitatea și interconectivitatea conceptelor. Neutrosofia transcende gândirea binară tradițională prin examinarea interacțiunii dintre opoziții și neutralitățile/indeterminările dintre ele.

Neutrosofia este o extensie a mișcării numite *Paradoxism* [Smarandache 1980], în literatură/artă/știință/filozofie, a *dialecticii* [Hegel], [Marx] și a filosofiei antic-chineze *Yin Yang*[1] – deoarece ultimele trei școli au luat în considerare doar dinamica dintre opoziții, omițând neutralitățile/indeterminările acestora, care joacă un rol important în echilibrul dintre opoziții.

[1] Britannica, The Editors of Encyclopaedia. 'yinyang.' *Encyclopedia Britannica*, 12 Feb. 2024. Online: https://www.britannica.com/topic/yinyang. Accesat: 30 mai 2024.

Prin identificarea și studiul părților comune în lucruri necomune și a părților necomune în lucruri comune, neutrosofia dezvăluie complexitatea și interconectivitatea inerente în diferite idei și fenomene.

Acest articol prezintă pe scurt cele două principii fundamentale ale cercetării neutrosofice în științele umaniste, apoi exemplifică aceste principii neutrosofice și sugerează aplicații, ilustrând modul în care neutrosofia oferă o înțelegere nuanțată a lumii.

2. Două principii de bază ale neutrosofiei

2.1. Căutând părți comune în lucruri necomune

Acest principiu presupune studiul conceptelor opuse care împărtășesc (unele) elemente comune. De exemplu:

- **Ideologii Politice**: *Capitalismul*[2] [Hickel] și *Socialismul*[3] [Cole] pretind ambele că scopul lor este bunăstarea societății și prosperitatea economică, deși metodele propuse diferă.
- **Teorii Psihologice**: *Psihanaliza* lui Freud[4] [Freud] și *Psihologia Analitică* a lui Jung[5] [Jung] se concentrează ambele pe mintea inconștientă, dar au fundamente teoretice și metodologii distincte.

2.2. Căutând părți necomune în lucruri comune

Acest principiu propune cercetarea conceptelor similare sau echivalente care conțin elemente distincte sau opuse.

- **Forme de Guvernare**: *Democrația*[6] și *Democrația Reprezentativă*[7] pun ambele accent pe rolul poporului, dar democrația reprezentativă implică oficiali aleși, în timp ce democrația directă implică participarea directă a cetățenilor. [Landemore]
- **Forme de Gândire**: *Empirismul* [Gupta] vs. *Alchimia* [Ferguson]: Ambele caută înțelegerea lumii, dar *alchimia*[8] nu are metodologia și susținerea empirică a științei[9].

[2] Britannica, The Editors of Encyclopaedia. 'What is capitalism?.' *Encyclopedia Britannica*, 24 Feb. 2023. Online: https://www.britannica.com/question/What-is-capitalism. Accesat: 27 mai 2024.
[3] Britannica, The Editors of Encyclopaedia. 'What is socialism?.' *Encyclopedia Britannica*, 11 Apr. 2022. Online: https://www.britannica.com/question/What-is-socialism. Accesat: 27 mai 2024.
[4] Jay, Martin Evan. 'Sigmund Freud.' *Encyclopedia Britannica*, 20 May. 2024. Online: https://www.britannica.com/biography/Sigmund-Freud. Accesat: 28 mai 2024.
[5] Fordham, Frieda. 'Carl Jung.' *Encyclopedia Britannica*, 18 Mar. 2024. Online: https://www.britannica.com/biography/Carl-Jung. Accesat: 28 mai 2024.
[6] Shapiro, Ian, Dahl, Robert A. and Froomkin, David. 'democracy.' *Encyclopedia Britannica*, 6 May. 2024. Online: https://www.britannica.com/topic/democracy. Accesat: 29 mai 2024.
[7] Raikar, Sanat Pai. 'representative democracy.' *Encyclopedia Britannica*, 29 Feb. 2024. Online: https://www.britannica.com/topic/representative-democracy. Accesat 29 mai 2024.
[8] Britannica, The Editors of Encyclopaedia. 'alchemy (pseudoscience).' *Encyclopedia Britannica*, 29 Mar. 2024. Online: https://www.britannica.com/topic/alchemy. Accesat: 29 mai 2024.
[9] Britannica, The Editors of Encyclopaedia. 'science.' *Encyclopedia Britannica*, 25 May. 2024. Online: https://www.britannica.com/science/science. Accesat: 29 mai 2024.

3. Investigație în Științele Sociale

3.1. Dialectica Hegeliană

În *dialectica* hegeliană,[10] ideile și realitatea se dezvoltă prin interacțiunea contrariilor. O idee inițială (*teză*) este contracarată de o idee opusă (*antiteză*), ducând la o rezolvare (*sinteză*) care integrează elemente din ambele. [Hegel]

3.2. Structuralism și Post-Structuralism

Aceste teorii explorează așa-numita construcție și de-construcție a sensului.

- *Structuralism*: Lévi-Strauss examinează modul în care elementele dintr-o cultură sau un sistem sunt interconectate. [Lévi-Strauss]
- *Post-Structuralism*: Derrida se concentrează pe instabilitatea acestor structuri, subliniind lacunele sau contradicțiile în sens și interpretare. [Derrida]

3.3. Teoria Sistemelor Sociale

Teoria Sistemelor în Sociologie analizează modul în care diferitele părți ale unui sistem interacționează și formează un întreg. De exemplu, Luhmann vede societatea ca un set complex de comunicări și interacțiuni, în care chiar și elementele opuse fac parte din funcționarea generală a sistemului. [Luhmann]

3.4. Intenția Paradoxală

În Psihologie, *Gândirea Paradoxală* [Frankl] implică recunoașterea și integrarea gândurilor sau comportamentelor contradictorii. Metoda încurajează pacienții să se angajeze activ în comportamentele de care se tem, pentru a reduce anxietatea asociată cu acestea.

4. Aplicații în contexte contemporane

4.1. Migrația: Aculturarea

În societățile multiculturale occidentale de astăzi [Vani, Mangan], valorile comune ale localnicilor coexistă cu practici culturale unice și diferite ale migranților, ducând la diverse rezultate individuale [Berry]. *Aculturarea*,[11] care rezultă din contactul intercultural, oferă patru rezultate diferite în grade variabile: integrare, asimilare, separare și marginalizare.

Să aplicăm un cadru neutrosophic. *Integrarea* (procesul de schimb bilateral) implică angajarea și identificarea cu ambele culturi. *Asimilarea* (procesul de schimb unilateral) presupune adoptarea culturii țării gazdă în timp ce se respinge cultura proprie. *Separarea* (procesul de schimb zero) implică identificarea cu propria cultură de origine și interacționarea exclusiv cu propriul grup. *Marginalizarea* (procesul de schimb negativ) reprezintă o (cvasi)totală lipsă de identificare cu ambele culturi.

Având în vedere importanța tot mai mare a migrațiilor globale și rolul esențial al integrării la locul de muncă în adaptare, există o nevoie presantă de concentrare asupra

[10] Knox, T. Malcolm. 'Georg Wilhelm Friedrich Hegel.' *Encyclopedia Britannica*, 23 Apr. 2024. Online: https://www.britannica.com/biography/Georg-Wilhelm-Friedrich-Hegel. Accesat: 29 mai 2024.
[11] Berry J. W. (1990). "Psychology of acculturation." În: J. Berman (Ed.), "Cross-cultural perspectives: Nebraska Symposium on Motivation", Vol. 37, 201–234. Lincoln: University of Nebraska Press.

procesului de socializare[12] a migranților în cadrul organizațiilor, nu doar în ceea ce privește cultura.[13]

S-ar putea extinde abordarea lui Benson [Benson], care se bazează pe o înțelegere dinamică conform căreia o organizație este modelată de procesele istorice de construcție socială, care sunt în continuă mișcare. Benson a formalizat[14] patru principii-cheie de analiză (construcția socială, contextul, contradicția și praxis-ul), care ─ utilizând neutrosofia ca instrument ─ pot fi fiecare subdivizate în diverse fațete.

În orice proces intercultural, este necesară o meta-analiză a proceselor de adaptare,[15] iar un instrument perspicace este oferit de un punct de vedere neutrosofic care ar putea descoperi diverși factori, în cazul dat ─ factori care ar putea atenua inegalitățile cu care se confruntă migranții în socializare.

4.2. Etică: Pro-alegere vs. Pro-viață

Opțiunile pro-alegere și pro-viață sunt două puncte de vedere contradictorii, adesea reduse la o opoziție totală. O perspectivă neutrosofică asupra obiectivelor lor comune ar putea sugera o bază posibilă pentru un dialog constructiv și eforturi colaborative.

4.2.1. Mișcarea pro-alegere

Mișcarea *pro-alegere* își are originile la începutul secolului XX, cu activiști precum Margaret Sanger care au militat pentru accesul femeilor la contracepție și au fondat American Birth Control League[16] în 1921. În jurul anului 1970, au apărut provocări legale semnificative și eforturi pentru reformarea legilor restrictive privind avortul în Statele Unite. Organizații precum National Association for the Repeal of Abortion Laws [NARAL], fondată în 1969, au fost esențiale în promovarea dreptului femeilor de a alege. Momentul pivotal pentru mișcarea pro-alegere a fost cazul Roe v. Wade,[17] care a extins dreptul constituțional la intimitate pentru decizia unei femei de a avorta, decizie care a galvanizat atât activiștii pro-alegere, cât și pe cei pro-viață. Astăzi, mișcarea pro-alegere se intersectează adesea cu alte mișcări sociale, inclusiv cu drepturile LGBTQ+.

4.2.2. Mișcarea pro-viață

Mișcarea *pro-viață* originează în opoziția catolică față de mișcarea pro-alegere în anii 1950 [Munson]. Ca răspuns la explozia mișcării feministe și la eforturile de liberalizare a legilor privind avortul, grupurile pro-viață au format organizații structurate, de exemplu, National Right to Life Committee în 1968 [NRLC]. În anii care au urmat procesului Roe v. Wade, mișcarea pro-viață și-a extins strategiile pentru a include lobby-ul pentru legi restrictive

[12] McGahan A. M. (2020). "Immigration and impassioned management scholarship." *Journal of Management Inquiry* 29(1), 111-114. https://doi.org/10.1177/1056492619877617

[13] Omanović, V., & Langley, A. (2023). "Assimilation, Integration or Inclusion? A Dialectical Perspective on the Organizational Socialization of Migrants." *Journal of Management Inquiry* 32(1), 76-97. https://doi.org/10.1177/10564926211063777

[14] Benson J. K. (1983). "A dialectical method for the study of organizations." În: Morgan G. (Ed.), "Beyond method: Strategies for social research", Sage Publications, 331-346.

[15] Nguyen, A.-M. T.D., Benet-Martínez, V. (2013). "Biculturalism and adjustment: A meta-analysis." *Journal of Cross-Cultural Psychology* 44(1), 122–159. DOI: 10.1177/0022022111435097.

[16] Moses, Theodora R. 'American Birth Control League.' *Encyclopedia Britannica*, 20 Mar. 2023. https://www.britannica.com/topic/American-Birth-Control-League. Accesat 29 mai 2024.

[17] Roe v. Wade, 410 U.S. 113 (1973). https://supreme.justia.com/cases/federal/us/410/113/.

privind avortul, care au fost parțial anulate pentru un moment de o nouă decizie a Curții Supreme în cazul Planned Parenthood v. Casey.[18] Recent, mișcarea pro-viață a câștigat o influență politică semnificativă. Deși se concentrează în principal pe avort, mișcarea pro-viață se intersectează adesea cu alte cauze conservatoare, inclusiv opoziția față de eutanasie.

4.2.3. În căutarea unui teren comun

Găsirea unui teren comun între pozițiile *pro-alegere* și *pro-viață*[19] poate fi dificilă din cauza viziunilor lor fundamental opuse asupra avortului. Totuși, ambele părți pot fi de acord cu următoarele afirmații: [Spitzer et al.]

- Ambele părți pot susține măsuri pentru reducerea sarcinilor neintenționate, cum ar fi educația sexuală cuprinzătoare și accesul crescut la contracepție.
- Ambele părți pot încuraja adopția ca o opțiune viabilă pentru femeile care nu doresc sau nu pot crește un copil.
- Ambele părți pot încuraja facilitarea procesului de adopție, făcându-l mai accesibil, mai puțin stigmatizat și mai ieftin.
- Ambele părți pot fi de acord cu importanța sprijinirii femeilor și familiilor, inclusiv prin acces mai bun la îngrijiri medicale, concedii parentale mai lungi și servicii de îngrijire a copiilor mai accesibile.
- Ambele părți pot fi de acord cu importanța protejării siguranței femeilor.
- Ambele părți pot colabora pentru îmbunătățirea oportunităților educaționale și economice pentru femei, pentru a le permite luarea deciziilor privind viețile lor reproductive.

4.3. Psihologie: Terapia cognitiv-comportamentală vs. Terapia psihodinamică

Terapia Cognitiv-Comportamentală[20] (CBT) se concentrează pe gândurile și comportamentele prezente, în timp ce *Terapia Psihodinamică*[21] (PDT) explorează procesele inconștiente și experiențele din trecut — ambele abordări având ca scop alievarea suferinței psihologice.

Deși ambele terapii par eficiente în abordarea problemelor de sănătate mintală, ele diferă în orientările teoretice, obiectivele, tehnicile și durata lor. Totuși, ele pot fi, de asemenea, complementare atunci când sunt utilizate împreună.

Să facem o comparație rapidă:

4.3.1. Fundamente

- Bazată pe modelul cognitiv, Terapia Cognitiv-Comportamentală se concentrează pe relația dintre gânduri, sentimente și comportamente. Accentuează identificarea și provocarea tiparelor de gândire negative sau

[18] Planned Parenthood of Southeastern Pa. v. Casey, 505 U.S. 833 (1992), https://supreme.justia.com/cases/federal/us/505/833/. De asemenea, https://www.oyez.org/cases/1991/91-744.
[19] Britannica, The Editors of Encyclopaedia. 'Pro and Con: Abortion.' *Encyclopedia Britannica*, 27 Dec. 2021, https://www.britannica.com/story/pro-and-con-abortion. Accesat: 29 mai 2024.
[20] Moulds, M., Grisham, J., & Graham, B. (2022). "Cognitive Behavioral Therapy for Anxiety." *Oxford Research Encyclopedia of Psychology*. Online: https://oxfordre.com/psychology/view/10.1093/acrefore/9780190236557.001.0001/acrefore-9780190236557-e-331. Accesat: 29 mai 2024.
[21] Crits-Christoph, P. (1992). "The efficacy of brief dynamic psychotherapy: A meta-analysis." *American Journal of Psychiatry* 149(2):151–158.

neadaptative și înlocuirea acestora cu unele mai adaptative. *CBT este orientată spre prezent și se axează pe obiective specifice*.

- Bazată pe principii psihanalitice, Terapia Psihodinamică explorează modul în care conflictele inconștiente și experiențele timpurii influențează gândurile, sentimentele și comportamentele actuale. Scopul este de a aduce materialul inconștient în conștientizare pentru a promova introspecția și vindecarea. *PDT tinde să fie exploratorie și orientată spre aprofundare.*

4.3.2. Tehnici terapeutice

- Tehnicile în CBT includ restructurarea cognitivă,[22] experimente comportamentale, terapia de expunere[23] și exerciții de consolidare a abilităților, cum ar fi tehnici de relaxare și de rezolvare a problemelor.
- Tehnicile PDT includ asocierea liberă,[24] analiza viselor, interpretarea transferului și contratransferului și explorarea experiențelor copilăriei și a relațiilor cu alții.

4.3.3. Relații terapeutice

- Terapeutul și pacientul lucrează împreună pentru a stabili obiective specifice și se angajează activ în sarcinile pentru acasă între sesiunile de CBT.
- În PDT, terapeutul servește ca interpret și ghid neutru, facilitând explorarea emoțiilor și conflictelor mai profunde.

4.3.4. Durată și focus

- CBT este de obicei pe *termen scurt* (de la câteva săptămâni la câteva luni) și se concentrează pe abordarea unor simptome sau probleme specifice.
- PDT este de obicei pe *termen mai lung* (durează câteva luni până la ani) și se concentrează pe explorarea problemelor și tiparelor emoționale subiacente.

Concluzii

Neutrosofia oferă un cadru pentru înțelegerea interacțiunilor vaste și a conexiunilor unor concepte tradițional văzute ca opuse și a neutralităților/indeterminării lor. Aplicând acest cadru la științele umaniste, se obțin perspective mai profunde asupra relațiilor nuanțate dintre idei, teorii și practici. Această abordare poate conduce la o înțelegere integrativă a cunoștințelor și experienței umane. Neutrosofia transcende limitele și găsește, în orice domeniu de cunoaștere, idei comune pentru școli de gândire diferite și, reciproc, idei necomune pentru școli de gândire comune.

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[22] Identificarea și contestarea gândurilor negative.
[23] Expunere graduală la stimuli de teamă.
[24] Încurajarea clienților să vorbească liber, fără autocenzură.

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Beyond Negation and Excluded Middle: An exploration to Embrace the Otherness Beyond Classical Logic System and into Neutrosophic Logic

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Abstract

As part of our small contribution in dialogue toward better peace development and reconciliation studies, and following Toffler & Toffler's *War and Antiwar* (1993), the present article delves into a realm of logic beyond the traditional confines of negation and the excluded middle principle, exploring the nuances of "*Otherness*" that transcend classical and Nagatomo logics. Departing from the foundational premises of classical Aristotelian logic systems, this exploration ventures into alternative realms of reasoning, specifically examining Neutrosophic Logic and Klein bottle logic (cf. Smarandache, 2005). The study challenges conventional boundaries and explores the implications of embracing paradoxes and self-reference in logic systems, aiming to redefine approaches to understanding truth and reasoning. The paper investigates how these alternative logics open avenues for philosophical inquiry, redefining entropy, and potentially influencing innovative perspectives in free energy systems. Through this exploration, it seeks to expand the discourse on logic, welcoming a broader spectrum of thought beyond established frameworks; and we also discuss shortly a number of possible implementations including in risk management and also Klein bottle entropy redefinition (Tang *et al*, 2018).

Keywords: Classical logic system; Nagatomo Logic; Lukasiewicz; Fuzzy logic; Klein bottle; Neutrosophic Logic

1. Introduction:

As part of our small contribution in dialogue toward better peace development and reconciliation studies, and following Toffler & Toffler's *War and Antiwar* (1993), the present article delves into a realm of logic beyond the traditional confines of negation and the excluded middle principle, exploring the nuances of "*Otherness*" that transcend classical and Nagatomo logics.

It is known, that classical Aristotelian logic has undeniably influenced Western thought profoundly. However, its limitations, particularly within the realm of negation and the excluded middle principle, have been subject to critical examination by various scholars. For instance, Paul Hiebert, in his article "*The Flaws of Excluded Middle*" (1981), sheds light on these limitations and invites us to consider the broader spectrum of logical reasoning beyond these principles.

The excluded middle principle asserts that a statement must either be true or false, with no middle ground or alternative possibilities. While this principle has been foundational in classical logic, its applicability in all contexts has been questioned. Hiebert, among others, points out that some situations may not neatly fit into such a simplistic framework. Real-world scenarios often present complexities and nuances that cannot be confined to a simple true/false dichotomy.

Consider the concept of negation, which implies the opposite of a proposition. Classical logic assumes that if a statement is false, its negation must be true, and vice versa. However, in certain cases, the negation of a proposition might not necessarily lead to its direct opposite truth value. For instance, in fuzzy logic or quantum mechanics, the truth value of a statement can exist in degrees between absolute truth and absolute falsity, challenging the rigid nature of classical logic.

Furthermore, Hiebert (1981) emphasizes that human experiences and cultural contexts often transcend the constraints of classical logic. Contextual factors, subjective viewpoints, and evolving perspectives contribute to a more complex understanding of truth and reasoning.
To overcome the limitations of classical logic, contemporary approaches like modal logic, paraconsistent logic, and non-classical logics have emerged. These systems acknowledge the limitations of the excluded middle principle and negation, offering alternative frameworks to address the intricacies of reasoning in different domains.
In essence, while it is known that classical Aristotelian logic has been foundational in shaping Western thought, acknowledging its limitations opens doors to exploring richer and more nuanced forms of reasoning. Hiebert's insights remind us to embrace diverse logical frameworks that better accommodate the complexities of our world, transcending the confines of strict negation and the excluded middle principle.

Results:

Beyond Classical logic

Let's delve into the three different logical systems as mentioned above: multivalued logic, fuzzy logic, and Nagatomo logic, which all offer approaches beyond the strict true/false dichotomy.

A. *Multivalued Logic* (Polish Logician - Łukasiewicz, 1918): This logic system extends classical logic by allowing more than just two truth values (true and false). Łukasiewicz introduced a three-valued logic system, which typically includes the truth values of "true," "false," and an additional value often termed "indeterminate" or "neither true nor false." This extension permits reasoning in scenarios where propositions might not have a clear-cut true or false value, acknowledging uncertainties or incomplete information.
B. *Fuzzy Logic* (Zadeh): Fuzzy logic, introduced by Lotfi Zadeh in the 1960s, aims to handle vagueness and uncertainty in reasoning. Unlike classical logic's binary true/false assignments, fuzzy logic allows for truth values between completely true (1) and completely false (0). It deals with degrees of truth, enabling a more nuanced representation of concepts that are not strictly black or white but exist in shades of gray. This approach finds applications in various fields, especially in systems where imprecise data or human-like reasoning is involved, such as artificial intelligence, control systems, and decision-making processes.
C. *Nagatomo Logic*: it is an alternative to Aristotelian classical logic that introduces a more extensive set of truth values and rules. It aims to represent reasoning in contexts where strict bivalence might be limiting. Nagatomo's logic extends beyond the true/false framework by incorporating multiple truth values, allowing for a more flexible representation of propositions that may not neatly fit into classical binary logic. This 'logic' is called the 'logic of not'. It is stated in a propositional form: '*A is not A, therefore it is A*'.[1] Since this formulation is contradictory or paradoxical when it is read in light of Aristotelean logic, one might dismiss it as nonsensical. In order to show that it is neither nonsensical nor meaningless, the paper will articulate the philosophical reasons why the *Sutra* makes its position in this contradictory form. (Nagatomo, 1979; Nagatomo, 2000; Tanabe, 2016).
D. Smarandache's *Neutrosophic Logic* is another alternative based on examining Zadeh's Fuzzy Logic. He once wrote that according to this theory every idea <A> tends to be neutralized and balanced by <antiA> and <nonA> ideas - as a state of equilibrium. In a classical way <A>, <neutA>, <antiA> are disjoint two by two. But, since in many cases the borders between notions are vague, imprecise, Sorites, it is possible that <A>, <neutA>, <antiA> (and <nonA> of course) have common parts two by two, or even all three of them as well. He also extends the Law of Included Multiple-Middles to the Law of Infinitely-Many-Middles, the Law of Included Multiple-Middle (as extension of the Law of Included Middle) (<A>; <neutA1>, <neutA2>, …, <neutAn>; <antiA>). Also, he discusses Aristotle's Syllogism, Principle of Identity, and Principle of Non-Contradiction. Elsewhere, Smarandache Refined / Split the Neutrosophic Components (T, I, F) into Neutrosophic SubComponents (T1, T2, ...; I1, I2, ...; F1, F2, ...) (Smarandache, 2005; Smarandache, 2023).

Each of these systems expands the scope of logical reasoning beyond the traditional binary framework, acknowledging the inherent complexities and nuances present in real-world scenarios. By introducing additional truth values or accommodating degrees of truth, these systems offer more sophisticated tools for modeling and dealing with uncertainties, vague concepts, and situations where the true/false dichotomy falls short.
These alternative logics have found applications in various fields where precision is essential, but strict binary logic might not capture the richness of real-world scenarios. By incorporating negation and embracing a broader range of truth values, they provide valuable frameworks for handling complex and ambiguous information in a way that classical logic cannot.

[1] Source: https://philpapers.org/rec/NAGTLO (accessed online, 1st Dec. 2023).

Exploring limitation of fuzzy logic and Nagatomo logic

Exploring the limitations of fuzzy logic and Nagatomo logic within Mathematica could involve creating scenarios where these systems might struggle or where their outcomes differ significantly from classical logic/Boolean logic. While Mathematica might not directly support Nagatomo logic as it's a specialized system, we can illustrate limitations of fuzzy logic within Mathematica's capabilities. Let's consider an example where fuzzy logic's handling of truth values could diverge significantly from classical logic:

Example Scenario - Fuzzy Logic Limitation:

Imagine a scenario where we're evaluating the temperature of water and determining whether it's "hot," "warm," or "cold" based on different temperature ranges. Fuzzy logic allows for gradual transitions between these linguistic values based on degrees of truth.

```
(* Define temperature ranges *)

hot[x_] := Piecewise[{{1, x >= 80}, {(x - 70)/10, 70 <= x < 80}, {0, x < 70}}]

warm[x_] := Piecewise[{{(x - 60)/10, 60 <= x < 70}, {1, 70 <= x < 80}, {(90 - x)/10,       80 <= x <
90}, {0, x < 60 || x >= 90}}]

cold[x_] := Piecewise[{{1, x < 60}, {(70 - x)/10, 60 <= x < 70}, {0, x >= 70}}]

(* Evaluate temperature *)

temperature = 75; (* Example temperature *)

{hot[temperature], warm[temperature], cold[temperature]}
```

In this code, the temperature is set to 75. The functions hot, warm, and cold represent the fuzzy definitions for these temperature ranges. Running this code will provide the degrees to which the given temperature belongs to each category: hot, warm, and cold.

The limitation here lies in the subjectivity of defining these linguistic values. While fuzzy logic allows for a smooth transition between categories based on degrees of truth, the exact boundaries and membership functions are somewhat arbitrary and can vary based on interpretation. This subjectivity might lead to different categorizations based on how these linguistic values are defined.

For Nagatomo logic, which incorporates multiple truth values beyond the true/false framework, implementing it directly in Mathematica might be more complex, as it's not a standard logic system within the Mathematica environment. This system typically involves specialized truth values and rules that extend beyond the typical binary or fuzzy logic.

By illustrating scenarios like the one above within Mathematica package, we can highlight the potential limitations of fuzzy logic, especially in its reliance on subjective linguistic values and interpretations. Exploring how these systems handle ambiguous or uncertain situations can showcase where they might struggle or provide outcomes that diverge significantly from classical logic/Boolean logic.

Another alternative way to logic

For instance, in Nagatomo it is written *A is non-A therefore A*, but it can be thought of as also implying Klein-bottle logic or self-referential logic. As it is known from history, in 1882 Felix Klein imagined sewing two Möbius Loops together to create a single sided bottle with no boundary. Its inside is its outside; it contains itself.

Take a rectangle and join one pair of opposite sides -- you'll now have a cylinder. Now join the other pair of sides with a half-twist. That last step isn't possible in our universe. A true Klein Bottle requires 4-dimensions because the surface has to pass through itself without a hole. It's closed and non-orientable, so a symbol on its surface can be slid around on it and reappear backwards at the same place.[2]

The concept of alternative logics expands far beyond classical and fuzzy logics. Self-referential or paradoxical logics, such as Klein-bottle logic, delve into intriguing realms where propositions can reference themselves, challenging traditional logical structures. While such logics might not have direct applicability in everyday human

[2] Source: https://www.kleinbottle.com/whats_a_klein_bottle.htm

interactions or biology systems in their purest forms, they offer fascinating theoretical constructs that push the boundaries of conventional reasoning (cf. Rapoport, 2013).

-Klein-Bottle Logic: Exploring Self-Reference and Paradox

Klein-bottle logic, inspired by the mathematical concept of a Klein bottle—a non-orientable surface—introduces self-reference and paradoxes within logical systems. One of its propositions, often referred to as "*A is non-A, therefore A*," embodies self-reference and contradiction simultaneously.

- Mathematica Exploration:

While Mathematica doesn't have a built-in Klein-bottle logic framework, we can illustrate a self-referential paradox using Mathematica's symbolic manipulation capabilities.

Self-Referential Paradox Illustration:

```
(* Self-referential paradox *)

paradox = Not[paradox]
```

In this simple code, we attempt to create a self-referential paradox by defining a statement (paradox) that negates itself. However, in a standard logical framework, this leads to an irresolvable paradox—the statement asserts its own negation, creating a contradiction.

Example in Real-Life Systems

A. A few Implications in Real-Life:
The direct application of Klein-bottle logic in day-to-day human interactions or biological systems might seem abstract. However, the exploration of self-reference and paradoxes has implications in understanding complexity, recursion, and the limits of formal systems.

B. Complexity and Recursive Systems:
Real-life biological systems often exhibit recursive and self-referential properties, such as DNA encoding proteins, neural networks, or feedback loops in biological processes. While not directly applying Klein-bottle logic, these systems share elements of self-reference and complexity that challenge traditional logical frameworks.

C. Limits of Formal Systems:
Klein-bottle logic, despite its theoretical nature, highlights the limitations of formal systems when dealing with self-reference and paradoxes. Understanding these limits is crucial in various fields, including philosophy, computer science (in handling recursive algorithms), and even psychology (in studying self-reference and cognitive processes).

Discussion: Further possible implications to risk management, entropy, free energy systems:

Self-referential Klein-bottle logic, known for its paradoxical and self-referential nature, might seem abstract when discussing implications for redefining entropy and exploring free energy systems. However, its conceptual framework challenges traditional logical structures and could offer intriguing avenues for reconsidering established scientific principles like entropy and the limitations set by Carnot's theorem in thermodynamics.

A. Implication of Neutrosophic Logic to Risk management
In a world characterized by Volatility, Uncertainty, Complexity, and Ambiguity (VUCA), traditional risk assessment and management approaches often fall short. The inherent limitations of classical logic/Boolean logic, which relies on clear-cut "true" or "false" values, can fail to capture the nuanced and often ambiguous nature of risk in complex systems. Enter neutrosophic logic, a powerful tool developed by one of us (FS) that offers a promising alternative for navigating the uncertainties of the VUCA world.

Neutrosophic logic, by introducing a third truth value, "*indeterminate*," can represent incomplete or unknown information in a better way. This allows for a more accurate representation of reality, where certainty is often elusive and information may be incomplete or contradictory. This is particularly relevant in VUCA environments, where rapid change, complex interactions, and limited information make risk assessment and management challenging (cf. Patrick, 2022).

Several key features of neutrosophic logic hold significant promise for risk assessment and management:

- Modeling Uncertainty: Neutrosophic sets can effectively capture and represent uncertainty associated with risk factors and their potential impacts. This allows for a more comprehensive and realistic assessment of risk scenarios.
- Handling Incomplete Information: In situations with limited data or conflicting information, neutrosophic logic can still provide valuable insights into potential risks. By explicitly acknowledging the "indeterminate" aspects of risk, decision-makers can avoid making inaccurate or misleading conclusions.
- Facilitating Decision-Making: Neutrosophic logic tools and operators can help analyze complex risk scenarios, identify critical risk factors, and prioritize mitigation strategies. This can lead to more informed and effective decision-making under uncertainty.
- Improving Communication: Neutrosophic logic provides a common language for stakeholders with diverse perspectives to discuss and analyze risk. This can foster collaboration and improve the overall effectiveness of risk management efforts.

Several studies have already demonstrated the potential of Neutrosophic Logic in various risk assessment and management applications. These include supply chain risk management, financial risk assessment, safety modelling of complex systems, and risk assessment in the food-store supply chain. While still in its early stages of adoption, neutrosophic logic holds significant promise for transforming the way we approach risk in the VUCA world. By providing a more nuanced and flexible framework for dealing with uncertainty, ambiguity, and incomplete information, neutrosophic logic can equip organizations and individuals with the tools needed to navigate the challenges of a rapidly changing and unpredictable environment.

However, further research and development are needed to fully unlock the potential of neutrosophic logic in risk assessment and management. This includes the development of standardized methodologies, user-friendly tools, and educational programs to promote awareness and understanding of this powerful approach.

B. Redefining Entropy through Self-Reference; Klein-Bottle Logic and Entropy:
Entropic principles, foundational in thermodynamics, describe the measure of disorder or randomness in a system. Traditional understandings of entropy revolve around statistical mechanics, where disorder tends to increase over time in isolated systems—a concept tied to the second law of thermodynamics (cf. Tang *et al*, 2018).

Klein-bottle logic introduces self-reference and paradoxes. While seemingly unrelated to thermodynamics, this type of logic touches upon the concept of self-reference and contradiction—elements that challenge conventional reasoning (cf. Tang *et al*, 2018).

Implication: Exploring entropy through a self-referential lens might prompt reconsideration of the conventional understanding of disorder in systems. Could self-reference and paradoxes offer new insights into the nature of disorder or randomness in complex systems?

C. Beyond Carnot's Theorem and Free Energy:
Carnot's theorem, a cornerstone of thermodynamics, sets theoretical limits on the efficiency of heat engines. In principle, Carnot's theorem states that: Heat engines that are working between two heat reservoirs are less efficient than the Carnot heat engine that is operating between the same reservoirs. Irrespective of the operation details, every Carnot engine is efficient between two heat reservoirs.[3] It establishes that no engine can surpass the efficiency of a reversible engine operating between the same heat reservoirs.

D. Klein-Bottle Logic and Free Energy Systems:
While Klein-bottle logic doesn't directly relate to Carnot's theorem, its examination of self-reference challenges established paradigms. This prompts consideration: could the paradoxical nature of self-reference inspire novel approaches to circumvent theoretical limits in energy systems?

Implication: Exploring alternative logics might inspire fresh perspectives on energy systems, potentially redefining theoretical boundaries set by Carnot's theorem. Could unconventional reasoning spark innovations in creating more efficient energy conversion systems?

Concluding remark:

The present article, inspired partly by Toffler & Toffler's *War and antiwar*, delves into a realm of logic beyond the traditional confines of negation and the excluded middle principle, exploring the nuances of "*Otherness*" that

[3] Source: https://byjus.com/physics/carnots-theorem

transcend classical and Nagatomo logics. Departing from the foundational premises of classical logic systems, this exploration ventures into alternative realms of reasoning, specifically examining Neutrosophic Logic and Klein bottle logic.We discuss shortly of implications of neutrosophic logic in risk management and risk assessment especially in VUCA world. In conclusion, the implications of Smarandache's Neutrosophic Logic for risk assessment and management are quite profound. By offering a framework that can effectively deal with the complexities of the VUCA world, neutrosophic logic has the potential to revolutionize how we assess, manage, and mitigate risk in the 21st century.

Several studies have already demonstrated the potential of Neutrosophic Logic in various risk assessment and management applications. These include supply chain risk management, financial risk assessment, safety modeling of complex systems, and risk assessment in the food-store supply chain. While still in its early stages of adoption, neutrosophic logic holds significant promise for transforming the way we approach risk in the VUCA world. As research and development continue, we can expect to see neutrosophic logic become an increasingly valuable tool for organizations and individuals seeking to thrive in an ever-changing world.

Moreover, Klein-bottle logic, with its self-referential and paradoxical nature, extends the boundaries of conventional reasoning. While not directly applicable in practical human interactions or biology, its exploration sheds light on the limitations and complexities of formal logical systems, offering insights into recursive structures and the boundaries of what can be logically expressed.

In Mathematica package, while there are of course difficulties to directly implement a full-fledged Klein-bottle logic, the simple demonstration of self-referential paradoxes showcases the challenges posed by such concepts in traditional logical frameworks. Exploring these alternative logics opens doors to understanding the intricacies and limitations of formal reasoning systems, despite their abstract nature.

The application of self-referential Klein-bottle logic to redefining entropy and exploring free energy systems represents a speculative yet intriguing avenue of inquiry. While abstract in its nature, the challenges posed by self-reference and paradoxes could stimulate unconventional thinking in understanding disorder in systems and potentially redefining the limitations set by established theorems in thermodynamics.

While directly applying Klein-bottle logic to redefine entropy or challenge Carnot's theorem remains a conceptual exercise, the exploration of alternative logics prompts us to question established scientific principles and encourages creative thinking in pushing the boundaries of traditional scientific thought. In the meantime, it is worth noting here that Klein bottle entropy has been discussed by Tu *et al,* and also Tang *et al*. (2018).

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FS, VC

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Upside-Down Logics: Falsification of the Truth and Truthification of the False

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Abstract

This paper is about funny science, recreational mathematics, upside-down thinking, or contradictory reasoning (to think backward). Since a statement in some conditions may be true, in other conditions false, and a third type of conditions partially true and partially false. The paper presents for the first time two types of Upside-Down Logic, the first one is falsification of the Truth (when a true statement is transformed into a false one), and the second one is the opposite: Truthification of the False (when a false statement is transformed into a true one) - within the frame of Recreational Neutrosophy. All transformations from <A> to <antiA> or vice versa should be real, making sense in our real world. Kind of magic logic! Falsification and Truthification are mostly used in the Social Sciences (Anthropology, Archaeology, Economics, Geography, History, Law, Linguistics, Politics, Psychology, Sociology), Philosophy, etc. excelling in Politics:

Falsifying the Truth of Enemies

> [We seek to diminish the enemy's positive side to the point of cancellation and to increase the enemy's negative side to the point of exaggeration;] and

Truthifying the False of the Friends

> [The opposite: We seek to decrease the friend's negative side to the point of cancellation, and to increase the friend's positive side to the point of exaggeration.].

Keywords: Neutrosophy, Contradictory Reasoning, Upside-Down Thinking, Recreational Neutrosophy, Recreational Mathematics.

1 | Neutrosophy

Neutrosophy is a philosophical framework and logic-based system introduced by Florentin Smarandache in 1998 [1, 2], which studies the relationship between opposites, contradictions, and their neutral or indeterminate parts, with an aim to explore and understand the ambiguity, inconsistency, and indeterminacy inherent in many concepts and systems.

Neutrosophy's emphasis on neutrality, paradoxes, and indeterminacy makes it relevant for exploring real-world scenarios where truth is complex and multifaceted. It has applications in fields like artificial intelligence, decision-making, and problem-solving, offering a way to handle uncertainty and contradiction.

Neutrosophy is based on the dynamics between the opposites <A>, <antiA>, and their neutralities <neutA>, where <A> is an item (concept, attribute, idea, theory, etc.) and <antiA> its opposite, while <neutA> is the neutrals or indeterminacies between them.

For example: <A> = truth, <antiA> = false, and <neutA> = neutral, or indeterminacy between truth and false (for example 70% truth and 30% false).

Neutrosophy can be seen as a generalization of dualism, Yin Yang in Ancient Chinese philosophy or dialectics, offering a broader and more flexible framework for understanding contradictions and relationships among opposites.

The neutral part <neutA> may be, for example, partially <A>, and partially <antiA>. Some neutrality may have a higher degree of truth than falsehood, others a higher degree of falsehood than truth. In the worst scenario, as indeterminacy in between the opposite one may have a partial degree of <A>, partial degree of <neutA>, and partial degree of <antiA>.

2 | Procedures and Transformations Used in Upside-Down Logics [4-6]

Neutrosophy is a philosophical framework and logic-based system introduced by Florentin Smarandache in 1998 [1, 2], which studies the relationship between opposites, contradictions, and their neutral or indeterminate parts, to explore and understand the ambiguity, inconsistency, and indeterminacy inherent in many concepts and systems.

Truthification (Truth-ification) means: to make something true; we introduce this word. Both, Falsification of the Truth & Truthification of the False, are carried out through the same strategies:

- By changing the space or the time of the problem;
- By changing the attributes of the elements in the space;
- By changing the logic (procedure, tactic, method) of approaching the problem;
- By interpreting the opposite meaning;
- Reversed logic;
- By transforming the linguist expressions from the figurative sense to the proper sense, or vice-versa;
- By finding the sense of the non-sense, or the non-sense of the sense;
- Word games, puzzles;
- Riddles;
- Jokes;
- Etc.

Of course, these methods and practical strategies should be real, they should make sense in our everyday life.

2.1 | Changing the Space or Time of a Problem

This involves shifting the context or perspective to gain new insights. Let us exercise an example of changing the timeframe: Problems can often seem daunting because of time constraints. By imagining the problem in a different timeframe, you can explore new approaches. For example, if you're working on a project with a tight deadline, imagine how you'd tackle it with more time. This exercise can reveal shortcuts or techniques that can be applied even within a limited timeframe.

Changing the space or time of the problem involves manipulating the context or setting of an issue to create a different narrative. By altering where or when an event is said to have occurred, one can shift its perceived truth.

2.2 | Changing Attributes of Elements in a Space

This technique involves altering certain characteristics to create a different outcome. For example, if you're solving a puzzle involving color-coded items, changing the color scheme could lead to a new solution.

Changing the attributes or characteristics of the problem involves modifying certain elements or traits of an issue to make it seem more plausible or to cast doubt on its accuracy. For example, changing key details about a person or event can make a fabricated story seem more credible, or make a true story appear dubious.

2.3 | Interpreting in Reverse

This method involves taking a concept and flipping it, often leading to humorous or unexpected outcomes.

(i) An example could be the classic double-sense of a word, "What has keys but can't open locks?" The answer is a piano, interpreting the word "keys" in a non-traditional way.

Changing the Logic (Procedure, Tactic, Method) of Approaching the Problem. This strategy focuses on altering the reasoning or method used to analyze an issue. By introducing different logic or misusing data, one can create confusion or lead to false conclusions.

(ii) Another logic: How can we have ten divided by two equals to zero?

Answer:

Ten cookies divided by two kids are eaten and nothing has remained!

2.4 | Literal and Figurative Interpretations

This technique involves taking figurative language literally or vice versa. A joke that uses this is, "I told my friend to break a leg, and he actually fell!" Here, the phrase "break a leg," which usually means "good luck," is taken literally.

2.5 | Riddles

Riddles often rely on misleading information or double meanings to challenge the solver.

(i) A classic example is, "What has to be broken before you can use it?" The answer is "an egg," where the word "broken" is used in a different context.

(ii) Spell out the number NINE.

Solution:

(In this case "spell" means to list all little (vertical, oblique, and horizontal) bars that compose the word NINE, or eleven bars!)

(iii) I am invisible but can be measured. I affect everyone and everything that is anything. I span the universe and change from place to place. What am I?

Solution:

I am "gravity".

2.6 | Jokes

Jokes use a variety of techniques to create humor, including wordplay, puns, or unexpected twists.

(i) An example of a pun-based joke is, "Why don't scientists trust atoms? Because they make up everything!" This plays on the double meaning of "make up."

(ii) How would a mathematician measure the intensity of an earthquake on a meteor as in the movie Armageddon?

Solution:

It is impossible to have an earthquake on a meteor!

(iii) 15 Hunters Went Bear Hunting. One Killed 2 Bears. How Many Bears Have One Killed?

Solution:

Two. ("One" is the name of one of the hunters.)

(iv) $\frac{w}{2} = u$. Find a logic for this equality.

Solution:

The letter "w" is double "u", or "uu", then dividing "uu" by 2 one gets one "u".

2.7 | Funny Problems

(i) Calculate the volume of a square (!)

Solution:

Volume = Area of the Base × Height = Side² × 0 = 0! We look at the square as an extreme case of a parallelepiped whose height is null.

(ii) $? \times 7 = 2$ (?)

Solution:

Of course, $\frac{2}{7} \times 7 = 2$ (!)

(iii) If you have a couple of threes and divide them in half, why do you end up with 4 pieces?

Solution: ~~33~~

2.8 | The Lucky Mathematics! [3]

If, by apparently a wrong calculation (method, algorithm, operation, etc.) one arrives at the right answer, that is called a Lucky Calculation (Method, Algorithm, Operation, etc.)!

The apparently wrong calculation (method, algorithm, operation, etc.) should be funny (somehow similar to a correct one, producing confusion and a smile)!

(i) What is a hungry man's multiplication factor?

Solution:

8×8, because "eight times eight" is pronounced the same as "ate time ate"!

(ii) How can you add 3 with 3 and get 8?

Solution:

$\in \ni = 8$

{Let's laugh: ha, ha, ha!}

2.9 | Falsification and Truthification in Soft Sciences

They are mostly used in the Social Sciences (Anthropology, Archaeology, Economics, Geography, History, Law, Linguistics, Politics, Psychology, Sociology), but also in Philosophy, etc.

Excelling in Politics:

- Falsifying the Truth of enemies; and
- Truthifying the False of the Friends.

Real Examples frequently broadcasted on the TV:

Let's assume a project of law is proposed to Congress to be approved. People who oppose it are labeled as enemies by the leaders, while those who support it are labeled as friends.

(i) Let's assume that there is a demonstration of several thousand people in the city center against this project of law.

Falsification of the Truth:

The TV station broadcast: A few dozen people demonstrate in the city center against this project of law.

(ii) Let's assume that there is a counterdemonstration of a few people in the city center in support of this project of law.

Truthification of the False:

The TV station broadcast: Thousands of people demonstrate in the city center in support of this project of law.

2.10 | More Methods and Strategies the Readers may Design Themselves

3 | Real Examples of Falsification of the Truth

Neutrosophy is a philosophical framework and logic-based system introduced by Florentin Smarandache in 1998 [1, 2], which studies the relationship between opposites, contradictions, and their neutral or indeterminate parts, with an aim to explore and understand the ambiguity, inconsistency, and indeterminacy inherent in many concepts and systems.

1) 1 = 1 (True)

Falsification of the above:

1 meter = 1 kilometer {*False*, by introducing the elements' attributes (measurement units)}

2) 2 + 3 = 5 (True)

Falsification of the above:

2 + 3 = 5 in base 10, but changing the base to 5 one gets:

2 + 3 = 10 in base 5, or $2_5 + 3_5 = 10_5$.

3) Ten birds are flying up in the sky. A hunter shoots three of them. How many birds remain?

Answer:

10 – 3 = 7 birds, those who are still flying [the space is the sky].

Three falsifications of the above [by changing the space of the elements (birds)].

3.1 Ten birds are on a fence. A hunter shoots three of them. How many birds remain?

Answer: The space is the fence.

None, because the three dead birds fell off the fence and the other seven flew away!

3.2 Ten birds are in a meadow. A hunter shoots three of them. How many birds remain?

Answer: The space is the meadow.

Three birds, the dead birds remaining on the meadow, because the other seven flew away!

3.3 Ten birds are in a cage. A hunter shoots three of them. How many birds remain?

Answer: The space is the cage.

Ten birds, dead and alive, because none could get out of the cage!

4 | Real Examples of Truthification of the False

1) 2 = 1 (False)

   Solution (Truthification of the above):

   2 pints = 1 quart {True, by introducing the elements' attributes (measurement units)}

2) A man weights the following weights on the following dates.

6/1/70	**150 lbs.**
6/3/70	0 lbs.
6/5/70	25 lbs.
6/7/70	0 lbs.
6/9/70	145 lbs.

How is it possible?

Solution (Truthification):

Man is an astronaut who went to the Moon and back.

Outer-space weightlessness: 0 lbs.

$\frac{1}{6}$ of his Earth Gravity (150 lbs), or Gravity of Moon: 25 lbs.

3) From two false hypotheses get a true statement.

   Grass is edible. False

   Edible things are green. False

   Therefore, grass is green. True

4) "When a cup of milk is added to a cup of popcorn then only one cup of mixture will result because the cup of popcorn will very nearly absorb a whole cup of milk without spillage. So, in the last case, we have 1 + 1 = 1. It is impossible in the conventional arithmetic but it is true for some non-Diophantine arithmetic." [7].

5) Prove that the equation $x = x + 2$ has two distinct solutions.

   Answer: This is an inconsistent equation in the set of finite real numbers, but in the set of infinities it is not: the solutions are $x = \pm\infty$.

5 | Conclusion

This paper presented for the first time two types of Upside-Down Logic, the first one: Falsification of the Truth (when a true statement is transformed into a false one), and the second one is the opposite: Truthification of the False (when a false statement is transformed into a true one) - within the frame of Recreational Neutrosophy. All transformations from <A> to <antiA> or vice versa should be real, making sense in our real world. Kind of magic logic!

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Data Availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that there is no conflict of interest in the research.

Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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Logici răsturnate: falsificarea adevărului și adevărizarea falsului

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Abstract

Acest articol se ocupă de știința amuzantă, matematica recreativă, gândirea pe dos sau raționamentul contradictoriu (a gândi invers). O afirmație într-o anumită condiție poate fi adevărată, în alte condiții falsă și într-un al treilea tip de condiții parțial adevărată și parțial falsă. Articolul prezintă pentru prima dată două tipuri de logică răsturnată: primul este falsificarea adevărului (când o afirmație adevărată este transformată într-una falsă), iar al doilea este opusul: adevărizarea (verosimilizarea) falsului (când o afirmație falsă este transformată într-una adevărată) - în cadrul neutrosofiei recreative. Toate transformările din <A> în <antiA> sau invers ar trebui să fie reale, având sens în lumea noastră reală. O logică de tip magie! Falsificarea și Adevărizarea sunt utilizate în principal în Științele Sociale (Antropologie, Arheologie, Economie, Geografie, Istorie, Drept, Lingvistică, Politică, Psihologie, Sociologie), Filosofie etc., excelând în Politică:

- Falsificarea Adevărului Inamicilor [Ne străduim să diminuăm partea pozitivă a inamicilor până la punctul de anulare și să amplificăm partea negativă a inamicilor până la punctul de exagerare;] și
- Adevărizarea Falsului Prietenilor [Opusul: Ne străduim să scădem partea negativă a prietenilor până la punctul de anulare și să amplificăm partea pozitivă a prietenilor până la punctul de exagerare.].

Cuvinte cheie

neutrosofie; raționament contradictoriu; gândire pe dos; neutrosofie recreativă; matematică recreativă.

1. Neutrosofia

Neutrosofia este un cadru filozofic și un sistem logic, introdusă de Florentin Smarandache în 1998 [1, 2], care studiază relația dintre opoziții, contradicții și părțile lor neutre sau nedeterminate, cu scopul de a explora și înțelege ambiguitatea, inconsistența și nedeterminarea inerente în multe concepte și sisteme.

Accentul neutrosofiei pe neutralitate, paradoxuri și nedeterminare o face relevantă pentru explorarea scenariilor din lumea reală în care adevărul este complex și multifațetat. Are

aplicații în domenii precum inteligența artificială, luarea deciziilor și rezolvarea problemelor, oferind o modalitate de a gestiona incertitudinea și contradicția.

Neutrosofia se bazează pe dinamica dintre opozițiile <A>, <antiA> și neutralitățile lor <neutA>, unde <A> este un element (concept, atribut, idee, teorie etc.) și <antiA> opusul său, în timp ce <neutA> reprezintă neutralitățile sau nedeterminările dintre acestea.

De exemplu: <A> = adevăr, <antiA> = fals, și <neutA> = neutru sau nedeterminare între adevăr și fals (de exemplu, 70% adevăr și 30% fals).

Neutrosofia poate fi văzută ca o generalizare a dualismului, a conceptului Yin Yang din filosofia chineză antică sau a dialecticii, oferind un cadru mai larg și mai flexibil pentru înțelegerea contradicțiilor și relațiilor dintre opoziții.

Partea neutră <neutA> poate fi, de exemplu, parțial <A> și parțial <antiA>. Unele neutralități pot avea un grad mai mare de adevăr decât de falsitate, altele un grad mai mare de falsitate decât de adevăr. În cel mai rău scenariu, ca nedeterminare între opoziții, o neutralitate poate avea un grad parțial de <A>, un grad parțial de <neutA> și un grad parțial de <antiA>.

2. Procedee și transformări utilizate în logica răsturnată [4-6]

Adevărizarea (propunem acest cuvânt: *Truth-ification*) înseamnă: a face ca ceva să fie adevărat. Atât Falsificarea Adevărului cât și Adevărizarea Falsului sunt realizate prin aceleași strategii:

- Prin schimbarea spațiului sau a timpului problemei;
- Prin schimbarea atributelor elementelor din spațiu;
- Prin schimbarea logicii (procedurii, tacticii, metodei) de abordare a problemei;
- Prin interpretarea sensului opus;
- Logică inversă (pe dos);
- Prin transformarea expresiilor lingvistice din sensul figurativ în sensul propriu, sau viceversa;
- Prin găsirea sensului nonsensului sau a nonsensului sensului;
- Jocuri de cuvinte, puzzle-uri;
- Ghicitori;
- Glume;
- Etc.

Desigur, aceste metode și strategii practice ar trebui să fie reale, să aibă sens în viața noastră de zi cu zi.

2.1. Schimbarea spațiului sau a timpului unei probleme

Acest lucru implică schimbarea contextului pentru a obține noi perspective. Să exersăm un exemplu de schimbare a intervalului de timp: problemele pot părea adesea copleșitoare din cauza constrângerilor de timp. Imaginându-ți problema într-un interval de timp diferit, poți explora noi abordări. De exemplu, dacă lucrezi la un proiect cu un termen limită strict, imaginează-ți cum l-ai aborda având mai mult timp. Acest exercițiu poate dezvălui scurtături sau tehnici care pot fi aplicate chiar și într-un interval de timp limitat.

Schimbarea spațiului sau a timpului problemei implică manipularea contextului sau a setării unei probleme pentru a crea o narațiune diferită. Prin schimbarea locului sau a

momentului în care se presupune că a avut loc un eveniment, se poate schimba adevărul perceput al acestuia.

2.2. Schimbarea atributelor elementelor într-un spațiu

Această tehnică implică modificarea anumitor caracteristici pentru a crea un rezultat diferit. De exemplu, dacă rezolvi un puzzle care implică articole codificate pe culori, schimbarea schemei de culori ar putea duce la o nouă soluție.

Schimbarea atributelor sau caracteristicilor problemei implică modificarea unor elemente sau trăsături ale unei chestiuni pentru a o face să pară mai plauzibilă sau pentru a pune la îndoială acuratețea acesteia. De exemplu, schimbarea detaliilor cheie despre o persoană sau un eveniment poate face ca o poveste inventată să pară mai credibilă sau poate face o poveste adevărată să pară îndoielnică.

2.3. Interpretarea inversă

Această metodă implică considerarea unui concept și răsturnarea lui, ducând adesea la rezultate amuzante sau neașteptate.

(i) Un exemplu ar putea fi clasicul dublu sens al unui cuvânt: "Ce are cheie, dar nu poate deschide lacăte?" Răspunsul poate fi: un pian, interpretând cuvântul "cheie" într-un mod non-tradițional.

Schimbarea logicii (procedurii, tacticii, metodei) de abordare a problemei.

Această strategie se concentrează pe modificarea raționamentului sau metodei folosite pentru a analiza o problemă. Prin introducerea unei logici diferite sau utilizarea greșită a datelor, se pot crea confuzii sau se poate ajunge la concluzii false.

(ii) O altă logică: Cum putem avea zece împărțit la doi egal cu zero?

Răspuns: Zece biscuiți împărțiți între doi copii sunt mâncați și nu a rămas nimic!

2.4. Interpretări literale și figurative

Această tehnică implică înțelegerea limbajului figurativ în mod literal sau invers. Un exemplu de glumă care folosește această tehnică este: "I-am spus prietenului meu să-și rupă un picior *(break a leg!)*, și chiar a căzut!" Aici, expresia "*break a leg*" (care în mod obișnuit înseamnă "noroc") este interpretată literal.

2.5. Ghicitori

Ghicitorile se bazează adesea pe informații înșelătoare sau sensuri duble pentru a provoca răspunsul.

(i) Un exemplu clasic este: "Ce trebuie să fie spart înainte să poți să-l folosești?" Răspunsul este "un ou," unde cuvântul "spart" este folosit într-un context diferit.

(ii) Scrieți numărul 9 (*NINE*, în engleză).

Soluție: (În acest caz, "a scrie" înseamnă să notezi toate barele mici (verticale, oblice și orizontale) care compun cuvântul *NINE*, adică unsprezece bare!)

(iii) Sunt invizibil, dar pot fi măsurat. Afectez pe toată lumea și tot ce este *ceva*. Mă întind în univers și variez de la un loc la altul. Cine sunt eu?

Soluție: Sunt "gravitația."

2.6. Glume

Glumele folosesc o varietate de tehnici pentru a crea umor, inclusiv jocuri de cuvinte sau întorsături neașteptate.

(i) Un exemplu de glumă bazată pe joc de cuvinte este: "*Why don't scientists trust atoms? Because they make up everything!*" Gluma derivă din dublul sens al verbului "*to make up*" în limba engleză:

- A forma sau a constitui: În acest sens, "*to make up*" înseamnă a compune sau a constitui ceva. De exemplu: "*Atoms make up everything*" (Atomii formează totul).
- A inventa sau a fabula: În acest sens, "*to make up*" înseamnă a crea sau a inventa o poveste sau o explicație care nu este neapărat adevărată. De exemplu: "*He made up a story*" (A inventat o poveste).

(ii) Cum ar măsura un matematician intensitatea unui cutremur pe un meteor, așa cum este în filmul Armageddon?

Soluție:

Este imposibil să ai un cutremur pe un meteor!

(iii) 15 vânători au plecat la vânătoare de urși. Unul a omorât 2 urși. Câți urși a omorât unul?

Soluție:

Doi. ("Unul" este numele unuia dintre vânători.)

(iii) $\frac{w}{2} = u$. Găsește o logică pentru această egalitate.

Soluție:

Litera "w" este dublul literei "u", sau "uu", iar împărțind "uu" la 2, se obține un "u".

2.7. Probleme distractive

i) Calculează volumul unui pătrat (!)

Soluție:

Volum = Aria Bazei × Înălțime = Latura² × 0 = 0!

Privim pătratul ca un caz extrem al unui paralelipiped al cărui înălțime este nulă.

ii) ? × 7 = 2 (?)

Soluție:

Desigur, $\frac{2}{7} \times 7 = 2$ (!)

iii) Dacă ai o pereche de trei și le împarți în jumătate, de ce ajungi cu 4 bucăți?

Soluție:

~~33~~

2.8. Matematica norocoasă [3]

Dacă, printr-un calcul aparent greșit (metodă, algoritm, operație etc.), se ajunge la răspunsul corect, aceasta se numește Calcul Norocos (Metodă, Algoritm, Operație etc.)!

Calculul aparent greșit (metodă, algoritm, operație etc.) ar trebui să fie amuzant (într-un fel asemănător cu unul corect, generând confuzie și un zâmbet)!

i) Care este factorul de multiplicare al unui om flămând?

Soluție (este necesară înțelegerea limbii engleze):

8×8, deoarece “eight times eight” se pronunță la fel ca “ate time ate”!

i) Cum poți adăuga 3 cu 3 și să obții 8?

Soluție:

∈∋ = 8

{Să râdem: ha, ha, ha!}

2.9. Falsificare și Adevărizare în Științele Sociale

Ele sunt utilizate în principal în Științele Sociale (Antropologie, Arheologie, Economie, Geografie, Istorie, Drept, Lingvistică, Politică, Psihologie, Sociologie), dar și în Filosofie etc.

Excelând în Politică:

- Falsificarea Adevărului inamicilor; și
- Adevărizarea Falsului prietenilor.

Exemple reale frecvent difuzate la televizor:

Să presupunem că un proiect de lege este propus Congresului spre aprobare. Oamenii care se opun sunt etichetați ca inamici de către lideri, în timp ce cei care susțin proiectul sunt etichetați ca prieteni.

i) Să presupunem că există o demonstrație de câteva mii de oameni în centrul orașului împotriva acestui proiect de lege.

Falsificarea Adevărului:

Postul de televiziune a transmis: Câteva zeci de oameni demonstrează în centrul orașului împotriva acestui proiect de lege.

Să presupunem că există o contrademonstrație de câțiva oameni în centrul orașului în sprijinul acestui proiect de lege.

Adevărizarea Falsului:

Postul de televiziune a transmis: Mii de oameni demonstrează în centrul orașului împotriva acestui proiect de lege.

2.10. Cititorii pot concepe ei înșiși mai multe metode și strategii

3. Exemple de falsificare a adevărului

3.1. 1 = 1 (Adevărat)

Falsificarea afirmației:

1 metru = 1 kilometru {*Fals*, prin introducerea atributelor elementelor (unități de măsură)}

3.2. 2 + 3 = 5 (Adevărat)

Falsificarea afirmației:

2 + 3 = 5 în baza 10, dar schimbând în baza 5, se obține:

2 + 3 = 10 în baza 5, ori $2_5 + 3_5 = 10_5$.

3.3. Zece păsări zboară pe cer. Un vânător trage în trei dintre ele. Câte păsări rămân?

Răspuns:

10 – 3 = 7 păsări, cele care încă zboară [spațiul este cerul].

Trei falsificări ale afirmației de mai sus [prin schimbarea spațiului elementelor (păsărilor)]:

3.3.1. Zece păsări sunt pe un gard. Un vânător împușcă trei dintre ele. Câte păsări rămân?

Răspuns: Spațiul este gardul.

Niciuna, pentru că cele trei păsări moarte au căzut de pe gard, iar celelalte șapte au zburat!

3.3.2. Zece păsări sunt într-o poiană. Un vânător împușcă trei dintre ei. Câte păsări rămân?

Răspuns: Spațiul este pajiștea.

Trei păsări, păsările moarte rămânând pe pajiște, căci celelalte șapte au zburat!

3.3.3. Zece păsări sunt într-o cușcă. Un vânător împușcă trei dintre ei. Câte păsări rămân?

Răspuns: Spațiul este cușca.

Zece păsări, moarte și vii, pentru că niciuna nu putea ieși din cușcă!

4. Exemple reale de adevărizare a falsului

4.1. 2 = 1 (Fals)

Soluție (Adevărizarea afirmației):

2 *pints* = 1 *quart* {Adevărat, prin introducerea atributelor elementelor (unități de măsură)}

4.2. Un bărbat cântărește următoarele greutăți în următoarele date:.

6/1/70	68,18 kg
6/3/70	0 kg
6/5/70	11,34 kg.
6/7/70	0 kg
6/9/70	65,77 kg

Cum este posibil?

Soluție (Adevărizare):

Omul este un astronaut care a călătorit pe Lună și s-a întors. În spațiu, greutatea este 0 lg din cauza imponderabilității. Pe Lună, gravitația este de $\frac{1}{6}$ din cea de pe Pământ (68,18 kg), deci greutatea sa acolo ar fi de 11,34 kg.

4.3. Din două ipoteze false se obține o afirmație adevărată.

Iarba este comestibilă. (Fals)

Lucrurile comestibile sunt verzi. (Fals)

Prin urmare, iarba este verde. (Adevărat)

4.4. "Când o cană de lapte este adăugată la o cană de popcorn, rezultă doar o cană de amestec, deoarece cana de popcorn va absorbi aproape întreaga cană de lapte fără vărsare. Așadar, în acest caz, avem 1 + 1 = 1. Acest lucru este imposibil în aritmetica convențională, dar este adevărat pentru unele aritmetici non-diofantine." [7].

4.5. Demonstrează că ecuația $x = x + 2$ are două soluții distincte.

Răspuns: Aceasta este o ecuație inconsistentă în mulțimea numerelor reale finite, dar în mulțimea infinităților nu este: soluțiile sunt $x = \pm\infty$.

Concluzii

Acest articol a prezentat pentru prima dată două tipuri de *logică răsturnată*: ***Falsificarea Adevărului*** (când o afirmație adevărată este transformată într-una falsă) și ***Adevărizarea Falsului*** (când o afirmație falsă este transformată într-una adevărată) – în cadrul *neutrosofiei recreative*. Toate transformările de la <A> la <antiA> sau invers ar trebui să fie reale, având sens în lumea noastră reală. Un fel de logică magică!

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Note on Partial Falsifiability of Fuzzy and Fuzzy-Extension Hypotheses

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Abstract

In this short note we adjust the (classical) Falsifiability of a hypothesis that may be 100% true as in classical logic, to the field of multi-valued logics, where there are partialities of truth-value, of indeterminacy, and of false-value of a hypothesis, and we call it Partial Falsifiability.

Keywords: Partial Falsifiability, Fuzzy-Extension Hypotheses, Classical Falsifiability.

1 | Falsifiability

A (classical, i.e. non multi-valued) hypothesis is considered credible when it is falsifiable (or refutable), meaning there is a possibility it could be proven inconsistent. Falsifiability, a concept introduced by Austrian philosopher of science Karl Popper (1902-1994), asserts that a theory must be falsifiable to be classified as scientific.

Karl Popper's concept of falsification [1] argues that the purpose of scientific inquiry is not to validate hypotheses, but rather to rigorously examine them and determine under what conditions they can be proven false. According to the falsification principle, a valid scientific theory must generate hypotheses that can potentially be disproven by empirical evidence or experimental outcomes. In contrast to verification, which seeks to affirm theoretical predictions, falsification is about decisively disproving them.

In scientific methodology, testability and replicability are widely accepted principles. However, the idea of falsifiability has been met with varying levels of acceptance.

Abstract and precise mathematical and scientific statements are not inherently falsifiable, as noted by Carnap in 1937. For example, if a theory is proven false, it is generally discarded.

2 | Partial Falsibiality

But what about multi-valued logical hypotheses, which allow for partial degrees of truth, indeterminacy, and falsehood, as for example, in neutrosophic logic? How do we test and make such hypotheses falsifiable with these kinds of complexities?

Until now, hypotheses have been considered classical (i.e., propositions that were 100% true or nearly so), but now we are extending this to hypotheses represented by propositions from multi-valued logic.

(i) Logical interpretation

If a neutrosophic logic hypothesis $\mathrm{NLH}(t, i, f)$ is falsifiable, where $t, i, f \in [0, 1]$ are degress of truth, indeterminacy, and falsehood respectively, then there should exist some space, time, and conditions within which the above neutrosophic hypothesis is negated, getting $\neg\mathrm{NLH}(f, 1 - i, t)$.

(ii) Probability interpretation

Let $\mathrm{NPH}(t, i, f)$ be a neutrosophic probabilistic hypothesis, where $t, i, f \in [0, 1]$ represent the chance that this hypothesis occurs, indeterminate-chance that the hypothesis occurs or not, and the chance that the hypothesis does not occur, respectively.

$\mathrm{NPH}(t, i, f,)$ is falsifiable if, similarly, there exist some space, time, and conditions under which the NPH is negated, getting: $\neg\mathrm{NPH}(f, 1 - i, t)$.

3 | Conclusion

In this short note we made a distinction between classical logical hypotheses and multi-valued logical hypotheses, and we introduced the concept of Partial Falsifiability as adaptation of the Popper's Falsifiability from classical logic to the multi-valued logics. We only used the neutrosophic logic, but any fuzzy and fuzzy extension logics [2] could be used, resulting in the same partiality of the falsifiability.

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Data Availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that there is no conflict of interest in the research.

Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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A few little steps beyond Knuth's Boolean Logic Table with Neutrosophic Logic: A Paradigm Shift in Uncertain Computation

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Abstract

The present article delves into the extension of Knuth's fundamental Boolean logic table to accommodate the complexities of indeterminate truth values through the integration of neutrosophic logic (Smarandache & Christianto, 2008). Neutrosophic logic, rooted in Florentin Smarandache's groundbreaking work on Neutrosophic Logic (cf. Smarandache, 2005, and his other works), introduces an additional truth value, 'indeterminate,' enabling a more comprehensive framework to analyze uncertainties inherent in computational systems. By bridging the gap between traditional boolean operations and the indeterminacy present in various real-world scenarios, this extension redefines logic tables, introducing neutrosophic operators that capture nuances beyond the binary realm. Through a thorough exploration of neutrosophic logic's principles and its implications in computational paradigms, this study proposes a novel approach to logic design that accommodates uncertain, imprecise, and incomplete information. This paradigm shift in logic tables not only broadens the spectrum of computing methodologies but also holds promise in fields such as decision-making systems and data analytics. This article amalgamates insights from over twelve key references encompassing seminal works in boolean logic, neutrosophic logic, and their applications in diverse scientific and computational domains, aiming to pave the way for a more robust and adaptable logic framework in computation.

Keywords: Knuth's Boolean logic table; Neutrosophic logic table; Neutrosophic Logic; uncertainties inherent in computational systems; adaptable logic framework.

1. Introduction

Knuth's 16 boolean functions, which he outlines in "*The Art of Computer Programming*," are fundamental in understanding the behavior of logical operations. These functions categorize all possible truth tables for two binary variables (inputs) and one binary result (output). Each function represents a unique combination of inputs and outputs.

Regarding their implications:

A. Computational Logic: These functions form the basis of Boolean algebra and logic gates in digital circuit design. They are crucial in designing computer systems and algorithms. For instance, logic gates like AND, OR, XOR, NAND, NOR, etc., are representations of specific boolean functions that form the building blocks of digital circuits.

B. Game Theory and Decision Making: In game theory, Boolean logic and truth tables are used to model decision-making processes and strategies. They help in analyzing the possible outcomes of different choices and actions, which is essential in understanding strategic moves in games.

C. Complex Systems and Control: Boolean functions find applications in modeling complex systems, such as in control systems, where decision-making processes are critical. They help define conditions and rules that control the behavior of these systems.
D. Cryptography: Boolean logic is fundamental in cryptography for designing encryption algorithms and protocols. Operations like XOR play a pivotal role in encryption and decryption processes.
Understanding these Boolean functions is essential for a broad range of fields, from computer science and mathematics to engineering, gaming, and even philosophy, as they form the foundation of logical thinking and computational processes.

Results:

Our proposition: Extending Knuth's Boolean logic table with Neutrosophic Logic:

Extending Knuth's Boolean logic operators to neutrosophic logic operators involves bridging the gap between traditional binary logic and neutrosophic logic, which deals with indeterminacy and uncertainty. Neutrosophic logic introduces a third truth value, "indeterminate," besides the typical true and false values in classical logic.
In principle, as Smarandache wrote in one of his books, the symbolic neutrosophic operators, allow us working with the letter T, I, F for conjunction, disjunction, negation, implication. Therefore, in Neutrosophic Logic, one has the following neutrosophic truth-value table for the
neutrosophic negation. So, we have to consider that the negation of I is I, while the negations of T and F are similar as in classical logic. The objective part (circled literal components in the above table) remains as in classical logic, but when indeterminacy I interferes, the neutrosophic expert may choose the most fit prevalence order. There are also cases when the expert may choose, for various reasons, to entangle the classical logic in the objective part. In this case, the prevalence order will be totally subjective (cf. Smarandache, *Symbolic Neutrosophic Theory*, 2015, pp. 169-172).
To connect Knuth's boolean logic operators with neutrosophic logic operators:
Expanding Truth Values: Neutrosophic logic incorporates the idea of indeterminacy, allowing for truth values to be characterized not just as true or false but also as indeterminate. This extension involves defining new logical operators (like AND, OR, NOT) that consider this additional truth value.
Neutrosophic Logic Operators: In neutrosophic logic, operators like neutrosophic AND, OR, NOT, etc., are defined to accommodate the indeterminate truth value. These operators handle situations where the truth values are imprecise, uncertain, or contradictory.
Mapping to Knuth's Operators: It's possible to map the operations of neutrosophic logic onto Knuth's boolean logic by considering how the additional truth value "indeterminate" interacts with the existing boolean operators. This mapping would involve defining rules for combining indeterminate values with true and false values.
Altering Functionality: Neutrosophic logic operators might behave differently from traditional boolean operators due to the inclusion of the "indeterminate" truth value. They may have altered truth tables or rules for handling indeterminate values that differ from Knuth's boolean logic.
While extending Knuth's boolean logic to neutrosophic logic is conceptually feasible, it's a complex endeavor that requires careful consideration of how the additional truth value impacts logical operations. It involves defining new operators, establishing their properties, and ensuring consistency within the framework of neutrosophic logic.

Application 1: Extending Knuth's Boolean logic table in football/soccer game

Soccer, a sport celebrated for its complexity and unpredictability, often defies simplistic analysis due to the dynamic interplay of player decisions, strategies, and uncertain outcomes. Traditional logic, rooted in binary truth values, offers a limited framework to dissect the intricacies of this game. However, the extension of Knuth's boolean logic to neutrosophic logic presents an intriguing avenue to revolutionize the understanding of soccer dynamics.

The Indeterminate Element in Soccer Logic:

Neutrosophic logic introduces an indeterminate truth value, acknowledging the uncertain, imprecise, and ambiguous nature inherent in soccer matches. In a game where player performance, weather conditions, strategy

effectiveness, and referee decisions intertwine, neutrosophic logic provides a more holistic approach to analyze scenarios that aren't distinctly true or false.

Mapping Knuth's Operators to Soccer Realities:

Adapting Knuth's boolean operators to the realm of soccer involves defining neutrosophic AND, OR, NOT, and other operators that accommodate indeterminate values. For instance, the neutrosophic AND could represent a player's decision-making process in an uncertain environment, where multiple factors influence their actions beyond a binary choice.

Modeling Match Scenarios:

Neutrosophic logic allows for the creation of models that consider uncertain elements in soccer games. It enables the simulation of scenarios where indeterminate factors, such as crowd influence, psychological states of players, or unexpected weather changes, significantly affect match dynamics.

Predictive Insights and Decision Support:

The application of neutrosophic logic in soccer analysis enhances predictive capabilities, providing coaches, analysts, and teams with more nuanced insights. It aids in formulating adaptable strategies that account for the indeterminate nature of the game.

Application 2: Knuth's Boolean logic table for quantum data compression (very efficient compression)

Creating a quantum data compression algorithm based on Knuth's extended boolean logic table to achieve a compression ratio of 1:1000 is quite a great task. Implementing such an algorithm typically involves leveraging advanced quantum computing techniques and specific compression strategies, which might not be directly translatable to Mathematica code, especially within the constraints of this platform. However, the following is an outline a general approach and steps you might consider to design such an algorithm:

Approach:

Quantum State Encoding: Convert the classical data into a quantum state by encoding it in a quantum superposition of states. This encoding usually involves mapping classical data to quantum states.

Quantum Operations: Apply quantum gates and operations to manipulate the quantum state according to the algorithm based on Knuth's extended boolean logic table. This step would involve utilizing quantum logic gates to perform computations.

Compression Procedure: Design a compression procedure leveraging the quantum state information to compress the data efficiently. This may involve exploiting quantum properties to reduce redundancies in the data representation.

Decoding: Revert the quantum state back to classical data, extracting the compressed information accurately.

Example Steps (Simplified):

```
(* Assume 'classicalData' is the classical data to be encoded *)
classicalData = {0, 1, 0, 1, 1, 0, 0, 1};
(* Define quantum states based on classical data *)
quantumState = Table[If[classicalData[[i]] == 0, Quantum`PureState`Zero[],
Quantum`PureState`One[]], {i, Length[classicalData]}];
```

The above code initializes a quantum state based on classical data. However, the subsequent steps involving quantum gates, compression procedure, and decoding are highly specialized and complex, beyond the scope of a simple code demonstration here.

Now, let us extend task a bit : *how we can compress data more efficiently down to 1:10000….*

Implementing a quantum data compression algorithm based on Knuth's extended boolean logic table using Mathematica could involve a complex set of operations. Creating an efficient compression algorithm down to 1:10000 compression ratio would require detailed knowledge of quantum computing and compression techniques beyond the standard functionalities available in Mathematica.

While we can provide a simple code of using Mathematica for basic quantum computing operations, such as creating quantum circuits or performing simple quantum operations, implementing a sophisticated compression algorithm may not be feasible within the constraints of this platform.

Example:

```
(* Importing the necessary Quantum Computing package *)
Needs["Quantum`Notation`"]
(* Define qubits *)
```

```
qubits = 4;
(* Create a quantum circuit *)
circuit = CreateCircuit[qubits];
(* Apply Hadamard gate to all qubits *)
Do[
ApplyHadamard[circuit, i],
{i, qubits}
];
(* Apply controlled-not gates *)
Do[
ApplyGate[circuit, ControlledNot, {i, qubits}],
{i, qubits - 1}
];
(* Display the circuit *)
circuit // ToBoxes // DisplayForm
```

2. Discussion:

In a preceding section, we have discussed how it is possible to figure out extension of Knuth's Boolean logic to neutrosophic logic in the context of soccer offers a promising avenue to advance the understanding and analysis of this intricate sport. By embracing the indeterminate truth value, soccer analysis can transcend binary constraints, leading to more comprehensive insights, enhanced decision-making, and a deeper appreciation of the multifaceted logic governing the beautiful game.

Hence, in the subsequent section, the above code creates a simple quantum circuit composed of Hadamard gates applied to all qubits and controlled-not gates between consecutive qubits. Implementing the compression algorithm to achieve a specific compression ratio would require a detailed understanding of quantum computing, compression techniques, and the Knuth's extended Boolean logic table. Moreover, achieving such compression ratios involves intricate algorithms and optimizations, which might not fit into a concise Mathematica code snippet.

For complex quantum algorithms and efficient compression techniques, it's recommended to explore quantum computing libraries and platforms (like Qiskit, Cirq, or others) that offer extensive tools and support for implementing quantum algorithms and data compression techniques. They often provide a more suitable environment for such advanced computations.

Concluding remark:

The present article delves into the extension of Knuth's fundamental Boolean logic table to accommodate the complexities of indeterminate truth values through the integration of neutrosophic logic (Smarandache & Christianto, 2008). Neutrosophic logic, rooted in Florentin Smarandache's groundbreaking work on Neutrosophic Logic (cf. Smarandache, 2005, and his other works), introduces an additional truth value, 'indeterminate,' enabling a more comprehensive framework to analyze uncertainties inherent in computational systems. By bridging the gap between traditional Boolean operations and the indeterminacy present in various real-world scenarios, this extension redefines logic tables, introducing neutrosophic operators that capture nuances beyond the binary realm.

The extension of Knuth's Boolean logic to neutrosophic logic in the context of soccer offers a promising avenue to advance the understanding and analysis of this intricate sport. By embracing the indeterminate truth value, soccer analysis can transcend binary constraints, leading to more comprehensive insights, enhanced decision-making, and a deeper appreciation of the multifaceted logic governing the beautiful game.

For implementing a complex compression algorithm based on Knuth's extended Boolean logic table and achieving a compression ratio of 1:10000 or less than that, it would require a detailed understanding of quantum algorithms, their implementations, and their integration with data compression techniques, which exceeds the scope of this platform.

For advanced quantum computing algorithms and intricate compression techniques, specialized quantum computing platforms, libraries, or environments would be more suitable. You might want to explore languages

specifically designed for quantum computing, like Cirq or Qiskit, which provide extensive functionalities and tools for implementing quantum algorithms and circuits.

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Law of Included Infinitely-Many-Middles within the frame of Neutrosophy

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Abstract: In this paper, we extend, for the first time, the Law of Included Multiple-Middles to the Law of Infinitely-Many-Middles. And we present several practical applications. Also, we discuss Aristotle's Syllogism, Principle of Identity, and Principle of NonContradiction.

Keywords: Excluded Middle; Included Middle; Included Multiple-Middles; Included Infinitely-Many-Middles; Syllogism; Many-Valued Syllogism; Identity; NonIdentity; NonContradiction, Anti-NonContradiction.

A. Short History

We present below the evolution from the Law of Excluded Middle to the Law of Included Infinitely-Many-Multiples.

1. Law of Excluded Middle

The **Law of Excluded Middle** was enounced by the Ancient Greek philosopher Aristotle (384 - 322 BC) on his opus on logic and reasoning [1, 2] that was based on analysis and dichotomy:

> "There cannot be an indeterminate between contraries, but of one subject we must either affirm or deny anyone predicate".

Therefore, a proportion is either 100% true or 100% false, as in Boolean logic. Or, an element either belong 100% to a set, or does not belong 100% to the set (as in the classical set theory).

2. Law of Included Middle

The **Law of Included Middle** is the denial of previous, and it supports the idea that between contraries there may be a middle. It is based on trichotomy. Several philosophers and logicians developed it, such as Stephane Lupasco's logic of contradiction, using the non-standard logic, followed by Basarab Nicolescu's levels of reality, and J.-J. Wunenburger. Gonseth pleads for a low necessity in using the logic of contradiction. [5]

With the introduction of modern sets and logics, such as fuzzy set/logic (Zadeh, 1965), intuitionistic fuzzy set/logic (Atanassov, 1983), neutrosophic set/logic/probability (Smarandache, 1995), the Law of Included Middle became evident and useful in our everyday life where we deal with approximate partial membership/non-membership/truth/falsehood while in neutrosophic probability besides the chance of occurrence of an event, there has been added the middle term: indeterminate-chance of occurrence or not.

Neutrosophic set and logic explicitly presented the *middle term I (indeterminacy or neutrality)* in between the opposite terms (membership/truth), and *F* (non-membership/falsehood).

3. Law of Included Multiple-Middles

The **Law of Included Multiple-Middles** is an extension of the previous, and it was enounced by Smarandache [4] in 2014.

Neutrosophy [6] is a branch of philosophy that studies the dynamics of the opposites <A> and <antiA> together with their neutrality <neutA>, where <A> is an item (idea, proposition, theory, etc.), <antiA> is its opposite, while <neutA> is the neutrality in between them (i.e. neither <A> nor <antiA>).

Of course, we are referring to the neutrosophic triads <A>, <neutA>, <antiA> that make sense in our real world.

Neutrosophy, together with Neutrosophic Set/logic/probability, have been refined [7] in 2013, by refining splitting/multiplicating <A>, <neutA>, <antiA> as follows:

<A> as $<A_1>$, $<A_2>$, ..., $<A_p>$;

<neutA> as $<neutA_1>$, $<neutA_2>$, ..., $<neutA_r>$;

and <antiA> as $<antiA_1>$, $<antiA_2>$, ..., $<antiA_s>$;

where $p, r, s \geq 0$ are integers,

and at least one of p, r, s is $\geq$ in order to ensure that at least one neutrosophic component amongst <A>, <neutA>, <antiA> is refined/split/multiplicated.

This definition also permits the refinement of fuzzy set/logic (for $p \geq 2$ and $r = s = 0$), and of intuitionistic fuzzy set/logic (for $p \geq 1$, $r = 0$, $s \geq 1$ and at least one of p or s is ≥ 2).

By taking $p = 1$, $r \geq 2$, and $s = 1$, we defined the Law of Included Multiple-Middles:

Between the opposites <A> and <antiA> there are included multiple-middles: $<neutA_1>$, $<neutA_2>$, ..., $<neutA_r>$.

It is of course based on a multichotomical analysis.

i) Between the opposite colors White and Black there are many colors such as: yellow, rose, red, blue, etc.

ii) Pentagonal Neutrosophic logic, where each proposition is characterized by five degrees of truth, such as (T, C, V, U, F)

where the opposites are:

T=degree of truth and F=degree of falsehood,

and the three included-middles are:

C=degree of contradiction

V=degree of vagueness

U=degree of unknowingness

For example, the logical proposition:

P=Artificial intelligence will take over the world, evaluated by experts, this proposition may be 40% true (T), 20% contradictory (C), 30% vague (V), 60% unknown (U) and 50% false (F).

P (0.4, 0.2, 0.3, 0.6, 0.5).

4. Law of Included Infinitely-Many-Middles

In between the opposites <A> and <antiA> there are infinitely many middles, denoted by

$<neutA_i>$, i =1, 2, ..., ∞.

(i) Practical Example

Between the White and Black colors there are infinitely-many nuances of colors.

(ii) Between 100% True and 100% False, there are included infinitely many middles, which are truth-values of the form: d% True and (1-d)% False, thus a logical proposition may be, for example:

1% True and 99% False, 2% True and 98% False, etc.

(iii) Similarly, between 100% membership and 100% non-membership, there are included infinitely many middles of the form: d% membership and (1-d%) non-membership.

5. Syllogism

Aristotle studied it:

if "A→B" and "B→C" are totally true, then "A→C" is also totally true.

This is in classical logic.

6. Many-Valued Syllogism

In many-valued logics, where "A→B" and "B→C" are partially true, then "A→C" is partially true as well.

7. Principle of NonContradiction

It was enounced by Aristotle [1, 2], that <A> and <antiA> cannot be true at the same time:

> "the same attribute cannot at the same time belong and not belong to the same subject and in the same respect",

and

> "it is impossible for anyone to believe the same thing to be and not to be, as something Heraclitus says".

8. Principle of Anti NonContradiction

We name this principle as "Anti Contradiction" that occur in the many-valued logics in order to distinguish it from the Principle of NonContradiction (also called Contradiction).

The above principle, related to the Law of Excluded Middle, does not work any longer in the modern theories. Again, with the introduction of modern set theories, it is possible to have both, degree of belonging and degree of not-belonging simultaneously of an element to a set, for example John (0.6, 0.4), meaning that John belongs (works) only 60% for his company and 40% does not. And similarly with respect to the modern logics, where a logical proposition may be partially true and partially false.

In fuzzy and fuzzy extension theories (except neutrosophic theories), <A> and <antiA> may be partially (not totally) true at the same time.

In fuzzy logic, if a proposition P is 50% true, then its negation ¬P is also 100%-50% = 50% true.

In neutrosophic logic, if a proposition has the truth-value P is $(a, 0.5, a)$, where $0 \le a \le 1$, then its negation ¬P is also $(a, 1 - 0.5, a) = (a, 0.5, a)$.

In neutrosophic theories, <A> and <antiA> may be partially or totally true at the same time. For example, a paradox is proposition that is 100% true and 100% false at the same time, therefore <A> and <antiA> are totally true at the same time.

9. Principle of Identity

As enounced by Aristotle:

A=A (an item is equal to itself).

This is true if one considers the item <A> under the same parameters that characterize it, and having the same corresponding values:

$$A(P_1 = v_1, P_2 = v_2, \ldots, P_n = v_n) = A(P_1 = v_1, P_2 = v_2, \ldots, P_n = v_n).$$

10. Principle of NonIdentity

The Principle of Identity, by Aristotle, that A = A, works when the entity A is compared to itself with respect to the same parameters that characterize A, with each parameter measured at the same scale and on the same time.

But, if the parameters that characterize A are different, or their corresponding values are different, then one has non-equality.

For example, if A=Andrew, then Andrew(at age 5) ≠ Andrew(at age 70) physically, intellectually, and psychically.

As such, one may also define a Principle of NonIdentity, when A is different from A in at least one circumstance. And, in general, an item (person, animal, object, etc.) is not equal to itself at different times:

item (at time *t1*) $\neq$ item (at time *t*2).

B. Conclusion

We have presented the Law of Excluded Middle by Aristotle, then the Law of Included Middle, Law of Included Multiple-Middles, and we introduced for the first time the Law of Included Infinitely-Many-Middles.

Afterwards, several comments we made on Aristotle's Syllogism, Principle of NonContradiction, and Principle of Identity, that, in the many-valued logics, may have degrees of partial truth and partial falsehood even partial indeterminacy - depending on each application.

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Legea Terțului-Multiplu-Infinit Inclus în cadrul neutrosofiei

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Abstract

În această lucrare, extindem, pentru prima dată, Legea Terțului-Multiplu Inclus la Legea Terțului-Multiplu-Infinit Inclus și prezentăm câteva aplicații practice. De asemenea, discutăm despre silogismul lui Aristotel, principiul identității și principiul noncontradicției.

Cuvinte cheie

Terț Exclus; Terț inclus; Terț-Multiplu Inclus; Terțului-Multiplu-Infinit Inclus; Silogism; Silogism multivalent; Identitate; NonIdentitate; NonContradicție; Anti-NonContradicție.

1. Scurt istoric

Prezentăm mai jos evoluția de la Legea Terțului Exclus la Legea Terțului-Multiplu-Infinit Inclus.

2. Legea Terțului Exclus

Legea Terțului Exclus a fost enunțată de filozoful grec antic Aristotel (384 - 322 î.Hr.) în opera sa despre logică și raționament [1, 2], fundamentată pe analiză și dihotomie:

"Nu poate exista un indeterminat între contrarii, ci despre un subiect trebuie să afirmăm sau să negăm orice predicat".

Prin urmare, o propoziție este fie 100% adevărată, fie 100% falsă, ca în logica booleană. Sau, un element fie aparține 100% unei mulțimi, fie nu aparține 100% acelei mulțimi (ca în teoria clasică a mulțimilor).

3. Legea Terțului Inclus

Legea Terțului Inclus este negarea celei anterioare și susține ideea că între contrarii poate exista un mijloc (un terț). Se bazează pe trihotomie. Mai mulți filosofi și logicieni au dezvoltat-o, cum ar fi logica contradicției a lui Stephane Lupasco, folosind logica non-standard, urmată de nivelele de realitate ale lui Basarab Nicolescu și J.-J. Wunenburger. Gonseth pledează pentru o necesitate redusă în utilizarea logicii contradicției. [5]

Odată cu introducerea teoriilor moderne ale mulțimilor și logicii, cum ar fi mulțimile/logica fuzzy (Zadeh, 1965), mulțimile/logica fuzzy intuiționistă (Atanassov, 1983), și mulțimile/logica/probabilitatea neutrosofică (Smarandache, 1995), Legea Terțului Inclus a devenit evidentă și utilă în viața noastră de zi cu zi, unde ne confruntăm cu apartenența/neapartenența parțială aproximativă la adevăr/fals. În probabilitatea neutrosofică, pe lângă șansa de apariție a unui eveniment, a fost adăugat un termen mediu: șansa nedeterminată de apariție sau de neapariție a acestuia.

Mulțimile și logica neutrosofică au prezentat explicit *termenul mediu I* (nedeterminare sau neutralitate) între termenii opuși (apartenență/adevăr) și *F* (neapartenență/falsitate).

4. Legea Terțului-Multiplu Inclus

Legea Terțului-Multiplu Inclus este o extensie a celei anterioare și a fost enunțată de Smarandache [4] în 2014.

Neutrosofia [6] este o ramură a filozofiei care studiază dinamica opozițiilor <A> și <antiA> împreună cu neutralitatea lor <neutA>, unde <A> este un element (idee, propoziție, teorie etc.), <antiA> este opusul său, iar <neutA> reprezintă neutralitatea dintre acestea (adică, nici <A>, nici <antiA>).

Desigur, ne referim la triadele neutrosofice <A>, <neutA>, <antiA>, care au sens în lumea noastră reală.

Neutrosofia, împreună cu mulțimea/logica/probabilitatea neutrosofică, au fost rafinate [7] în 2013, prin rafinarea împărțirii/multiplicării <A>, <neutA>, <antiA> după cum urmează:

<A> ca $<A_1>$, $<A_2>$, ..., $<A_p>$;

<neutA> ca $<neutA_1>$, $<neutA_2>$, ..., $<neutA_r>$;

și <antiA> ca $<antiA_1>$, $<antiA_2>$, ..., $<antiA_s>$;

unde $p, r, s \geq 0$ sunt numere întregi,

și cel puțin unul dintre p, r, s este ≥ 1, pentru a ne asigura că cel puțin unul dintre componentele neutrosofice dintre <A>, <neutA>, <antiA> este rafinat/împărțit/multiplicat.

Această definiție permite de asemenea rafinarea mulțimilor/logicii fuzzy (pentru $p \geq 2$ și $r=s=0$), și a mulțimilor/logicii fuzzy intuiționiste (pentru $p \geq 1$, $r=0$, $s \geq 1$ și cel puțin unul dintre p sau s este ≥ 2).

Prin alegerea lui $p = 1$, $r \geq 2$, și $s = 1$, am definit Legea Terțului-Multiplu Inclus:

Între opozițiile <A> și <antiA> sunt incluse multiple nuanțe terțe: $<neutA_1>$, $<neutA_2>$, ..., $<neutA_r>$.

Aceasta se bazează, desigur, pe o analiză multihotomică.

i) Între culorile opuse Alb și Negru există multe culori, cum ar fi: galben, roz, roșu, albastru etc.

ii) Logica Neutrosofică Pentagonală, în care fiecare propoziție este caracterizată de cinci grade de adevăr, cum ar fi (*T, C, V, U, F*) unde opozițiile sunt:

T= gradul de adevăr și *F*=gradul de fals, și cele trei nuanțe de mijloc incluse sunt:

C=gradul de contradicție *V*=gradul de vag *U*=gradul de necunoaștere.

De exemplu, propoziția logică:

P= Inteligența artificială va prelua controlul asupra lumii, evaluată de experți, această propoziție poate fi 40% adevărată (*T*), 20% contradictorie (*C*), 30% vagă (*V*), 60% necunoscută (*U*) și 50% falsă (*F*).

P (0.4, 0.2, 0.3, 0.6, 0.5).

5. Legea Terțului-Multiplu-Infinit Inclus

Între opozițiile <A> și <antiA> există infinit de multe nuanțe terțe, denotate prin <neutA_i>, i =1, 2, ..., ∞.

(i) Exemplu practic.

Între culorile Alb și Negru există infinit de multe nuanțe de culori.

(ii) Între 100% Adevărat și 100% Fals, există infinit de multe nuanțe terțe, care sunt valori de adevăr de forma: *d*% Adevărat și (1-*d*)% Fals, astfel încât o propoziție logică poate fi, de exemplu: 1% Adevărat și 99% Fals, 2% Adevărat și 98% Fals, etc.

(iii) În mod similar, între 100% apartenență și 100% neapartenență, există infinit de multe nuanțe de tipul: *d*% apartenență și (1-*d*%) neapartenență.

6. Silogism

Aristotel a enunțat următorul silogism: dacă "A→B" și "B→C" sunt complet adevărate, atunci "A→C" este de asemenea complet adevărat. Acest lucru este valabil în logica clasică.

7. Silogism multivalent

În logica multivalentă, unde "A→B" și "B→C" sunt parțial adevărate, atunci "A→C" este, de asemenea, parțial adevărat.

8. Principiul NonContradicției

A fost enunțat de Aristotel [1, 2] și afirmă că <A> și <antiA> nu pot fi adevărate în același timp:

"Același atribut nu poate aparține și nu poate să nu aparțină în același timp aceluiași subiect și în același sens".

Și:

"Este imposibil ca cineva să creadă că același lucru este și nu este, așa cum spune Heraclit".

9. Principiul Anti NonContradicției

Îl numim Principiul "Anti Contradicției" în logica multivalentă pentru a-l distinge de Principiul Non-Contradicției.

Principiul de mai sus, legat de Legea Terțului Exclus, nu mai funcționează în teoriile moderne. Odată cu introducerea teoriei moderne a mulțimilor, este posibil ca un element să aibă simultan un grad de apartenență și un grad de neapartenență la o mulțime; de exemplu,

John (0,6, 0,4), ceea ce înseamnă că John aparține (lucrează) doar 60% pentru compania sa și 40% nu. În mod similar, în logica modernă, o propoziție logică poate fi parțial adevărată și parțial falsă.

În teoriile fuzzy și în teoriile de extensie fuzzy (cu excepția teoriilor neutrosofice), <A> și <antiA> pot fi parțial (nu total) adevărate în același timp.

În logica fuzzy, dacă o propoziție P este 50% adevărată, atunci negația sa ¬P este, de asemenea, 100% - 50% = 50% adevărată.

În logica neutrosofică, dacă o propoziție are valoarea de adevăr P ca (a, 0.5, a), unde $0 \leq a \leq 1$, atunci negația sa ¬P este, de asemenea, (a, 1 – 0.5, a) = (a, 0.5, a).

În teoriile neutrosofice, <A> și <antiA> pot fi parțial sau total adevărate în același timp. De exemplu, un paradox este o propoziție care este 100% adevărată și 100% falsă în același timp, astfel <A> și <antiA> sunt complet adevărate în același timp.

10. Principiul Identității

Așa cum a enunțat Aristotel:

A=A (un element este egal cu sine însuși).

Acest lucru este adevărat dacă se consideră elementul <A> sub aceleași parametri care îl caracterizează și având aceleași valori corespunzătoare:

$A(P_1 = v_1, P_2 = v_2, ..., P_n = v_n) = A(P_1 = v_1, P_2 = v_2, ..., P_n = v_n)$.

11. Principiul NonIdentității

Principiul Identității, enunțat de Aristotel, afirmă că A=A. Acesta se aplică atunci când entitatea A este comparată cu ea însăși în raport cu aceiași parametri care o caracterizează, fiecare parametru fiind măsurat pe aceeași scară și în același timp.

Dar, dacă parametrii care caracterizează A sunt diferiți sau valorile lor corespunzătoare sunt diferite, atunci se întâlnește o non-egalitate.

De exemplu, dacă A = Andrew, atunci Andrew (la vârsta de 5 ani) ≠ Andrew (la vârsta de 70 de ani) din punct de vedere fizic, intelectual și psihic.

Astfel, se poate defini și un Principiu al NonIdentității, atunci când A este diferit de A în cel puțin o circumstanță. Și, în general, un element (persoană, animal, obiect etc.) nu este egal cu el însuși în momente diferite: element (în momentul t_1) ≠ element (în momentul t_2).

Concluzii

Am prezentat Legea Terțului Exclus a lui Aristotel, apoi Legea Terțului Inclus, Legea Terțului-Multiplu Inclus și am introdus pentru prima dată Legea Terțului-Multiplu-Infinit-Inclus. Am propus câteva comentarii pe marginea silogismului lui Aristotel, apopi am discutat Principiul noncontradicției și Principiul identității, care, în logica multivalentă, pot avea grade de adevăr parțial și falsitate parțială, dar chiar și de nedeterminare parțială ─ în funcție de fiecare aplicație.

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De la complejidad a la neutrosofía: una propuesta pedagógica desde la n-aléctica y la cosmovisión latinoamericana

From complexity to neutrosophy: a pedagogical proposal from n-alectics and the Latin American worldview

Da complexidade à neutrosofia: uma proposta pedagógica a partir da n-alética e da cosmovisão latino-americana

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RESUMEN

Introducción: El artículo examina las limitaciones de la teoría de la complejidad de Edgar Morin en pedagogía, señalando su carácter abstracto, su anclaje en la lógica dialéctica binaria y su universalismo que dificulta su aplicación en contextos específicos como el latinoamericano. Materiales y métodos: Como propuesta superadora, se articula un marco teórico integrador a partir del análisis crítico de literatura, comparando los fundamentos de la complejidad moriniana con la neutrosofía y la n-aléctica de Florentin Smarandache, y dialogando con conceptos de la filosofía y cosmovisión latinoamericana (como el pensamiento decolonial, el Buen Vivir y la interculturalidad crítica). Resultados: El análisis comparativo revela que la lógica neutrosófica (que opera con los componentes Verdad, Indeterminación y Falsedad) y su derivación n-aléctica (que analiza la dinámica entre una entidad, su opuesto y su neutral) ofrecen un andamiaje conceptual más robusto. Este marco, en diálogo con el pensamiento latinoamericano, permite gestionar la incertidumbre, la contradicción y la neutralidad inherentes a los procesos educativos. Discusión: Se esbozan propuestas pedagógicas concretas para un diseño curricular n-aléctico, una práctica docente como mediación intercultural y una evaluación neutrosófica, incluyendo la metodología del Análisis Cualitativo Comparado Neutrosófico (nQCA) y el uso de prompts estructurados para Modelos de Lenguaje (LLMs) que fomenten el pensamiento no binario. Conclusiones: Se fundamenta así una pedagogía neutrosófica latinoamericana, epistemológicamente plural, políticamente decolonial y éticamente relacional, que supera las aporías de la complejidad moriniana y se presenta como una herramienta transformadora para formar sujetos capaces de habitar un mundo incierto y plural.

Palabras clave: neutrosofía, n-aléctica, pedagogía, teoría de la complejidad, filosofía latinoamericana, pensamiento decolonial.

ABSTRACT

Introduction: This article examines the limitations of Edgar Morin's complexity theory in pedagogy, highlighting its abstract nature, its grounding in binary dialectical logic, and its universalism, which hinders its application in specific contexts such as Latin America. Materials and methods: As a proposed solution, an integrative theoretical framework

is articulated based on a critical analysis of literature, comparing the foundations of Morin's complexity theory with Florentin Smarandache's neutrosophy and n-alectics, and engaging with concepts from Latin American philosophy and worldview (such as decolonial thought, Buen Vivir, and critical interculturality). Results: The comparative analysis reveals that neutrosophical logic (which operates with the components Truth, Indeterminacy, and Falsity) and its n-alectic derivation (which analyzes the dynamics between an entity, its opposite, and its neutral) offer a more robust conceptual framework. This framework, in dialogue with Latin American thought, allows for managing the uncertainty, contradiction, and neutrality inherent in educational processes. Discussion: Concrete pedagogical proposals are outlined for an n-alectic curriculum design, a teaching practice based on intercultural mediation, and a neutrosophic assessment, including the methodology of Neutrosophic Comparative Qualitative Analysis (nQCA) and the use of structured prompts for Language Models (LLMs) that foster non-binary thinking. Conclusions: This establishes a Latin American neutrosophic pedagogy that is epistemologically plural, politically decolonial, and ethically relational, overcoming the aporias of Morin's complexity theory and presenting itself as a transformative tool for forming subjects capable of inhabiting an uncertain and plural world.
Keywords: neutrosophy, n-alectic, pedagogy, complexity theory, Latin American philosophy, decolonial thought.

RESUMO

Introdução: Este artigo examina as limitações da teoria da complexidade de Edgar Morin na pedagogia, destacando sua natureza abstrata, seu fundamento na lógica dialética binária e seu universalismo, que dificulta sua aplicação em contextos específicos como o da América Latina. Materiais e métodos: Como solução proposta, articula-se um arcabouço teórico integrativo baseado em uma análise crítica da literatura, comparando os fundamentos da teoria da complexidade de Morin com a neutrosofia e a n-alética de Florentin Smarandache, e dialogando com conceitos da filosofia e cosmovisão latino-americanas (como o pensamento decolonial, o Buen Vivir e a interculturalidade crítica). Resultados: A análise comparativa revela que a lógica neutrosófica (que opera com os componentes Verdade, Indeterminação e Falsidade) e sua derivação n-alética (que analisa a dinâmica entre uma entidade, seu oposto e seu neutro) oferecem um arcabouço conceitual mais robusto. Esse arcabouço, em diálogo com o pensamento latino-americano, permite lidar com a incerteza, a contradição e a neutralidade inerentes aos processos educativos. Discussão: São apresentadas propostas pedagógicas concretas para um currículo n-alético, uma prática de ensino baseada na mediação intercultural e uma avaliação neutrosófica, incluindo a metodologia da Análise Qualitativa Comparativa Neutrosófica (nQCA) e o uso de estímulos estruturados para Modelos de Linguagem (MLs) que fomentam o pensamento não binário. Conclusões: Isso estabelece uma pedagogia neutrosófica latino-americana epistemologicamente plural, politicamente decolonial e eticamente relacional, superando as aporias da teoria da complexidade de Morin e apresentando-se como uma ferramenta transformadora para a formação de sujeitos capazes de habitar um mundo incerto e plural.
Palavras-chave: neutrosofia, n-alético, pedagogia, teoria da complexidade, filosofia latino-americana, pensamento decolonial.

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Introducción

La crisis de los paradigmas educativos modernos, anclados en la simplificación, la fragmentación del conocimiento y una visión lineal del progreso, ha impulsado la búsqueda de nuevos marcos epistémicos que permitan comprender y habitar la creciente complejidad del mundo contemporáneo. En este contexto, la teoría de la complejidad, popularizada por el filósofo y sociólogo francés Edgar Morin, ha emergido como una de las propuestas más influyentes en las últimas décadas (Morin, 1999). Su llamado a "religar" los saberes, a través de principios como el dialógico, el recursivo y el hologramático, ha inspirado a educadores de todo el mundo a repensar los fines y los medios de la educación (Alhadeff-Jones, 2009).

Sin embargo, a pesar de su innegable valor diagnóstico, la propuesta de Morin presenta limitaciones significativas cuando se intenta traducirla en una praxis pedagógica concreta y transformadora. La crítica principal, que este artículo suscribe, es que la complejidad moriniana, si bien identifica los límites del pensamiento simplificador, no logra trascender completamente la lógica binaria y dialéctica de la tradición occidental (Batista-Hernández, Leyva-Vázquez, & Batista-Barallobre, 2025) . Además, su carácter eminentemente abstracto y universalista dificulta su anclaje en contextos culturales y políticos específicos, como el latinoamericano, marcado por la herida colonial y la lucha por la descolonización del saber (Quijano, 2000).

Frente a este panorama, el presente trabajo postula que una articulación entre la neutrosofía y la n-aléctica, desarrolladas por el matemático y filósofo Florentin Smarandache, y los aportes de la cosmovisión y la filosofía

latinoamericana, puede ofrecer un horizonte superador (Smarandache, 2014). La neutrosofía, como una nueva rama de la filosofía que estudia el origen, la naturaleza y el alcance de las neutralidades, introduce un componente fundamental que la dialéctica clásica y la complejidad de Morin omiten: la indeterminación (I) como un estado autónomo entre la verdad (T) y la falsedad (F) (Smarandache, 2016). De esta lógica tricotómica se deriva la n-aléctica, una superación de la dialéctica que, en lugar de buscar la síntesis superadora de los opuestos, explora la dinámica entre una entidad <A>, su opuesto <anti-A> y su neutral <neut-A> (Smarandache, 2013).

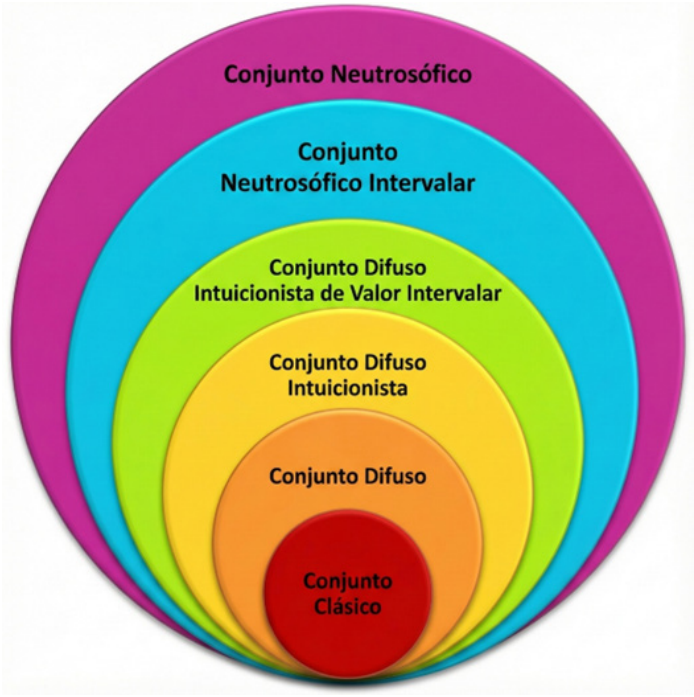

Figura 1 Jerarquía de generalización desde los conjuntos clásicos hasta los conjuntos neutrosóficos

Este andamiaje conceptual, al ser puesto en diálogo con la riqueza de la filosofía y la cosmovisión latinoamericana —con conceptos como el "Buen Vivir", la comunalidad y la interculturalidad crítica—, permite construir una propuesta pedagógica que no solo reconoce la complejidad, sino que la abraza en sus múltiples dimensiones: epistémica, política, ética y estética (Walsh, 2009). Se argumenta que esta "pedagogía neutrosófica latinoamericana" está mejor equipada para lidiar con la incertidumbre, promover el diálogo de saberes y formar ciudadanos con un pensamiento crítico y creativo, capaces de navegar las contradicciones del siglo XXI sin sucumbir a la parálisis o al dogmatismo.

El artículo se estructura en cinco partes. En la primera, se presenta una revisión crítica de la teoría de la complejidad de Morin y sus aplicaciones en la pedagogía, destacando sus principales aportes y limitaciones. En la segunda, se introducen los fundamentos de la neutrosofía y la n-aléctica, así como los conceptos clave de la filosofía latinoamericana pertinente. La tercera sección desarrolla un análisis comparativo entre ambos marcos teóricos, demostrando cómo la perspectiva neutrosófica-latinoamericana supera las aporías de la complejidad. La cuarta sección presenta propuestas pedagógicas concretas, incluyendo la metodología del nQCA. Finalmente, se explora la aplicación de la neutrosofía en sistemas de IA y LLMs como herramientas pedagógicas.

La Teoría de la Complejidad de Edgar Morin: aportes y límites en la pedagogía

La obra de Edgar Morin, y en particular su "Método", representa uno de los esfuerzos intelectuales más sistemáticos y ambiciosos del siglo XX por construir una epistemología a la altura de los desafíos de nuestro tiempo (Morin, 1990). Frente a lo que denomina el "paradigma de la simplificación" —caracterizado por la disyunción, la reducción y la abstracción—, Morin propone un "pensamiento complejo" que busca "religar" aquello que el pensamiento clásico separó: el objeto de su contexto, las partes del todo, lo humano de lo biológico, la ciencia de la filosofía (Morin, 2004).

En el ámbito educativo, la recepción de estas ideas ha sido entusiasta. La propuesta de Morin de "enseñar la condición humana", "enseñar la identidad terrenal" o "enseñar la comprensión" resuena profundamente con las aspiraciones de una pedagogía que busca superar la fragmentación curricular y formar ciudadanos globales, críticos y solidarios (Morin, 2001). Los siete saberes necesarios para la educación del futuro se han convertido en un referente ineludible para reformas educativas en diversas partes del mundo (UNESCO,

2001). El pensamiento complejo invita a la escuela a abandonar la transmisión de conocimientos inertes y a embarcarse en la aventura de construir un conocimiento pertinente, que dé cuenta de los contextos, lo global, lo multidimensional y, en definitiva, lo complejo.

Los principios del pensamiento complejo ofrecen herramientas conceptuales para esta tarea. El principio dialógico permite mantener la dualidad en el seno de la unidad, asociando términos a la vez complementarios y antagónicos. El principio de recursividad organizacional rompe con la idea lineal de causa-efecto, mostrando cómo los productos y los efectos son, al mismo tiempo, causas y productores de aquello que los produce. Finalmente, el principio hologramático nos recuerda que no solo la parte está en el todo, sino que el todo está también en la parte (Morin, 2004). Estos principios, aplicados a la pedagogía, sugieren un diseño curricular no lineal, una evaluación que valore el proceso tanto como el resultado, y una relación docente-alumno basada en el diálogo y la co-construcción del saber.

Sin embargo, a pesar de estos valiosos aportes, la propuesta moriniana no está exenta de críticas y limitaciones, especialmente cuando se intenta llevarla al terreno de la práctica pedagógica en contextos no europeos. Una de las críticas más recurrentes es su carácter metafórico y poco operacionalizable. Morin describe la complejidad, la celebra, pero no ofrece una lógica formal ni un método riguroso para navegarla. Su pensamiento, a menudo denso y elusivo, corre el riesgo de convertirse en un "comodín retórico" que justifica cualquier práctica vagamente holística sin un verdadero rigor epistémico (Reynoso, 2008).

Una segunda limitación, más profunda, es su anclaje en la tradición dialéctica hegeliana. Aunque Morin busca superar la dialéctica clásica, su pensamiento sigue atrapado en la lógica de los contrarios. La complejidad moriniana es, en esencia, una dialéctica de la dialéctica, un juego de oposiciones (orden/desorden, unidad/diversidad, autonomía/dependencia) que, si bien reconoce la tensión, sigue operando en un plano bidimensional. Como veremos, esta estructura binaria es insuficiente para captar la riqueza de lo real, que a menudo se manifiesta no solo como oposición, sino también como neutralidad, ambigüedad e indeterminación (Smarandache, Vázquez, Cevallos-Torres, & Barco, 2025).

Finalmente, el universalismo abstracto de Morin, a pesar de su defensa de la diversidad, presenta problemas desde una perspectiva latinoamericana. Su "condición humana" e "identidad terrenal", si bien bienintencionadas, corren el riesgo de invisibilizar las profundas asimetrías de poder y las heridas históricas que marcan a los pueblos del Sur Global (Quijano, 2000). La complejidad de Morin, al no incorporar una crítica explícita al colonialismo y al eurocentrismo, puede terminar siendo funcional a un nuevo tipo de globalización hegemónica que, bajo el manto de la "unidad en la diversidad", perpetúa las estructuras de dominación existentes. Es aquí donde el diálogo con la filosofía latinoamericana se vuelve no solo pertinente, sino urgente.

Neutrosofía, N-aléctica y Filosofía Latinoamericana: Fundamentos para una Pedagogía Decolonial

Frente a las limitaciones de la complejidad moriniana, emerge un nuevo horizonte de pensamiento desde la obra de Florentin Smarandache y su articulación con la tradición crítica latinoamericana. Este apartado introduce los conceptos fundamentales de la neutrosofía y la n-aléctica, para luego ponerlos en diálogo con la cosmovisión y la filosofía de nuestra región.

De la Lógica Difusa a la Lógica Neutrosófica

La neutrosofía, propuesta por Smarandache en 1980, es una nueva rama de la filosofía que generaliza la dialéctica hegeliana (Smarandache, 2014). Su punto de partida es la superación de las lógicas bivalentes (verdadero/falso) y de la lógica difusa. Mientras que la lógica difusa introdujo grados de verdad y falsedad, la lógica neutrosófica da un paso más allá al incorporar la indeterminación (I) como un componente autónomo y fundamental (Smarandache, 2016). Así, toda proposición <A> se analiza en un espacio tridimensional (T, I, F), donde T representa el grado de verdad, I el grado de indeterminación o neutralidad, y F el grado de falsedad. Estos tres componentes son, en principio, independientes entre sí, y su suma puede ser menor, igual o mayor que 1, lo que permite modelar la incertidumbre, la ambigüedad, la vaguedad y la contradicción de una manera mucho más precisa que las lógicas anteriores (Smarandache, 2013).

La neutrosofía es una rama de la filosofía que estudia el origen, la naturaleza y el alcance de las neutralidades, así como sus interacciones con diferentes espectros ideacionales. Considera una proposición, teoría, evento, concepto o entidad <A> en relación con su opuesto <antiA> y su neutral <neutA> (Smarandache, 2014).

Esta concepción tiene implicaciones profundas para la pedagogía. Un proceso de aprendizaje no puede ser reducido a una simple adquisición de verdades (T=1, F=0). El estudiante transita constantemente por

estados de indeterminación (I): la duda, la pregunta sin respuesta, la intuición no verificada, la coexistencia de múltiples perspectivas. La pedagogía tradicional, obsesionada con la certeza y la respuesta correcta, tiende a castigar o ignorar este espacio de indeterminación. Una pedagogía neutrosófica, por el contrario, lo valora como un momento esencial y potente del proceso de conocer.

La N-aléctica: Más Allá de la Síntesis

De la lógica neutrosófica se deriva la n-aléctica, que es una generalización de la dialéctica (Smarandache, 2013). Mientras que la dialéctica clásica se centra en la dinámica entre una tesis <A> y su antítesis <anti-A>, buscando una síntesis superadora, la n-aléctica analiza la dinámica entre <A>, <anti-A> y <neut-A> (la neutralidad entre los opuestos). Esta neutralidad no es pasividad, sino un espacio activo de coexistencia, diálogo y transformación. Además, la n-aléctica puede ser refinada para incluir múltiples dimensiones de verdad, falsedad e indeterminación (T1, T2...; I1, I2...; F1, F2...), lo que permite un análisis mucho más granular de los fenómenos complejos.

En términos pedagógicos, la n-aléctica nos invita a superar el modelo del debate adversarial, donde una postura debe triunfar sobre otra. En su lugar, propone un modelo de diálogo donde las posiciones opuestas (<A> y <anti-A>) son puestas en relación no solo entre sí, sino también con las posibles zonas de neutralidad y acuerdo parcial (<neut-A>). Esto es fundamental para una educación para la paz y la ciudadanía, donde el objetivo no es eliminar el conflicto, sino aprender a gestionarlo de manera creativa y no violenta.

Diálogos con la Cosmovisión y Filosofía Latinoamericana

La propuesta neutrosófica y n-aléctica encuentra un eco sorprendente y potente en la tradición de pensamiento latinoamericano. La crítica al binarismo y al pensamiento eurocéntrico ha sido una constante en la filosofía de la liberación y en los estudios decoloniales (Quijano, 2000). Autores como Enrique Dussel, Rodolfo Kusch o Aníbal Quijano han insistido en la necesidad de pensar desde "el otro lado de la frontera", desde la exterioridad del sistema-mundo moderno/colonial (Dussel, 2009).

La cosmovisión andina del "Buen Vivir" (Sumak Kawsay), por ejemplo, ofrece una alternativa radical al modelo de desarrollo occidental (Gudynas, 2011). No se trata de un "vivir mejor" individual y competitivo, sino de un "buen convivir" en armonía con la comunidad, la naturaleza y el cosmos . Este pensamiento relacional y complementario, que no busca la síntesis sino el equilibrio dinámico de las partes, es profundamente n-aléctico. La relación entre el ser humano (runa) y la naturaleza (Pachamama) no es de oposición ni de dominio, sino de reciprocidad (ayni) (Kusch, 2000).

Asimismo, la insistencia de la filosofía latinoamericana en la interculturalidad crítica como proyecto político y epistémico dialoga directamente con la propuesta neutrosófica (Walsh, 2009). La interculturalidad no es un simple multiculturalismo donde las culturas coexisten como entidades separadas. Es un proyecto que busca construir puentes de diálogo en condiciones de simetría, reconociendo las asimetrías de poder existentes y trabajando para desmontarlas (Walsh, 2010). La "neutralidad" n-aléctica puede ser entendida aquí como ese "tercer espacio" de enunciación donde el diálogo intercultural se hace posible, un espacio que no pertenece ni a la cultura hegemónica (<A>) ni a la cultura subalterna (<anti-A>), sino que emerge de su encuentro conflictivo y creativo.

En resumen, la articulación de la neutrosofía y la n-aléctica con la filosofía latinoamericana nos proporciona un marco conceptual robusto para una pedagogía decolonial. Esta pedagogía se fundamenta en el reconocimiento de la pluralidad de saberes, la valoración de la indeterminación como potencia epistémica y la búsqueda de un diálogo intercultural que supere las trampas del pensamiento binario y eurocéntrico.

Materiales y métodos

El artículo se fundamenta en un análisis teórico-crítico que toma como material principal el corpus de obras de Edgar Morin sobre la teoría de la complejidad y de Florentin Smarandache sobre neutrosofía y n-aléctica. A estos se suma un corpus complementario de filosofía y pensamiento latinoamericano, con autores como Aníbal Quijano, Catherine Walsh y Enrique Dussel, que aportan los conceptos de decolonialidad, Buen Vivir e interculturalidad crítica. Estos materiales textuales constituyen la base documental sobre la que se construye la reflexión.

El método central empleado es el análisis comparativo de marcos teóricos. Mediante una revisión crítica de

la literatura, se contrastan los postulados de la complejidad moriniana con los de la propuesta neutrosófica, identificando las limitaciones de la primera —como su lógica dialéctica binaria y su universalismo abstracto— y las potencialidades de la segunda para gestionar la indeterminación y la neutralidad en los procesos educativos. Este contraste se sintetiza en una tabla comparativa que organiza las diferencias en ejes clave como la lógica de base, el tratamiento de la contradicción y el rol de la incertidumbre.

A partir de este análisis, se aplica un diálogo teórico-interdisciplinario como método de articulación creativa. Este procedimiento integra de manera constructiva la lógica trivalente de la neutrosofía (Verdad, Indeterminación, Falsedad) y la estructura n-aléctica con los principios del pensamiento decolonial y la cosmovisión latinoamericana. El objetivo es trascender la mera crítica para fundar un nuevo marco pedagógico, superador de las aporías identificadas en la teoría de la complejidad.

Finalmente, el método conduce a una deducción y propuesta teórica aplicada. A partir de la síntesis conceptual lograda, se deducen y esbozan aplicaciones pedagógicas concretas. Estas incluyen principios para un diseño curricular n-aléctico, lineamientos para una práctica docente como mediación intercultural y propuestas para una evaluación neutrosófica. Además, se proponen herramientas metodológicas innovadoras, como el Análisis Cualitativo Comparado Neutrosófico (nQCA) y el diseño de prompts estructurados para Modelos de Lenguaje, configurando así una contribución metodológica tangible derivada del análisis teórico.

Resultados

Análisis Comparativo: De la Complejidad Dialéctica a la N-aléctica Decolonial

Una vez presentados ambos marcos teóricos, es posible realizar un análisis comparativo que ponga de manifiesto la potencia de la perspectiva neutrosófica-latinoamericana para superar las limitaciones de la teoría de la complejidad de Morin en el campo pedagógico. Este análisis se centrará en tres ejes fundamentales: el tratamiento de la contradicción, el lugar de la indeterminación y el abordaje de la universalidad.

Tabla 1. Comparación entre la Teoría de la Complejidad de Morin y la Perspectiva Neutrosófica-Latinoamericana en el Campo Pedagógico

Característica	Teoría de la Complejidad (Morin)	Perspectiva Neutrosófica-Latinoamericana	Implicaciones Pedagógicas
Lógica de base	Dialéctica. Se enfoca en la tensión y el diálogo entre opuestos (orden/desorden).	N-aléctica. Analiza la dinámica entre opuestos y sus zonas de neutralidad (<A>, <antiA>, <neutA>).	Superación del debate adversarial. Búsqueda de espacios comunes y diálogo intercultural.
Tratamiento de la contradicción	La contradicción es un motor del pensamiento, pero se busca una meta-unidad dialógica que la contenga.	La contradicción puede coexistir con la no-contradicción y la indeterminación. Se aceptan estados paraconsistentes.	Se legitima la coexistencia de múltiples perspectivas, incluso contradictorias, sin la urgencia de una síntesis forzada.
Rol de la incertidumbre	Se reconoce la incertidumbre como un límite del conocimiento, un "agujero negro" que hay que navegar.	La indeterminación (I) es un componente ontológico y epistémico fundamental, un espacio de potencialidad y creación.	La duda, la pregunta y la ambigüedad se convierten en herramientas pedagógicas centrales, no en obstáculos a superar.
Universalidad	Propone un universalismo abstracto y reconciliador ("condición humana", "identidad terrenal").	Cuestiona los universales abstractos y propone un "pluriversalismo transmoderno" anclado en el diálogo decolonial.	La educación se sitúa geopolíticamente, reconociendo la colonialidad del saber y promoviendo un diálogo de saberes Sur-Norte.
Praxis	Ofrece principios generales y abstractos, de difícil operacionalización.	Proporciona una lógica formalizable y herramientas conceptuales (T, I, F) que permiten un análisis más riguroso.	Permite diseñar estrategias didácticas y evaluativas más precisas para gestionar la complejidad en el aula.

Como se observa en la tabla, la principal diferencia radica en el paso de una lógica dialéctica a una n-aléctica. Mientras Morin, heredero de la tradición europea, sigue pensando en términos de pares opuestos cuya tensión debe ser gestionada, la propuesta de Smarandache, al introducir el componente de neutralidad (<neut-A>), abre el juego. Ya no se trata solo de la lucha entre <A> y <anti-A>, sino de la exploración de ese vasto territorio intermedio. Este territorio no es un vacío, sino un campo fértil para el diálogo, la negociación y la emergencia de nuevas realidades. Pedagógicamente, esto significa pasar de una lógica del "o lo uno o lo otro" (o incluso del "y lo uno y lo otro" en el mejor de los casos morinianos) a una lógica del "quizás", del "depende", del "ni lo uno ni lo otro, sino algo diferente".

En segundo lugar, el tratamiento de la incertidumbre es radicalmente distinto. Para Morin, la incertidumbre es algo a ser "afrontado", una especie de enemigo inevitable. Para la neutrosofía, la indeterminación (I) es constitutiva de la realidad y del conocimiento. No es un defecto, sino una potencia (Pi Crespo et al., 2019). Una pedagogía neutrosófica no busca eliminar la indeterminación con respuestas definitivas, sino que enseña a habitarla, a explorarla y a utilizarla como motor para la creatividad y el pensamiento crítico. Se trata de formar estudiantes que no teman al "no saber", sino que lo vean como el punto de partida de toda aventura intelectual.

Finalmente, el anclaje en la filosofía latinoamericana permite a la propuesta neutrosófica evitar el universalismo abstracto de Morin. La n-aléctica, al ser puesta en diálogo con el pensamiento decolonial, se convierte en una herramienta para la interculturalidad crítica. El espacio <neut-A> puede ser teorizado como el "tercer espacio" o la "frontera" donde se encuentran y se tensionan los saberes hegemónicos y los saberes subalternos. El objetivo de una pedagogía n-aléctica decolonial no es simplemente que los estudiantes "comprendan" la diversidad cultural desde una posición externa, sino que experimenten la tensión epistémica y política de esa diversidad, y que tomen posición en ella. Se trata de una educación que no solo reconoce la complejidad del mundo, sino que asume la complejidad de su propia posición en las relaciones de poder que lo estructuran.

Hacia una Pedagogía Neutrosófica Latinoamericana: Propuestas Concretas

La articulación teórica desarrollada hasta aquí no tendría valor si no fuera capaz de inspirar prácticas pedagógicas transformadoras. En esta sección, se esbozan algunas propuestas concretas, sin pretensión de exhaustividad, para aplicar el enfoque neutrosófico-latinoamericano en el diseño curricular, la práctica docente y la evaluación.

Diseño Curricular N-aléctico

Un currículo diseñado desde esta perspectiva debería superar la tradicional organización por disciplinas estancas. En su lugar, se podría organizar en torno a "nudos problémicos" o "preguntas generadoras" que sean socialmente relevantes y epistémicamente complejas. Por ejemplo, en lugar de estudiar "la historia de la conquista de América" como un contenido cerrado, se podría plantear la pregunta: "¿Qué significó (y significa hoy) el encuentro entre Europa y América?".

El abordaje de esta pregunta se haría de forma n-aléctica:

- <A> (Tesis): Se analizarían las fuentes y narrativas hegemónicas, la visión de los vencedores, el discurso de la "civilización" y el "progreso".
- <anti-A> (Antítesis): Se investigarían las fuentes y narrativas de los pueblos originarios, la visión de los vencidos, el trauma de la conquista y la resistencia cultural.
- <neut-A> (Neutralidad/Indeterminación): Se explorarían las zonas grises y ambiguas: el mestizaje, el sincretismo religioso, las figuras de los traductores y mediadores culturales, las alianzas cambiantes, las contradicciones internas de cada bando. Se analizarían también las interpretaciones historiográficas diversas y en conflicto.

El objetivo no sería llegar a una "síntesis" final, sino que los estudiantes pudieran construir su propia comprensión matizada del fenómeno, expresada en términos neutrosóficos: "Entiendo que la conquista fue en un X% un acto de violencia genocida (T), pero también en un Y% un proceso de hibridación cultural (F), y reconozco que hay un Z% de aspectos que aún no comprendo o que son inherentemente ambiguos (I)".

Práctica Docente como Mediación Intercultural

El rol del docente en una pedagogía neutrosófica no es el de un transmisor de verdades, ni siquiera el de un "facilitador" de consensos. Es, más bien, el de un mediador intercultural y epistémico. Su tarea es crear un espacio de aula seguro donde puedan emerger y dialogar múltiples perspectivas, incluyendo aquellas que son contradictorias o indeterminadas.

Esto implica:

- Gestionar la incertidumbre: En lugar de responder a todas las preguntas, el docente debe saber sostener la pregunta, devolverla al grupo, y animar a la exploración de múltiples respuestas posibles. Debe modelar una actitud de humildad epistémica, reconociendo los límites de su propio saber.
- Visibilizar la geopolítica del conocimiento: El docente debe ayudar a los estudiantes a identificar desde

dónde habla cada texto, cada autor, cada teoría. ¿Quién habla? ¿Desde qué lugar geográfico, social, cultural? ¿Qué intereses defiende? ¿Qué silencios oculta?

- Fomentar el pensamiento fronterizo: Se deben proponer actividades que obliguen a los estudiantes a "cruzar la frontera" epistémica, a ponerse en el lugar del otro. Por ejemplo, a través de juegos de rol, debates simulados, análisis de casos desde diferentes perspectivas culturales, o la invitación a miembros de diferentes comunidades para que compartan sus saberes.

Evaluación Neutrosófica

La evaluación es quizás el ámbito donde la lógica binaria (aprobado/reprobado, correcto/incorrecto) está más arraigada. Una evaluación neutrosófica debería ser capaz de capturar la complejidad del aprendizaje.

Esto podría traducirse en:

- Portafolios de aprendizaje: En lugar de un único examen final, el estudiante podría compilar un portafolio que muestre su proceso a lo largo del tiempo: sus preguntas iniciales, sus borradores, sus errores y cómo los superó, sus reflexiones sobre sus propios cambios de perspectiva.
- Autoevaluación y coevaluación n-aléctica: Se podría pedir a los estudiantes que evalúen su propio trabajo y el de sus compañeros usando criterios neutrosóficos. Por ejemplo: "En este ensayo, creo que mi argumento es un 70% convincente (T), pero reconozco que hay un 20% de puntos débiles o no desarrollados (F), y un 10% de ideas que son exploratorias y que aún no sé si son válidas (I)".
- Evaluación de la capacidad de formular preguntas: Se podría evaluar no solo la capacidad de los estudiantes para dar respuestas, sino también su habilidad para formular preguntas pertinentes, complejas y bien fundamentadas. Una buena pregunta puede ser un indicador de aprendizaje más valioso que una respuesta memorizada.

De la Evaluación Estadística a la Causalidad Configuracional: Del fsQCA al nQCA

Una de las áreas donde la pedagogía neutrosófica puede ofrecer un avance metodológico tangible es en la investigación educativa y la evaluación de sistemas. Tradicionalmente, el análisis del éxito o fracaso educativo se ha basado en métodos estadísticos que buscan correlaciones lineales entre variables aisladas (e.g., nivel socioeconómico y rendimiento académico) (Cilesiz, 2020). Este enfoque reduccionista es incapaz de capturar la complejidad de los fenómenos educativos, donde diferentes combinaciones de factores pueden llevar a un mismo resultado (equifinalidad).

Una alternativa más sofisticada es el Análisis Cualitativo Comparado de Conjuntos Difusos (fsQCA), una metodología que permite identificar configuraciones causales complejas (Ragin, 2008). Sin embargo, el fsQCA, al basarse en la lógica difusa, sigue operando en el eje verdad/falsedad (pertenencia/no-pertenencia a un conjunto).

La propuesta neutrosófica va un paso más allá, sugiriendo la transición hacia un Análisis Cualitativo Comparado Neutrosófico (nQCA). Esta metodología, aún en desarrollo, incorporaría la dimensión de la indeterminación (I) en el análisis configuracional. En lugar de simplemente calibrar el grado de pertenencia de un caso a un conjunto (e.g., "escuela con alto involucramiento parental"), el nQCA permitiría calibrar el grado de pertenencia (T), el grado de no-pertenencia (F) y, crucialmente, el grado de indeterminación o ambigüedad (I).

Por ejemplo, al analizar el éxito educativo de una escuela, podríamos encontrar una configuración causal como la siguiente:

ALTO_FINANCIAMIENTO * ~bajo_ratio_profesor_alumno * ALTA_AUTONOMIA_CURRICULAR * INDETERMINADO_LIDERAZGO_DIRECTIVO-> EXITO_ACADEMICO

Esta expresión indicaría que el éxito académico se produce en escuelas con alto financiamiento, un ratio profesor-alumno que no es bajo (es decir, puede ser medio o alto), alta autonomía curricular, y donde el tipo de liderazgo directivo es ambiguo o indeterminado (pudiendo ser a veces autoritario, a veces democrático, o simplemente no siendo un factor claramente definible). El nQCA permitiría tratar esa indeterminación no como un "ruido" en los datos, sino como parte integrante de la configuración causal (Vázquez & Smarandache, 2025).

Esta propuesta metodológica implica un cambio radical: en lugar de buscar "la" causa del éxito educativo, se buscan las múltiples configuraciones causales neutrosóficas que pueden conducir a él .

Discusión

Aplicación de la Neutrosofía en Sistemas de IA y LLMs: Hacia una Pedagogía Aumentada

Un ámbito emergente y de gran relevancia para la pedagogía contemporánea es la integración de la lógica neutrosófica en el trabajo con Modelos de Lenguaje de Gran Escala (LLMs). Los LLMs como ChatGPT, Claude o Gemini se han convertido en herramientas pedagógicas cada vez más utilizadas en contextos educativos (Yu et al., 2024). Sin embargo, su aplicación acrítica puede reforzar el pensamiento binario que buscamos superar. La neutrosofía ofrece un marco para "entrenar" a estos sistemas a generar análisis más matizados y complejos.

El Problema: LLMs y el Pensamiento Binario

Los LLMs, entrenados principalmente en corpus textuales de tradición occidental, tienden a reproducir patrones de pensamiento binario: verdadero/falso, correcto/incorrecto, bueno/malo (Liu et al., 2025). Cuando un estudiante pregunta a un LLM "¿Es la inteligencia artificial una amenaza?", la respuesta típica oscila entre dos posiciones extremas, sin explorar adecuadamente la zona de indeterminación donde residen las preguntas más importantes. Esta limitación no es técnica, sino epistemológica: los LLMs reflejan la lógica de los datos con los que fueron entrenados.

La Solución: Prompts Neutrosóficos para LLMs

Una estrategia pedagógica innovadora es diseñar prompts que instruyan explícitamente al LLM a analizar temas complejos utilizando la tríada neutrosófica (T, I, F). El siguiente es un ejemplo de un prompt neutrosófico que puede ser utilizado con cualquier LLM:

Rol: Actúa como un Profesor Experto en Lógica Neutrosófica y Pensamiento Crítico.

Objetivo: Ayudar a los estudiantes a analizar un tema complejo evitando el pensamiento binario (blanco/negro) y utilizando la tríada neutrosófica (T, I, F).

Tarea: Voy a proporcionarte una afirmación o un evento histórico/científico. Tu trabajo es generar un análisis educativo que desglose el tema en tres componentes:

Grado de Verdad (T): Presenta las evidencias sólidas, hechos comprobados y argumentos que validan la afirmación.

Grado de Falsedad (F): Presenta los contraargumentos, errores comunes, o contextos donde la afirmación falla estrepitosamente.

Grado de Indeterminación (I): Analiza qué aspectos son ambiguos, qué información falta, qué variables son subjetivas o neutrales, y dónde existe paradoja.

Formato de Salida:

Usa un tono académico pero accesible.

Al final, asigna una valoración simbólica estimativa (por ejemplo: T: 0.6, I: 0.3, F: 0.1) para ilustrar el peso de cada componente (no tiene que ser matemático exacto, sino conceptual).

Termina con una "Pregunta Provocadora" que obligue al estudiante a investigar la zona de Indeterminación.

Caso de Aplicación: Análisis Neutrosófico del Teletrabajo

Para ilustrar cómo funciona esta metodología en la práctica, consideremos el análisis de la afirmación: "El teletrabajo (home office) es superior al trabajo presencial para la productividad y el bienestar."

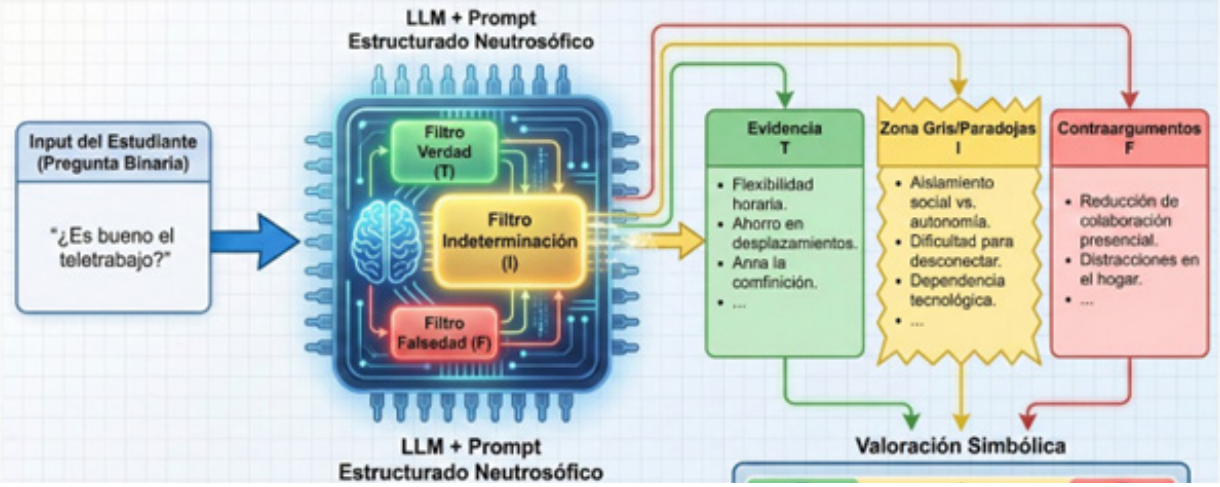

Figura 2. Flujo de procesamiento de un prompt estructurado neutrosófico en un LLM

Al aplicar el prompt neutrosófico a un LLM, obtenemos un análisis estructurado que evita la falsa dicotomía:

Componente de Verdad (T) - Eficiencia y Autonomía

Es verdaderamente cierto que se recuperan horas al eliminar el desplazamiento (commute), lo que matemáticamente aumenta el tiempo disponible. Para tareas que requieren alta concentración individual (programación, redacción), el aislamiento del hogar suele ser superior al ruido de una oficina abierta. Además, el teletrabajo permite acceder a talento global sin barreras geográficas, democratizando las oportunidades laborales.

Componente de Falsedad (F) - Aislamiento y Desconexión

Es falso que la innovación colaborativa ocurra igual de bien por videoconferencia. Se pierden los "momentos de pasillo" donde surgen muchas ideas creativas espontáneamente. El teletrabajo ha incrementado problemas ergonómicos y sedentarismo al reducir la movilidad natural del día. Para empleados junior, el teletrabajo es inferior; aprenden menos por ósmosis al no ver trabajar a sus mentores.

Componente de Indeterminación (I) - La Zona Gris y Híbrida

La paradoja de la disponibilidad: ¿Se trabaja menos o más? La frontera se vuelve indeterminada. Muchos empleados experimentan "burnout digital" porque nunca "salen" de la oficina, aunque estén en casa. La productividad es indeterminada porque depende de variables externas a la empresa: ¿El empleado tiene una oficina silenciosa o trabaja en la mesa de la cocina con niños pequeños? Lo que para un introvertido es "paz" (T), para un extrovertido es "ansiedad" (F). La "superioridad" del modelo es subjetiva al individuo.

Valoración Simbólica Estimativa

(T(0.40),I(0.45),F(0.15))

La Indeterminación es la variable más alta porque el éxito del teletrabajo depende casi exclusivamente de la autodisciplina y el entorno del individuo, no del modelo en sí.

Pregunta Provocadora:

Dado que la productividad es indeterminada y varía según el perfil psicológico del empleado, ¿deberían las empresas dejar de imponer políticas universales (todos vienen o todos se quedan) y pasar a contratos "neutrosóficos" personalizados por objetivos?

Implicaciones Pedagógicas

Esta aplicación de la neutrosofía con LLMs tiene profundas implicaciones pedagógicas:

- Superación del Pensamiento Binario: Los estudiantes aprenden a reconocer que la mayoría de los problemas complejos no tienen respuestas simples de verdadero/falso, sino que requieren un análisis multidimensional .
- Valorización de la Indeterminación: Al asignar un peso significativo a la componente I, se enseña a los estudiantes que la incertidumbre no es un defecto, sino una característica legítima del conocimiento .
- Pensamiento Crítico Aumentado: Los LLMs, guiados por prompts neutrosóficos, se convierten en herramientas para desarrollar pensamiento crítico, no en sustitutos de él .
- Democratización del Análisis Complejo: Esta metodología hace accesible el análisis de sistemas complejos a estudiantes de cualquier nivel, desde secundaria hasta postgrado.
- Preparación para la Incertidumbre: En un mundo caracterizado por la ambigüedad y la complejidad, esta pedagogía prepara a los estudiantes para tomar decisiones informadas en contextos de indeterminación.

Conclusiones

La pedagogía, en su búsqueda constante por formar seres humanos capaces de comprender y transformar su realidad, se encuentra en una encrucijada. Los modelos basados en la simplificación y la certeza han demostrado ser insuficientes para un mundo caracterizado por la interconexión, la incertidumbre y el conflicto. La teoría de la complejidad de Edgar Morin representó un paso crucial al diagnosticar esta insuficiencia y proponer un

pensamiento que religa (Morin, 2001). Sin embargo, como hemos argumentado, su propuesta, anclada en una dialéctica europea, no ofrece las herramientas lógicas ni políticas para ir más allá del diagnóstico y construir una praxis pedagógica verdaderamente emancipadora, especialmente en contextos marcados por la herida colonial como el latinoamericano.

Este artículo ha propuesto un camino alternativo y superador: la articulación de la neutrosofía y la n-aléctica de Florentin Smarandache con la riqueza de la cosmovisión y la filosofía crítica de América Latina (Smarandache, 2014). La introducción del componente de indeterminación (I) como un elemento central, y el paso de una lógica dialéctica a una n-aléctica, nos permite no solo pensar la complejidad, sino habitarla de una manera más rica y potente. Ya no se trata de gestionar la tensión entre opuestos, sino de explorar el vasto y fértil territorio de la neutralidad, la ambigüedad y la multiplicidad.

Al poner este marco conceptual en diálogo con el "Buen Vivir", la interculturalidad crítica y el pensamiento decolonial, hemos esbozado los fundamentos de una pedagogía neutrosófica latinoamericana. Esta pedagogía es:

- Epistémicamente plural: Reconoce la validez de múltiples saberes y lógicas, más allá del canon occidental (Walsh, 2009).
- Políticamente decolonial: Cuestiona la geopolítica del conocimiento y trabaja para desmontar las estructuras de poder que subyacen en la educación (Quijano, 2000).
- Éticamente relacional: Promueve una ética del cuidado, la reciprocidad y la responsabilidad con la comunidad y la naturaleza (Kusch, 2000).
- Prácticamente transformadora: Ofrece herramientas concretas para rediseñar el currículo, la práctica docente y la evaluación, orientándolas hacia la formación de un pensamiento crítico, creativo y capaz de navegar la incertidumbre.

Superar la complejidad de Morin no significa descartarla, sino integrarla en un marco más amplio y potente. La neutrosofía y la n-aléctica no niegan la dialéctica, la generalizan. De manera similar, una pedagogía neutrosófica latinoamericana no ignora los aportes del pensamiento complejo, pero los sitúa, los critica y los enriquece desde la exterioridad de la frontera colonial. El camino está abierto. La construcción de una educación que abrace radicalmente la incertidumbre, la diversidad y la justicia social es el gran desafío de nuestro tiempo, y la perspectiva aquí esbozada espera ser una contribución significativa en esa dirección.

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THE FLUID MARGINS OF PHYSICS & COSMOLOGY

Modern Physics and Hyperrealism-Antirealism Tendency

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Abstract

This paper delves into the philosophical implications of modern physics, particularly theoretical physics, and its tendency towards hyperrealism-antirealism. Inspired by Baudrillard's concept of hyperreality, we argue that certain aspects of contemporary physics, such as the interpretation of quantum mechanics and the pursuit of theories of everything, exhibit characteristics of a hyperreal state. This hyperreal state is characterized by the detachment from reality, the dominance of simulation over the real, and the loss of a clear distinction between the real and the imaginary. We explore how this trend has led to a departure from a balanced realist perspective, hindering the pursuit of a complete and coherent understanding of the physical world.

Keywords: Modern Physics; Philosophical; Hyperrealism-antirealism; Hyperreality.

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1 | Introduction

Modern physics, with its exploration of the quantum realm and the cosmos, has pushed the boundaries of human understanding. However, along with its groundbreaking discoveries, it has also introduced profound philosophical questions. One such question concerns the nature of reality itself. While physics has traditionally sought to describe the real world objectively, certain interpretations and theoretical frameworks have led to a departure from this realist perspective.

This paper examines the emergence of a hyperrealist-antirealist tendency within modern physics, drawing inspiration from Baudrillard's concept of hyperreality. Hyperreality, as described by Baudrillard, is a condition in which the simulation of reality becomes more real than reality itself. In the context of physics, this can be observed in the increasing reliance on mathematical models and abstract concepts, which often overshadow the underlying physical reality.

We argue that certain interpretations of quantum mechanics, such as the Copenhagen interpretation, and the pursuit of theories of everything, such as string theory, exhibit characteristics of a hyperreal state. These interpretations and theories often involve counterintuitive and seemingly paradoxical concepts, which can lead to a loss of touch with the tangible world. The focus on mathematical elegance and theoretical consistency, while essential for scientific progress, can sometimes overshadow the need for empirical verification and a clear connection to observable phenomena.

By exploring the implications of this hyperrealist-antirealist trend, this paper aims to highlight the importance of a balanced realist perspective in physics. A healthy balance between realism and idealism is crucial for a comprehensive understanding of the physical world. By recognizing the limitations of current theories and interpretations, and by fostering a critical approach to scientific inquiry, we can strive for a more grounded and meaningful understanding of reality.

2 | Hyperreality and Physics

Hyperreality, a concept popularized by the French philosopher Jean Baudrillard, describes a condition in which the simulation of reality becomes more real than reality itself. In the context of physics, this can manifest in various ways. For instance, some interpretations of quantum mechanics, such as the Copenhagen interpretation, prioritize mathematical formalism over physical intuition. While this approach has been incredibly successful in making accurate predictions, it can also lead to a detachment from the underlying reality of the quantum world.

Furthermore, the pursuit of grand unified theories, such as string theory, often involves highly abstract mathematical constructs that are difficult to test experimentally. While these theories may offer elegant solutions to fundamental problems in physics, they can also lead to a situation where the theoretical framework becomes more important than the empirical evidence.

3 | Hyperreality and the Quantum Conundrum: A Baudrillardian Critique of Modern Physics

Jean Baudrillard, the renowned French philosopher, introduced the concept of hyperreality, a condition in which the simulation of reality becomes more real than reality itself. This concept, while seemingly abstract, has profound implications for understanding the philosophical underpinnings of modern physics, particularly quantum mechanics.

Quantum mechanics, one of the most successful scientific theories of all time, has also been one of the most philosophically perplexing. Its counterintuitive predictions and probabilistic nature have led to a variety of interpretations, each with its own set of metaphysical assumptions.

The Copenhagen interpretation, for instance, while providing a practical framework for quantum mechanics, has been criticized for its operationalist stance. It suggests that we should not inquire into the underlying reality of quantum phenomena but rather focus on the measurable outcomes of experiments. This approach, while useful for making predictions, can be seen as a form of hyperreality, where the mathematical formalism and experimental results take precedence over any attempt to understand the underlying physical reality.

4 | The Hyperreal Tendency in Theoretical Physics

The tendency towards hyperreality in theoretical physics can be traced back to the influence of German idealism, particularly the work of Immanuel Kant and Georg Wilhelm Friedrich Hegel. These philosophers emphasized the role of the mind in shaping our understanding of reality. In a similar vein, many physicists seem to prioritize the mathematical elegance and logical consistency of their theories over their empirical adequacy.

This hyperrealist tendency can be seen in the pursuit of theories of everything, such as string theory. While string theory offers a mathematically sophisticated framework for unifying the forces of nature, it has yet to make any experimentally verifiable predictions. Nevertheless, many physicists continue to invest significant effort in developing this theory, often at the expense of more empirically grounded research.

The dominance of hyperreality in physics can have several negative consequences. First, it can lead to a loss of touch with the physical world. By focusing on abstract mathematical models and theoretical constructs, physicists may neglect the importance of empirical observation and experimentation. Second, it can stifle

creativity and innovation. When the emphasis is on adhering to established paradigms and mathematical formalisms, it becomes difficult to think outside the box and explore new ideas.

To counter this trend, it is essential to maintain a healthy balance between theoretical speculation and empirical investigation. Physicists should strive to develop theories that are not only mathematically elegant but also physically meaningful. They should also be open to the possibility that our current understanding of reality may be incomplete or even fundamentally flawed.

By recognizing the dangers of hyperreality and embracing a more critical and open-minded approach, physicists can help to ensure that their discipline remains grounded in the physical world and continues to make significant contributions to our understanding of the universe.

Jorge Luis Borges, the renowned Argentine writer, presented a chilling tale in his short story "*Tlön, Uqbar, Orbis Tertius.*" In this narrative, a secret society discovers a fictional planet, Tlön, and its inhabitants, who possess a unique worldview, one that prioritizes idealism over empiricism. This fictional world, with its peculiar laws of physics and metaphysics, challenges our understanding of reality and raises questions about the limits of human knowledge.

While Borges' tale is a work of fiction, it offers a sobering reflection on the potential dangers of excessive idealism and the role of empirical evidence in scientific inquiry. In the realm of physics, particularly in the realm of theoretical physics, there is a growing concern that a similar trend toward hyperreality may be taking hold.

5 | The Risk of a Fictional Universe

By prioritizing theoretical elegance and mathematical consistency over empirical verification, physicists risk creating a fictional universe, much like the planet Tlön. This could lead to a situation where scientists become so engrossed in their theoretical models that they lose sight of the real world. In such a scenario, the pursuit of knowledge could become an end in itself, rather than a means to understand the natural world.

To prevent this from happening, it is essential to maintain a healthy balance between theoretical speculation and empirical observation. Physicists should always strive to ground their theories in experimental data and to critically evaluate the assumptions underlying their models. By doing so, they can avoid the pitfalls of hyperreality and ensure that their work remains firmly rooted in the physical world.

In conclusion, while theoretical physics is a powerful tool for understanding the universe, it is crucial to use it responsibly. By recognizing the limitations of our current knowledge and by remaining open to new ideas and empirical evidence, we can avoid the dangers of hyperreality and continue to make progress in our understanding of the natural world.

6 | Another Example

As a case example, let us tell the story of a group of young experimental physicists who at the time frequently discussed with us their results on the real shape of electrons. While initial experiments seemed quite promising, after a few months they told us that their supervisor suggested that they not conduct more experiments on electron shape anymore, instead, they shall take a more "theoretical" learning path. As a result, later on, we read a draft paper of them trying to describe electrons in terms of mini-blackholes or sort of that. That is quite devastating in comparison to their promising initial experiment results.

7 | More Hidden Problems Causing Hyperrealism-antirealism: Obscurantism and Bourbakiism

It is known that there are more hidden problems, for instance, Nicolas Bourbaki (French: [nikɔla buʁbaki]) is the collective pseudonym of a group of mathematicians, predominantly French alumni of the École normale

supérieure (ENS). Founded in 1934–1935, the Bourbaki group originally intended to prepare a new textbook in analysis.

Shortly speaking, the rise of hyperrealism-antirealism in theoretical physics is a complex issue with roots in various philosophical and methodological factors. One such factor is the increasing influence of obscurantism and Bourbakiism in mathematical physics.

Obscurantism, a tendency to make something unnecessarily complex or difficult to understand, has become a pervasive problem in modern physics. Some physicists seem to take pride in the complexity of their theories as if the difficulty of understanding them is a measure of their intellectual depth. This trend can be seen in the proliferation of highly technical jargon and arcane mathematical formalism, which can obscure the underlying physical reality.

Bourbakiism, named after the collective pseudonym of a group of French mathematicians, refers to a highly formal and axiomatic approach to mathematics. While this approach can be useful for establishing rigorous foundations, it can also lead to a loss of intuition and a focus on abstract structures over concrete reality. In the realm of physics, Bourbakiism can manifest in the excessive use of mathematical formalism, which can obscure the physical meaning of equations and lead to a disconnect between theory and experiment.

8 | The Dangers of Obscurantism and Bourbakiism

Both obscurantism and Bourbakiism can contribute to the rise of hyperrealism-antirealism in physics. By obscuring the physical meaning of theories and prioritizing mathematical formalism over empirical evidence, these trends can lead to a situation where physicists become more concerned with the elegance of their theories than with their ability to explain the real world. This can result in the development of highly abstract and speculative theories that are difficult to test experimentally, and which may ultimately prove to be misguided.

To combat the influence of obscurantism and Bourbakiism, it is essential to prioritize clarity and simplicity in scientific communication. Physicists should strive to explain their ideas in clear and concise language, avoiding unnecessary jargon and technicalities. They should also be willing to engage in open and honest debate with their colleagues and to be critical of their assumptions and beliefs.

By adopting a more critical and empirical approach to physics, we can help to ensure that the field remains grounded in reality and continues to make significant contributions to our understanding of the universe.

9 | Discussion

9.1 | How to Consider a Spectrum from Healthy Realism to Antirealism from a Neutrosophic Logic Perspective

As we discussed in previous sections, Modern physics, with its exploration of the quantum realm and the cosmos, has pushed the boundaries of human understanding. However, it has also introduced profound philosophical questions about the nature of reality. These questions have led to a spectrum of approaches, ranging from hyperrealism to antirealism, with healthy realism as a middle ground.

A novel perspective to understand this spectrum is through the lens of Neutrosophic Logic, a logical system that deals with indeterminacy and uncertainty. This approach can help us to appreciate the nuanced interplay of realism and antirealism in various physical theories.

The Spectrum of Approaches

1. **Hyperrealism:**
    - Characterized by: A strong emphasis on empirical evidence and a skepticism towards speculative theories.

- Example: Early quantum mechanics, which focused on the observable properties of particles.
- Neutrosophic Perspective: A predominantly T (True) component, with minimal I (Indeterminate) and F (False) components.

2. **Healthy Realism:**
   - Characterized by: A balanced approach that acknowledges the limitations of our knowledge while striving for a realistic understanding of the world.
   - Example: Classical mechanics, which provides a robust framework for understanding macroscopic phenomena.
   - Neutrosophic Perspective: A balanced combination of T, I, and F components, reflecting the interplay of knowledge, uncertainty, and ignorance.
3. **Hyperrealism and Antirealism:**
   - Characterized by: A tendency to prioritize theoretical elegance and mathematical consistency over empirical evidence.
   - Example: String theory, which postulates the existence of tiny, vibrating strings as the fundamental building blocks of the universe, but lacks experimental verification.
   - Neutrosophic Perspective: A predominantly I and F component, with minimal T component, indicating a high degree of uncertainty and speculation.

A Neutrosophic View of Quantum Mechanics

Quantum mechanics, perhaps the most perplexing theory in physics, offers a fascinating case study in the interplay of realism and antirealism. The Copenhagen interpretation, for instance, is often criticized for its antirealist stance, as it suggests that quantum states do not correspond to physical reality until they are measured. However, other interpretations, such as the many-worlds interpretation, offer a more realist perspective, positing that all possible outcomes of a quantum measurement occur in different universes.

A Neutrosophic analysis of quantum mechanics would acknowledge the inherent indeterminacy of the quantum world. It would recognize that both realist and antirealist interpretations have their merits and limitations. By embracing the indeterminacy of quantum phenomena, we can avoid the pitfalls of both hyperrealism and antirealism.

9.2 | How to Consider a Spectrum from Healthy Realism to Antirealism from a Philosophical Anthropology Perspective

Philosophical anthropology, a field that explores the nature of humans and their place in the world, provides a valuable lens through which to examine the spectrum of scientific approaches, from healthy realism to antirealism. This perspective emphasizes the cultural, social, and psychological factors that shape scientific inquiry, and suggests that the choice of a particular approach may be influenced by broader cultural and historical contexts.

9.2.1 | Cultural Factors and Scientific Approach

Different cultures have distinct attitudes toward authority, tradition, and innovation. These cultural factors can significantly impact the way scientists approach their work. For instance, in cultures that value tradition and authority, there may be a tendency towards a more conservative, realist approach to science. In contrast, cultures that embrace innovation and challenge the status quo may be more receptive to antirealist and speculative ideas.

A classic example of this cultural influence can be seen in the different approaches to scientific inquiry in Britain, Germany, Russia, and Italy. As the saying goes:

- Britain: Everything is permitted except what is forbidden.
- Germany: Everything is forbidden except what is permitted.
- Russia: Everything is forbidden, including what is permitted.
- Italy: Everything is permitted, including what is forbidden.

These cultural stereotypes, while exaggerated, highlight the different attitudes towards rules and regulations in these countries. These attitudes can also be applied to the realm of scientific inquiry. A British scientist may be more inclined to take risks and explore unconventional ideas, while a German scientist may be more cautious and adhere to established norms.

9.2.2 | The Role of the Scientific Community

The scientific community itself is a cultural entity with its norms, values, and traditions. These norms can influence the way scientists approach their work, and can also shape the broader cultural attitudes towards science. For example, the rise of hyperrealism in certain fields of physics may be partly attributed to the influence of a particular school of thought or a specific cultural milieu.

The history of science is replete with examples of ideas that were initially considered heretical or absurd but eventually became accepted as mainstream. The discovery of heliocentrism, Newtonian dynamics, and quantum mechanics are just a few examples of scientific breakthroughs that challenged the prevailing worldview. These examples demonstrate the importance of a culture of open-mindedness and intellectual freedom, which can encourage innovation.

10 | Concluding Remark

The dominance of hyperreality in physics can have several negative consequences. First, it can lead to a loss of touch with the physical world. By focusing on abstract mathematical models and theoretical constructs, physicists may neglect the importance of empirical observation and experimentation. Second, it can stifle creativity and innovation. When the emphasis is on adhering to established paradigms and mathematical formalisms, it becomes difficult to think outside the box and explore new ideas.

To counter this trend, it is essential to maintain a healthy balance between theoretical speculation and empirical investigation. Physicists should strive to develop theories that are not only mathematically elegant but also physically meaningful. They should also be open to the possibility that our current understanding of reality may be incomplete or even fundamentally flawed.

By adopting a Neutrosophic perspective, we can gain a deeper understanding of the complex interplay between realism and antirealism in modern physics. This approach allows us to appreciate the nuanced nature of scientific inquiry and to avoid dogmatic adherence to any particular worldview. As we continue to explore the frontiers of knowledge, it is essential to maintain a healthy balance between empirical evidence and theoretical speculation. By doing so, we can ensure that physics remains grounded in reality and continues to make significant contributions to our understanding of the universe.

Last but not least, we shall also consider that by considering the cultural, social, and psychological factors that shape scientific inquiry, we can gain a deeper understanding of the spectrum of approaches, from healthy realism to antirealism. A philosophical anthropology perspective can help us to appreciate the complex interplay between individual and collective factors, and to recognize the importance of a balanced approach to scientific investigation. By embracing a diversity of perspectives and encouraging open-minded inquiry, we can ensure that science continues to thrive and contribute to the betterment of humanity.

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Conflicts of Interest

The authors declare that there is no conflict of interest in the research.

Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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The Principles of (Partial Locality, Partial Indeterminacy, Partial NonLocality) and (Multi Locality, Multi Indeterminacy, Multi NonLocality)

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Abstract: This article introduces new neutrosophic principles aiming to extend and generalize the concepts of locality and nonlocality by addressing scenarios involving indeterminacy, together with partiality and multitude, in physics, mechanics, cosmology, biology, medicine, chemistry, economics, ecology, sociology. Locality refers to interactions or processes confined within a limited region of space or time. But there may be a Total (100%) Locality, or a Partial Locality (less than 100% and greater than 0%). The effects are constrained to the immediate environment. Contrariwise, NonLocality refers to interactions or connections between entities separated by space or time. The changes in one location have instantaneous effects on another. Similarly, there may be a Total (100%) NonLocality, or a Partial NonLocality (less than 100% and greater than 0%). Total (100%) or Partial (less than 100% and greater than 0%) Indeterminacy may arise from hidden variables and from environment. For instance, it may involve nonlocal connections between objects that are only partially entangled or influence each other in limited ways, rather than exhibiting complete freedom. The Principle of Partial Locality, Partial Indeterminacy, and Partial NonLocality implies an interplay of locality, indeterminacy, and nonlocality acting in a dynamic neutrosophic system. A generalization of (Locality, Indeterminacy, NonLocality) is the (MultiLocality, MultiIndeterminacy, MultiNonLocality). Practical examples from different fields are provided.

Keywords:
locality, indeterminacy, nonlocality, multilocality, multiindeterminacy, multinonlocality, multiaction at a distance, universal gravitation, superposition, superluminal, quantum physics, quantum computing, quantum field theory, quantum information, quantum entanglement, Bell Inequality, hidden variables, dark energy, dark matter, electromagnetic forces, Bohmian mechanics, Copenhagen interpretation, cell (in biology), paracrine, CPU caches, Latent Dirichlet Allocation, curling, pollination, trophic cascade.

1. Introduction

Let's start by briefly discuss —with simple, quasi-random examples— the concepts of locality and non-locality, and then explain and illustrate some theoretical hidden variables parameters in scientific models.

In ecosystems, the interactions within and between ecosystems can show both local and nonlocal characteristics. Migratory species travel sometimes vast distances[1] between different ecosystems, yet their migration patterns connect those distant ecosystems, meaning their presence and behaviors have nonlocal effects and exert widespread consequences on food webs, predator-prey relationships, or nutrient cycling across regions, whilst daily interactions within a specific local habitat, such as

[1] In Tanzania's Serengeti region, plains animals, especially wildebeests (*Connochaetes taurinus*) and zebras, migrate over 1,600 kilometers (1,000 miles) seasonally [Dorst 2024].

feeding, mating, and nesting behaviors, are local.[2] Keystone species[3] have disproportionate effects on their ecosystems relative to their abundance.[4] This type of nonlocal effects (known as the 'trophic cascade'[5]) illustrate how changes in one species can ripple throughout the ecosystem, shaping environmental conditions and affecting other species that are far removed from the original source of change.

The effects of a collision between two objects are local to the region of contact. When two curling[6] stones[7] collide, the forces exerted and the resulting changes in motion are confined to the immediate area where they touch. Velocity and direction, as any other interactions and resulting effects, are purely local to the point of impact.[8]

The production of car parts in a local factory involves local labor, resources, and economic interactions. The final assembly of a car often involves sourcing parts from various countries. Disruptions in one part of the supply chain (e.g., a factory shutdown in one country) can have significant nonlocal effects, impacting production schedules and economic conditions in other parts of the world.[9]

Hidden variables are theoretical parameters or entities in scientific models that are not directly observable but are postulated to exist in order to explain certain phenomena [Kochen et al. 1975].

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[2] Such as the herds of caribou (*Rangifer tarandus*) that settle during the summer in the barrens, which are relatively flat wastelands with sparse vegetation, while, in July, they begin an irregular southward migration, and they spend the winter in the taiga, where each herd moves according to local conditions, lacking a well-defined pattern [Dorst 2024].

[3] Elephants (*genus Loxodonta*), for example, are considered as "ecosystem engineers" because they modify their environment by dispersing seeds, uprooting trees, or creating water holes, stimulating "plant richness to peak (…), setting up a creative browsing-grazing tension and a patchwork of habitats" [Western et al. 2021].

[4] The reintroduction of gray wolves (*Canis lupus*) in Yellowstone National Park in 1995/1996 after a 70-year absence has led to cascading effects on the entire ecosystem, including the recovery of aspen trees, which provide habitats for birds and other species. Wolves are apex predators that hunt large herbivores like elk. By reducing elk populations, overgrazing is prevented, and vegetation is allowed to recover [Ripple et al. 2012; Farquhar 2023].

[5] American zoologist Robert Paine coined the term in 1980's to describe changes in food webs caused by manipulations of predators. Others employed the term to describe changes in aquatic ecosystems arising from factors such as sudden increases or dramatic declines in predatory fishes, caused, e.g., by overfishing [Carpenter 2023].

[6] Curling is an ice-played game akin to lawn bowls, where two teams – each consisting of four players – compete, sliding round stones across the ice toward a target (button). The stones are concave at the bottom and have a handle on top. The button is located at the center of a circle marked with concentric rings (house). The aim of the game is for each team to position their stones as close to the center as possible. For more info on this game, see Britannica – The Editors of Encyclopedia. "curling". Encyclopedia Britannica, 11 Apr. 2024, https://www.britannica.com/sports/curling. Accessed 26 June 2024.

[7] Curling regulations specify that a curling stone can weigh up to 20 kilograms, with most stones typically weighing between 17 and 20 kg. Additionally, the maximum permissible circumference of a stone is 36 inches (910 mm), while the minimum height is 4.5 inches (110 mm). The substantial weight allows the stone to glide the length of the rink without slowing down too soon.

[8] The curl distance for a curling stone, given a typical angular velocity (four rotations over 28.35 meters in 23 seconds, equivalent to 1.09 rad/s or 62.6°/s), ranges from approximately 0.5 to 1.5 meters when aimed at a circular target 28.35 meters away on ice. Various factors influence these curl distances, including the ice surface conditions (such as temperature, pebble density, and the size and shape of the pebbles) and the characteristics of the curling stones themselves. [Kameda et al. 2020]

[9] In recent decades, economic globalization has led to a substantial shift of manufacturing to regions with lower perceived costs, notably moving production to China and other Asian countries. Although this approach initially seemed economically sensible, its significant downsides have become apparent in the past three years. Consequently, many industries, especially the automotive sector, are now embracing a "local for local" production strategy, particularly with the shift towards electric vehicles. [Abuelsamid 2022]

Bohmian mechanics[10] offers an interpretation of quantum mechanics by introducing hidden variables to translate quantum phenomena "in an objective way using deterministic dynamics" [Dabin 2009, 40], unlike the standard inherently probabilistic Copenhagen interpretation. According to this theory, particles possess definite positions at all times even when unobserved. These positions are hidden variables guided by a "pilot wave" [Goldstein 2024]. Valentini has expanded this theory to incorporate signal nonlocality, enabling entanglement to function as an independent communication channel, and thus eliminating the need for a secondary classical "key" signal to "unlock" the message encoded in the entanglement [Valentini 1991, 2009].

Two concepts proposed in cosmology to explain observable phenomena such as galaxies' rotation curves and the universe's rapid expansion, and deduced from gravitational effects on visible stuff and the universe's large-scale structure, are *dark matter* and *dark energy*.[11] They are also hidden variables. The observable universe consists of *matter*, which occupies only 5% of the cosmos. The remaining 95% is made up of *dark matter* (27%) and *dark energy* (68%), mysterious substances still under investigation by scientists.[12] Normal matter, composed of protons, neutrons, and electrons, can be seen directly or through telescopes. Dark matter, however, does not interact with light, making it invisible.

Latent variables are employed to model topics in machine learning. Latent Dirichlet Allocation (LDA) is a three-level hierarchical Bayesian model that identifies hidden subjects in a set of documents. It depicts each item in a collection as a finite mixture over a set of topics, and each topic as an infinite mixture over a set of topic probabilities [Blei et al. 2003].

In the construction of measurement models for psychological data (psychometrics), *latent traits* —describing veiled psychological abilities or skills like motivation, or anxiety, so on— are hidden variables, being not immediately quantifiable, but only inferred from outcomes of genetic profiles and psychological tests. [Hambleton&Cook 1977, 75].

Certain *genetic mutations* (alterations) —that are are not readily visible, but can be found by genetic testing— raise the risk of an individual of developing cancer or other serious health issues.[13]

In the study of *infectious diseases*, *asymptomatic carriers* are individuals who harbor a pathogen without showing symptoms and can transmit the disease to others. The presence of these carriers is a hidden variable in epidemiological models, complicating efforts to track and control the spread of infections.[14]

2. The Principles of Locality, NonLocality, and Neutrosophic Locality

2.1. ***The Principle of Locality***

The Principle of Locality means that an object is influenced directly only by its immediate surroundings.

Let's briefly review a few cases from various fields.

[10] Also known as the de Broglie-Bohm theory, proposed by Louis de Broglie in 1927 and reshaped by David Bohm in 1952.

[11] The concepts were proposed in the 20th century by Fritz Zwicky, and confirmed by Vera Rubin in the 1970s.

[12] Percentages estimated by the European Space Agency (ESA), https://www.esa.int/Science_Exploration/Space_Science/What_are_dark_matter_and_dark_energy. Dark matter and dark energy stories are updated continuously on United States National Aeronautics and Space Administration (NASA) website, https://science.nasa.gov/universe/dark-matter-dark-energy/.

[13] See, for example, a case study about risks and mechanisms of cancer in women with inherited susceptibility to epithelial ovarian cancer [Shulman 2011].

[14] See, for example, a case study about Coronaviruses SARS-CoV, MERS-CoV, and SARS-CoV-2 in asymptomatic pediatric population [Aleebrahim-Dehkordi et al. 2021].

2.1.1. *Computer Science*: CPU Cache

In computer architecture, locality is a key principle in the design of memory hierarchies. For example, if a particular memory location is accessed, it is likely to be accessed again in the near future (this is called *temporal locality*). Also, if a particular memory location is accessed, nearby memory locations are likely to be accessed soon (and this is called *spatial locality*). CPU caches[15] exploit these principles by locally storing frequently accessed data, close to the processor, to minimize access time and improve performance.

2.1.2. *Biology: Cellular Processes*

Cell signaling often operates on a local level. For example, paracrine[16] signaling involves the release of signaling molecules[17] from one cell that affect nearby target cells. These molecules typically do not travel far from their point of release. Their effects are localized to the immediate cellular environment.

2.1.3. *Ecology: Habitat Interactions*

In many ecosystems, pollination[18] is a local process. Bees, butterflies, and other pollinators typically collect and transfer pollen within a specific area. The interactions between plants and their pollinators occur in close proximity, and the effects[19] are localized to that region [Kevan 2001; 2020].

2.1.4. *Medicine: Local Anesthesia*

Local anesthetics are used to numb a part of the body during minor surgical procedures. When a local anesthetic is administered, its effects are confined to the targeted area, blocking nerve signals and preventing pain in that specific region while the patient remains unaffected elsewhere.

2.1.5. *Chemistry: Chemical Reactions*

In a simple acid-base neutralization reaction,[20] the interactions and resultant effects (like the formation of water and salt) occur at the molecular level in the immediate vicinity where the reactants come into contact. The reaction is localized to the solution where the chemicals are mixed.

2.1.6. *Sociology: Community Dynamics*

Neighborhood Watch Programs[21] are examples of local initiatives where community members collaborate to monitor and improve safety within their immediate area. The actions and effects[22] of these programs are confined to the neighborhood level.

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[15] A short but comprehensive introduction in CPU Cache offers *Britannica* – The Editors of Encyclopaedia. "cache memory". Encyclopedia Britannica, 20 May. 2021, https://www.britannica.com/technology/cache-memory. Accessed 23 June 2024.

[16] More on the paracrine control can be read on *Britannica* – Utiger, Robert D. "human endocrine system". Encyclopedia Britannica, 17 Apr. 2019, https://www.britannica.com/science/human-endocrine-system. Accessed 24 June 2024.

[17] Like growth factors.

[18] A basic but extended entry about pollination can be read here: The Editors of Scholarly Community Encyclopedia "Efficient Pollination Technology of Crops", https://encyclopedia.pub/entry/40852. Accessed 20 June 2024.

[19] Successful pollination and subsequent seed production.

[20] Such as mixing hydrochloric acid (HCl) with sodium hydroxide (NaOH).

[21] Read more about the US program and statistics of crime prevention through neighborhood collaboration on the National Neighborhood Watch (a Division of the National Sheriffs' Association) website: https://www.nnw.org.

[22] These would be reducing crime rates and increasing community cohesion.

2.1.7. *Economics: Local Markets*

Farmers' markets operate on a local scale, involving transactions between local farmers and consumers. The economic activities are confined to a specific geographic area, and the effects (exchange of goods, pricing, and consumer satisfaction) are localized to the community where the market takes place.

2.2. ***The Principle of Non-Locality***

The Principle of Non-Locality (Instantaneity), or Action at a Distance, means that an object is influenced by another object without being in physical contact (e.g., gravity, Colomb's Law, Electric Forces, and so on).

2.2.1. *Physics: Quantum Entanglement*

In quantum mechanics, entangled particles exhibit nonlocal behavior. When two particles become entangled, the state of one particle is directly related to the state of the other, regardless of the distance separating them. If you measure the spin of one entangled particle, the spin of the other particle is instantly known, even if they are light-years apart. This phenomenon has been experimentally confirmed and is a cornerstone of quantum mechanics, defying classical intuition.[23]

2.2.2. *Classical Physics: Gravitational and Electromagnetic Forces (pre-relativity)*

Gravitational and electromagnetic forces were thought to operate at a distance instantaneously prior to the postulation of relativity. Two masses exert an attractive force on one other regardless of distance, with no apparent intermediate impact, according to Newton's law of universal gravitation.[24] Equally, Coulomb's inverse-square law[25] is an experimental physical law that determines the amount of force between two electrically charged particles at rest.

2.2.3. *Computer Science: Blockchain Technology*

The verification and recording of transactions in blockchain systems demonstrates nonlocal interactions. Many nodes must validate a transaction that is broadcasted to the network. The consensus process[26] ensures that all nodes agree on the blockchain's current state. Proving nonlocality, the addition of a new block or other changes to the blockchain are immediately returned throughout the network.

2.2.5. *Genetics: Epigenetic Modifications*

Epigenetic changes[27] can have nonlocal effects across generations. Environmental factors like diet, stress, and exposure to toxins can cause epigenetic modifications that do not change the DNA sequence but affect gene expression. These changes can be passed down to offspring, influencing their development and health.

2.2.6. Ecology: Ecosystem Dynamics

Migrations of a vast numbers of individuals from diverse taxa occur worldwide. It plays a crucial role in transporting nutrients, and other organisms, as they forage and are preyed upon. The

[23] See, for example, a paper on the exploration of the relationship between quantum nonlocality and entanglement [Cao et al. 2023], or on quantum entanglement without nonlocal causation [Pettini 2023].

[24] Britannica, The Editors of Encyclopaedia. "Newton's law of gravitation". Encyclopedia Britannica, 12 Apr. 2024, https://www.britannica.com/science/Newtons-law-of-gravitation. Accessed 2 August 2024.

[25] Britannica, The Editors of Encyclopaedia. "Coulomb's law". Encyclopedia Britannica, 25 Jul. 2024, https://www.britannica.com/science/Coulombs-law. Accessed 2 August 2024.

[26] Such as *Proof of Work* or *Proof of Stake*, used to verify new cryptocurrency transactions [Napoletano 2023].

[27] Such as DNA methylation [Moore 2013].

movement of migrating species across extensive spatial scales influences the ecosystems through which these animals travel. Hidden variables include the genetic traits or physiological states of migratory species that determine their migration patterns and behavior. [Cohen et al. 2020], [Bauer et al. 2014], [Garcia et al. 2024]

2.3. Principle of Partial Locality, Partial Indeterminacy (because of the hidden variables that may influence the object), and Partial Non-Locality

We propose for the first time the Principle of Partial Locality, Partial Indeterminacy (because of the hidden variables that may influence the object), and Partial Non-Locality, which we call the Principle of Neutrosophic Locality.

The Aharonov-Bohm effect, which is a quantum mechanic phenomen, proved that: an electrically charged particle is affected by an electromagnetic potential that is totally outside of the particle (the particle being situated in a region where the intensity of the magnetic field is zero).

This Non-Locality event, first observed by the Soviets, was later on investigated by other secret services in order to control the Human Psyche from a distance.

The Action at a Distance may be:

i. Continuous action;
ii. Quantized action, or action by quantas, which are small subdivisions of physical processes and phenomena of a particular system.

We may call it infinitdecimally discrete action.

3. Degree of Locality, Degree of NonLocality, and Degree of Indeterminacy (neither Locality, nor NonLocality)

3.1. Degree of Locality

Locality means that an object is directly influenced only by its immediate surroundings. The Degree of Locality measures the degree to which a system or interaction follows this principle. A high degree of locality indicates that there is little to no effect from far-off particles or events, meaning that the interactions are very limited.

3.2. Degree of NonLocality

The phenomenon known as NonLocality describes the ability of particles that are distant in space to instantly display linked actions by a Degree of NonLocality in the system. Strong correlations between far-off particles or systems indicate a high degree of nonlocality.

3.3. Degree of Indeterminacy (neither Locality nor NonLocality)

There are situations when uncertainty or lack of definitive behavior falter in terms of locality and nonlocality, therefore the system does not exhibit clear characteristics of either local or nonlocal interactions. The Degree of Indeterminacy quantifies the degree to which the behavior of a system defies simple classification as exclusively local or completely nonlocal. When a system displays behaviors that are unclear, inconsistent, or that don't easily fit into the categories of locality or nonlocality, it is said to have a high degree of indeterminacy.

3.4. Measuring These Degrees

To quantify these degrees in practice, one typically relies on experimental setups that test the correlations between particles or systems, such as:

- **Bell Inequality Test** is a "real-world physics experiment designed to test the theory of quantum mechanics" against the concept of local realism[28]
- Theories of Local Hidden Variables aim "to attribute 'hidden' definite outcomes to any potential measurement on a quantum system" [Bertlman 2023]
- **Neutrosophic Statistical Analysis** [Smarandache 2013] evaluates indeterminacy by analyzing the variance and distribution of measurement outcomes to see if they defy categorization into local or nonlocal.

4. Recapitulation, generalization, and applications

Quantum Non-Locality states that a quantum particle instantaneously knows the state of – and correlates the behavior with – other quantum particle.

Quantum Entanglement asserts that two entangled particles correlate their behaviors simultaneously (therefore faster than light), as such our faster-than-light speed hypothesis has been confirmed.

John Clauser, an American physicist, in 1970's, and Alain Aspect, a French physicist, in 1980's, has proved that the entanglement is real, and, as a consequence, the superluminal speed is real.

Faraday said that some medium transmits the electrical force (not action at a distance).

Quantum Teleportation is the transfer from sender to receiver of quantum information, whose basic unit is the qubit.

The Copenhagen Interpretation of "wave-particle duality" that "the electron travels as a wave and is detected as a particle", and "an electron is in more places at the same time" are popular (mis)interpretations.

The behavior of a particle is described by a wave fluctuation (ψ) that has the shape of an equation.

The wave-particle duality is actually the indeterminate part of the neutrosophic form of the electron, because it fluctuates between – or is a superposition of – the opposites (wave vs. particle).

Locality (or Local Causality) was named by Bell; others called it later local realism.

Bell's Inequality: In Quantum Mechanics, when electrons are sent across the magnetic field, half of the electrons get deflected towards to the right, and the other held to the left.

Violation of Bell's Inequality: In Experiments the predictions of local realistic models disagree with those of quantum mechanics.

Realism means the assumption that measurement outcomes are well-defined prior to and independent of the measurements.

We now introduce for the first time Partial Locality Causality, Partial NonLocal Causality, and Partial Indeterminate Causality.

*

Partial Locality means that an object is influenced only partially by its immediate surroundings.

Similarly, Partial NonLocality means that an object is only partially influenced by another object without being in physical contact.

Partial Indeterminacy means that it is not clear that an object is influenced either by its immediate surroundings, or by another object without being in physical contact, or by both.

Such behavior is exhibited by an ambiguous, vague, incomplete, or inconsistent system.

*

Quantum Physics defy the Classical Physics, because the behavior of erattic atomic and subatomic particle.

The degrees of locality, nonlocality, and indeterminacy may be measured for each system, upon the interactions between touching objects (Locality), non-touching objects (NonLocality), and the influence of hidden variables (Indeterminacy) on these interferences.

[28] See *Scholarly Community Encyclopedia*, "Bell Test Experiments.". Available online: https://encyclopedia.pub/entry/36569 (accessed on 02 August 2024).

Quantum Physics: The correlation between particles can be tested by experiments. At the quantum level, the degree of NonLocality may be measured by checking in what degree the test results violate the Bell Inequalities.

Degree of Indeterminacy by using statistical analysis: comparing the means, mediums, standard deviations, variance, distributions of the outcomes.

Degree of Locality: Predictions versus real measurement outcomes. That may be compared to the models of hidden variables.

Entangled Particles: In quantum physics, the state of one particle has an instantaneous effect on the state of its entangled particle, no matter how far away they are of each other. This is NonLocality.

By measuring the spin of one particle, the spin of its entangled particle is immediately known, defying the classical physical intuition.

Epigenetics study the heritable traits, that do not change the DNA sequence, and may result from normal development or from environmental conditions, such as stress, tocins, diet, etc. DNA methylation, as a strong epigenetic change, has nonlocal effects across generations, therefore passed down to offsprings.

Ecosystems: The presence or absence of a species in an ecosystem has cascading effects on the whole chain of species in that ecosystem. These are NonLocal effects. Local effects: feeding, mating, nesting together, or predatory-prey relationships, or plants that are pollinated by insects. NonLocal and Local Effects occur by the interaction between the ecosystems. For examples, the whales travel long distances from a sea (or ocean) zone to another water zone, connecting various ecosystems (NonLocality). Similarly the migratory birds.

Blockchain Systems: Social Networks, Brain Networks (neuronal interactions), Financial/Economic/Commercial/Cultural/Political etc. Networks function as blockchain systems, where the NonLocal interactions facilitate the transactions between multiple nodes.

If two nodes are adjacent, the communication between them is Local, but for non-adjacent nodes it is a NonLocal interaction.

Each node has two types of functionalities: Local and NonLocal simultaneously.

In general, in an open system, its elements has practically local interactions, partially nonlocal interactions, partially indeterminate interaction when the hidden variables of the environment influence the decoherence process.

5. MultiAction at a Distance and (MultiLocality, MultiIndeterminacy, MultiNonLocality)

5.1. MultiNonLocality caused by MultiAction at a Distance in Newton's Law of Universal Gravitation and in Superposition

Electromagnetic and gravitational forces instantaneously act at a distance. Newton's Law of Universal Gravitation states that, in the universe, a particle attracts another particle with a force that is direct proportional with the product of their masses, and inversely proportional with the square of the distance between them.

For objects, their masses are considered concentrated at their centers of gravity.

$F = G \cdot \frac{m_1 m_2}{r^2}$,

where F = attraction force; G = the universal gravitational constant; m_1, m_2 = the masses of the two particles (or objects); r = the distance between the two particles (or objects).

Herein too, we have a MultiAction at a Distance, for example many objects at various altitudes starting to fall on the Eart at the same time. SO, the Earth has an action (attraction) at a distance on each object separately, or MultiNonLocality.

5.2. MultiNonLocality caused by MultiAction at a Distance in Superposition

The Superposition Principle enables us to calculate the force exerted on a charge by considering the effects of multiple other charges. To illustrate, let's consider three charged particles. Using Coulomb's Law, we can determine the force between any pair of these particles. According to the

Superposition Principle, the total force acting on any single charge is the vector sum of the individual forces exerted by each of the other charges, as if each of these charges were the only one present. This means that we add up the forces from each charge independently to find the overall force on the target charge.

5.3. Coulomb's Law in scalar form

The Coulomb's Law in scalar form is the following:

$$|F_{12}| = k_e \cdot \frac{|q_1| \cdot |q_2|}{r_{12}^2} = \frac{|q_1| \cdot |q_2|}{4\pi\varepsilon_0 \cdot r_{12}^2}$$

where F_{12} = the electrostatic (attractive or repulsive) force between point charges q_1 and q_2

q_1, q_2 = point charges

r_{12} = distance between charges q_1 and q_2

k_e = Coulomb's constant

$k_e = \frac{1}{4\pi\varepsilon_0}$, where ε_0 is the electric constant

$|\ |$ means absolute value.

Coulomb's Law states that the absolute value of electrostatic force between two charged particles (points) is directly proportional to the product of the magnitudes square of the distance between them (similar to the Newton's Law of Universal Gravitation).

Opposite charges attract each other, while the like charges repel each other.

5.4. Superposition Formula

For multiple charges, the total force on a particular charge is the vector sum of the forces exerted by all other charges.

$$F_i = k_e \sum j \neq i \frac{q_i q_j}{r_{ij}^2} \widehat{r_{ij}}$$

where r_{ij} is the unit vector from q_i and q_j.

The Law of Superposition is an extension of Coulomb's Law from two to three or more point charges, for linear bilateral networks.

On a single point charge many forces act from the other point charges:

Therefore, one has some multi-action at a distance, and correspondingly MultiNonLocality.

5.5. *MultiLocality* results from a group of things that touch and influence together two by two.

5.6. While *MultiIndeterminacy* is produced by hidden variables, unclear environments, uncertainty between many Local and NonLocal interactions, as a result of MultiActions locally and MultiActions at a Distance.

Conclusion

In this article, we introduced a new neutrosophic principle (The Principle of Partial Locality, Partial Indeterminacy, and Partial Non-Locality) that extends and generalizes the concepts of locality and nonlocality by addressing scenarios involving indeterminacy. Locality pertains to interactions confined within a limited space or time, with effects restricted to the immediate environment. In contrast, nonlocality refers to interactions between entities separated by space or time, where changes in one location instantly affect another without intermediaries. Indeterminacy involves hidden variables, such as nonlocal connections between partially entangled objects. This principle works within a dynamic neutrosophic system.

The Principle of Partial Locality, Partial Indeterminacy, and Partial NonLocality implies an interplay of locality, indeterminacy, and nonlocality acting in a dynamic neutrosophic system. A

generalization of (Locality, Indeterminacy, NonLocality) is the (MultiLocality, MultiIndeterminacy, MultiNonLocality).

MultiLocality results from a group of things that touch and influence together two by two. *MultiIndeterminacy* is produced by hidden variables, unclear environments, uncertainty between many Local and NonLocal interactions, because of MultiActions locally and MultiActions at a Distance. While *MultiNonLocality* is caused by a MultiAction at a Distance. Practical examples from different fields were provided.

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A New Hypothesis of Spin Supercurrent as Plausible Mechanism of Biological Nonlocal Interaction, Synchronicity, Quantum Communication

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Abstract

We start with citing a seminal paper by Josephson-Pallikari-Viras, that biological entities can be assumed to be able to communicate nonlocally, i.e., instantaneously. However, they also admit that the underlying mechanism of such an entangled communication is not clear yet from the wave mechanical equations. Similar arguments have been pointed out by several authors, citing that quantum equations themselves have not described anything on a possible mechanism of quantum-type interaction between two biological entities. This chapter intends to fill that research gap by suggesting a new hypothesis of spin supercurrent as a physical mechanism, based on the assumption of macroquantum condensate having nonlocal effects. Moreover, we also draw several potential applications including superconductor quasi-crystalline structure of space and plausible new method of quantum communication. Such an argument is outlined herein partly based on our personal encounter with astrophysical quantization in the past 17 years or so.

Keywords: biological nonlocal interaction, quantum nonlocality, entanglement, spin supercurrent, superfluid dynamics, superconductor quasicrystalline, quantum communication

1. Introduction

In a seminal paper by Prof. Brian Josephson—Pallikari-Viras, they argued that despite quantum nonlocal interaction tends to be undetected by statistical averaging, but by assuming macroquantum system, biological entities can be assumed to be able to communicate nonlocally, i.e. instantaneously. However, they also admit that the underlying *mechanism* of such an entangled communication is not clear yet from the wave mechanical equations [1, 2].

Actually, it is known for a long time that quantum physics allows quantum correlations—common reliance of attributes of wave capacity of supposed entangled quantum substances while there is space partition. This may include a phenomenon called the near-field antenna effect, i.e. the presence close to radio wire (a wavering electric dipole) of superluminally spreading electromagnetic field [3].

Nonetheless, various arguments have been pointed out by several authors, citing that quantum equations themselves have not described anything on the possible mechanism of quantum-type interaction between two biological entities.

In this chapter, we will discuss some existing literature and then we come up with a new hypothesis that spin supercurrent provides the sought-after physical mechanism for biological nonlocal interaction, synchronicity, and plausible new quantum communication method.

2. Literature survey

2.1 Wave mechanics equations

In quantum mechanics, the depiction of action of the field-free magnetic vector potential depends on Schrödinger's equations without presenting any actual interaction. As the activity of the field-free magnetic vector likely takes place in space where the electromagnetic field is missing, this potential has both non-electric and non-attractive nature. While there are researchers who did try to develop an electric representation of quantum wave mechanics, such as Gabriel Kron, but it did not give new results, as far as our knowledge. Moreover, in our previous book, Shpenkov and Kreidik have shown that Weyl provided cut-off to solutions of the original Schrodinger equation (3D), to achieve a quite good agreement with experimental data at the time. It is clear that in most textbooks on QM, whenever the authors discuss solutions of spherical Schrodinger equations, they rarely compare the results with actual experimental data, because they know, there is no agreement at all between spherical wave mechanics and experiment. It should be clear, that despite fairytale stories have been circulated to invoke certain mystical elements to wave mechanics origin, the fact is, it was a failed attempt since the beginning [4].

2.2 Classical EM theory approach

As Boldyreva wrote, which can be paraphrased as follows: "EM hypothesis portrays field-free magnetic vector potential. In traditional electrodynamics, the magnetic field of acceptance B is resolved by condition curl = B curl A , where A will be an attractive vector potential. In protecting of attractive field, 0 = B, the accompanying may happen: $0 \neq A$. This case is alluded to as the without field vector potential. Magnetic vector potential has its very own actual significance. In 1949, Erenberg furthermore Siday anticipated the capacity of attractive vector potential to impact straightforwardly the attributes of quantum substances despite the fact that there is no electromagnetic field at the area of the elements. In 1959, the chance of such an impact was considered by Aharonov and Bohm. Hence, an extraordinary number of tests have been directed which affirmed the hypothesis" [3].

2.3 Macroquantum condensate hypothesis

Here allow us to mention our chapter in a journal of Foundation Louis de Broglie 2006, suggesting that astrophysical quantization can be explained for instance by assuming macroquantum condensate of astrophysical bodies [5].

Provided such astrophysical macroquantum effects can be accepted, then it seems not so hard to suppose that under certain circumstances biological nonlocality interaction can happen, once we assume similar macroquantum condensates.

2.4 Spin supercurrent in superfluid helium

What we can read in some recent papers by Liudmila Boldyreva, she puts forward an argument of the existence of spin supercurrent to mediate biological quantum-type interactions [3].

Boldyreva wrote among other things, which can be paraphrased as follows:

> *"This work proposes basically another way to deal with portrayal of the above-thought about peculiarities: specifically, it is shown that it is conceivable to portray these peculiarities as far as such actual interaction as spin supercurrent. The twist supercurrent arises between objects having turn, and its activity will in general make equivalent the individual characteristics of precession of twists of collaborating objects. (Note that Yuri Bunkov, Vladimir Dmitriev and Igor Fomin were granted the Fritz London Memorial Prize in 2008 for the investigations of spin supercurrents in superfluid 3 He-B)."*

In this model, quantum correlations between quantum entities may be performed by spin supercurrent emerging between virtual photons (virtual particles pairs) created by those quantum entities.

2.5 Carl Jung's synchronicity

Limar wrote a review on the possible link between Carl Jung's concept of synchronicity with quantum non-local effect, known as entanglement. He argues in favor of cellular level or DNA level quantum type interaction, such as meiosis etc. Nonetheless, he also admits that many paper streams on this subject are plagued by the non-existence (as yet) of the physiological or physical mechanism of such non-local interaction [6].

2.6 Research gap

Similar arguments have been pointed out by several authors, citing that quantum equations themselves have not described anything on the possible mechanism of quantum-type interaction or communication between two biological entities. This chapter intends to fill that research gap by suggesting a new hypothesis of spin supercurrent as the physical mechanism, based on the assumption of macroquantum condensate having nonlocal effects. Moreover, we also draw several potential applications including superconductor quasi-crystalline structure of space and plausible new method of quantum communication. Such an argument is outlined herein partly based on our personal encounter with astrophysical quantization in the past 17 years or so.

3. A new hypothesis

If now we put all the above findings from macroquantum condensate (or close to superfluid 3 He) to spin supercurrent, hence, we come up with a new hypothesis, that we will state here for the first time:

3.1 Hypothesis

"There is spin supercurrent to be observed to mediate interaction between biological entities, between consciousness which known as synchronicity (in Jungian term), and also to provide quasi-crystalline structure of space, and in turn, it allows a new model of quantum-type nonlocal communication."

3.2 Simple physical model

According to Bunkov and Volovik, the superfluid current of twists—turn supercurrent—is another agent of superfluid flows, for example, the superfluid current of mass and molecules in superfluid 4He; superfluid current of electric charge in superconductors [7].

According to Boldyreva, such spin supercurrent mechanism can be helpful to mediate biological nonlocal interaction, can be modeled as follows:

$$J_z = g_1(a_1 - a_2) + g_2(t_1 - t_2), \tag{1}$$

where g_1 and g_2 are coefficients depending on deflection angles and the properties of the medium where spin supercurrent emerges. Turn supercurrent is certainly not an electric or attractive interaction and therefore it is not protected by electromagnetic screens [3].

Moreover, Boldyreva also argues that such a spin supercurrent interaction can find implications in alternative medicine, which can be paraphrased as follows: "...a deterministic portrayal of the association is utilized, which continues as per the laws overseeing the conduct of sub-atomic fluid when the temperature of the last option is near outright zero (the properties of superfluid 3He-B). This methodology concurs with E. Schrödinger's perspective communicated in his book '*What is life?*', i.e. "The living organic entity is by all accounts a naturally visible framework which to a limited extent of its conduct ways to deal with that absolutely mechanical (as differentiated with thermodynamical) direct to which all frameworks tend, as the temperature moves toward outright zero and the atomic problem is taken out" [8].

4. Sideways and rationale

4.1 Our personal encounter with macroquantum condensate astrophysics

This segment permits us to recount an account of our experience with macroquantum condensate in astrophysics. Everything started by a to some degree "reasonable deduction" (or readers might call it: einfuhlung), when one of us (VC) got an old book by Nozieres and Pines [9], on superfluid Bose liquid. He inquired: Let us see what this

book can bring to the domain of astronomy and cosmology. Before long, he tracked down many fascinating discoveries with regards to the writing, from W.H. Zurek to Grigory Volovik and so forth. That is the start of our undertaking for more than 18 years up to this point, coming about a few papers in a series [5, 10–14]. The soonest paper called "Cantorian superfluid vortex hypothesis" was distributed in January 2004, where VC presented a forecast of potential areas of three new circles of planetoids on the external side of Pluto. Then, at that point, after 2 years, VC distributed a paper in AFLB [5], where he laid out what are potential clarifications of macroquantum impacts in astronomy (for example, noticed likewise by Tifft and furthermore Virginia Trimble and so forth). One of the contentions in that AFLB paper is macroquantum condensate, for example, conceivable quantum impact actuated by BEC or superfluid-type medium [5].

More recently, we (VC, FS, YU) come up with an argument of cosmological entanglement supposing such a macroquantum effect is real.

4.2 Observational evidence

4.2.1 Quantization of planetary orbit distances in the solar system

In this section, we will review the work and results by us, during the past 17 years or so. The basic assumption here is that the Solar System's planetary orbits are quantized. But how do their orbits behave? Do they follow Titius-Bode's law? Our answer can be summarized as follows (**Figure 1**):

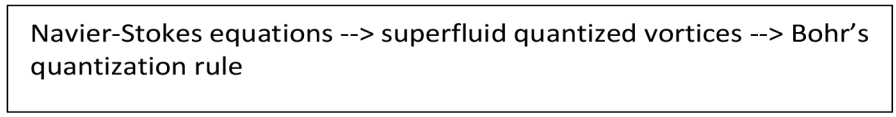

Figure 1.
From NS turbulence to quantized vortices.

And it seems that the proposed model is slightly better compared to Nottale-Schumacher's gravitational Schrödinger model and also Titius-Bode's empirical law [1, 11].

The evidence of quantization of planetary orbit distances seem to suggest to wave mechanics model at a large scale [5, 10–14]. See also Peter Coles [15].

4.2.2 Observational finding on cosmological entanglement

Interestingly there is a recent report from MIT suggesting that ancient quasars support such quantum entanglement at large-scale phenomena. In an article, it is reported about the possibility of cosmological entanglement [16], which can be paraphrased as follows:

"In 2014, ... the William Herschel Telescope and the Telescopio Nazionale Galileo, both located on the equal mountain and separated via about a kilometer. One telescope focused on a particular quasar ..... Meanwhile, researchers at a station located between the two telescopes created pairs of entangled photons and beamed particles from each pair in contrary directions toward every telescope" [17].

Therefore, such a discovery has opened up a new way to look at the Universe: *an entangled Cosmos* [18, 19].

4.2.3 Newtonian action at a distance: Smarandache's

Hypothesis expresses that there is no speed limit of anything, including light and particles [20]. Eric Weisstein likewise composed ramifications of Smarandache's Hypothesis [21], which can be summarized as follows: "...the speed of light c is as of now not a biggest at which estimations can be sent and that abstract speeds of data or mass switch can occur. These confirmations fly notwithstanding every idea and investigation, as they misuse both Einstein's exceptional rule of relativity and causality and don't have any test support. It is genuine that current preliminaries have confirmed the presence of positive sorts of quantifiable superluminal quirks....." [21].

While the thought is very basic and in view of known speculation of quantum mechanics, called Einstein-Podolski-Rosen bridge, actually such a superluminal material science appears to be still difficult to acknowledge by the greater part of physicists. Beginning around 2011, there was a clear astounding outcome as declared by the OPERA group. Regardless, hardly any months after the fact, it was disavowed on the ground of mistakes in dealing with the estimation.

Permit us to offer not many remarks on such a clear inability to identify quicker than light speed as follows: Despite those discussions over the OPERA results, we believed that a seriously persuading test has been finished by Alain Aspect and so on; he had the option to show that quantum non-territory association is genuine. In 1980, Alain Aspect played out the first EPR try (Einstein-Podolski-Rosen) which demonstrated the presence of room nonlocality (Aspect 1982). Alain Aspect and his group at Orsay, Paris, led three Bell tests utilizing calcium course, i.e. the first and last utilized the CH74 disparity. The second was the first use of the CHSH imbalance.

The third (and generally well known) was organized with the end goal that the decision between the two settings on each side was made during the flight of the photons (as initially proposed by John Bell). A few experimenters demonstrated a comparative outcome until the distance of more than 90 km.

So, the thought of "spooky action at a distance" is a genuinely actual peculiarity. In addition, activity a way off was at that point referenced in Newton's *Principia Mathematica.* Regardless of obviously Einstein was attempting to make each of Newton's demeanors into nothing, our result suggests that the Maxwell equations in classical electrodynamics have "spooky interaction at a distance" type of interactions (as it has also been proven for Coulomb potential), which might be noticed both at limited scope tests just as in a cosmological scale, as ongoing confirmations show similar effect at a distance in relation to Smarandache's hypothesis.

4.2.4 Evidence of Cooper-pair tunneling in nuclei is likely to indicate superfluid vacuum model instead of gluon

In a recent report published in *Phys. Rev. C*, Potel et al. wrote on a breakthrough on the subject was made through the study of one- and two-neutron transfer reactions with heavy-ion collisions in inverse and direct kinematics, enabled by the use of magnetic and γ-ray spectrometers, which suggest that there can be Cooper-pair tunneling in nuclei [22]. In retrospect, this finding seems to indicate that the superfluid vacuum model can be a better approach than the gluon model as in the Standard Model. See also [23, 24].

Besides, the superfluid nuclear matter hypothesis is known for a quite long time, especially going beyond BCS theory, cf. Walecka, Matsuzaki, Lombardo *etc.* [25, 26].

4.2.5 Initial evidence on the synchronicity between patient and doctor

In a 2008 article, Alex Hankey argues in favor of Macroscopic Quantum Coherence in Patient-Practitioner-Remedy Entanglement. An interesting remark in his article goes, which can be paraphrased as follows: "A different relationship length implies that the quantum cognizance's initially infinitesimal connection length currently becomes naturally visible. We reason that, for the most part, at every basic precariousness (remembering input dangers for natural administrative frameworks), quantum vacillation fields display plainly visible quantum cognizance" [27].

Although he did not come up yet with a clear physical mechanism of such macro-quantum coherence, one can arrive at a similar hypothesis of spin (supercurrent) interaction like Boldyreva's, as it is known in biological phenomena. See Likhtenshtein [28].

4.2.6 Initial evidence on galactic synchronicity

Although it is known that "One of the cornerstones of inflationary cosmology is that primordial density fluctuations have a quantum mechanical origin," as Kanno & Soda wrote, however, most physicists consider that such quantum mechanical effects disappear in CMB data due to decoherence [29].

We have discussed before that cosmological entanglement has been observed, which in turn, it can be attributed to the superfluid turbulent interstellar medium.

Presently, there is a new striking report by Charlotte Olsen et al., proposing that 36 cosmic systems appear to have "facilitated" in a such way that they seem to give synchronized stars arrangement. From Olsen et al. paper, they do not give a potential hypothetical explanation [30].

Notwithstanding, by theorizing such a twist supercurrent system likewise can occur at cosmic scale in view of superfluid interstellar medium, we can concoct a "potential" clarification, that such a lucid star arrangement is because of some sort of "*galactic synchronicity*." We know that such a term is not accessible yet in present cosmological vocabularies, however, we can predict that time for that term will come as well, as there is likewise a book, proposing that synchronicity is probably going to show up all around in Cosmos [31].

4.2.7 Other experimental results

Other reports seem to indicate that there are reasons to believe such a quantum effect between consciousness, mind-matter interaction, and also Aharonov-Bohm type interaction in the superfluid vortex [16, 32, 33]. Suter et al. also provide other experiment evidence, as they wrote in the abstract: "The data unambiguously show that Bzclearly deviates from an exponential law and represent the first direct, model independent proof for a nonlocal response in a superconductor" [34].

5. Discussion: four plausible applications in various fields

a. A new theoretical model of high-temperature superconductivity may lead to extremely efficient energy generation and transmission

b. A new type of electronic device.

c. Superconductor quasi-crystalline vacua hypothesis.

d. The plausible new method of quantum communication.

The explanation for each of the aforementioned plausible applications will be discussed shortly below:

5.1 A new theoretical model of high-temperature superconductivity may lead to extremely efficient energy generation and transmission

It is known that a superconductor permits the flow of current without resistance. The conventional way of thinking about the transition from normal to superconducting is called the Bardeen-Cooper-Schrieffer (BCS) theory. But last year, H. Koizumi, a researcher at Tsukuba University has announced a new theoretical model of high-temperature superconductivity, which may lead to extremely efficient energy generation and transmission. Instead of focusing on the pairing of charged particles, this new theory uses the mathematical tool called the Berry connection. This value computes a twisting of space where electrons travel. In the standard BCS theory, the origin of superconductivity is electron pairing. In this new theory, the supercurrent is identified as the dissipation less flow of the paired electrons [35]. We will discuss later; we may come up with an alternative method of quantum communication based on such Berry connection.

5.2 A new type of electronic device

Hua Chen et al. wrote experimental evidence which can lead to a new type of electronic device based on spin supercurrent, according to their abstract which can be paraphrased as follows: "In slight film ferromagnets with amazing simple plane anisotropy, the part of absolute twist perpendicular to the simple plane is a decent quantum number and the relating turn supercurrent can stream without scattering. In this Letter we clarify how turn supercurrents couple spatially remote turn blending vertical vehicle channels, in any event, when simple plane anisotropy is flawed, and examine the likelihood that this impact can be utilized to manufacture new kinds of electronic device" [36, 37].

5.3 Superconductor quasicrystalline vacua hypothesis

As we discussed in a forthcoming paper [38], we discuss on the possibility that the space consists of discrete cells, to become cells composed of superconductor quasi-crystalline. We put forth a new hypothesis that the discrete cellular structure of space consists of cells of superconductor quasi-crystalline. It is argued that the definition of quasicrystals should not include the requirement that they possess an axis of symmetry that is forbidden in periodic crystals. The term "quasicrystal" should simply be regarded as an abbreviation for "quasiperiodic crystal," possibly with two provisos.

To sum up, quasicrystals display a non-periodic, yet ordered, arrangement of atoms. They contain a small set of local environments which reappear again and again, albeit not in a periodic fashion. Their structure is not random either, since the diffraction pattern shows sharp Bragg peaks, although their symmetry is noncrystallographic,

with the n-fold symmetries (n = 5, 8, 10, ...) stemming from the fact that these local environments occur with n equiprobable orientations. A recent discovery suggests that quasi-crystalline has a superconductive phase at a very low temperature [39].

It may resemble Finkelstein's hypercrystalline model of vacuum. What if the quasicrystalline model is not in semiconductor solid...but a superconductor quasicrystalline? We may call it: super-crystalline structure of 3D space.

Quasicrystalline solid is also good because it brings in more than three dimensions, which may be very relevant. This would also bring in Finkelstein and Penrose and some of Frank Tony Smith's investigations. The next item to consider is a super-quasi-crystalline solid (SQC).

Because of its fractal properties, we can expect that the SQC can extend down to the structure of space, similar to what Finkelstein envisaged [40].

The quasicrystal structure of space may be composed of solid matter or soft matter, of which its general dynamics have been outlined by Fan et al. [41]. Plausible assembly of soft matter quasicrystal is shown in **Figure 2**:

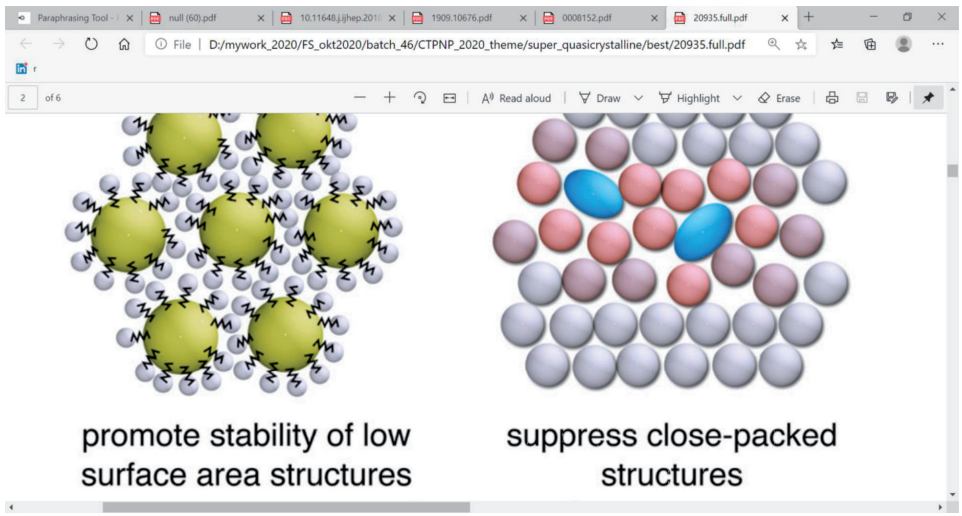

Figure 2.
Plausible assembly of soft-matter quasi-crystal.

It exhibits close-packed structures. This dense-packed structure of space should be verified with experiment. A few observables:

5.4 Natural quasicrystal in rock:

Steinhardt and Bindi [42] argued a unique hypothesis, proposing that quasicrystals might conceivably be pretty much as hearty and steady as gems, maybe, in any event, framing normally. These contemplations roused a very long term look for a characteristic quasicrystal finishing in the revelation of icosahedrite (Al63Cu24Fe13), an icosahedral quasicrystal found in a stone example made fundamentally out of khatyrkite (translucent (Cu,Zn)Al2) named as coming from the Koryak Mountains of far eastern Russia. In their paper, they contended that the examination demonstrates the example to be of an extraterrestrial beginning (**Figure 3**).

Figure 3.
Light image of entire MSNF specimen prior to sampling, Khatyrka CV3 meteorite [42].

Moreover, some papers argue that such a rock may be of manmade origin, as Bindi et al. noted:

> *"The proof for the presence of the quasicrystal deliberately work in the stone is thusly predominantly solid. Be that as it may, the perception of intermetallic compounds with copper and iron, which requires a profoundly lessening climate, is profoundly bewildering. It raises the likelihood that the example started from slag or another anthropogenic interaction. Nonetheless, the example was found in a far-off locale exceptionally a long way from any modern action." [43]*

While we admit that it would need further studies, as we see it such a hypothetical origin of meteorites and rock from extraterrestrial or manmade origin remains puzzling. It may be more possible to argue in favor that the quasicrystalline that happens in nature was caused by the structure of space itself is composed of SQC.

5.4.1 Natural quasicrystals in the solar system

Luca Bindi and also Matthias Meier et al. seem to suggest that quasicrystals have a cosmic origin [44, 45]. While such a hypothesis is quite reasonable, allow us to add a possibility that such a cosmic origin might yield from hidden structure of space itself. Such a hypothetical origin may be more "workable" than most quantum gravity hypotheses [46].

What is more interesting here is that Sakai has presented superconductor effects of quasicrystals [47].

5.4.2 A plausible new method of quantum communication

Inspired partly by Koizumi's research at Tsukuba Univ., we may come up with an alternative method of quantum communication based on such Berry

connection. Prof. M.V. Berry is widely known for his research, but mostly for his theorem called Berry phase and Berry connection, which are often linked to the Aharonov-Bohm effect.

While there are various researchers who have come up with a number of possible quantum information or quantum internet methods, such as Pomorski & Staszewski, who wrote: "The idea of quantum web was.

displayed in this work. The more definite picture requires considering different impacts as decoherence processes that drive the quantum position-base qubit out of its cognizance just as decoherence processes that annihilate the cognizance of qEC (quantum Electromagnetic Cavity) [48].

Nonetheless, just yesterday an idea came to us, inspired by Berry connection and also crystalline structure of space. Actually, Tesla came up first with an idea to propagate telecommunication with the help of the telluric field of Earth, but alas it was canceled because Marconi obtained the patent first before him.

As we mentioned above, Sakai presented evidence of superconductivity of quasi-crystal, therefore provided, we accept the super-quasi-crystalline model of 3D space, we may come up with an idea that can be considered as quantum communication built on the crystalline structure of space itself. The outline of our idea is as follows, as we began to read papers by Prof. Michael V. Berry from UK (**Figure 4**):

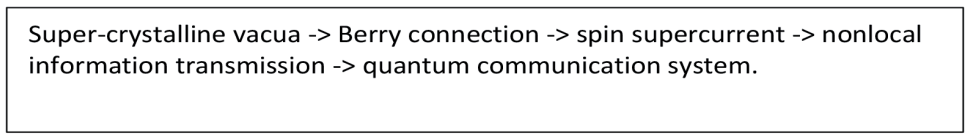
Super-crystalline vacua -> Berry connection -> spin supercurrent -> nonlocal information transmission -> quantum communication system.

Figure 4.
A process flow of quantum communication based on Berry connection.

If we can prove this can work, at least a conceptual design, then may it can be a quite viable alternative to the 5G cellular network.

Moreover, such an assumption of superconductor crystalline structure of space, it seems to find support from our descriptive model of the Solar System in terms of superconductors. See our recent paper [49, 50].

6. Concluding remarks

In this chapter, we discuss how conventional wave mechanics does not provide a physical mechanism which is supposed to mediate nonlocal biological interaction, as discussed by Josephson-Pallikari Viras and others. Based on the hypothesis and also research findings by Yuri Bunkov, L. Boldyreva et al., we submit wholeheartedly that a new hypothesis of spin supercurrent is the sought-after physical mechanism, based on the assumption of macroquantum condensate having nonlocal effects.

In the last section, we discuss four plausible applications of such a scheme, including: a. A new theoretical model of high-temperature superconductivity may lead to extremely efficient energy generation and transmission; b. A new type of electronic device; c. Superconductor quasi-crystalline vacua hypothesis; d. Plausible new method of quantum communication. Clearly, more research is recommended to verify further what we outlined herein.

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Two-dimensional steady squeezing flow over a vertical porous channel with free convective heat/mass transfer and invariable suction

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Zeeshan, Muhammad, et al. "Two-dimensional steady squeezing flow over a vertical porous channel with free convective heat/mass transfer and invariable suction." Case Studies in Thermal Engineering, 60, 2024, 104800. https://doi.org/10.1016/j.csite.2024.104800.

A B S T R A C T

This research reports on the combined effects of heat and mass transfer (HMT) under the influences of the Soret and Dufour in natural convection steady 2D magnetohydrodynamic flow through the boundary layer in a porous vertical tube or duct. The current study is motivated by the significant applications of HMT in engineering processes such as casting and welding. The goal of this framework is to explore the assisting and opposing movements with HMT above a vertical porous channel under the influence of invariant suction and fluid dissipation which have not been reported in the earlier studies. The governing flow equations in terms of partial differential equations (PDEs) are altered to dimensionless ordinary differential equations (ODEs) by using dimensionless variables. Employing the BVP4C approach, the leading equations are solved numerically. The dual nature of solutions has been observed due to suction. Stability exploration has been provided to confirm a stable solution. A comparison between published and current studies shows outstanding agreement. Key parameter effects on flow characteristics are visually offered using graphs as well as tables. It is noteworthy that the influence of Soret effects becomes apparent in a suspended mixture of particles and fluids. These phenomena can be attributed to temperature differences, whereby the motion of fluid particles in the warmest region with the maximum energy level causes the particles to migrate towards the coldest region. Notably, the flow speed rate at the left plate is initially high and slowly diminishes near the right plate.

Keywords:
Assisting and opposing flow
Stability
bvp4c
Suction effect
MHD squeezing flow
Porosity
Heat mass transfer

1. Introduction

The simultaneous movement of HMT through a saturated, permeable medium, driven by natural convection, is extremely important in many sectors of engineering, including geothermal energy and geophysical manufacturing. Additionally, heat and mass transportation mutually influence one another, leading to the impacts of cross-diffusion. Mass transport induced by heat gradients is re-

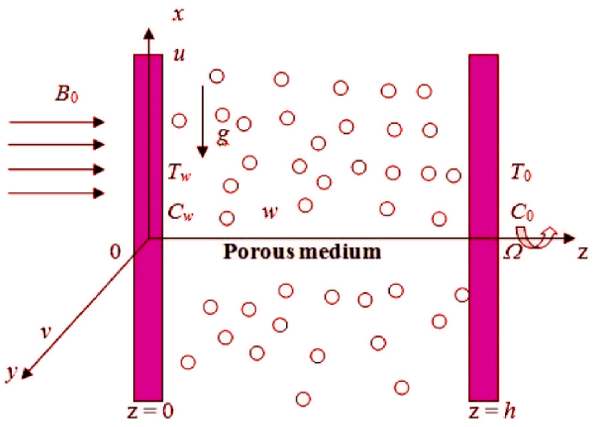

Fig. 1. Physical model.

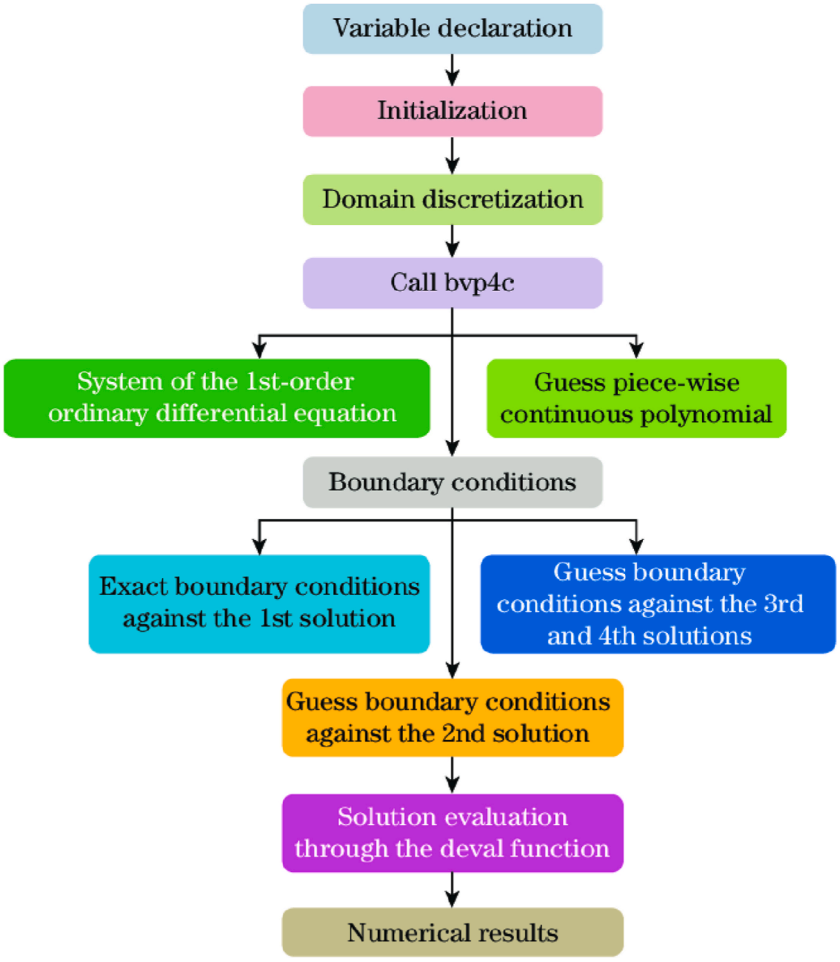
Variable declaration
Initialization
Domain discretization
Call bvp4c
System of the 1st-order ordinary differential equation
Guess piece-wise continuous polynomial
Boundary conditions
Exact boundary conditions against the 1st solution
Guess boundary conditions against the 3rd and 4th solutions
Guess boundary conditions against the 2nd solution
Solution evaluation through the deval function
Numerical results

Fig. 2a. BVP4C procedure.

ferred to as Soret effects, while thermal transport caused by concentration gradients is referred to as Dufour effects. In order to achieve optimal cost and energy efficiency, enhancing heat transport plays a crucial role. The current advancements within the realm of technological and scientific developments are elevating the demand for machines that possess unusual characteristics, encompassing superior performance, speed, precise rolling, and an extended lifespan. Consequently, researchers and technologists have gathered to focus on the heat management of thermal transfer equipment. The investigation of combined thermal and mass conveyance, coupled with convection from the air in a fluid-saturated medium with pores, is of great interest in numerous industries, including hy-

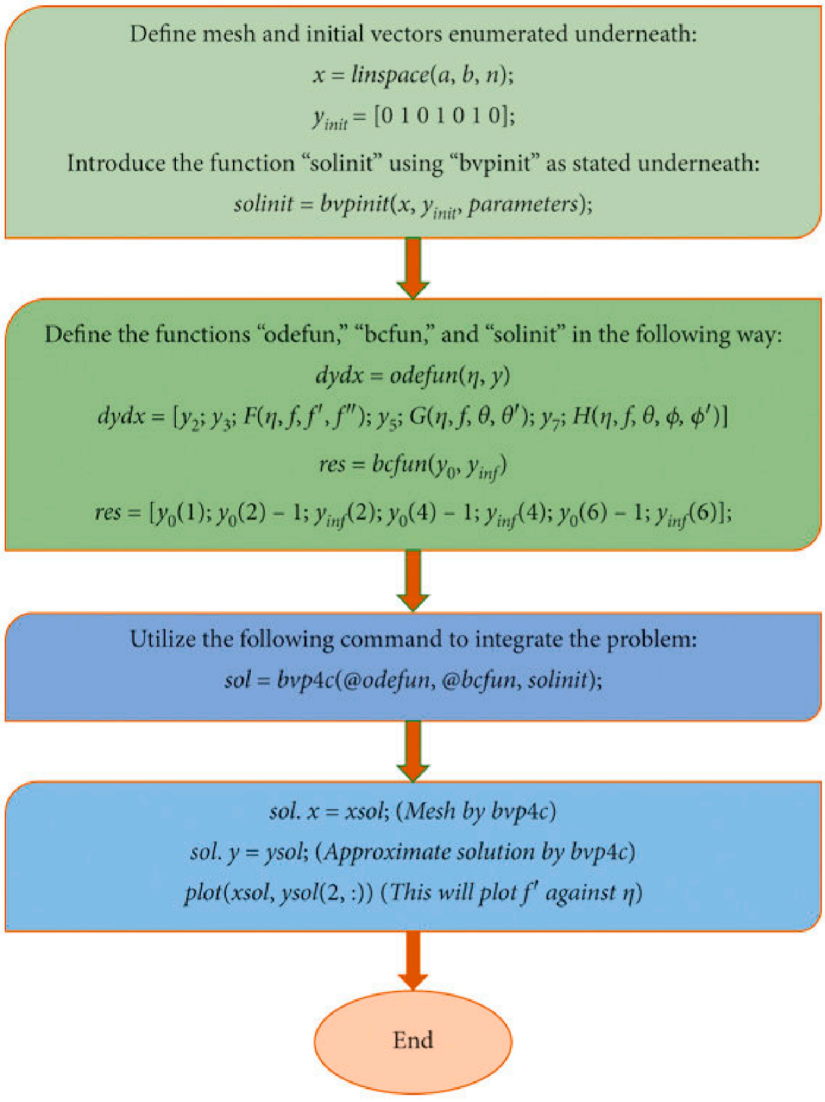
Define mesh and initial vectors enumerated underneath:
x = linspace(a, b, n);
y_init = [0 1 0 1 0 1 0];
Introduce the function "solinit" using "bvpinit" as stated underneath:
solinit = bvpinit(x, y_init, parameters);
Define the functions "odefun," "bcfun," and "solinit" in the following way:
dydx = odefun(η, y)
dydx = [y2; y3; F(η, f, f′, f″); y5; G(η, f, θ, θ′); y7; H(η, f, θ, ϕ, ϕ′)]
res = bcfun(y0, y_inf)
res = [y0(1); y0(2) − 1; y_inf(2); y0(4) − 1; y_inf(4); y0(6) − 1; y_inf(6)];
Utilize the following command to integrate the problem:
sol = bvp4c(@odefun, @bcfun, solinit);
sol. x = xsol; (Mesh by bvp4c)
sol. y = ysol; (Approximate solution by bvp4c)
plot(xsol, ysol(2, :)) (This will plot f′ against η)
End

Fig. 2b. BVP4C code procedure.

Table 1
Confirmation of numerical code by associating with available work for velocity when (Gm = 2,Sc = 0.3;Pr = 0.61,Sr = 1.2,S = 3,Ec = 0.03,Du = 0.02,Kc = 0.5).

M	*K*	*Gr*	J	Uwanta and Halima [31]	Present Results *R* = 0
0.3	0.2	3	1	0.325547	0.325436
0.6				0.201146	0.201135
1.0				0.088741	0.088631
	0.3			0.436584	0.436473
	0.5			0.557419	0.557307
		5		0.644156	0.6440547
		7		0.902115	0.902052
			2	0.452147	0.452057
			4	0.599010	0.599020

drogeology, earth science, electronic cooling through fans, exploitation of ground thermal power., petroleum storage, and the development of metal alloys and their molecular energy facilities. An expanded variation of the present series has been examined by contemporary researchers [1,2].

In the past few years, substantial consideration has been focused on investigating hydromagnetic flows for the purpose of thermal and mass transportation. This research is motivated by the relevance of these flows to the fields of physics, aerospace research, and chemistry technology. Palani and Srikanth [3] conducted a study on the hydromagnetic movement of an electric-conductive liquid

Table 2
Smallest eigenvalues γ against λ_1.

λ_1	γ	
	First solution	Second solution
−1.3318	0.00625	−0.00531
−1.330	0.05130	−0.1750
−1.325	0.1760	−0.2336
−1.3	0.37871	−0.4415

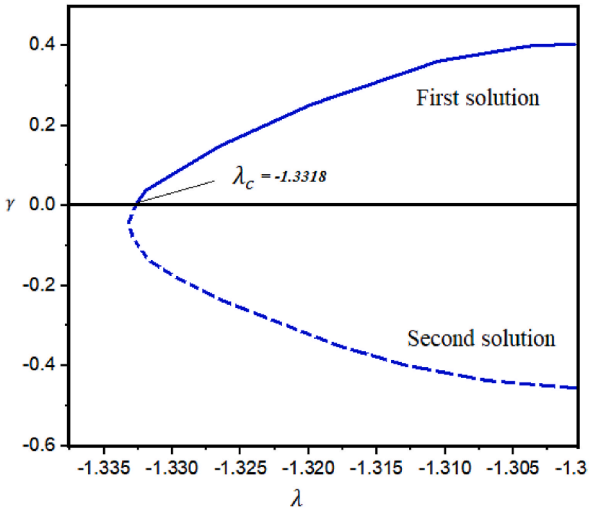

Fig. 3. γ against λ.

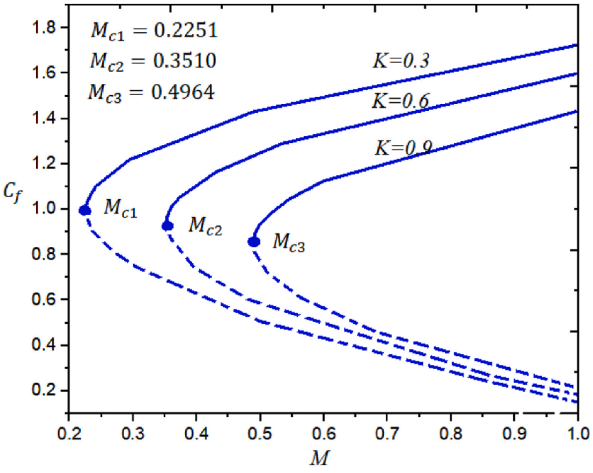

Fig. 4. Impact of K on C_f.

along a semi-infinite perpendicular sheet with Lorentz force. Makinde [4] investigated how a magnetohydrodynamic boundary layer flows past a vertically moving sheet, transporting thermal energy and mass. Similarly, Duwairi [5] analyzed the properties of viscosity with Joule heating on convection flow via a dispersed isothermal surface. The influence of fluid dispersion is frequently defined by the Eckert number, and this plays a crucial role in the dynamics flow in the nuclear industry [6]. This also has a significant impact on natural airflow into a variety of large-scale processes or for larger celestial objects. The effect of suction on boundary transport across sheets has a greater impact on manufacturing operations and has been frequently studied by numerous prominent researchers. Numerous prominent researchers frequently study the effect of suction on boundary transport across sheets, which has a significant impact on manufacturing operations. Uwanta [7] conducted an investigation into the effects of chemical mix reactions and exposure on the movement of mass and heat through a semi-boundless perpendicular porous panel that is subject to constant flow and dispersion. Mansour et al. [8] investigated the effects of chemical substance reactions and fluid disappearance on hydromagnetic diffusion. Kandasamy et al. [9] described the reaction of compounds, the flow of HMT under the heat source's pressure, and the end products of ab-

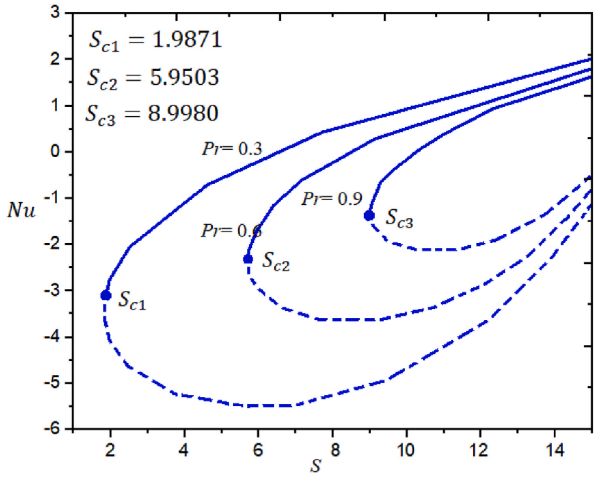

Fig. 5. Impression of *Pr* on *Nu*.

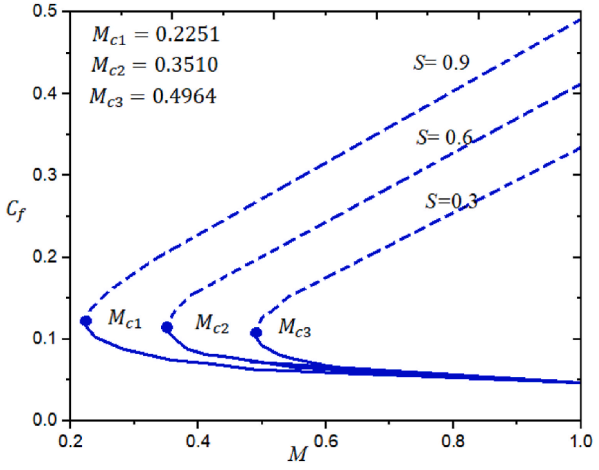

Fig. 6. Impression of *S* on C_f.

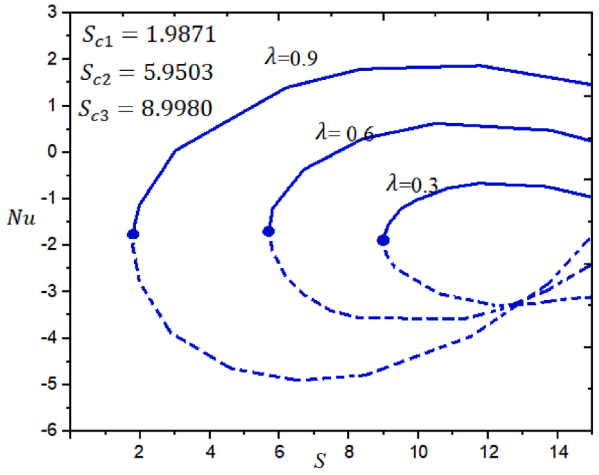

Fig. 7. Impression of λ on *Nu*.

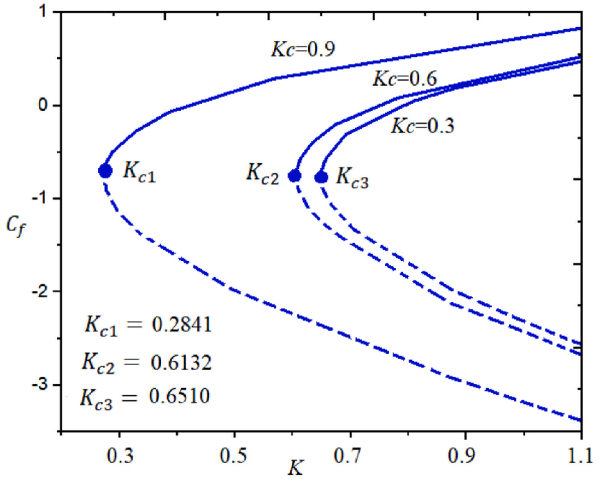

Fig. 8a. Impression of *Kc* on C_f.

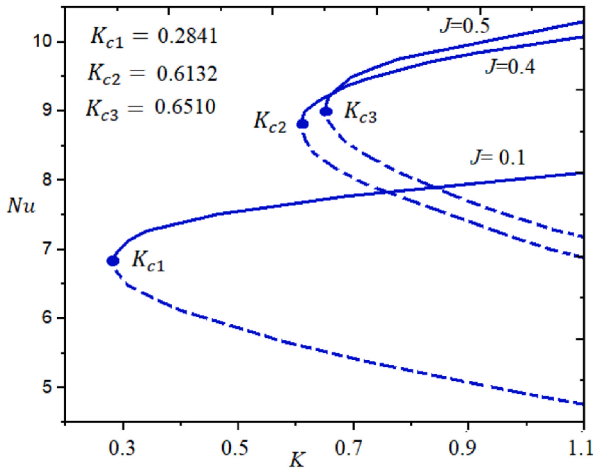

Fig. 8b. Impression of *J* on *Nu*.

sorption resulting from absorption or vaccination events. Govardhan et al. [10] investigated the natural convection of HMT over isothermal stretched surface in the presence of magnetic field and viscous dissipation.

Krishna and Chamkha [11] conducted a thorough investigation into the various impacts of thermo-diffusion, radiation-absorption, Hall, and ion-slip on the flow characteristics of nano-fluids over a semi-infinite permeable moving plate underneath the stimulus of a constant heat source. Shamshuddin et al. [12] scrutinized the secondary flow of magnetized squeezing movement of nanofluid via a rotating framework. A numerical solution has been obtained for this purpose. Shamshuddin et al. [13] numerically scrutinized the MHD bioconvection squeezing. Similarly, Shamshuddin et al. [14] used the variation parameter approach to investigate the MHD non-Fourier squeezing stream in the Riga channel with viscous and thermal effect. Khaleque and Samad [15] outlined the results of emissions, generation of heat, and dense dissipation for magnetohydrodynamic natural transfer flow over a stretched plate. Eckert and Drake [16] recently established that the Dufour effect is of considerable significance, emphasizing its importance. The impression of the Soret and Dufour on warmth and mass conveyance was analyzed by Postelnicu [17]. Theoretical investigations on the Soret and Dufour influence on combined HMT across an upright cone revolving in an atmospheric fluid with a magnetic field and chemical interactions were conducted by numerous researchers [18–21].

Chamkha [22] studied the hydro-magnetic convective transport of nanofluid between two parallel porous discs, observing the Hall effect. Krishna et al. [23] explored the consequences of Hall, including ion-slip on the revolving motion of nanofluids in time-dependent magnetohydrodynamic convection flow. Krishna [24] investigated the propagation of heat during the constant hydromagnetic flow of Cu and Al2O3 nanofluids across a porous material surface. Krishna et al. [25] investigated how Hall and ion slip affect non-steady hydromagnetic natural convective rotating movement via pore-filled mediums. Shahzad et al. [26] scrutinized the Sutterby nanofluid in solar-HVAC for energy storage optimization using the KKl correlation model. Jamshed et al. [27] examined the Maxwell nanofluid model through PTSC with thermal performance. Shamshuddin et al. [28] explored the bio-convection movement using Casson hybrid nanofluid over stretched sheets with mixed convection. The impact of thermal emission in non-steady magneto-hydrodynamic unrestricted convective rotating movement of Jeffrey's fluid across a boundless upward absorbing sheet at stepped

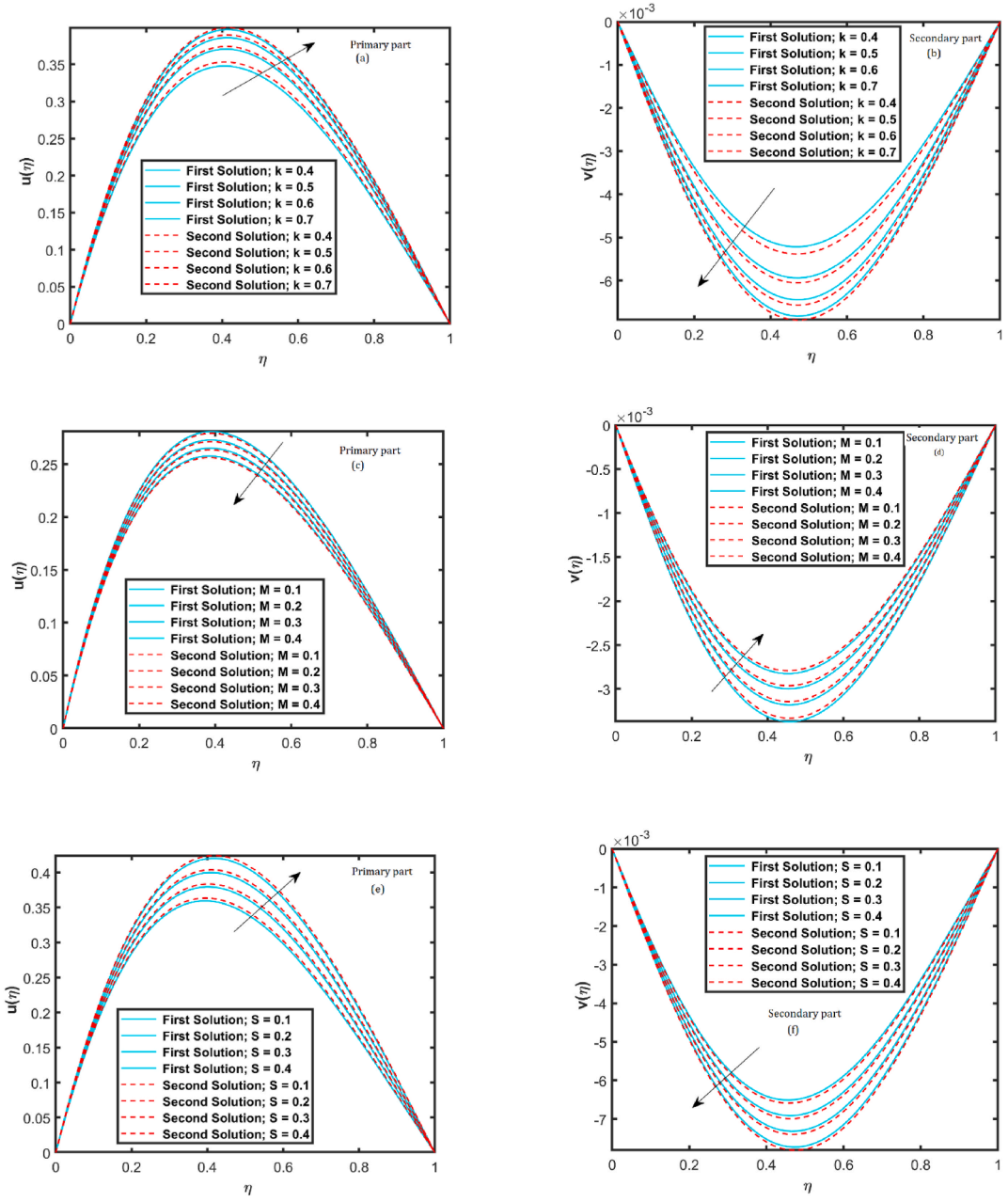

Fig. 9. (a–f). Comparison of Primary part solution (a, c, e) and Secondary part solution (b, d, f) for different values of *K*,*M*, and S.

wall temperatures was studied by Krishna [29]. Krishna [30] studied the effects of emission, chemical reactions, Hall, and ion slip with MHD effect over a semi-infinite permeable surface. Krishna [32] studied the Hall and ion slip effects in MHD flow of Jeffrey fluid across a rotating surface.

Rehman et al. [33] applied a group theoretic approach to implement a Lie point of transformation. This Lie point of conversion is then utilized to characterize the mathematical formulas in terms of self-governing variables. Ali et al. [34] deliberated the consequences of magnetohydrodynamically generated heat, absorption, and slip on Newtonian liquid movement, along with the chemical interaction caused by the gyrating disk. The constant 2-D magneto-hydrodynamic movement of a 2nd-order chemical reaction by means of a porous medium, including HMT, was examined by Rehman and Malik [35] in their study. The magneto hydrodynamic stagnation zone movement of non-Newtonian liquid caused by an extending surface with mixed convection and Joule heating impacts was studied by Rehman et al. [36]. Rehman et al. [37] deliberated the heat-transmitting characteristics of the Williamson fluid in the manifestation of a magnetic effect. In their study, Rehman et al. [38] examined non-Newtonian fluid movement in a revolving magneto hydrodynamic system with a thermal source over a rigid disk.

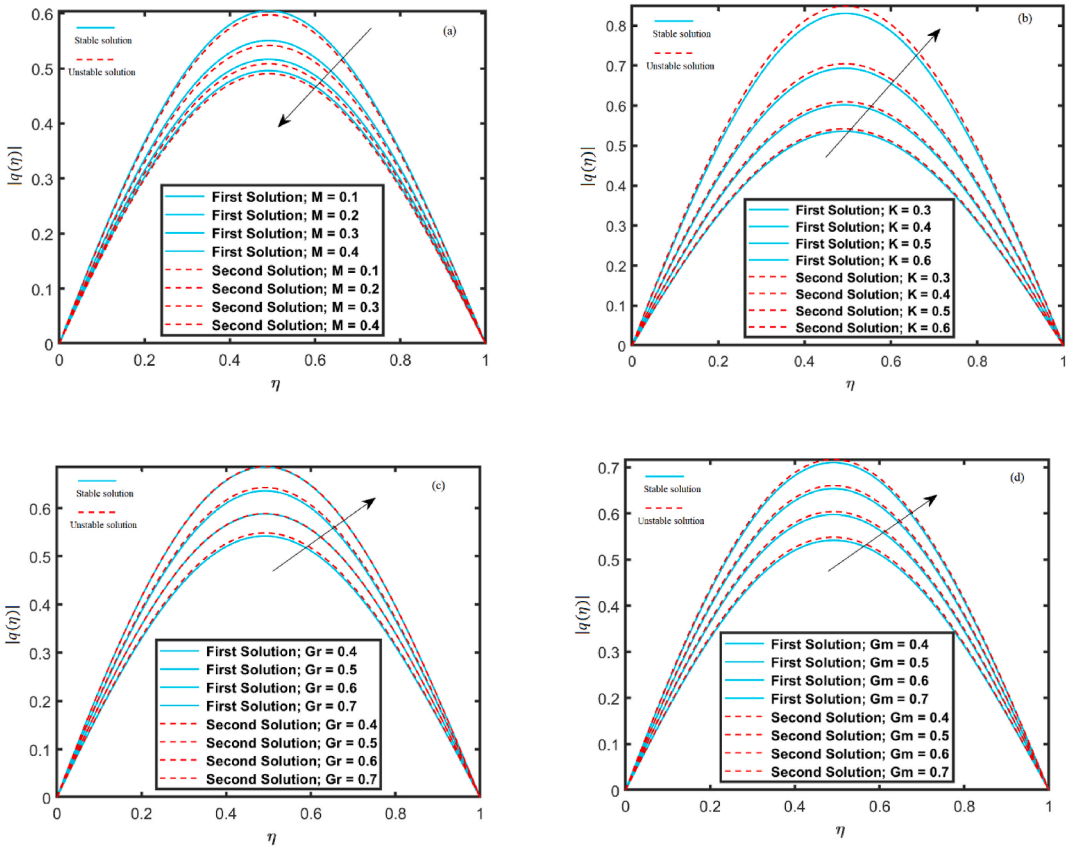
(a)
Stable solution
Unstable solution
First Solution; M = 0.1
First Solution; M = 0.2
First Solution; M = 0.3
First Solution; M = 0.4
Second Solution; M = 0.1
Second Solution; M = 0.2
Second Solution; M = 0.3
Second Solution; M = 0.4
(b)
First Solution; K = 0.3
First Solution; K = 0.4
First Solution; K = 0.5
First Solution; K = 0.6
Second Solution; K = 0.3
Second Solution; K = 0.4
Second Solution; K = 0.5
Second Solution; K = 0.6
(c)
First Solution; Gr = 0.4
First Solution; Gr = 0.5
First Solution; Gr = 0.6
First Solution; Gr = 0.7
Second Solution; Gr = 0.4
Second Solution; Gr = 0.5
Second Solution; Gr = 0.6
Second Solution; Gr = 0.7
(d)
Unstable solution
First Solution; Gm = 0.4
First Solution; Gm = 0.5
First Solution; Gm = 0.6
First Solution; Gm = 0.7
Second Solution; Gm = 0.4
Second Solution; Gm = 0.5
Second Solution; Gm = 0.6
Second Solution; Gm = 0.7
|q(η)|
η

Fig. 10. (a–d). Velocity variation versus *M*,*K*,*Gr*, and *Gm*.

The impression of heat dissipation and the production of thermal energy over an unsteady magnetohydrodynamically flow of thermal and electrically conductive nanofluids across a fluctuating perpendicular plate in an absorbent medium was inspected by Sheikholeslami et al. [39]. Kataria and Mittal [40,41] examined the mathematical representation of the movement, thermal, and mass transmission in the time-dependent magnetized convective movement of nanofluid through a vertically oscillating plate. The HMT of nanofluid containing Al_2O_3 within two parallel plates under a magnetized absorbent medium was deliberated by Sheikholeslami et al. [42]. Li et al. [43] utilized a mesoscopic approach for evaluating the hydromagnetic flow movement of a nanofluid in a porous duct. Patel et al. [44] inspected the magnetohydrodynamic behavior of a micropolar base nanofluid that incorporates the microstructure and inertial attributes of the frame particles. Mittal and Patel [45] studied the 2D combined convective hydromagnetic stagnation zone movement of Casson fluid through a medium with pores using an infinite plate. Mittal et al. [46] investigated a combined convective micropolar-ferrofluid movement, including viscous dispersion. Mittal and Kataria [47] explored the movement of 3-dimensional nanofluids with Brownian motion and radiation. Mittal [48] studied the transfer of mass and thermal energy in three-dimensional magneto-hydrodynamic nanofluid flow via parallel plates. Mittal [49] investigated how a magnetic field affects the movement of nanofluid among two equal-distance plates.

The unsteady thermally magnetized movement of a Casson hybrid nanofluid across a vertical permeable sheet was studied by Krishna et al. [50,51]. Krishna et al. [52] inspected the impacts of heat emission, chemical species, and Hall on the double-diffusive, unsteady magneto-hydrodynamic natural convective revolving movement of a micropolar fluid above a semi-infinite permeable surface with convective boundary characteristics. Krishna and Chamkha [53–56] studied the ion-slip significance for the magneto-hydrodynamic convective movement of Jeffrey's fluid. The three-dimension Navier-Stokes equation for Newtonian fluid over an elastic sphere is celebrated in Ref. [57]. Alharbi et al. [58] studied heat transfer and entropy generation along flat surfaces.

In view of the above literature, the aim of this framework is to explore the assisting and opposing flow for time-independent 2D flow with HMT via a vertical porous channel under the influence of invariant suction and fluid dissipation, which have not been reported in the earlier studies. The PDEs are altered to dimensionless ODEs via suitable transformation and computationally solved by utilizing the bvp4c approach in MATLAB [59]. Stability assessment is conducted in order to ascertain a stable resolution through the calculation of eigenvalues. Furthermore, this notion demonstrates immense practicality within the realm of chemical engineering.

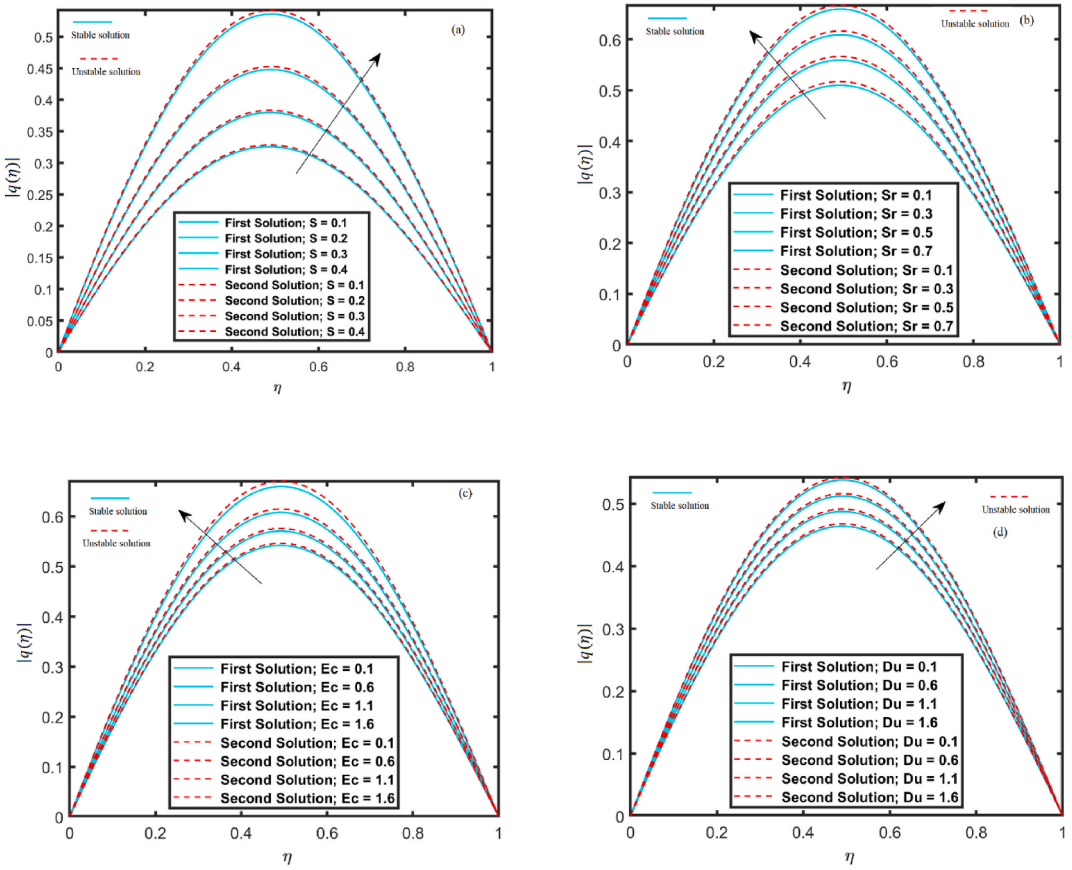
(a)
Stable solution
Unstable solution
First Solution; S = 0.1
First Solution; S = 0.2
First Solution; S = 0.3
First Solution; S = 0.4
Second Solution; S = 0.1
Second Solution; S = 0.2
Second Solution; S = 0.3
Second Solution; S = 0.4
(b)
First Solution; Sr = 0.1
First Solution; Sr = 0.3
First Solution; Sr = 0.5
First Solution; Sr = 0.7
Second Solution; Sr = 0.1
Second Solution; Sr = 0.3
Second Solution; Sr = 0.5
Second Solution; Sr = 0.7
(c)
First Solution; Ec = 0.1
First Solution; Ec = 0.6
First Solution; Ec = 1.1
First Solution; Ec = 1.6
Second Solution; Ec = 0.1
Second Solution; Ec = 0.6
Second Solution; Ec = 1.1
Second Solution; Ec = 1.6
(d)
First Solution; Du = 0.1
First Solution; Du = 0.6
First Solution; Du = 1.1
First Solution; Du = 1.6
Second Solution; Du = 0.1
Second Solution; Du = 0.6
Second Solution; Du = 1.1
Second Solution; Du = 1.6
|q(η)|
η

Fig. 11. (a–b). Velocity variation versus *S*,*Sr*,*Ec* and *Du*.

2. Mathematical modeling

The study investigates the assisting and opposing flow for time-dependent 2D flow combined HMT over a vertical absorbent channel underneath the inspiration of invariant suction and fluid dissipation. The current structure flow is discussed in Fig. 1. The $x-$ axis is positioned along the finite plate and aligned with the vertical free stream velocity, while the $z-$ axis is oriented perpendicular to the plate. It is supposed that all fluid properties remain constant. A small-intensity magnetic field is generated in that $z-$ direction. The fluid is considered to have slight conductivity, resulting in a magnetic Reynolds number significantly less than unity. As a result, the induced magnetic field is disregarded when compared to the functional magnetic field.

In light of these assumptions, the leading flow equations can be stated as follows:

$$\frac{\partial w}{\partial z} = 0, \tag{1}$$

$$w\frac{\partial u}{\partial z} - 2\Omega v = \nu\frac{\partial^2 u}{\partial z^2} - \frac{\sigma B_0^2 u}{\rho} - \frac{\nu}{k}u + g\beta_T\left(T - T_0\right) + g\beta_C\left(C - C_0\right), \tag{2}$$

$$w\frac{\partial v}{\partial z} + 2\Omega u = \nu\frac{\partial^2 v}{\partial z^2} - \frac{\sigma B_0^2 v}{\rho} - \frac{\nu}{k}v, \tag{3}$$

$$w\frac{\partial T}{\partial z} = \frac{1}{\rho C_p}\frac{\partial}{\partial z}\left(K_T\frac{\partial T}{\partial z}\right) + \frac{D_M}{\rho C_p}\frac{\partial^2 C}{\partial z^2} + J_1\left(u^2 + v^2\right) + \frac{1}{\rho C_p}\left[\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2\right], \tag{4}$$

$$w\frac{\partial C}{\partial z} = D\frac{\partial^2 C}{\partial z^2} + D_T\frac{\partial^2 \theta}{\partial z^2} - K_l\left(C - C_0\right). \tag{5}$$

The BCs are defined as,

$$u = v = 0, T = T_w, C = C_w, \text{at } z = 0 \tag{6}$$

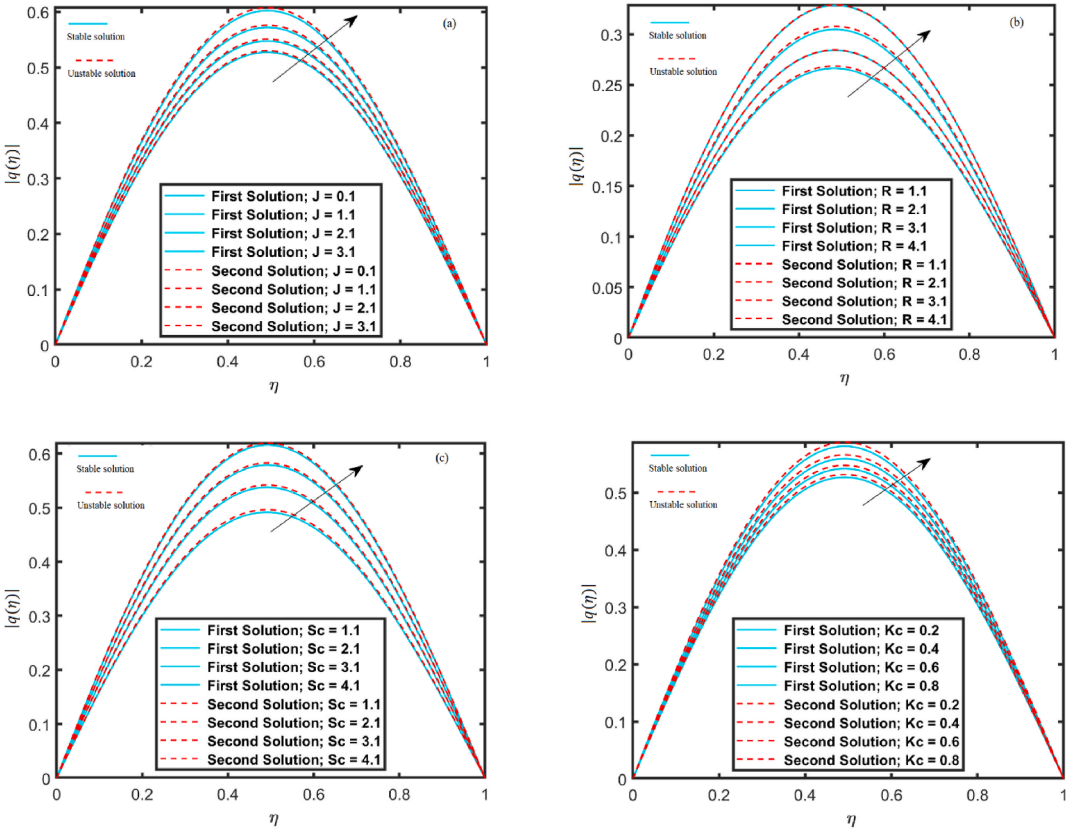

Fig. 12. (a–d). Velocity variation versus *J*,*R*,*Sc* and *Kc*.

$$u = v = 0, T = T_0, C = C_0, \text{at } z = h \tag{7}$$

The equation of continuity clearly states that the velocity of suction is invariant. To integrate equation (1), the velocity of suction is:

$$w = -w_0 \tag{8}$$

Here w_0 represents a nonzero positive constant suction velocity scale. The negative symbol designates "suction to the plate", and $w_0 > 0$ indicates "suction normal to the plate". The 4^{th} and 5^{th} components in equation (2) characterize thermal and solutal buoyancy impacts separately. The conductivity of heat is:

$$K_T = k_0 \left[1 + \alpha \left(T_w - T_0\right)\right] \tag{9}$$

The constant α is determined by the fluid's properties.
Incorporating the subsequent non-dimensional variables

$$u^* = \frac{u}{u_0}, v^* = \frac{v}{u_0}, z^* = \frac{z}{h}, \theta = \frac{T - T_0}{T_w - T_0}, \phi = \frac{C - C_0}{C_w - C_0}, K = \frac{ku_0}{\nu h}, \lambda = \alpha(T_w - T_0), R^2 = \frac{\Omega h}{u_0},$$
$$M^2 = \frac{\sigma B_0^2 h}{\rho u_0}, \Pr = \frac{u_0 \rho C_p}{k_0}, \mathrm{Sc} = \frac{u_0}{D}, \mathrm{Gr} = \frac{hg\beta_T \left(T_w - T_0\right)}{u_0^2}, \mathrm{Gm} = \frac{hg\beta_C C_w - C_0)}{u_0^2},$$
$$\mathrm{J} = \frac{J_1 h}{u_0 \left(T_w - T_0\right)}, \mathrm{Kc} = \frac{K_l h}{u_0}. \tag{10}$$

Using dimensionless variables, the flow Eqs. (2)–(8) reduces to

$$S\frac{\partial u}{\partial z} + 2Rv = -\frac{\partial^2 u}{\partial z^2} + \left(M^2 + \frac{1}{K}\right)u - \mathrm{Gr}\theta - \mathrm{Gm}\,\phi$$

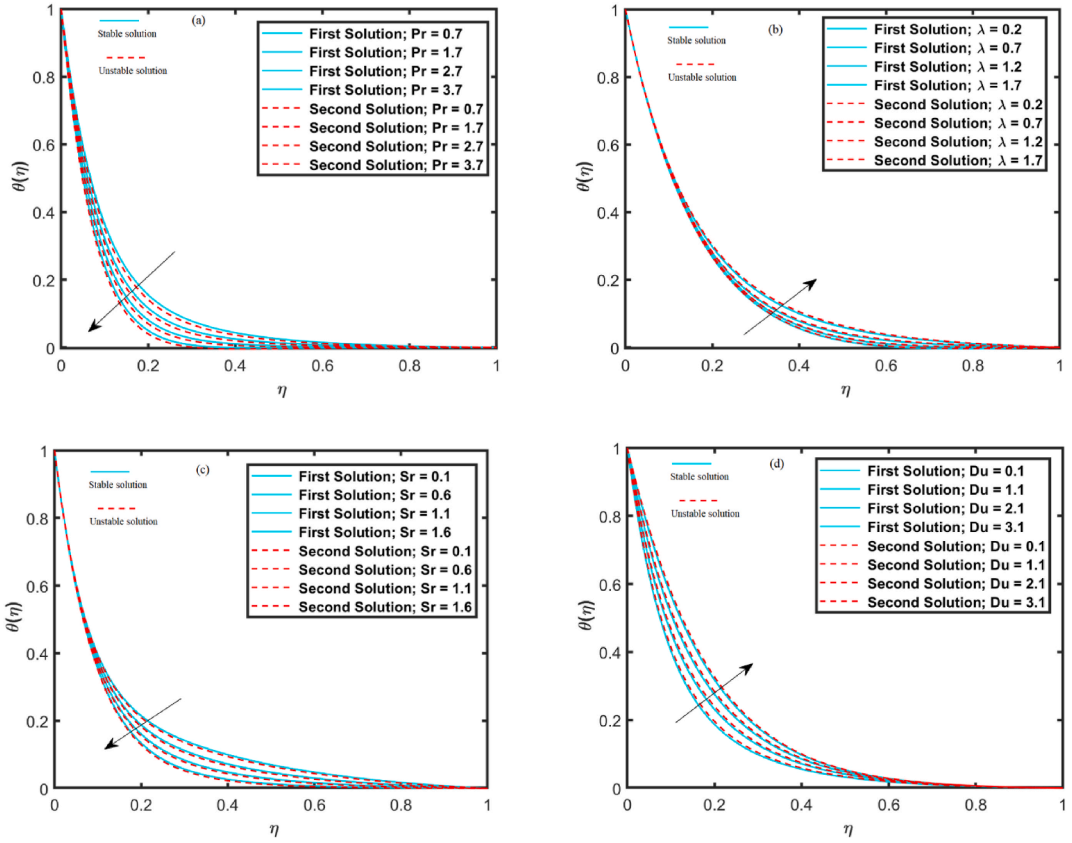
(a)
Stable solution
Unstable solution
First Solution; Pr = 0.7
First Solution; Pr = 1.7
First Solution; Pr = 2.7
First Solution; Pr = 3.7
Second Solution; Pr = 0.7
Second Solution; Pr = 1.7
Second Solution; Pr = 2.7
Second Solution; Pr = 3.7
(b)
First Solution; λ = 0.2
First Solution; λ = 0.7
First Solution; λ = 1.2
First Solution; λ = 1.7
Second Solution; λ = 0.2
Second Solution; λ = 0.7
Second Solution; λ = 1.2
Second Solution; λ = 1.7
(c)
First Solution; Sr = 0.1
First Solution; Sr = 0.6
First Solution; Sr = 1.1
First Solution; Sr = 1.6
Second Solution; Sr = 0.1
Second Solution; Sr = 0.6
Second Solution; Sr = 1.1
Second Solution; Sr = 1.6
(d)
First Solution; Du = 0.1
First Solution; Du = 1.1
First Solution; Du = 2.1
First Solution; Du = 3.1
Second Solution; Du = 0.1
Second Solution; Du = 1.1
Second Solution; Du = 2.1
Second Solution; Du = 3.1
θ(η)
η

Fig. 13 (a–d). Temperature variation versus *Pr*,*λ*, *Sr* and *Du*.

$$S\frac{\partial v}{\partial z} - 2Ru = -\frac{\partial^2 v}{\partial z^2} + \left(M^2 + \frac{1}{K}\right)v \tag{11}$$

Assuming $q = u + iv$, Eqs. (8) and (9) become

$$S\frac{\partial q}{\partial z} + \frac{\partial^2 q}{\partial z^2} - \left(M^2 + 2iR^2 + \frac{1}{K}\right)q + \mathrm{Gr}\theta + \mathrm{Gm}\phi = 0, \tag{12}$$

$$S\frac{\partial \theta}{\partial z} + \frac{\lambda}{\mathrm{Pr}}\left(\frac{\partial \theta}{\partial z}\right)^2 - \frac{1+\lambda}{\mathrm{Pr}}\frac{\partial^2 \theta}{\partial z^2} + \mathrm{Du}\frac{\partial^2 \phi}{\partial z^2} + \mathrm{J}q^2 + \mathrm{Ec}\left(\frac{\partial q}{\partial z}\right)^2 = 0, \tag{13}$$

$$S\frac{\partial \phi}{\partial z} + \frac{1}{\mathrm{Sc}}\frac{\partial^2 \phi}{\partial z^2} + \mathrm{Sr}\frac{\partial^2 \theta}{\partial z^2} + \mathrm{Kc}\phi = 0. \tag{14}$$

The non-dimensional BCs are,

$$q = 0, \theta = 1, \phi = 1, at\ z = 0, \tag{15}$$

$$q = 0, \theta = 0, \phi = 0, at\ z = 1. \tag{16}$$

The important physical characteristics for this sort of BLF are

$$C_f = \left(\frac{\partial q}{\partial z}\right)_{z=0}, Nu = \left(\frac{\partial \theta}{\partial z}\right)_{z=0} \text{ and } Sh = \left(\frac{\partial \phi}{\partial z}\right)_{z=0} \tag{17}$$

3. Bvp4c solutions and code validation

Since the exact model involves a couple of differential equations that are nonlinear. So, numerical simulation is necessary for the problem's resolution. As a result, the non-linear PDEs are first converted to dimensionless form by using dimensionless conversion. The nonlinear ODEs are then converted to first-order ODEs by presenting new variables, and then solved by using the bvp4c Matlab code. By entering new variables, we alter the modified system (12)–(17) into a first-order system by using the variables.

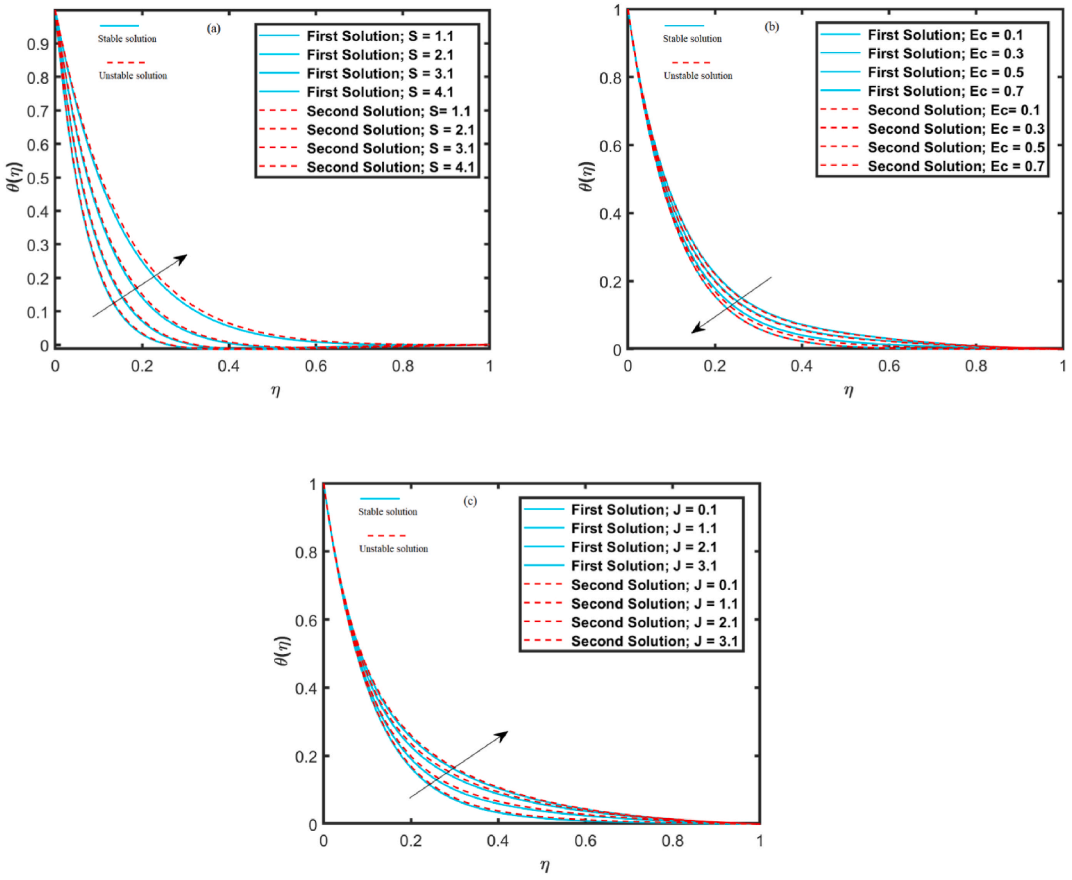

Fig. 14 (a–c). Temperature variation versus *S*,*Ec*, and *J*.

$$q = y_1, q' = y_2, \theta = y_3, \theta' = y_4, \phi = y_5 \text{ and } \phi' = y_6. \tag{18}$$

Hence, the equations become as:

$$y_2' = \left(M^2 + 2iR^2 + \frac{1}{K}\right) y_1 - Sy_2 - Gry_3 - Gmy_5, \tag{19}$$

$$\frac{1+\lambda}{\mathrm{Pr}} y_4' = Sy_4 + \frac{\lambda}{\mathrm{Pr}} (\mathrm{y}_4)^2 + \mathrm{Duy}_6' + J(\mathrm{y}_1)^2 + \mathrm{Ec}(y_2)^2, \tag{20}$$

$$\frac{1}{\mathrm{Sc}} y_6' = -Sry_4' - Kcy_5 - y_6. \tag{21}$$

and the BCs are

$$y_1 = 0, y_3 = 1, y_5 = 1, at\ z = 0, \tag{22}$$

$$y_1 = 0, y_3 = 0, y_5 = 0, at\ z = 1. \tag{23}$$

The method bvp4c is a finite-difference code that uses the three-stage Labatto IIIa formula. Generally, it is a collocation approach that is fourth-order accurate across the time frame in which the computation is integrated. The ongoing solution's residual is used to facilitate preventing errors and mesh choice in this case. The approximate tolerance was established at 10^{-7}. To obtain the desired numerical answers for a problem with many solutions (upper and lower branches), an "excellent" initial estimation is necessary. The estimate should fulfil the supplied boundary constraints and demonstrate the nature of the solution. Even with initial starting predictions, the BVP4C approach converges to the first answer, making it simple to discover the first solution. However, the second approach for extending a sheet for $\lambda < 0$ is more difficult to estimate accurately. This approach, referred to as 'continuation', has been extensively detailed in Ref. [57]. In this technique, we substitute the 'infinity' at Eq. (12) with an appropriate finite quantity based on the numerical values of the appropriate parameters. To achieve convergence, we begin with a small value (e.g., $\eta = \eta\infty = 0$) and

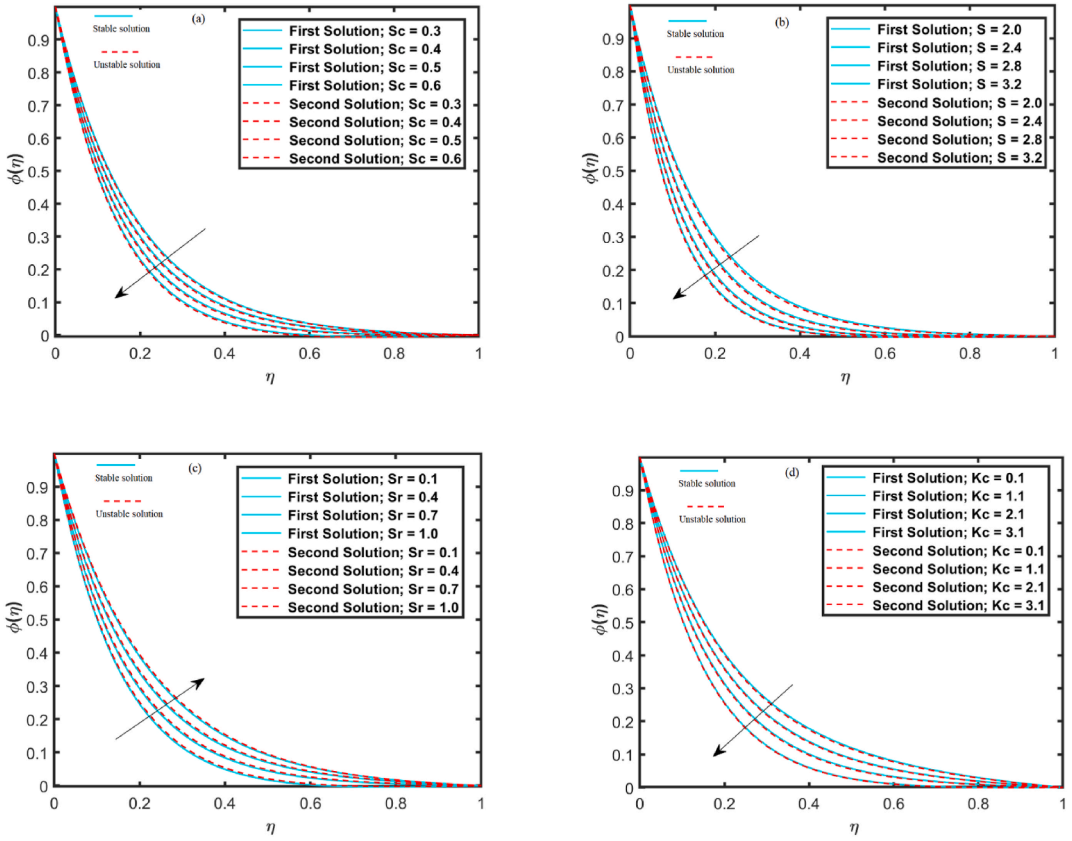
(a)
Stable solution
Unstable solution
First Solution; Sc = 0.3
First Solution; Sc = 0.4
First Solution; Sc = 0.5
First Solution; Sc = 0.6
Second Solution; Sc = 0.3
Second Solution; Sc = 0.4
Second Solution; Sc = 0.5
Second Solution; Sc = 0.6
(b)
First Solution; S = 2.0
First Solution; S = 2.4
First Solution; S = 2.8
First Solution; S = 3.2
Second Solution; S = 2.0
Second Solution; S = 2.4
Second Solution; S = 2.8
Second Solution; S = 3.2
(c)
First Solution; Sr = 0.1
First Solution; Sr = 0.4
First Solution; Sr = 0.7
First Solution; Sr = 1.0
Second Solution; Sr = 0.1
Second Solution; Sr = 0.4
Second Solution; Sr = 0.7
Second Solution; Sr = 1.0
(d)
First Solution; Kc = 0.1
First Solution; Kc = 1.1
First Solution; Kc = 2.1
First Solution; Kc = 3.1
Second Solution; Kc = 0.1
Second Solution; Kc = 1.1
Second Solution; Kc = 2.1
Second Solution; Kc = 3.1
$\phi(\eta)$
η

Fig. 15. (a–d). Concentration variation versus *Sc*,*S*,*Sr* and *Kc*.

Table 3
Variation of Skin friction.

M	S	Ist solution $\left(\frac{\partial u}{\partial z}\right)_{z=0}$	2nd solution $\left(\frac{\partial u}{\partial z}\right)_{z=0}$	Ist solution $\left(\frac{\partial v}{\partial z}\right)_{z=0}$	2nd solution $\left(\frac{\partial v}{\partial z}\right)_{z=0}$
0.3	0.7	1.3418	1.3425	−0.32570	−0.32609
0.4		1.3483	1.3489	−0.32132	−0.32180
0.5		1.3537	1.3541	−0.31610	−0.31666
0.6		1.3579	1.3582	−0.31011	−0.31074
0.3	0.7	1.3200	1.3209	−0.3295	−0.32978
	0.8	1.3298	1.3307	−0.33224	−0.3325
	0.9	1.3393	1.3403	−0.33475	−0.33499
	1.0	1.3487	1.3496	−0.33703	−0.33725

gradually increase the value of η∞ until it reaches a suitable high number. The selected finite values for $\eta \to \infty$ are $\eta_\infty = 1$ for the first solution and second solution. The graphical illustration of the bvp4c and its code explanation are depicted in Fig. 2(a and b).

To confirm the validity and trustworthiness of our numerical approach, we compared the findings obtained to those previously demonstrated by Uwanta and Halima [31]. In Table 1, the outcomes for non-dimensional velocity are compared with those acquired by Uwanta and Halima [31] who examined the Soret and Dufour influence on applied magnetized convective HMT with constant suction through a non-rotating vertical channel, and excellent agreement is established. Consequently, it can be inferred that our numerical code, BVP4C, accurately predicts the behavior of the fluid stream.

4. Stability, results and discussion

This research focuses on the influences of Soret and Dufour on natural convection combined with heat and mass transmission in 2D steady flow through the boundary layer. The examination takes place in a porous vertical tube or duct with the effects of continuous suction and the dissipation of viscous fluid. The basic flow model in terms of PDEs also contains the influence of magnetic field. The leading PDEs are altered to ordinary differential equations by using dimensionless variables. Employing the BVP4C technique, the leading equations are solved numerically. A comparison between published and current studies shows outstanding agreement. Key parameters such as *K*, *Gr*, *Gm*, *S*, *Ec*, *Du*, *J*, *Pr*, *Sr*, and *R* on the distributions of $q(\eta)$, $\theta(\eta)$, $\phi(\eta)$, C_f, *Nu*, and *Sh* are visually presented through graphs and tables.

Table 4
Variation of Sherwood number.

Sc	*S*	*Sr*	*Kc*	*Sh*
0.3	0.3	0.3	0.3	1.0646
0.6				1.1323
0.9				1.2032
1.2				1.2775
0.3	0.3			1.0646
	0.6			1.1147
	0.9			1.1661
	1.2			1.2188
	0.3	0.3		1.2119
		0.6		1.1628
		0.9		1.1137
		1.2		1.0646
		0.3	0.3	1.0646
			0.6	1.0344
			0.9	1.0039
			1.2	0.97293

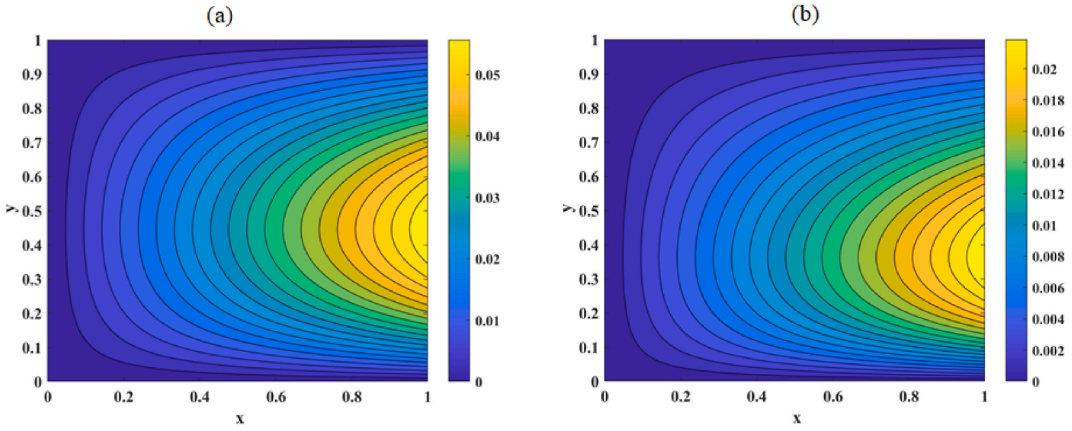

Fig. 16. Streamlines for (a) $Sr = 0.3$ (b) $Sr = 0.6$.

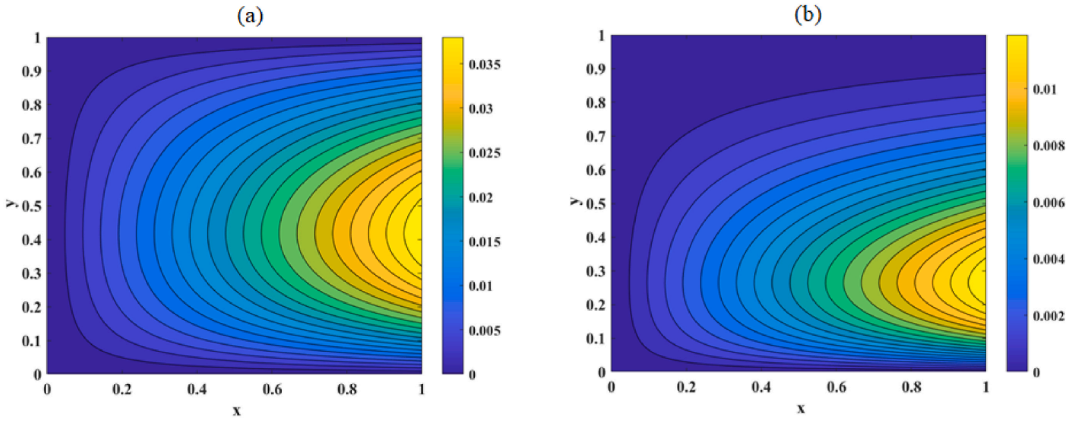

Fig. 17. Streamlines for (a) $Du = 0.3$ (b) $Du = 0.6$.

Numerical analysis reveals dual solutions. It is crucial to determine a stable solution. Naturally, the upper branch is denoted as the physical branch due to its adherence to the boundary assumptions. Nonetheless, the significance of the stability of the lower branch should not be overlooked and can be examined through the implementation of stability procedures. Furthermore, this approach has been demonstrated in the homework conducted by [37,38]. Hence, there is no need to reiterate this argument here. Further, the research demonstrates that the presence of two solutions is contingent upon the suction parameter. The importance of this current study lies in comprehending critical values and the range within which dual solutions exist. Referring to Weidman et al. [37] and Merkin

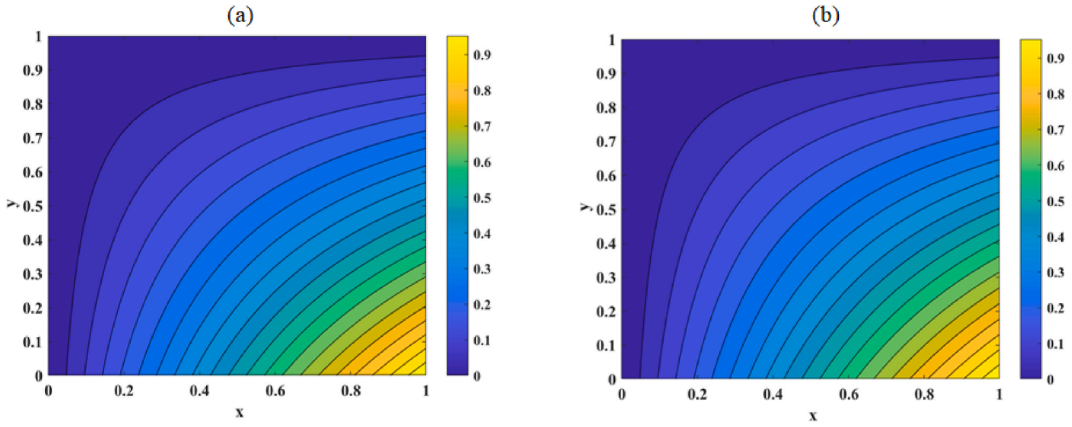

Fig. 18. Contour visualization of temperature (a) $J = 0.3$ (b) $J = 0.6$.

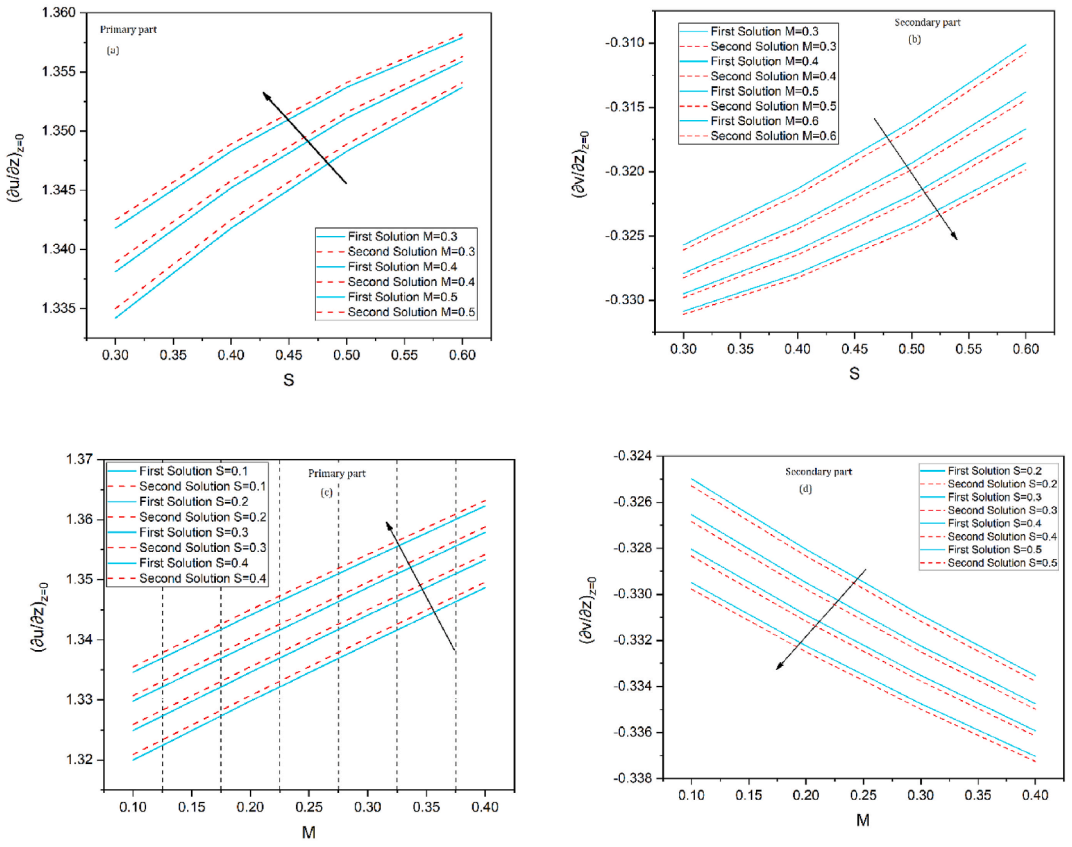

Fig. 19. Variation of Skin friction (a, c) Primary part (b, d) Secondary part.

[38], $e^{-\gamma\tau} \to 0\ 0$ as $\tau \to \infty$ for $\gamma > 0$ and $e^{-\gamma\tau} \to \infty$ for $\gamma < 0$. This consequence recommends that, over time, the first branch (FB) remains reliable, while the second branch (SB) exhibits instability and is hence not appropriate for long-term physical illustration.

Upon careful observation, it can be asserted that the upper branch exhibits greater stability in light of γ. The positive indication of γ guarantees the reliability of the solution and imitates the outcomes of decline and growth. Conversely, the negation symbol γ indicates instability. The stable and unstable phenomena are reported in Table 2 and Fig. 3.

Figs. 4 and 5 depict the effect of K and Pr on C_f and Nu the accumulative magnitude of M and S factors, respectively. This demonstrates that there may be multiple solutions within a certain range of $\geq M_{ci}$; $i = 1,2,3$ and no solution in the range $M < M_{ci}$. From Fig. 4, it is revealed that C_f decreases as the magnitude of K is increased from 0.3 to 0.9. The critical values for K = 0.3, 0.6, and 0.9 are $M_{c1} = 0.2251, M_{c2} = 0.3510$, and $M_{c1} = 0.4964$, respectively. As the quantities of the porous factor K enhances, velocity at the porous walls and thickness of the momentum boundary layer (MBL) enhances, and the resultant C_f decreases. It is noteworthy to

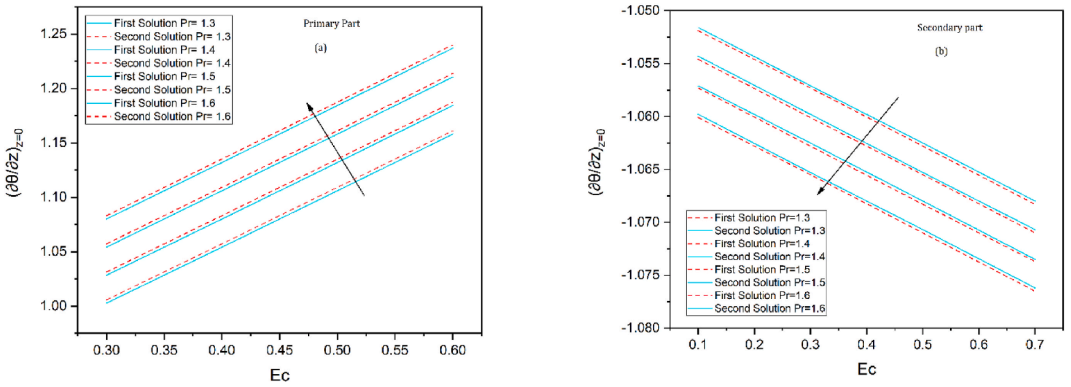

Fig. 20. Variation of Nusselt number for different values of *Pr* (a) Primary part (b) Secondary part.

mention here that the skin friction in the first solution decreases while increases in the second solution. Same influence is reported for the Prandtl number *Pr* on the variation of C_f with the disparity of suction factor *S*. As the magnitude of *Pr* enhances from 0.3 to 0.0, the critical values also increase. The corresponding critical values for $Pr = 0.3$, 0.6, and 0.9 are $S_{c1} = 1.9871, S_{c1} = 5.9503$, and $S_{c1} = 8.9980$, respectively.

Figs. 6 and 7 explore the variation of suction parameter and thermal conductivity quantity λ on the variation of C_f and *Nu* with the increasing magnitude of *M* and *S*, respectively. Fig. 6 reports that C_f enhaces as the magnitude of the suction parameter increases. A significant increment is observed in the range $0.2 < M \leq 0.5$ for the first solution, while the behavious almost the same in the region $0.5 < M \leq 1.0$. Similarly, Fig. 7 reports a direct relation between *Nu* and λ for the first solution, while an inverse relation is depicted for the second solution. The rising magnitude of λ trend enhances the Nusselt number in the first branch while a decreasing trend is demonstrated in the second branch. As the quantities of λ enhance from 0.3 to 0.9, the corresponding critical values $S_{c1} = 1.9871$, $S_{c1} = 5.9503$, and $S_{c1} = 8.9980$ increase.

The variation of C_f and *Nu* with the variation of *Kc* and *J* is reported in Fig. 8(a) & Fig. 8(b), respectively, with the increasing magnitude of *K*. For both Figures, the critical values are $K_{c1} = 0.2841, K_{c1} = 0.6132$, $K_{c1} = 0.6510$ as the quantities of *Kc* and *J* enhance from 0.3 to 0.9 and 0.1 to 0.5, respectively. It is mentioned to note here that C_f enhances in the first branch while declines in the second branch as the values of *K* augmented from 0.3 to 0.9. But, the *Nu* upsurges in both branches as the values of *J* are increased from 0.1 to 0.5.

The forthcoming section imitates the inspiration of dissimilar physical factors on the flow features, i.e., on the dimensionless $q(\eta)$, $\theta(\eta)$, $\phi(\eta)$, C_f, *Nu*, and *Sh*. For clarification of the consequences, obtained statistical values are strategized graphically, and physically. Therefore, the heat transportation characteristics of the flow through porous media are thoroughly examined and illustrated in Figs. 9–15 and in Tables 3–4. Additionally, stream plots for $q(\eta)$, $\theta(\eta)$, and $\phi(\eta)$ are elaborated through graphs (see Figs. 16–18). It remains consistent throughout $M = 0.5, K = 0.3, R = 1, \lambda = 0.3, \mathrm{Gr} = 3, \mathrm{Gm} = 5, \mathrm{Sr} = 0.3, \mathrm{Sc} = 0.3, J = 1, \mathrm{Pr} = 0.71, \mathrm{Du} = 0.3, \mathrm{Kc} = 0.5$, and is scattered over a range of options.

Fig. 9(a–f) are plotted for the primary and secondary part solutions. In the primary part solution (PPS) $u(\eta)$, the first solution is denoted by solid lines, and the second solution is denoted by dashed lines, and similarly for the secondary part solution (SPS) $v(\eta)$. For different parametric values, the graphs are plotted to see the impact on PPS and SPS. From these figures, it is observed that both PPS and SPS have opposite effects. It is scrutinized that the flow rate enhances with the growing values of *k* for both the first and second solutions of PPS (see Fig. 9(a)) while decreasing in the SPS (see Fig. 9(b)). From Fig. 9c and d, it is investigated that the PPS of the fluid flow show decreasing behavior with increasing values of the magnetic parameter *M*. It is due to the Lorentz force, which resists the flow. Since, due to the application of a magnetic field, Lorenz force occurs, which is a resistive force. But the opposite trend is observed for the SPS. Similarly, for the parameter *S*, the opposite trend is investigated for both PPS and SPS, as shown in Fig. 9e and f.

Fig. 10(a–b) investigates the consequences of the magnetic field parameter *M*, porosity factor *K*, Grashof number *Gr*, and modified Grashof number *Gm* on the resulting velocity profile. The profiles have a parabolic type. The fluid velocity diminishes as the magnitude of *M* enhances. It is due to the Lorentz force, which develops due to a magnetic field. The influence of a permeable factor *K* on the velocity field is opposite. Clearly, from Fig. 10 (b), it is revealed that $q(\eta)$ enhances with increasing magnitude of *K* on porous wall which also leads the thickness of MBL. Fig. 10c and d shows the effect of *Gr* and *Gm* on $q(\eta)$. Improved thermal and solute buoyancy forces lead to increased fluid velocity. For the above figures, dual behavior is observed. The velocity of the second solution is higher than the first solutions in both cases. The velocity curves show an escalating trend with increasing quantities of *Gr* and *Gm*, also increase the MBL thickness.

The variation of factors such as suction parameter *S*, Soret parameter *Sr*, Eckert number *Ec*, and Dofour parameter *Du* on the variation of dimensionless velocity is described in Fig. 11(a–d). Fig. 11(a) shows the velocity escalation as the suction parameters S increase. The same trend is evident with increasing Soret parameter *Sr* (Fig. 11(b)). The *Sr* factor describes the effect of heat gradients that cause significant mass diffusion repercussions. Increasing *Ec* and *Du* factors improve the MBL, leading to increased velocity across the entire fluid domain (Fig. 11(c and d)).

Figs. 12a and b shows how increasing the Joule heating variable J or the rotation variable R affects the velocity inside the boundary layer. Increasing J and/or R values lead to an increase in velocity and MBL thickness. Fig. 12(c) shows how the velocity changes based on various Schmidt number Sc values. Increasing Sc leads to an increase in velocity and MBL thickness. Fig. 12(d) shows the consequence of Kc factor on velocity, which is similar to the effect of the rotation factor. Therefore, it has been discovered that negative chemical reactions reduce the resistive Lorentz force resistance which enhances fluid velocity across the channel.

Fig. 13(a–d) and 14(a-c) illustrate the effect of the Prandtl number Pr, factor λ, Sr, Du, S, Ec, and factor J, on the temperature of the fluid. As seen in Fig. 13(a) and (c), an increase in Pr or Sr lowers the fluid heat across the fluid channel. Fig. 13(b) demonstrates that increasing the thermal conductivity parameter λ leads to a rise in fluid temperature. This is due to heat transmission from the fluid, which uses up its own energy. Fig. 13(d) shows that increasing the Dufour parameter results in similar behavior. The Dufour parameter Du represents the thermal gradients in caloric energy release during the fluid motion. This is shown as enhancing the temperature inside the fluid. Fig. 14(a) displays that increasing S factor upsurges the temperature. Fig. 14(b) illustrates how the dissimilar values of Ec affect the temperature. The Eckert number Ec indicates the ratio of kinematic energy to the enthalpy, observing the heat transportation dispersion. Higher viscous heat dissipation leads to temperature reduction over the thermal boundary layer. Fig. 14(c) displays that increasing the Joule heating factor J increases the temperature as well as MBL thickness.

Fig. 15(a–d) illustrates the analysis of $\phi(\eta)$ (concentration distribution) using the factors Sc, S, Sr, and Kc. Fig. 15(a) shows that the $\phi(\eta)$ field decreases due to a broadening in Sc during the flow region. The Schmidt number Sc displays the ratio of kinematic viscosity to mass diffusion. It was discovered that increasing the quantities of Sc affected the concentration and BL thickness, which decreased due to increasing Sc. A similar feature is reported for the parameter S (Fig. 15(b)) on the concentration field. Similarly, concentration of volume fraction $\phi(\eta)$ escalations with a growing magnitude of S, whereas declining function of Kc as depicted in Fig. 15 (c and d).

The transient flow structure for the S and Du factors is shown in Fig. 16. The velocity is highest at the left plate and progressively drops as it approaches the opposite plate. The thickness of the border layer also stays unchanged. At the left plate, there is an extremely high velocity gradient. There is a noticeable shift in the velocity distribution when the Sr factor is raised. When the Du factor is increased, a similar behavior is perceived, and a high velocity area forms at the right plate (see Fig. 17). The variation in temperature across the plates with the modification of the Joule factor J is shown in Fig. 18. When $J = 0.3$, a small temperature BL forms, indicating initially reduced heat movement. Later, the BL thickens, and the temperature gradients spread across the channel. This suggests that heat is transferred by convection.

Fig. 19(a–d) and Table 3 illustrate the relationship between C_f and various factors such as M and S on both the primary and secondary skin frictions. It is reported that as the magnitude of M enhances skin friction grows in PPS (see Fig. 19(a)), while decreases in SPS (see Fig. 19(b)). Similarly, as the factor S increases, the level of C_f increases for both cases in PPS (see Fig. 19(c)), but decreases in SPS (see Fig. 19(d)). Fig. 20 represents the variation of the Nusselt number for numerous values of Pr regarding Ec. It is analyzed that when the quantities of Pr and Ec improve, the Nusselt number improves in PPS (see Fig. 20(a)), and the opposite trend is observed in SPS (see Fig. 20(b)). Moreover, Sh the diminishing function of Sr and Kc. Conversely, it is noted that Sh is augmented when Sc and S are enhanced as indicated in Fig. 19(c) and Table 4, which are similarly to the results prescribed by Uwanta and Halima [31].

5. Conclusions

The analysis examines the outcomes of the Soret and Dufour parameters in relation to the natural convection of heat mass transportation in a two-dimensional time-independent boundary layer flow across a vertical framework, taking into account viscous dissipation and constant suction. The basic flow equations in terms of partial differential equations (PDEs), also contain the influence of continued suction, magnetic field, and viscous dissipation, are converted to dimensionless form by using dimensionless variables. Employing the BVP4C technique, the leading equations are solved numerically. Dual branches of solutions have been observed due to suction. Stability exploration has been provided to confirm a stable solution. Based on the eigenvalue's calculation, the smallest eigenvalue represents a stable solution, while negative signifies an unstable solution. A comparison between published and current studies shows outstanding agreement. The influence of emerging factors on the flow features is investigated via graphs and tables. The positive indication of γ guarantees the stability of the branch and reflects the outcomes of decay and growth. Conversely, the negative values of γ indicate that the branch is unstable. It is demonstrated that there may be multiple solutions within a certain range of $\geq M_{ci}$; $i = 1,2,3$ and no solution in the range $M < M_{ci}$. The rising magnitude of λ trend to enhance the Nusselt number in the first branch, while a decreasing trend is demonstrated in the second branch. As the quantities of λ enhances from 0.3 to 0.9, the corresponding critical values $S_{c1} = 1.9871$, $S_{c1} = 5.9503$, and $S_{c1} = 8.9980$ also increase.

The velocity decreases as M increases. When R factor is increased, the Sr and Du factors are strengthened, leading to an escalation in velocity and MBL thickness. The thermal BL thickness decreases as the magnitudes of Pr, S, and Ec are reinforced. Both Sc and S reduce volume friction concentration. The C_f with Du and R factors. The Nu and Sh numbers show a decline behavior as Sr improves, while a reverse trend is reported for S. Moreover, it is important to highlight that this concept holds great practical significance within the domain of chemical engineering, further emphasizing its relevance and applicability in real-world scenarios.

CRediT authorship contribution statement

Zeeshan: Writing – original draft. **Waris Khan:** Software. **Taoufik Saidani:** Methodology. **Florentin Smarandache:** Investigation. **Muhammad Shahid Khan:** Formal analysis. **Hamdi Ayed:** Validation. **M. Modather M. Abdou:** Data curation.

Declaration of competing interest

The authors declare no conflict of interest.

Data availability

No data was used for the research described in the article.

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Partial Collisions of Unmater-Matter, Unmatter-Antimatter, and Unmatter1-Unmatter2 to Generate High Energy

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In this paper we present the possibility of partial collisions between unmatter with mat-ter, and unmatter with antimatter, and two or more different types of unmatters colliding between themselves to create high energy. In general, the collisions between unmatter with matter, or with antimatter, or with other type of unmatter, because being partial, they release less energy than the matter-unmatter collision which is a total collision. But the unmatter may be easier to produce in laboratory than antimatter.

1 Introduction

According to the CERN website [1], the universe is composed from 5% matter, 26.8% dark matter, and 68.2% dark energy.

The antimatter (also called dark matter) is composed from antiparticles.

The matter-antimatter asymmetry is enormous, contrary to the Big Bang Theory, which sustains that it should have been created equal amounts of matter and antimatter.

The unmatter-matter and unmatter-antimatter asymmetries should also be studied. At the Big Bang, if any, not only matter and antimatter would have been generated, but unmatter as well. Similarly, Unmatter Plasma [8, 9, 10] is a novel form of plasma, exclusively made of matter and its antimatter counterpart. It was first generated in the 2015 experiment.

The Big Bang Theory would have occurred 13.7 billion years ago, and the accelerated expansion of the universe would be due to dark matter — which contradicts the hypothesis of the initial explosion, according to which the universe should slow down...

The antimatter is considered a doppelganger of the matter. According to [2], "a gram of antimatter colliding with a gram of matter would produce as much energy as a nuclear bomb".

2 Elementary Particles

The smallest units of matter (i.e., not made up by other smaller units of matter) are called elementary particles. A particle has: charge, mass, and angular momentum (spin).

The building blocks of matter, or most elementary particles known today are:

6 quarks and 6 leptons

with their corresponding antiparticles

6 antiquarks and 6 antileptons

(see Table 1 and Table 2 on top of the next Page).

The quarks are bonded together by *gluons*, and the study of strong interactions between quarks is called Quantum Chromodynamics (QCD).

In an atom, the electrons are leptons, while the protons, neutrons, and pions are made up of quarks.

A *baryon* is made up of an odd number of quarks (usually three); while a meson is made up of an even number of quarks (usually a quark and an antiquark, therefore such a meson is the most elementary form of unmatter, let us call it ***unmatter-meson***.

A *pion* (or *pi-meson*) is an unstable subatomic particle of one of the following three forms: and each one is composed from a quark and an antiquark (as such, they are mesons, and therefore elementary forms of unmatter, let us call them ***unmatter-pions***.

The *hadron* is a particle made up of two or more quarks that, through a strong interaction, are hold together.

Antimatter is matter composed of antiparticles.

While the *antiparticle*, as partner of a particle, is matter with reversed charge, parity, and time of its corresponding particle.

The *photon particle* is its own photon antiparticle, but this is an exception. All other particles have different corresponding antiparticles.

In collision, particle and antiparticle annihilate each other and produce gamma photons, neutrinos, and other particles.

3 Standard Model

According to the Standard Model, there are 17 distinct subatomic particles:

12 fermions and 5 bosons.

The fermion is a particle that only in combination with its antiparticle can be created or destroyed. Like the electron.

While the boson is a particle that can be created and destroyed in arbitrary numbers. Like photon.

Boson is in general a force carrier particle.

The fundamental forces that act in the nature are: gravity, electromagnetism, strong force, and weak force.

4 Defining Unmatter

In short, unmatter is formed by matter and antimatter that bind together and it was introduced by Smarandache in 2004 on the CERN website [4], and developed later [5–17].

Quarks	up (u)	down (d)	charm (c)	strange (s)	top (t)	bottom (b)
Antiquarks	up	down	charm	strange	top	bottom

Table 1: Quarks and antiquarks.

Leptons	charged	antielectron	antimuon	antitaun
Antileptons	neutrals	anti-electron-neutrino	anti-muon-neutrino	anti-tau-neutrino

Table 2: Leptons and antileptons.

The building blocks of matter and antimatter (most elementary particles known today) are 6 quarks and 6 leptons; their 12 antiparticles also exist.

Then *unmatter* will be formed by at least a building block and at least an antibuilding block which can bind together.

Let us start from neutrosophy [3], which is a generalization of dialectics, i.e., not only the opposites are combined but also the neutralities. Why? Because when an idea is launched, a category of people will accept it, others will reject it, and a third one will ignore it (don't care). But the dynamics between these three categories changes, so somebody accepting it might later reject or ignore it, or an ignorant will accept it or reject it, and so on. Similarly the dynamicity of <A>, <antiA>, <neutA>, where <neutA> means neither <A> nor <antiA>, but in between (neutral). Neutrosophy considers a kind not of di-alectics but tri-alectics (based on three components: <A>, <antiA>, <neutA>).

Hence unmatter is a kind of intermediary (not referring to the charge) between matter and antimatter, i.e. neither one, nor the other.

Neutrosophic Logic (NL) is a generalization of fuzzy logic (especially of intuitionistic fuzzy logic) in which a proposition has a degree of truth, a degree of falsity, and a degree of neutrality (neither true nor false); in the normalized NL the sum of these degrees is 1.

5 Unmatter Atom

It is possible to define the unmatter in a more general way, using the exotic atom. The exotic atom is an atom obtained after substituting one or more particles by other particles of the same charge (constituents).

The classical unmatter atoms were formed by particles like (a) electrons, protons, and antineutrons, or (b) antielectrons, antiprotons, and neutrons.

In a more general definition, an unmatter atom is a system of particles as above, or such that one or more particles are replaces by other particles of the same charge.

Other categories would be (c) a matter atom with where one or more (but not all) of the electrons and/or protons are replaced by antimatter particles of the same corresponding charges, and (d) an antimatter atom such that one or more (but not all) of the antielectrons and/or antiprotons are replaced by matter particles of the same corresponding charges.

In a more composed system we can substitute a particle by an unmatter particle and form an unmatter atom.

Of course, not all of these combinations are stable, semi-stable, or quasi-stable, especially when their time to bind together might be longer than their lifespan.

6 Unmatter Charge

The charge of unmatter may be positive as in the pentaquark Theta-plus, 0 (as in the atom of positronium), or negative as in anti-Rho meson, i.e. uˆd, (M. Jordan).

7 Containment

The unmatter could be store as the antimatter atom "by atomic traps (based on electric or magnetic dipoles) and by lasers (magneto-optical traps and optical tweezers)" [18].

8 Matter and Antimatter

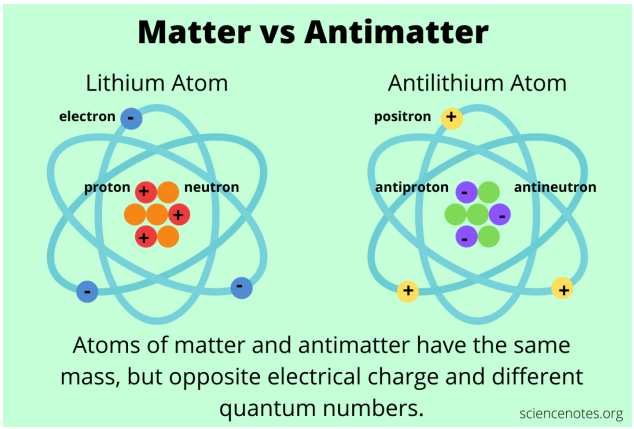

Fig. 1: Matter vs Antimatter. Courtesy of Anne Helmenstine, "What Is Antimatter? Definition and Examples" [18].

Lithium Atom:
electron1 (−), electron2 (−), electron3 (−);
proton1 (+), proton2 (+), proton3 (+);
neutron1 (0), neutron2 (0), neutron3 (0), neutron4 (0).

Lithium Antiatom:
positon1 (+), positon2 (+), positon3 (+);
antiproton1 (−), antiproton2 (−), antiproton3 (−);
antineutron1 (0), antineutron2 (0), antineutron3 (0), antineutron4 (0).

9 Proton and AntiProton

The Proton is made up of: up-quark, up-quark, down-quark; and AntiProton is made up of: anti-up-quark, anti-up-quark, anti-down-quark.

10 Real Examples of Unmatter

(i) Meson, which in general is made up of: quark and anti-quark, and as particular cases of mesons one has:
Pion1 = anti-up-quark, down-quark;
Pion2 = up-quark, anti-down-quark. Pion2 is the Anti-Pion1.

(ii) **Pentaquark**, which is made up of:
up-quark, up-quark, down-quark, down-quark, anti-strange-quark.

11 Partial Collisions of Unmatter-Unmatter

(i) Pion1 vs. Pentaquark: anti-up-quark vs. up-quark (only), since between down-quark and down-quark there is no collision.

(ii) Pion2 vs. Pentaquark: anti-down-quark vs. down-quark (only), since between up-quark and up-quark there is no collision.

12 Total Collision of Unmatter-Unmatter

Pion1 vs. Pion2: anti-up-quark vs. up-quark;
and down-quark vs. anti-down-quark.

13 Partial Collisions of Unmatter-Matter

(i) Pion1 vs. Proton: anti-up-quark vs. up-quark (only), since between down-quark and down-quark there is no collision.

(ii) Pion2 vs. Proton: anti-down-quark vs. down-quark (only), since between up-quark and up-quark there is no collision.

14 Partial Collisions of Unmatter-Antimatter

(i) Pion1 vs. Antiproton: down-quark vs. anti-down-quark (only), since between anti-up-quark and anti-up-quark there is no collision.

(ii) Pion2 vs. Antiproton: up-quark vs. anti-up-quark (only), since between anti-down-quark and anti-down-quark there is no collision.

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Ermakov Equations can be Derived from Zel'dovich Pancake, and they are Cold and Nonlocal through using Neutrosophic Venn Diagram

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Abstract

As we argue in the previous article [3], the labyrinthine worlds of Jorge Luis Borges are more than captivating narratives; they are portals to a deeper understanding of existence. By weaving elements of science-fiction fantasy with philosophical and ethical inquiries, Borges's short stories bridge the seemingly disparate realms of physics and the humanities, offering fertile ground for contemporary physics research. The present-day universe consists of galaxies, galaxy clusters, one-dimensional filaments and two-dimensional sheets or pancakes, all of which combine to form the cosmic web. The so called "Zeldovich pancakes", are very difficult to observe, because their overdensity is only slightly greater than the average density of the universe. Falco et al. presented a method to identify Zeldovich pancakes in observational data, and the method were used as a tool for estimating the mass of galaxy clusters [2]. Here we provide an outline from Zel'dovich pancake to Burgers equations to represent cosmic turbulence, and then from Burgers equations to Ermakov dynamics systems, which in turn they plausibly lead to nonlocal current.

Keywords: Neutrosophic Set; Venn Digram; Mathematics; Physics; Ermakov.

1 | Introduction: Through the Looking Glass of Borges or where Sci-Fi Fantasy and Physics Converge

As we argue in the previous article, the labyrinthine worlds of Jorge Luis Borges are more than captivating narratives; they are portals to a deeper understanding of existence. By weaving elements of science-fiction fantasy with philosophical and ethical inquiries, Borges's short stories bridge the seemingly disparate realms of physics and the humanities, offering fertile ground for contemporary physics research.

As we know, Borges's stories are rife with fantastical concepts - infinite libraries, forking timelines, cyclical realities. These elements resonate with the speculative nature of science fiction, where the boundaries of reality are constantly pushed. Yet, Borges doesn't merely indulge in fantastical flourishes. He imbues them with philosophical weight. In "Tlön, Uqbar, Orbis Tertius," a fictional world alters reality through the power

of belief. This challenges our perception of truth and objectivity, mirroring real-world debates in physics regarding the role of the observer in shaping reality (think quantum mechanics).

In conclusion, Borges's short stories offer a unique confluence of science-fiction fantasy, philosophical inquiry, and physics. By examining the essence of his work, we can see how these seemingly disparate fields can inspire each other. Through the looking glass of Borges's imagination, contemporary physics research might discover groundbreaking new avenues for exploration.

Here we provide an outline from Zel'dovich pancake to Burgers equations to represent cosmic turbulence, and then from Burgers equations to Ermakov dynamics systems.

1.1 | What is a Neutrosophic Venn Diagram?

Borges's stories can serve as springboards for new avenues in physics research. The concept of infinite libraries in "*The Library of Babel*" can be seen as an allegory for the vastness of the universe and the search for unifying theories. The story's exploration of an all-encompassing library with every possible book hints at the existence of a "theory of everything" that could explain all physical phenomena.

Furthermore, Borges's fascination with labyrinths can be seen as a metaphor for the complexities of physics itself. String theory, for instance, proposes a universe with extra dimensions, akin to a labyrinthine structure. By delving into Borgesian labyrinths, physicists might gain new perspectives on navigating the complexities of their research. These can be captured in the Neutrosophic Venn Diagram.

As with Neutrosophic Logic, it has been introduced by one of us (FS) what is termed as Neutrosophic Venn Diagram,[1] where the classical Venn diagram is generalized to a Neutrosophic Diagram, which deals with vague, inexact, ambiguous, ill-defined ideas, statements, notions, entities with unclear borders. In a neutrosophic Venn diagram, the traditional circles used in standard Venn diagrams are replaced with neutrosophic sets, which can include elements with degrees of truth, indeterminacy, and falsity. This allows for a more flexible representation of complex relationships between sets where the boundaries are not clearly defined and whose elements partially belong to the set, partially do not belong, and partially are indeterminate. Now, allow us to reconsider three different sets of fantasy stories, physics, and philosophy including ethics in a Neutrosophic Venn Diagram as follows in Figure 1:

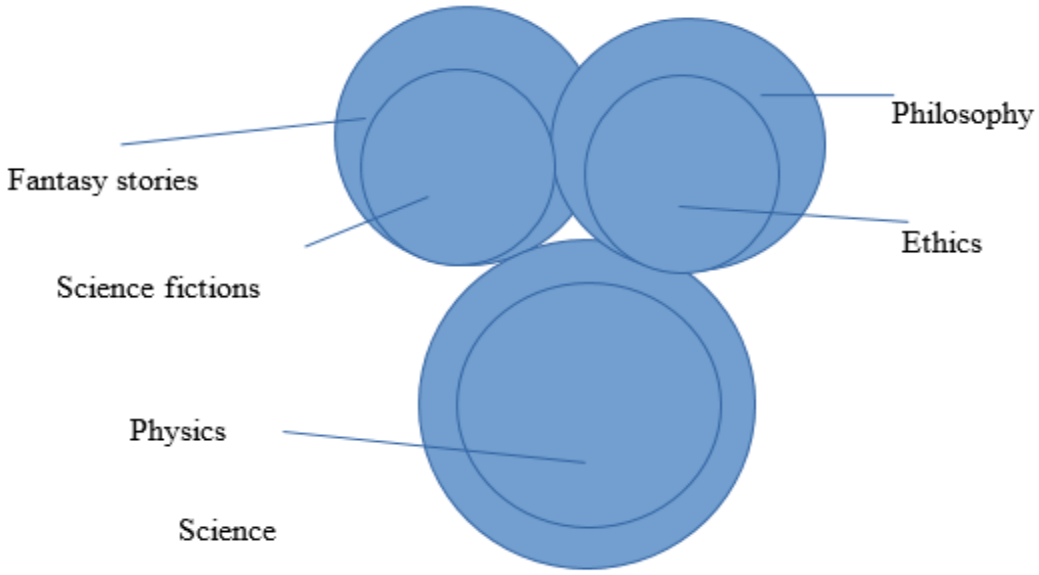

Figure 1. Neutrosophic sets depictions of science fiction, physics, and philosophy/ethics.

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A Neutrosophic Venn Diagram, on the other hand, is a visualization tool used in neutrosophic sets, and it is a Venn Diagram whose three sets (circles) have unclear/indeterminate borders and their intersections are also unclear. Neutrosophic Set deals with indeterminacy, ambiguity, and inconsistency. Here's how it expands on a regular Venn Diagram:

- **Standard Venn:** Represents sets (without indeterminacy), overlaps show elements belonging to both or all three sets.
- **Neutrosophic Venn:** Represents sets with indeterminacy, and adds shading or markings to depict three components:
- T1 (Truth): Degree of an element belonging to the set.
- T2 (Indeterminacy): Degree of element having uncertain belonging.
- T3 (Falsity): Degree of the element not belonging to the set.

Neutrosophic Venn Diagrams help visualize complex relationships where elements and their intersections can have varying degrees of truth, indeterminacy, and falsity.

1.2 | n-Dimensional Venn Diagram

Smarandache [4] has extended in 2010 the 3-dimensional Venn Diagram, based on three sets S_1, S_2, and S_3, to the n-dimensional Venn Diagram, based on n sets S_1, S_2, …, S_n algebraically, since geometrically it is not possible to visualize it, and used an **algebraic codification** of unions and intersections.

Let's first consider $1 \leq n \leq 9$, and the sets S_1, S_2, …, S_n.

Then one gets 2^n-1 disjoint parts resulting from the intersections of these n sets. Each part is encoded with decimal positive integers specifying only the sets it belongs to. Thus: part 1 means the part that belongs to S_1 (set 1) only, part 2 means the part that belongs to S_2 only, ..., and part n means the part that belongs to set S_n only.

Similarly, part 12 means that part which belongs to S1 and S2 only, i.e. to $S1 \cap S2$ only.

Also, for example, part 1237 means the part that belongs to the sets S_1, S_2, S_3, and S_7 only, i.e. to the intersection $S_1 \cap S_2 \cap S_3 \cap S_7$ only. And so on. This will help the construction of a base formed by all these disjoint parts and implementation in a computer program of each set from the power set $P(S_1 \dot{\cup} S_2 \dot{\cup} \ldots \dot{\cup} S_n)$ using a binary number.

The sets S_1, S_2, …, S_n, are intersected in all possible ways in a Venn diagram. Let $1 \leq k \leq n$ be an integer. Let's denote by $i_1 i_2 \ldots i_k$ the Venn diagram region/part that belongs to the sets S_{i1} and S_{i2} and … and S_{ik} only, for all k and all n. The part which is outside of all sets (i.e. the complement of the union of all sets) is noted by 0 (zero). Each Venn diagram will have 2^n disjoint parts, and each such disjoint part (except the above part 0) will be formed by combinations of k numbers from the numbers: 1, 2, 3, …, n.

1.3 | Unions and Intersections of Sets in the n-Dimensional Venn Diagram

This codification is user-friendly in algebraically simply doing unions and intersections.

The union of sets Sa, Sb, …, Sv is formed by all disjoint parts that have in their index either the number a, or the number b, …, or the number v.

While the intersection of Sa, Sb, …, Sv is formed by all disjoint parts that have in their index all numbers a, b, …, v.

For n = 3 and the above diagram: S1US23 = {1, 12, 13, 23, 123}, i.e. all disjoint parts that include in their indexes either the digit 1 or the digits 23; and $S1 \cap S2$ = {12, 123}, i.e. all disjoint parts that have in their index the digits 12.

1.4 | Remarks on Operations in n-Dimensional Venn Diagram

When n ≥ 10, one uses one space in between numbers: for example, if we want to represent the disjoint part which is the intersection of S3, S10, and S27 only, we use the notation [3 10 27], with blanks in between the set indexes.

Depending on preferences, one can use other characters different from the blank in between numbers, or one can use the numeration system in base n + 1, so each number/index will be represented by a unique character.

1.5 | Neutrosophic n-Dimensional Venn Diagram

This is a n-dimensional Venn Diagram whose sets are neutrosophic (i.e. they have some degrees of truth, indeterminacy, and falsehood) and whose borders are unclear and their intersections are unclear too.

1.6 | What is Zel'dovich pancake: Unveiling the Zel'dovich Pancake in the Early Universe

Have you ever wondered how galaxies came to be? The vast cosmic web, with its intricate filaments and clusters, holds the key to this mystery. One theory points to the intriguing concept of the Zel'dovich pancake, named after the renowned physicist Yakov Zel'dovich (cf. Gott, 2016).

Imagine this: The universe was initially a cold uniform soup of particles. But within this uniformity, there were tiny fluctuations in density. These fluctuations, according to Zel'dovich's approximation (developed in 1970), would act as seeds for future structure formation.

Here's where things get interesting. Zeldovich proposed that under the influence of gravity, these overdense regions, initially roughly ellipsoidal, would collapse along their shortest axis.

These Zel'dovich pancakes, though theoretical, are predicted to be vast, spanning supergalactic scales (far larger than our Milky Way galaxy). The immense size allows us to neglect the effects of pressure, making the model simpler.

The present-day universe consists of galaxies, galaxy clusters, one-dimensional filaments, and two-dimensional sheets or pancakes, all of which combine to form the cosmic web. The so-called" Zeldovich pancakes" are very difficult to observe, because their overdensity is only slightly greater than the average density of the universe. Falco et al. presented a method to identify Zeldovich pancakes in observational data, and the method was used as a tool for estimating the mass of galaxy clusters [2].

While the Zel'dovich approximation provides valuable insights, it has limitations. It doesn't account for smaller scales where pressure becomes significant. Modern cosmologists use sophisticated computer simulations that incorporate pressure and other complexities to get a more realistic picture. However, the concept of Zel'dovich pancakes remains a cornerstone in our understanding of large-scale structure formation. Astronomers continue to refine models and search for observational evidence to support the existence of these cosmic delicacies.

So, the next time you gaze at the night sky, remember the invisible pancakes lurking in the cosmic web, their potential a testament to the fascinating and dynamic nature of our universe.

2 |The Zel'dovich Pancake: A Building Block of the Cosmic Web

The Zel'dovich pancake model offers a crucial starting point for understanding the large-scale structure of the cosmos, known as the cosmic web. Here's how it sheds light on this intricate network (cf. Gott, 2016):

2.1 | Seed for Structure Formation

- The model proposes that tiny density fluctuations in the early, uniform universe became the seeds for future structures.
- Gravity pulls these overdense regions together, initiating their collapse.

2.2 | Birth of Pancakes

- Zel'dovich's idea is that these collapsing regions wouldn't simply shrink uniformly.
- Due to their initial ellipsoidal shape, gravity acts strongest along the shortest axis, causing them to flatten into vast, sheet-like structures – the Zel'dovich pancakes.

2.3 | These Pancakes are the Building Blocks of the Cosmic Web

- The model predicts that further collapse within these pancakes, along the remaining two axes, will lead to the formation of filaments – one-dimensional threads of matter.
- The intersections of these filaments become denser regions, eventually condensing into galaxy clusters, like giant knots in the web.

2.4 | A Simplified View

- It's important to remember that the Zel'dovich pancake model is a simplified view.
- It neglects pressure, which becomes important at smaller scales.

2.5 | Complementing Modern Simulations

- While not a complete picture, the model provides a valuable framework.
- Modern computer simulations incorporate pressure and other complexities, building upon the pancake concept to create a more realistic picture of the cosmic web.

Zel'dovich pancakes, although not directly observed, offer a theoretical foundation for understanding the large-scale structure formation in the cosmic web. Imagine the cosmic web as a giant spiderweb, with the Zel'dovich pancakes forming the large, flat sheets, the filaments as the connecting threads, and the galaxy clusters as the dense knots where the threads intersect.

By studying the Zel'dovich pancake model, cosmologists gain insights into how the uniform universe we see in the Cosmic Microwave Background radiation.

3 | An outline from Zel'dovich Pancake to Burgers Equations to Represent Cosmic Turbulence

a[t_] := ScaleFactor[t]; (* Scale factor as a function of time *) R[t_] := Perturbation[t]; (* Perturbation as a function of time *) (* Define Hubble parameter *) H[t_] = D[Log[a[t]], t]; (* Define peculiar velocity *) v[t, x_] = D[R[t], x]/a[t]; (* Continuity equation *) Div[v[t, x], {1}] + D[a[t], t]/a[t] == 0; (* Euler equation *) D[v[t, x], t] + H[t]*v[t, x] + v[t, x] D[v[t, x], x] == 0; (* Solve for v[t, x] - Zel'dovich approximation *) vSol[t, x_] = Simplify[v[t, x] /. Solve[{Div[v[t, x], {1}] + D[a[t], t]/a[t] == 0}, v[t, x]][1]]; (* Apply transformation to get density perturbation *) delta[t, x_] = R[t] - vSol[t, x]; (* Take derivative of density w.r.t. time *) deltaDot[t, x_] = D[delta[t, x], t]; (* Simplify using continuity equation *) deltaDot[t, x_] = Simplify[deltaDot[t, x] /. {D[vSol[t, x], t] -> - H[t]*vSol[t, x] - vSol[t, x] D[vSol[t, x], x]}]; (* Burgers equation *) deltaDot[t, x] + H[t]*delta[t, x] = (vSol[t, x])^2/a[t] == - H[t] R[t] + (R[t] D[R[t], x]/a[t})^2/a[t]; (* Print

result *) Print["The Burgers equation derived from Zel'dovich pancake collapse is:"] Print[deltaDot[t, x] + H[t]*delta[t, x] == - H[t] R[t] + (R[t] D[R[t], x]/a[t})^2/a[t]];

This code defines the scale factor, perturbation, Hubble parameter, and peculiar velocity. Then, it solves the continuity and Euler equations to obtain the Zel'dovich approximation for the peculiar velocity. Subsequently, it defines the density perturbation and its time derivative. Using the continuity equation, the time derivative is simplified. Finally, the Burgers equation is obtained by expressing the change in density as a function of the initial perturbation, the Hubble parameter, and the square of the peculiar velocity. **Note:**

- This code assumes a specific form for the scale factor and perturbation.
- Additional assumptions and simplifications are often made in the Zel'dovich pancake model.

4 | What are Burgers equations?

The Burgers' equation is fundamental in applied mathematics, particularly useful for describing phenomena involving **convection and diffusion**. It captures the interplay between two forces:

- **Convection:** Imagine pushing a wave of hot water through a cooler tank. The wavefront (hottest part) moves with the water's flow, while the heat itself diffuses, spreading out from the hot center. This is convection.
- **Diffusion:** Diffusion is the natural tendency of particles to spread out from areas of high concentration to areas of low concentration.

The Burgers' equation allows us to model this interplay mathematically. It's particularly useful for scenarios where the convection is non-linear, meaning the speed of the wavefront depends on its density.

4.1 | Burgers' Equation in Cosmology

So, how does this relate to the early universe? In the very first moments after it is supposed there were fluctuations. These fluctuations, while tiny, are believed to be the seeds for the large-scale structures we see today, like galaxies and clusters. The Burgers' equation becomes a powerful tool for studying the **non-linear evolution** of these density fluctuations. Here's why:

- **Non-linearity Matters:** In the early universe, gravity caused denser regions to attract even more matter, amplifying the fluctuations. This non-linear growth is crucial for understanding structure formation. The Burgers' equation, with its ability to handle non-linearity, becomes highly relevant.
- **Shock Formation:** As the universe expands, denser regions can become so dense that they undergo a rapid collapse, forming a **shock wave**. The Burgers' equation helps model the formation and evolution of these shock waves, which are important for understanding how structures like galaxies condense out of the smooth early universe.

While the Burgers' equation provides valuable insights, it's important to acknowledge its limitations. The model is simplified, neglecting factors like dark matter and radiation pressure that play a role in the real universe. Cosmologists use more sophisticated tools like **hydrodynamic simulations** that incorporate these complexities. However, the Burgers' equation remains a valuable starting point and a powerful tool for understanding the qualitative behavior of density fluctuations in the early universe.

The Burgers' equation, though seemingly abstract, offers a window into the dynamic processes that shaped the cosmos from its infancy. By studying this equation, cosmologists gain a deeper appreciation for the intricate dance of matter and energy that gave rise to the universe we inhabit today.

As the second step, we proceed to provide an outline from Burgers equations to Ermakov dynamics equations. Here's an outline of Mathematica code to derive the Ermakov equations from the Burgers equation for simple turbulence:

```
(* Define the Burgers equation *) burgersEq[u_[t, x_]] := D[u[t, x], t] + u[t, x] D[u[t, x], x] - nu*D[u[t, x], {x, 2}]; (* nu - viscosity *) (* Introduce Cole-Hopf transformation *) v[t, x_] := Log[u[t, x]/(nu + u[t, x])]; (* Apply transformation to Burgers equation *) D[v[t, x], t] + D[v[t, x], x]*D[v[t, x], x] = 1; (* Define first Ermakov equation *) ermakow1[v_[t, x_]] := D[v[t, x], t] + D[v[t, x], x]*D[v[t, x], x]; (* Define second Ermakov equation (optional) *) (* Due to the transformation, the second equation becomes an identity. Uncomment the following lines if needed for your specific application. *) {w[t, x_] = D[v[t, x], x], ermakow2[v_[t, x_]] = D[w[t, x], t] + D[v[t, x], x]*D[w[t, x], x]}; (* Print the results *) Print["The first Ermakov equation derived from the Burgers equation is:"] Print[ermakow1[v[t, x]] == 1]; (* Print the second Ermakov equation (if uncommented) *) (* Print["The second Ermakov equation is:"] *) (* Print[ermakow2[v[t, x]] == 0]; *)
```

This code first defines the Burgers equation with viscosity nu. Then, it introduces the Cole-Hopf transformation, which transforms the Burgers equation into the first Ermakov equation. The second Ermakov equation essentially becomes an identity due to the transformation.

In the third step, we would like to provide an outline *nonlocal current effect* from Ermakov equations. While Ermakov equations can be useful for turbulence modeling, they are not directly applicable to deriving the nonlocal current effect. Here's why:

- **Ermakov equations originate from the Burgers equation:** This equation describes fluid flow, where currents are local phenomena. Nonlocal currents, however, exhibit effects beyond the immediate vicinity of a point.
- **Nonlocal currents arise in specific contexts:** These effects are often seen in materials with complex band structures or strong correlations, not typically captured by the Burgers equation.

However, we can explore an alternative approach:

1. **Nonlocal conductivity models:** There are several existing models for nonlocal conductivity that we can explore in Mathematica. These might involve integral equations or memory functions to describe the nonlocal character of the current.
2. **Material-specific packages:** Depending on the material you're interested in (e.g., graphene, organic conductors), specific Mathematica packages might be available that incorporate nonlocal effects.

```
(* Define spatial variable *) x_/; x > 0:= x; (* Define local current density *) jLocal[E_, x_]:= sigma*E; (* sigma - local conductivity *) (* Define a placeholder function for nonlocal kernel *) kernel[x1_, x2_]:= (* Replace this with an appropriate kernel function describing the nonlocal effect *) Exp[-(x1 - x2)^2/lambda^2]; (* Placeholder - modify lambda for decay length *) (* Define integral for nonlocal current contribution *) jNonlocal[E_, x_]:= Integrate[kernel[x, x1]*jLocal[E, x1], {x1, -Infinity, Infinity}]; (* Total current density *) jTotal[E_, x_]:= jLocal[E, x] + jNonlocal[E_, x]; (* Sample electric field (replace with your actual profile) *) E[x_]:= Sin[x]; (* Plot the total current density *) jPlot = Plot[jTotal[E[x], x], {x, 0, 10}, PlotRange -> {-.5, 1.5}]; (* Show the plot *) jPlot
```

This code defines local and nonlocal current contributions (replace the placeholder kernel function with the appropriate model for your chosen material and effect). It then calculates the total current density and plots it for a sample electric field.

5 | Discussion: Nonlocal Currents from Ermakov's Playground: A Possible Leap in Interstellar Travel?

The vast stretches of interstellar space hold mysteries beyond our wildest dreams. One intriguing avenue of exploration involves harnessing the power of charged particles and their interactions. Here's where the concept of "nonlocal current effect derived from Ermakov equations" enters the scene, sparking both scientific curiosity and a touch of speculation. Ermakov equations describe the motion of a charged particle

in a specific type of electric and magnetic field configuration. These fields are carefully designed to create a stable "playground" where the particle's trajectory is predictable and repeatable. Studying these equations helps us understand the fundamental principles of charged particle motion.

Now, the "nonlocal current effect" within this context is a hypothetical phenomenon. It suggests that under specific conditions within Ermakov's configuration, the movement of charged particles could lead to a **surprising surge in current**. This surge wouldn't follow the usual linear relationship between voltage and current, hence the term "nonlocal." While the existence of this nonlocal, nonlinear effect remains unproven, the very possibility is exciting for understanding interstellar matter. Here's why:

- **Charged Plasma:** Interstellar space is filled with a hot, ionized gas called plasma. This plasma contains charged particles – electrons and ions.
- **Harnessing the Flow:** If the nonlinear current effect is real, it could offer a way to manipulate the flow of charged particles within interstellar plasma. This, in theory, could lead to the development of novel propulsion systems for spacecraft ventures.

6 | Concluding Remark: A Word of Caution

However, it's important to inject a healthy dose of caution. The leap from manipulating charged particle flow to achieving faster-than-light travel (like warp drives from science fiction) is immense. Ermakov's equations and the nonlinear current effect are currently theoretical concepts. Furthermore, achieving warp drive would likely require manipulating gravity, something beyond the scope of charged particle interactions.

The study of Ermakov equations and the hypothetical nonlinear current effect represent a scientific exploration, not a shortcut to interstellar travel. It's a pursuit of knowledge that could deepen our understanding of charged particle behavior and potentially lead to future propulsion advancements within the realm of physics.

For now, the vast expanse of space remains a frontier to be explored with a combination of scientific rigor and a touch of healthy imagination. As we delve deeper into the mysteries of charged particles and their interactions, who knows what exciting discoveries await us on the cosmic dance floor?

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Conflicts of Interest

The authors declare that there is no conflict of interest in the research.

Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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A Short Communication on Progress and Problems of ITER Fusion Project

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ABSTRACT In recent years, it becomes clear that ITER project in France, as one of the largest experimental fusion reactors underway, is far away from achieving net energy production. In this review article, we presented a short communication this week with Robert Neil Boyd, a senior physicist who happens to have his own working design of fusion reactor in the past. We hope that this transcript of our communication with him (as per 15-17th Nov. 2021) may be found useful for younger scientists.

INTRODUCTION

As we know, many people and scientists believe that fusion energy is our hope for clean source of energy in the future. But whether there is real ground for such a hope, or it is just another techno-fantasy, we shall find out. In recent years, it becomes clear that ITER project in France, as one of the largest experimental fusion reactors underway, is far away from achieving net energy production. According to W. Wayt Gibbs: "Major fusion projects such as ITER in France and NIF in the U.S. have consumed billions of dollars and are nowhere close to generating enough energy to even sustain their own operation

much less create commercial power. Smaller, simpler designs are now being explored, in some cases by private companies. Preliminary results have raised hopes that there might be more practical, less expensive paths to fusion power plants."[1] In this article, we presented a short communication this week with Robert Neil Boyd, PhD, a senior physicist who happens to have his own working design of fusion reactor in the past. He is now associated with Princeton Biotechnology Corp., New Jersey. We hope that this transcript of our communication with him (as per 15-17th Nov. 2021) may be found useful for younger scientists.

RECENT PROGRESS OF ITER PROJECT

As the story goes, for future power generation, fusion reactors have unique benefits. Unlike conventional nuclear reactors, fusion reactors cannot melt down and do not produce radioactive material that can be weaponized or that requires special disposal. Safety and environmental concerns with fusion reactors are minimal, and the deuterium and lithium required for fuel can be extracted from seawater. A fusion power plant can, in aspiration, be built at a competitive capital cost and have virtually no input cost beyond operating expenses.[2] For readers not quite familiar with the progress of ITER, allow us to quote W. Wayt Gibbs again: "Right now the pragmatists have people's attention because the academics have hit practical dead ends: enormous reactors that have clarified some fusion science but are not on track to pump electricity into the grid by midcentury. One example is the National Ignition Facility (NIF) at Lawrence Livermore National Laboratory. "NIF fires just a few hundred shots a year," Binderbauer says in his Austrian lilt. A power plant would have to fire tens of thousands of times a day. Two years ago Livermore pulled the plug on designing a prototype power plant. The second discouraging example is ITER, a 10-story-high machine under construction in France by a consortium of nations. It will rely on giant superconducting magnets to control a plasma burning at roughly 150 million degrees C for minutes at a time. Even if it succeeds, ITER will make no electricity."[1]

NEIL BOYD'S REMARK AND VC'S RESPONSE

Note: RNB stands for Robert Neil Boyd, and VC stands for one of these authors. RNB: "Similar and related events are seen in this Aureon-produced video: https://vimeo.com/453152469. The striations which arise from the plasma dis- charge are due to quantization of the electric field. Here is an example of a larger Natural version of the instrumented results of the Safire experiments, as observed in by astrophysical instruments, such as this Hubble telescope image of the "Cat's Eye" Nebula. The point of this is that if one looks closely at the full moon, which has been surrounded by a rainbow ring for more than 7 consecutive years, you can see similar circular radial behaviors of light as it arises from the surface of the moon. However, the electric field caused directly by solar and stellar radiation is not sufficiently large to cause such layered radiation sources.

...By the way, "*free electricity*" was discovered and proved, long ago, by Faraday during one of his experiments regarding electric and magnetic field behaviors. Faraday's result got stopped and it was never published in any of the journals of those days. But Faraday still recorded it and kept it among his personal effects, for posterity." VC:"Thank you. Thanks also for the story of Michael Faraday." RNB: "Plasmas exhibit "double layers" which form *Birkeland currents*, plasma "cells" and multiple boundary layers, due to charge separation, exactly as we can see in the Aureon video. This happens when all the electrons go to one side of the given volume, while the positive charges (typically protons) are separated away from the electrons at another wall in the given volume, usually because protons and positively charged atomic nuclei are much more massive than electrons, so they have a lot more inertia and resistance to moving, and tend not to move very much or very fast. The force developed between the positive and the negative portions are measurably huge, and

eventually the positive and negative charges will meet and neutralize, if nothing about the existing configuration changes. Eventually the plus charges will start moving slowly towards the negatively charge volume. Or the electrons will flash over to where the positive charges are. Many energetic events can happen, associated with that kind of behavior. The Aureon results demonstrate some of the above discussion. I started my research in plasma physics back in 1966 and continued until the late 1990s. I designed the third energy-gaining thermonuclear fusion reactor in history. It was built and tested by DoE and I was notified of the success of the design back in 1989. They did an in-house video interview with me, but very few have any awareness of it aside from me. Bogdan Maglich designed and tested the second energy-gaining fusion reactor. I corresponded personally with Maglich for a while. He was really frustrated because nobody would fund him to build a power plant using his fusion reactor design, which had been publicly proven. Philo Farnsworth, who designed most of the components of the old tube-style television, made the first energy-gaining fusion reactor, based on a "multipactor tube", a very unusual tube he had developed during his TV researches. In 1965 his "Fusor Mark IV" was producing 50 giga-neutrons per second, due to huge numbers of fusion reactions occurring in a fusion confinement volume the size of a grapefruit. The 4th demonstrated fusion reactor was designed and constructed by a group of college kids in someone's basement. They got a patent of the device. The patent was sold to Chrysler-Daimler and never heard from again. I was in touch with them for a while too. I guess they got some good income from selling their patent, but they stopped talking to me soon after the patent issued. So where are any of these functional fusion reactors?" VC: "Thank you for your story, and also Maglich and Farnsworth's designs for real fusion reactor. I am skeptical on ITER project. I heard that only a Japanese team wrote honestly that the real purpose of ITER project is to improve gain function, not to obtain net energy production."

OTHER POTENTIAL CREATIVE ENERGY BREAKTHROUGH - TOWARD HYDROGEN AGE

Allow us to make a quick note about the potential for breakthrough energy towards a hydrogen-based economy (cf. Capra, 2009), as follows: There are indeed some leaders of technology companies, such as Elon Musk, who are skeptical of the fuel cell, and call it: "*fool sell.* " It may be that fuel cell technology is not economical in today's context, but let us briefly mention that since the 1980s-1990s, efficient water electrolysis technology has been discovered by Dr. Andrija Puharich, and what's interesting is that a few months after his electrolysis patent was accepted by the US patent office, he seems to have inspired Stanley Meyer's experiments on his known *water-fueled car design* (and a more efficient electrolysis process).

Recently, there are various debates in online channels on Indonesian version of the alternative fuel research for the motorbikes, which has become one of the hot topics, from the findings of H. Aryanto Misel from Lemahabang, Cirebon (Indonesia), which he calls Nikuba (according to him from the Javanese expression, "*niku banyu*"; trans. it's water). There has been controversy over these findings, and this is quite reasonable, considering that the electrolysis method has long been known in the field of chemistry.

What is interesting is that standard textbooks state that the electrolysis process is not efficient, because it takes a lot of energy to separate H_2O into H-H-O. In addition, among mechanical engineers, one other issue is corrosion that must be prevented. Maybe that's why the Nikuba electrolysis design, practically designed by Aryanto, Misel uses stainless steel. In addition, to achieve an efficient electrolysis process, it seems that he did not use the usual catalysts such as NaCl etc., but he used a special homemade catalyst. This is an interesting innovation, compared to the available information regarding the findings of the electrolysis process by Dr Andrija Puharich, around the 80s, which opens up the possibility of efficient

processes, including: (a) ordinary electrolysis, (b) pulsed electrolysis, (c) gated-pulsed electrolysis with a type of source wave called a waveform. The fourth innovative breakthrough made by Stan Meyer is to add a fourth alternative method to achieve an efficient electrolysis process, namely: (d) resonance electrolysis. From discussions among others in cyberspace, there is information that perhaps what is meant by resonance here is not the resonant frequency of water molecules, which are on the order of MHz, but ordinary electric resonance in the range of 280 Hz or about 495 Hz.

Another interesting piece of information is that Stan Meyer triggered the resonance using a *bifilar coil source*, which can be traced back to Nikola Tesla's inventions. H. Aryanto's findings can be considered as the fifth innovation for an efficient electrolysis process because it uses a catalyst, so we can call it in more scientific terms: "catalytic-aided electrolysis generator." Of course it will be interesting, if the research can be continued further by developing alternative combinations between the catalytic method coupled with the bifilar coil / resonance: who knows maybe we will achieve a dramatic increase in efficiency, above 100% of the performance of current water electrolysis machines. Thus presumably the process of scientific discovery can continue. What is interesting is not only the concept of resonance which has long been known in physics, but also bifilar coil. Besides bifilar coil, there is also a so-called *Rodin coil*, which may be rarely known (cf. Boyd & Smarandache, 2022) [12]. Rodin coil has the potential not only to have a good effect on health (Schumann resonance), but can also be engineered to create a magnetic levitation machine. These things would be interesting if there are researchers who are interested in conducting experiments on aerial vehicles for the future.

Summarizing, in addition to developing a transportation ecosystem based on electrical energy (EV) as many countries begin to develop, it may also be time for us to prepare for the next stage of transformation: i.e. towards hydrogen energy age; cf. Capra, 2009 [13]. It is worth to mention here, that around two decades ago, Prof Pm. Kanarev from Kuban State University has proved that hydrogen economy era has come, especially with his finding of nuclear water electrolysis (Kanarev & Mizuno, 2001; Mizuno, 2001) [9,8].

DISCUSSIONS

As we pointed above, the Japanese ITER team is the only participating group to clearly and transparently state the power capability of ITER: "Will ITER make more energy than it consumes? . . . ITER is about equivalent to a zero (net) power reactor, when the plasma is burning."[4] Other than that, the problem is more accute, not only that Tokamak-based fusion includes complicated MHD problems which are highly nonlinear; cf. ref. [7], but also confusion in messages to public. As S. Krivit wrote: "Members of your fusion science community promoted ITER with language that made it appear to laypersons as designed for Q- engineering=10, rather than Q-fusion=10. They did this for three decades. Nobody said a word."[6] At this point, some readers may ask:"So, what are our choices for future energy?" We can cite T. Linden here, that there are: "relatively small and compact devices compared to tokamaks decreases the costs and building time of the reactors and this has allowed some private companies to enter the field, like EMC2, General Fusion, Helion Energy, Lockheed Martin and LPP Fusion."[3] Alternatively, one may also consider Prof. Ph. M. Kanarev's method of water electrolysis reactor.[10-11]

Last but not least, we can also argue that actually fusion process can happen even from classical electromagnetic considerations, see our paper at JCMNS, 2017 [8]. In other words, we may expect that fusion process can happen at relatively low temperature, such as from biological transmutation (see J. Biberian's papers in JCMNS).

CONCLUDING REMARKS

In this paper, we began by citing from W.

Wayt Gibbs, that actually our hope for plentiful energy based on fusion, is not based on actual progress of present experimental reactors. But as Linden, Gibbs, and Kanarev wrote, there are actually more hope on the so-called *'fusion underground*," or small fusion ex periments carried out by independent scientists. [1] We recall comment by a senior scientist at Russia, that there is a different culture of science between Russia and USA scientists; is that scientist in Russia is allowed to do small experiments at any type of inquiry that they like in their basement or houses more or less freely. Such a culture seems quite lacking, but it seems CF report by Pons-Fleischmann around 80s began to inspire other physicists, to do small basement-scale experiments at their own. Then such a small-scale experiment can be scaled up later on. All in all, for the time being, renewable energy sources (such as: WWS, geothermal, biofuels, etc.) may be expected to meet the world energy demands, at least partially, until that energy abundance days come. See for instance Mark Z. Jacobson's book [16]. Also ref. [15]. And in latter part, we discuss other potential energy breakthroughts, especially if we consider seriously going toward hydrogen age, cf. Pm. Kanarev (2001, 2002), F. Capra (2009).

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Three Novel Approaches to Deep Space: Interstellar Travel that Transcend the Limitations Imposed by the Rocket Equation

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Abstract

At the time of writing this abstract, we read about Betelgeuse star has exploded in the last few days. While there are various explanations and interpretations on how that event would have impacted this Earth's inhabitants, our interpretation asserts that there are several chains of stars that act to lock this Earth to the 3D realm, and the exploded Betelgeuse star can be considered as a signal from heaven that we, all Earth inhabitants, are allowed to be elevated to 5D consciousness and to explore the Deep Space beyond this solar system. Therefore, in the present paper, it is considered three novel approaches to Deep Space / interstellar travel that transcend the limitations imposed by the Tsiolkovsky rocket equation. Among other things, we explore the possibility of utilizing macro quantum tunneling as an alternative propulsion method. By inducing a state of quantum coherence in a macroscopic object, it is theorized that it could tunnel through barriers, bypassing the need for conventional propulsion systems. Furthermore, we investigate the potential role of spin supercurrents in facilitating this process. The paper delves into the theoretical underpinnings of macroquantum tunneling and spin supercurrents, discussing the challenges and opportunities associated with their application to space travel.

Keywords: 5D Consciousness; Betelgeuse Star; Macroquantum Tunneling; Alternative Propulsion Methods; Deep Space Travel.

1 | Limitations of Tsiolkovsky Equation

In the past decades, there has been growing interest in novel approaches beyond existing technologies, especially to explore the Deep Space and beyond [1-3]. In this regard, the Tsiolkovsky rocket equation, a cornerstone of astronautics, provides a fundamental relationship between a spacecraft's initial mass, final mass, exhaust velocity, and the change in velocity it can achieve. While this equation has been instrumental in numerous space missions, it also presents inherent limitations that hinder our ability to explore the vast expanse of the universe [1].

One significant limitation of the Tsiolkovsky equation is the requirement for carrying propellant. The more propellant a spacecraft carries, the heavier it becomes, necessitating even more propellant to accelerate that increased mass. This creates a feedback loop that limits the maximum achievable velocity. As a result, long-duration missions to distant destinations, such as Mars or beyond, become increasingly challenging and resource-intensive.

Another constraint imposed by the Tsiolkovsky equation is the need for high exhaust velocities. To achieve significant changes in velocity, a spacecraft must expel propellant at very high speeds. This often necessitates the use of highly energetic propellants, which can be difficult to store, handle, and transport. Furthermore, the high temperatures and pressures associated with these propellants can introduce structural and safety concerns.

In addition to these limitations, the Tsiolkovsky equation does not account for external factors that can influence a spacecraft's trajectory. Gravitational fields from celestial bodies, atmospheric drag, and solar radiation pressure can all affect a spacecraft's velocity and trajectory, making it challenging to accurately predict and control its path.

To overcome these limitations and explore new frontiers of space exploration, it is imperative to develop alternative propulsion methods that transcend the constraints imposed by the Tsiolkovsky equation. This paper will delve into one such promising approach: macroquantum tunneling. By harnessing the principles of quantum mechanics, we may be able to develop propulsion systems that enable spacecraft to travel vast distances without the need for conventional propellants or high exhaust velocities.

2 | Results

2.1 | An Alternative to Tsiolkovsky Equation: Towards Macroquantum Tunneling/EPR Bridge

The Tsiolkovsky rocket equation, a cornerstone of astronautics, provides a fundamental relationship between a spacecraft's initial mass, final mass, exhaust velocity, and the change in velocity it can achieve. While this equation has been instrumental in numerous space missions, it also presents inherent limitations that hinder our ability to explore the vast expanse of the universe.

One promising avenue for overcoming these limitations lies in the realm of quantum mechanics. Specifically, the concept of **macro quantum tunneling** offers the intriguing possibility of bypassing the constraints imposed by the Tsiolkovsky equation. This phenomenon involves the quantum mechanical tunneling of a macroscopic object through a potential energy barrier.

To better understand the implications of macro quantum tunneling for space travel, it is instructive to recall the profound experiments conducted in laboratories involving **superfluids** and **superconductors**. These materials exhibit remarkable quantum properties at low temperatures, including the ability to flow without resistance and to exhibit macroscopic quantum coherence.

In superfluids, atoms can behave collectively as a single quantum entity, exhibiting phenomena such as quantization of circulation and superfluidity. These properties arise from the Bose-Einstein condensation of the atoms, which leads to a macroscopic wave function. In superconductors, electrons form Cooper pairs, which can also behave as a single quantum entity. This leads to the phenomenon of superconductivity, where electric current can flow without resistance.

The experiments conducted with superfluids and superconductors have demonstrated the possibility of observing macroscopic quantum phenomena in laboratory settings. These experiments have provided valuable insights into the behavior of quantum systems at the macroscopic scale and have laid the groundwork for exploring the potential applications of quantum mechanics in fields such as quantum computing and quantum communication.

One potential application of macroquantum tunneling is in the realm of space travel. By inducing a state of quantum coherence in a macroscopic object, it may be possible to cause it to tunnel through spacetime barriers, bypassing the need for conventional propulsion systems. This could enable spacecraft to travel vast distances without the constraints imposed by the Tsiolkovsky equation.

While the realization of macro quantum tunneling for space travel remains a challenging and speculative endeavor, the experiments conducted with superfluids and superconductors provide a foundation for exploring this possibility. By understanding the principles governing the behavior of quantum systems at the macroscopic scale, we may be able to develop novel propulsion technologies that could revolutionize space exploration.

2.1.1 | Detecting Macroquantum Tunneling: Near-Field Physics and Spin Supercurrents

Following the aforementioned arguments concerning to possibility of macroquantum tunneling to be considered in the lab setting, we already discussed small-scale experiments; while our results are not quite convincing regarding traversability, what we reported concerns plausible quantum tunneling simulation for iced-water with simple measurement devices [4, 5].

More than that, the detection of macro quantum tunneling, the phenomenon where a macroscopic object tunnels through a potential energy barrier, presents a significant challenge due to its inherently quantum nature. However, recent advances in near-field physics and the study of spin supercurrents offer promising avenues for overcoming these obstacles [7].

2.1.2 | Near-Field Physics

Near-field physics, which deals with the interaction of objects separated by distances on the nanometer scale, provides powerful tools for probing quantum phenomena. Techniques such as atomic force microscopy (AFM) and scanning tunneling microscopy (STM) can be used to measure the forces and currents acting between a probe tip and a sample surface with unprecedented sensitivity [6].

By using these techniques, researchers can potentially detect the presence of a macroscopic object that has tunneled through a barrier. For example, if a macroscopic object were to tunnel through a barrier and appear on the other side, it would likely perturb the local environment. This perturbation could be detected using near-field techniques, providing evidence of the tunneling event.

2.1.3 | Spin Supercurrents

Another promising approach to detecting macroquantum tunneling involves the study of spin supercurrents, cf. our article at [6]. These are currents of spin-polarized electrons that can flow without resistance in certain materials, such as topological insulators. Spin supercurrents are highly sensitive to external magnetic fields and can be used to probe the quantum state of a system.

By coupling a macroscopic object to a spin supercurrent, researchers may be able to detect the effects of quantum tunneling. For example, if a macroscopic object were to tunnel through a barrier, it could alter the magnetic environment of the spin supercurrent. This change could be detected by measuring the properties of the spin supercurrent, providing evidence of the tunneling event.

2.1.4 | Hartman Effect / Estimation of Tunneling Time

Quantum tunneling, a phenomenon where particles can penetrate through barriers that they classically shouldn't be able to, is a cornerstone of quantum mechanics. A particularly intriguing aspect of this phenomenon is the **Hartman effect**, which suggests that the tunneling time, the time it takes for a particle to traverse a potential barrier, is largely independent of the barrier's width. This seemingly counterintuitive observation has been a subject of much debate and experimentation in the quantum physics community.

2.1.5 | The Experiment

To understand the Hartman effect, consider a simple experiment. A particle, such as an electron, is fired toward a potential barrier. Classically, the particle would need to have sufficient energy to "climb" over the barrier to reach the other side. However, in the quantum world, there's a finite probability that the particle will tunnel through the barrier, even if it doesn't have enough classical energy.

Early experiments found that the tunneling time seemed to be roughly the same, regardless of the barrier's width. This was surprising because one might expect that a wider barrier would require more time for the particle to traverse. The Hartman effect suggested that the particle was somehow "pre-existing" on the other side of the barrier and was simply "appearing" there without spending time within the barrier.

The Hartman effect has sparked considerable debate among physicists. Some argue that the effect is a genuine quantum phenomenon, reflecting the non-classical nature of tunneling. Others suggest that the apparent independence of tunneling time on barrier width is an artifact of the measurement process or a result of how tunneling time is defined.

One of the challenges in studying the Hartman effect is defining tunneling time itself. In classical physics, time is a well-defined quantity. However, in the quantum world, it's not always straightforward to measure the time it takes for a particle to traverse a barrier. Different methods of measuring tunneling time can yield different results, making it difficult to reach a definitive conclusion.

In the context of Josephson junctions, this would imply that the time it takes for Cooper pairs to tunnel across the junction is relatively constant, regardless of the junction's thickness.

2.1.6 | Using the OR Theorem

a. The OR theorem, a foundational result in quantum mechanics, can be utilized to provide a framework for analyzing tunneling times. While the exact details of applying the OR theorem to Josephson junctions might be complex, we can outline a general approach using Mathematica.

   Steps Involved:

   1. Define the Potential Barrier: We'll consider a simple rectangular potential barrier:
      - potential[x_] := Piecewise[{{0, x < 0}, {V, 0 <= x <= a}, {0, x > a}}]
   2. Set Up the Time-Dependent Schrödinger Equation: We'll use the time-dependent Schrödinger equation to describe the evolution of the wave function:
      - iℏ D[ψ[x, t], t] == -ℏ^2/(2m) D[ψ[x, t], {x, 2}] + potential[x] ψ[x, t]
   3. Specify Initial Wave Function: We'll use a Gaussian wave packet as the initial wave function
      - ψ[x, 0] := Exp[-(x - x0)^2/(2 σ^2)] Exp[I p0 x/ℏ]
   4. Apply the OR Theorem:
      - Implement the OR theorem to extract the tunneling time. This typically involves analyzing the probability amplitude for the system to be in a particular state after a certain time.
      - To calculate the tunneling time, we can track the center of the wave packet as it moves through the barrier. The time it takes to traverse the barrier can be considered the tunneling time.

Mathematica code (outline only):

```
Mathematica

(* Define parameters *)

ℏ = 1;

m = 1;

V = 10;

a = 5;

x0 = -10;
```

```
σ = 2;
p0 = 1;
(* Define the potential and initial wave function *)
potential[x_] := Piecewise[{{0, x < 0}, {V, 0 <= x <= a}, {0, x > a}}]
ψ[x, 0] := Exp[-(x - x0)^2/(2 σ^2)] Exp[I p0 x/ħ]
(* Set up the time-dependent Schrödinger equation *)
eq = I ħ D[ψ[x, t], t] == -ħ^2/(2 m) D[ψ[x, t], {x, 2}] + potential[x] ψ[x, t];
(* Numerical solution (adjust parameters as needed) *)
sol = NDSolve[{eq, ψ[x, 0] == ψ[x, 0]}, ψ[x, t], {x, -20, 20}, {t, 0, 20},
        Method -> "MethodOfLines",
        "SpatialDiscretization" -> {"TensorProductGrid", "MinPoints" -> 200}]
(* Visualize the wave function evolution *)
Animate[Plot[Abs[ψ[x, t] /. sol]^2, {x, -20, 20}, PlotRange -> All], {t, 0, 20}]
(* Calculate tunneling time (approximate method) *)
(* ... (Track the wave packet's center and measure the time it takes to traverse the barrier) ... *)
```

b. Provided we are allowed to hypothesize that the aether medium is composed of quasicrystalline tessellation [11, 12] or something like the crystalline phase of iced water as we discussed previously, we may consider an alternative way to come up with a Hartman tunneling time estimate.

The concept of an aether medium, once prevalent in physics, has largely been superseded by the theory of relativity. However, for this hypothetical exercise, let's assume that the aether is composed of a quasicrystalline tessellation.

2.1.7 | Challenges in Modeling a Quasicrystalline Aether

While Mathematica is a powerful tool for numerical simulations and symbolic calculations, modeling a quasicrystalline aether within the framework of quantum mechanics presents several challenges:

1. Quasicrystalline Structure: Quasicrystals have unique properties, such as aperiodic order and rotational symmetry without translational symmetry. Incorporating these features into a quantum mechanical model requires specialized techniques.
2. Define the Quasicrystalline Potential:

   potential[x_] := V0 (1 + Cos[2 π x/λ1] + Cos[2 π x/λ2]),

   where V0 is the potential amplitude, and λ1 and λ2 are incommensurate periods, creating a quasicrystalline structure.
3. Set Up the Time-Dependent Schrödinger Equation:

   This remains the same as in the previous

   iħ D[ψ[x, t], t] == -ħ^2/(2m) D[ψ[x, t], {x, 2}] + potential[x] ψ[x, t]
4. Specify Initial Wave Function:

   ψ[x, 0] := Exp[-(x - x0)^2/(2 σ^2)] Exp[I p0 x/ħ]

5. Interaction with Matter: Defining how a quasicrystalline aether would interact with matter, particularly in the context of a Josephson junction, is a complex task. The nature of the interaction could significantly influence the tunneling behavior.
6. Quantum Mechanical Formulation: Translating the concept of a quasicrystalline aether into a quantum mechanical formalism might require new theoretical frameworks or modifications to existing ones.

2.1.8 | Mathematica Code (outline only)

(* Define parameters *) $\hbar$ = 1; m = 1; V0 = 10; λ1 = √2; λ2 = √3; x0 = -10; σ = 2; p0 = 1; (* Define the quasicrystalline potential *) potential[x_] := V0 (1 + Cos[2 π x/λ1] + Cos[2 π x/λ2]) (* Set up the time-dependent Schrödinger equation *) eq = I $\hbar$ D[ψ[x, t], t] == -$\hbar$^2/(2m) D[ψ[x, t], {x, 2}] + potential[x] ψ[x, t]; (* Numerical solution (adjust parameters as needed) *) sol = NDSolve[{eq, ψ[x, 0] == ψ[x, 0]}, ψ[x, t], {x, -20, 20}, {t, 0, 20}, Method -> "MethodOfLines", "SpatialDiscretization" -> {"TensorProductGrid", "MinPoints" -> 200}] (* Visualize the wave function evolution *) Animate[Plot[Abs[ψ[x, t] /. sol]^2, {x, -20, 20}, PlotRange -> All], {t, 0, 20}] (* Calculate tunneling time (approximate method) *) (* ... (Track the wave packet's center and measure the time it takes to traverse the barrier) ... *).

2.1.9 | Considerations and Further Exploration

- Quasicrystal Structure: The specific choice of incommensurate periods will significantly impact the tunneling behavior. Experiment with different values to observe varying effects.
- Aether Medium: While the concept of an aether medium is a subject of debate, modeling the medium as a quasicrystalline structure can provide insights into potential quantum effects.
- Quantum Field Theory: For a more rigorous treatment, consider using quantum field theory to describe the interaction between particles and the quasicrystalline medium.
- Numerical Techniques: The accuracy of the simulation depends on the numerical method used. Experiment with different methods (e.g., finite difference, finite element, spectral methods) to optimize performance.

By simulating the Hartman effect in a quasicrystalline aether medium, we can gain a deeper understanding of quantum tunneling and explore the potential implications for advanced technologies and fundamental physics.

Note:

While this hypothetical code provides a starting point, it's important to emphasize that modeling the interaction between a quasicrystalline aether and a Josephson junction within a quantum mechanical framework is a complex and challenging problem. Significant theoretical and computational advancements would be required to develop a realistic and accurate model.

2.2 | Another Alternative Method to Deep Space Travel with Non-orientable Wormhole / Tunneling

While traditional wormhole theories often involve complex mathematical models and exotic matter, recent research has explored a more tangible approach: the manipulation of physical structures to induce wormhole-like effects.

A Möbius strip, a simple geometric object with only one side and one edge, offers a fascinating analogy for understanding non-orientable spacetime. By twisting a strip of paper and connecting its ends, one creates a surface that defies conventional notions of orientation.

2.2.1 | Crystal-Induced Wormholes: A Laboratory Experiment

Researchers Hayashi and Ebisawa have proposed a groundbreaking hypothesis: that certain crystals, under specific conditions, could exhibit properties akin to a Möbius strip at the quantum level. By carefully manipulating the crystal's structure and applying external stimuli, it may be possible to induce a localized curvature of spacetime, creating a microscopic wormhole [4-5, 13, 14].

2.2.2 | Superconductors: A Quantum Bridge

Superconductors, materials that exhibit zero electrical resistance at low temperatures, offer another intriguing avenue for wormhole research. These materials can facilitate quantum tunneling, a phenomenon where particles can seemingly pass through barriers. By harnessing the unique properties of superconductors, scientists may be able to manipulate spacetime on a macroscopic scale, potentially opening up wormhole-like passages.

2.2.3 | Challenges and Future Directions

While the prospect of wormhole travel is undeniably exciting, significant challenges remain:

- **Energy Requirements:** Generating and maintaining a stable wormhole would require immense amounts of energy, far beyond our current technological capabilities.
- **Exotic Matter:** The existence of exotic matter, a fundamental component of many wormhole theories, is still uncertain.
- **Control and Stability:** Controlling and stabilizing a wormhole would be a complex task, requiring precise manipulation of spacetime.

Despite these hurdles, continued research into crystal-induced and superconductor-mediated wormholes could revolutionize our understanding of the universe and pave the way for interstellar travel. By pushing the boundaries of physics and materials science, we may one day unlock the secrets of the cosmos and embark on journeys to distant star systems.

2.3 | Utilizing Similarity between Brain Neurons and Galaxy Clusters

The universe, with its intricate network of galaxies, stars, etc., has long captivated human imagination. As we delve deeper into the cosmos, we encounter questions about its origins, evolution, and potential for future exploration. Recent research has revealed intriguing parallels between the structure of the universe and the human brain, suggesting that these two complex systems may share fundamental principles [16, 17].

Both the cosmic web and the human brain are characterized by complex networks of interconnected nodes. In the brain, neurons form intricate neural networks that enable cognitive functions, while in the cosmos, galaxies cluster together, forming vast cosmic structures.

- **Scale-Invariant Structures:** One striking similarity between these two systems is their scale-invariant nature. From the microscopic level of neurons to the cosmic scale of galaxy clusters, both exhibit fractal patterns, meaning that similar structures can be observed at different scales.
- **Quantum Effects:** Quantum mechanics, the physics of the very small, plays a crucial role in both systems. Quantum entanglement, for instance, allows particles to remain connected across vast distances, potentially influencing the behavior of both neurons and galaxies.

2.3.1 | Harnessing the Cosmic Neural Quantum Effect

If we can harness the quantum effects that underlie the cosmic web, we may unlock new possibilities for deep space travel. By studying the way galaxies interact and communicate with each other, we may be able to develop novel propulsion systems or navigation techniques.

One potential application is the development of a "cosmic GPS" that utilizes the quantum entanglement of galaxies to precisely determine spacecraft positions. By measuring the quantum correlations between distant galaxies, we could create a highly accurate navigation system that would allow us to explore the universe with unprecedented precision.

Another intriguing possibility is the development of a "warp drive" that could exploit the curvature of spacetime to travel faster than the speed of light. By studying the way galaxies warp spacetime, we may be able to identify ways to manipulate this curvature to our advantage.

2.3.2 | Challenges and Future Directions

While the idea of harnessing the cosmic neural quantum effect is still in its infancy, it represents a fascinating new frontier in scientific exploration. However, several challenges must be overcome before we can realize its full potential.

- **Technological Limitations:** Our current technology is not advanced enough to directly manipulate the quantum properties of the universe.
- **Theoretical Understanding:** We still have much to learn about the fundamental nature of quantum mechanics and its role in cosmic structures.

Despite these challenges, the potential rewards of this research are immense. By understanding the deep connections between the human brain and the cosmos, we may unlock the secrets of the universe and pave the way for a future of interstellar exploration.

2.3.3 | Harnessing the Cosmic Neural Quantum Effect

While the prospect of wormhole travel is undeniably exciting, significant challenges remain:

3 | Discussion

Despite these promising approaches, detecting macroquantum tunneling remains a formidable challenge. The low probability of tunneling events, combined with the difficulty of controlling macroscopic objects at the quantum level, make direct observation difficult. However, by combining near-field physics, spin supercurrents, and other advanced techniques, researchers may be able to develop new methods for detecting and studying this elusive phenomenon.

Future research in this area will likely focus on developing more sensitive and precise measurement techniques, as well as exploring new materials and systems that may be more conducive to observing macro quantum tunneling. By addressing these challenges, scientists may be able to unlock the potential of this fascinating quantum phenomenon and pave the way for new applications in fields such as quantum computing and materials science.

One quite fascinating thing about the macro quantum tunneling effect is that Hartman tunneling time will not vary much especially for the large thickness of the barrier, and even if we consider macro quantum tunneling in the context of the ER=EPR bridge hypothesis, it probably would not involve too much time to pass the barrier to the edge of the EPR bridge (at least hypothetically). We discuss Hartman tunneling time in the following section.

As with the challenge to scale up from lab experiments on low-temperature wormhole tunneling to prototype scale, allow us to suggest that Salvatore Pais's patent with the title Piezoelectricity-induced high-temperature superconductor, perhaps can be used as a guideline to develop prototype-scale high-temperature superconductor for that purpose [15].

With regards to exploring the possibility of non-orientable wormhole tunneling via crystals or superconductors, we admit there are certain great hurdles. Nonetheless, continued research into crystal-induced and superconductor-mediated wormholes could revolutionize our understanding of the universe and

pave the way for interstellar travel. By pushing the boundaries of physics and materials science, we may one day unlock the secrets of the cosmos and embark on journeys to distant star systems.

4 | Concluding Remark

We have discussed in the present article, among other things that there is growing interest in novel approaches beyond existing technologies, especially in exploring the Deep Space and beyond [1-3]. In this regard, the Tsiolkovsky rocket equation, a cornerstone of astronautics, provides a fundamental relationship between a spacecraft's initial mass, final mass, exhaust velocity, and the change in velocity it can achieve. While this equation has been instrumental in numerous space missions, it also presents inherent limitations that hinder our ability to explore the vast expanse of the universe [1].

We also discussed the possible advantage of utilizing the macro quantum tunneling effect for instance what is known as the Josephson junction, which can be considered a phenomenon well-observed in lab settings, including in various low-temperature physics of superfluid and superconductor.

One interesting feature is called the Hartman effect, a known fact that Hartman tunneling time will not vary much especially for large thickness of barrier, and even if we consider macroquantum tunneling in the context of the ER=EPR bridge hypothesis, it probably would not involve too much time to pass the barrier to the edge of the EPR bridge (at least hypothetically).

Provided we are allowed to hypothesize that the aether medium is composed of quasicrystalline tessellation [11, 12] or something like the crystalline phase of iced water as we discussed previously, we may consider an alternative way to come up with Hartman tunneling time estimation, as well as in the usual time by defining quantum operator of time. [9, 10].

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Data Availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that there is no conflict of interest in the research.

Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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An Evidence-Based Approach to Set Theory Paradoxism: From Set Boundary to Mixed-Fermion-Boson Condensate Hypothesis

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Abstract: Physics thrives on precision, but paradoxes in set theory reveal limitations in our understanding of well-defined boundaries. Neutrosophic logic, challenging the *excluded middle* principle, introduces the concept of "betweenness" and partial belonging. This article explores among other things several possible avenues to resolve set theory paradoxism, including potential applications of neutrosophic logic in cosmology and particles, from set boundary, to the hypothetical "cosmosphere" boundary, to mixed fermion-boson condensate hypothesis. Embracing indeterminacy and fuzzy boundaries paves the way for a more holistic understanding of the universe's complexity.

Keywords: set theory paradoxism; boundaries of set; Neutrosophic Logic; betweenness; partial belonging; Lakoff & Nunez

1. Introduction

The concept of infinity has captivated mathematicians and philosophers for centuries, leading to groundbreaking discoveries and perplexing paradoxes. One such paradox stands tall: Russell's Paradox, a logical contradiction within set theory that threatened the very foundations of mathematics.[1] But what if the solution lay not in more complex axioms, but in a shift in our approach, moving from abstract symbol manipulation to an "*evidence-based*" framework grounded in human cognition?

Traditionally, attempts to resolve set theory paradoxism focused on constructing intricate axiom systems, like Zermelo-Fraenkel with Choice (ZFC). While successful in formalizing mathematics, these systems often feel removed from our intuitive understanding of infinity. Enter George Lakoff and Rafael Núñez, pioneers in cognitive science who propose a new perspective. They argue that mathematics, including set theory, is not an abstract, disembodied language, but rather a product of our embodied experience and conceptual knowledge. So, how can this cognitive lens help us tackle Russell's Paradox? Let's revisit the crux of the paradox: it arises when we consider the set of all sets that do not contain themselves. Does this set contain itself? If it does, it violates its own defining property. If it doesn't, then it contains all sets that don't contain themselves, including itself, leading to a contradiction.

From a cognitive perspective, the issue resides in our attempt to apply a single, uniform definition of "set" to all possible collections. In reality, our brains categorize and reason about different types of collections differently. Lakoff and Núñez propose that instead of a single "set" concept, we consider diverse conceptual categories like collections of physical objects, abstract

ideas, or potential actions. Each category comes with its own inherent constraints and logic, shaping how we reason about its members.

Applying this framework, we might recognize that the problematic "set of all sets that do not contain themselves" belongs to a category that is fundamentally self-referential and unbounded. Such a category may not be amenable to the same logical rules as collections of physical objects or finite sets. Recognizing this cognitive limitation, we can avoid the paradoxism by simply excluding such self-referential categories from our formal set theory, focusing instead on well-defined, grounded collections.

This "*evidence-based*" approach does not negate the value of formal systems like ZFC. Instead, it complements them by acknowledging the cognitive underpinnings of mathematics and emphasizing the importance of aligning formal structures with our intuitive understanding of the world. This can lead to a more "healthy" and accessible mathematics, less prone to paradoxes and closer to how humans naturally reason about quantity and infinity.[3]

Furthermore, this shift can open doors to exploring alternative set theories that better reflect different cognitive perspectives. Imagine set theories inspired by spatial reasoning, probability judgments, or even social interactions. Such explorations could not only enrich our understanding of infinity but also provide valuable insights into the cognitive diversity of mathematical thinking.

While Lakoff and Núñez's approach is in its early stages, it offers a promising avenue for addressing long-standing mathematical challenges. By embracing the evidence of our embodied cognition, we can move beyond abstract symbol manipulation and develop a more natural, "*evidence-based*" approach to mathematics, paving the way for a more inclusive and vibrant understanding of infinity.

2. Materials and Methods

The method used here is analysis and analogy with well-defined problems such as cell biology, diffusion and osmosis etc. toward rethinking of set theory paradoxism [6], especially in light of evidence-based physics and evidence-based mathematics. Recent literature which are relevant to the theme of this article have been cited.

3. Results

3.1. Approach #1: ***Cell model as boundary to any set, diffusion-osmosis interpretation to set paradox***

The shadow of Russell's Paradox(-ism) looms large over set theory, its logical contradiction threatening the very foundations of mathematics. Traditional solutions focus on complex axiom systems, but what if the answer resides not in abstract symbols, but in the concrete world of living cells? This article proposes a novel "cell model" inspired by Lakoff and Núñez's cognitive science approach, utilizing the concepts of diffusion and osmosis to shed light on the paradox.

Imagine a set as a living cell, bounded by a semipermeable membrane. This membrane regulates what enters and exits the set, just as a cell membrane controls the flow of molecules. Elements within the set are like nutrients inside the cell, while the surrounding "soup" represents potential members waiting to join.[1][2]

Now consider Russell's problematic set – the set of all sets that do not contain themselves. According to the cell model, this set's defining characteristic acts as a selective membrane. It allows sets that don't contain themselves to "diffuse" in (like nutrients), but it should also allow itself in, as it doesn't contain itself, leading to contradiction.

Here's where the concept of **osmosis** comes in. Osmosis describes the spontaneous movement of molecules across a semipermeable membrane to equalize concentrations. Applied to our cell

model, osmosis represents the inherent tendency of sets to be consistent and avoid paradoxes. When the problematic set tries to "diffuse" in, due to its self-referential nature, it triggers an "*osmotic pressure*" within the set. This pressure, analogous to the corrective force in osmosis, prevents the paradox by pushing the problematic set back out. Essentially, the set's defining characteristic itself acts as a barrier, preventing its own inclusion and maintaining consistency.

This model aligns with cognitive principles. Our minds naturally categorize and reason about collections differently, understanding physical objects differently from abstract ideas. The cell model reflects this, treating different types of sets as distinct "cells" with unique membranes and osmotic pressures.

This approach offers several advantages. It provides a more intuitive understanding of set boundaries and avoids complex formal machinery. It emphasizes the inherent limitations of self-referential sets, aligning with our cognitive constraints. Furthermore, it suggests alternative ways to think about set theory, inspired by biological processes like osmosis and cellular dynamics. While the cell model is a first step, further development is needed. Refining the analogy, exploring implications for different set types, and formalizing the osmotic pressure concept are crucial next steps.

In conclusion, the cell model and its osmosis interpretation offer a promising evidence-based approach to the set theory paradoxism. By grounding abstract concepts in the familiar world of living cells, we gain a new perspective, highlighting the importance of cognitive limitations and inherent dynamics within sets. This approach opens doors to a more intuitive and inclusive understanding of infinity, enriching both mathematics and our understanding of human thought.

3.2. Approach #2: ***Exploring Cognitive Constraints: A Categorical Approach to Set Theory Paradox***

This section explores how our cognitive limitations influence our understanding of infinity and contribute to paradoxes like Russell's paradox(-ism); cf. [3]. Drawing on the work of Lakoff and Núñez, you could:

- Analyze the cognitive categories we use to reason about collections, highlighting differences between physical objects, abstract ideas, and potentially unbounded sets.

- Explore how these categories shape our intuition and logic, leading to potential contradictions when applied to specific set definitions like "the set of all sets not containing themselves."

- Propose alternative set theories that respect these cognitive constraints, potentially by limiting self-referential definitions or introducing category-specific rules.

- Connect this approach to existing research in cognitive science and philosophy of mathematics, showcasing its evidence-based foundation.

This approach offers several advantages:

- Grounds the solution in evidence: It builds upon established research in cognitive science and avoids relying on unproven concepts like morphic fields.

- Addresses the root cause: It focuses on how our cognitive limitations contribute to the paradoxism, offering a deeper understanding of the issue.

- Connects to broader discussions: It aligns with ongoing research on embodied cognition and its impact on mathematics.

- Offers concrete solutions: It suggests alternative set theories grounded in cognitive constraints, contributing to the overall development of set theory.

It shall be kept in mind that, the goal of an *evidence-based approach* is to provide solutions supported by robust evidence and aligned with established scientific principles. By exploring well-researched areas like cognitive science and applying their insights to mathematical problems, we can contribute to a more robust and inclusive understanding of infinity and mathematics as a whole.[3]

3.3. *Approach #3:* ***Beyond the Excluded Middle: Exploring Neutrosophic Frontiers in Fermion-Boson systems***

Traditional physics excels in clear-cut definitions, but what if nature itself defies rigidity? This article explores the potential of neutrosophic logic, which goes beyond the "in" or "out" paradigm, to describe fuzzy boundaries and indeterminate states in the cosmos. We delve into intriguing possibilities like a partially defined border for the universe and hybrid particles exhibiting characteristics of both fermions and bosons. Accepting such "*betweenness*" challenges established paradigms and offers exciting avenues for future discoveries, leading us closer to a more nuanced picture of reality.

It is known, that aidst the elegance of its equations, paradoxes lurk, whispering of hidden complexities. One such riddle is the question of set theory paradoxism, where seemingly logical axioms lead to contradictory results. Traditionally, these paradoxes are resolved by upholding the "*excluded middle*" principle - every element either belongs to a set or doesn't. However, what if reality itself defies such crisp classifications?

This is where the intriguing idea of **Neutrosophic logic** emerges. It dares to challenge the rigidity of the excluded middle, introducing the notion of **partial belongingness** and **indeterminacy**. Instead of a traditional "in" or "out," elements can reside in a fuzzy "betweenness," exhibiting characteristics of both sets simultaneously. This opens a fascinating gateway to explore realms where traditional physics might reach its limits.

One intriguing application lies in the vast unknown beyond our familiar solar system. We know of the *heliosphere*, a thick "wall" of charged particles marking the boundary of our Sun's influence, as observed for instance by Voyager. Ref. [4] Could there be an analogous "***cosmosphere,***" a boundary to the observable Universe? If future observations reveal such a barrier, Neutrosophic logic could elegantly describe its nature. Objects within this boundary might exhibit degrees of both "inside-ness" and "outside-ness," existing in a state of partial belongingness.

Another promising avenue resides in the subatomic realm. The fundamental distinction between fermions and bosons, particles with differing statistics, forms a cornerstone of quantum mechanics. But what if, as hinted by **mixed fermion-boson statistics**, there also exist particles exhibiting **both fermionic and bosonic characteristics**? The prospect of observing a condensate corresponding to such a hybrid entity, transcending the traditional Bose-Einstein condensate, would challenge and even transform our very understanding of particle classification. That particular condensate may be called *mixed fermion-boson condensate* **(MFBC).** Ref. [5]

Exploring these Neutrosophic frontiers necessitates that we must embrace the possibility of **fuzzy boundaries, indeterminate states, and partial memberships**. This doesn't negate the value of traditional physics, but rather acknowledges its limitations in the face of the universe's inherent complexity.

The road ahead may be riddled with conceptual and experimental challenges. Yet, venturing into the uncharted territory of Neutrosophic physics holds immense potential. It could reshape our understanding of the cosmos, from its grandest boundaries to its quantum enigmas, leading us closer to a truly holistic picture of reality. So, let us cast aside the shackles of the excluded middle

and embrace the vibrant "betweenness" whispered by Neutrosophic logic. The Universe, in all its enigmatic splendor, awaits.

4. Applications and Concluding Remark

These authors have outlined several possible approaches to solve the set theory paradoxism, for instance by defining certain real boundary system to a given set, let say cell system. Such an *evidence-based physics* approach will allow us to figure out what can happen actually when certain entity goes in or goes out of any given cell through the boundary layer. Alternatively, we can figure out how Voyager space vehicle which goes through the boundary or thick wall of in the outer side of the Solar System, or it is often termed heliosphere, faces the edge or boundary of outer Solar System.[4] Similarly, we can hypothesize there is good likelihood that there is certain thick boundary in the foremost edge of the Universe, which may be termed *Cosmosphere*. Therefore in such a way, the meaning of set which contains all sets, i.e. the outer layer of the Universe that contains everything else inside the Universe can be figured out in astrophysics term.

We hope that more discussions of evidence-based physics approach to set theory paradoxism can be expected.

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Beyond Cryptic Equations: Reimagining Concepts in Physics Through Metaheuristics and Fantasy Stories using Neutrosophic Venn Diagram

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Abstract

Physics, the grand narrative of the universe, has long been viewed as a realm of cold, hard equations. But what if we looked beyond the formulas and considered a more imaginative origin for some of its concepts? This article explores the intriguing possibility that physics, and even cosmology, might share a surprising kinship with metaheuristics and fantastical fiction. Metaheuristics, a branch of computer science, deals with finding approximate solutions to complex problems. Perhaps the universe, in its vastness, employs a set of "rules" that lead to the most likely outcomes, much like an algorithm searching for the best solution within a vast space of possibilities. The connection strengthens when we consider the fantastical. Argentine writer Jorge Luis Borges, known for his thought-provoking short stories, often explored themes of infinity, labyrinths, and forking realities. In this article, we discuss, among other things, how to look at physics laws from an alternative fundamental viewpoint that is fluid dynamics perspective. As an example, we provide an outline for deriving the Newton gravitational law from the Kutta-Joukowski theorem, and then deriving the Kutta-Joukowski theorem from Bernoulli principles. In the meantime, it is known that vortex flows, related to solar convective turbulent dynamics at granular scales and their interplay with magnetic fields within intergranular lanes, occur abundantly on the solar surface and in the atmosphere above.

Keywords: Neutrosophic Set; Venn Digram; Mathematics; Physics.

Physics, the grand narrative of the universe, has long been viewed as a realm of cold, hard equations. But what if we looked beyond the formulas and considered a more imaginative origin for some of its concepts? This article explores the intriguing possibility that physics, and even cosmology, might share a surprising kinship with metaheuristics and fantastical fiction.

Metaheuristics, a branch of computer science, deals with finding approximate solutions to complex problems. Perhaps the universe, in its vastness, employs a set of "rules" that lead to the most likely outcomes, much like an algorithm searching for the best solution within a vast space of possibilities.

The connection strengthens when we consider the fantastical. Argentine writer Jorge Luis Borges, known for his thought-provoking short stories, often explored themes of infinity, labyrinths, and forking realities. These concepts bear a striking resemblance to ideas in modern cosmology or the possibility of an infinitely large universe. Could it be that the human mind, in its quest to understand the cosmos, naturally gravitates towards fantastical constructs that later find scientific backing?

Here's where the line blurs further. Metaheuristics often draw inspiration from natural phenomena, like simulated annealing mimicking the cooling process of metals. Could it be that our scientific understanding is a cyclical loop, where fantastical ideas inspire scientific inquiry, which then leads to new discoveries that further fuel our imaginations?

This is not to suggest that physics is mere fiction. Rather, it's a call to consider the creative spark that ignites scientific discovery. Perhaps the most fundamental laws of the universe are not just logical equations, but also a reflection of the human capacity for wonder and the creation of fantastical narratives.

As with Neutrosophic logic, it has been introduced by one of us (FS) what is termed as Neutrosophic Venn Diagram,[1] where the classical Venn diagram is generalised to a Neutrosophic Diagram, which deals with vague, inexact, ambiguous, ill-defined ideas, statements, notions, entities with unclear borders. In a neutrosophic Venn diagram, the traditional circles used in standard Venn diagrams are replaced with neutrosophic sets, which can include elements with a degree of truth, indeterminacy, and falsity. This allows for a more flexible representation of complex relationships between sets where the boundaries are not clearly defined. Now, allow us to reconsider three different sets of fantasy stories, physics and philosophy including ethics in a Neutrosophic Venn diagram as follows:

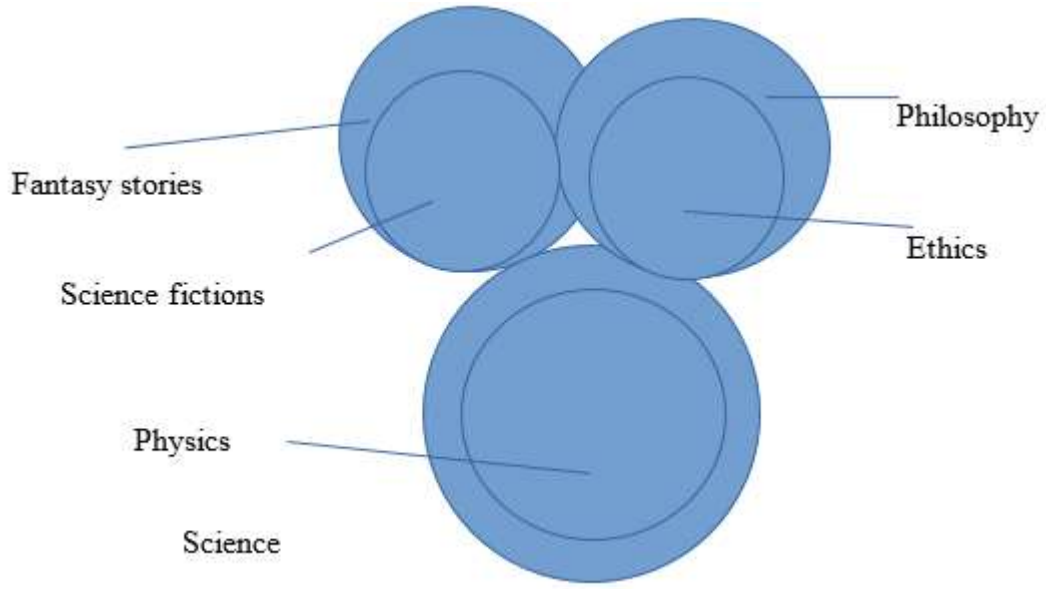

Figure 1. Neutrosophic sets depiction of science fictions, physics and philosophy/ethics.

The above diagram may help us to consider that sometimes physics and also science in general can work hand in hand along with science fiction, or with philosophy. And the intersection among the three is where physics sometimes is guided by philosophy exploration and science fiction to move forward, step by step. As it is often the case, such a broader view of physics belong to a wide category of science fiction can serve scientists to move beyond old boundaries and to explore the terra incognita, like the exploration to the Moon, etc.

This reframing has intriguing implications. It allows us to see physics not just as a rigid set of rules, but as a constantly evolving story, shaped by both observation and imagination. It encourages us to embrace the beauty

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and mystery of the cosmos, acknowledging that even the most advanced scientific theories are ultimately human attempts to understand the grand, fantastical reality that surrounds us.

2 | Reimagining Force and Fields: From Flow to Physics Laws

The great Chinese philosopher Mozi (c.470-391 BC) said, "*Force puts a body in motion*." Its modern statement is Newton's second law, where the body's acceleration is the effect, and the force is the cause. But what if these seemingly fundamental concepts – force and even field – could be reimagined through a different lens?

Intriguingly, even within the history of science itself, we find hints of fluidity. Isaac Newton, in his early forays into calculus, referred to it as "*fluxions*," suggesting a dynamic, ever-changing nature. This fluidity resonates with the concept of a **force field** – a region of space where an object experiences a push or pull.

Could we redefine these concepts by drawing inspiration from the idea of a **fluid**? Imagine forces not as singular pushes or pulls, but as gradients within a flowing medium. A stronger force would be like a steeper incline in this fluid, causing a greater change in an object's motion (acceleration) as it traverses it.

This reframing aligns with Mozi's notion of force initiating motion. Just as an object entering a flowing river experiences a change in its state (from rest to motion), an object entering a force field, interpreted as a *fluid-like medium*, would experience a change in its state of motion. The benefits of such a reimagining are twofold. First, it acknowledges the historical fluidity of scientific concepts. Second, it injects a sense of dynamism into our understanding of forces and fields. They become less like static entities and more like ever-shifting currents shaping the motion of objects within them.

This doesn't diminish the power of these concepts. Newton's second law remains a cornerstone of physics. However, by viewing forces and fields through the lens of fluidity, we open doors for deeper exploration. Perhaps this fluidity hints at a unified field theory encompassing all fundamental forces.

The journey of scientific discovery is often one of reimagining the familiar. By embracing fluidity, we can unlock a new perspective on these foundational concepts, propelling us further in our quest to understand the universe.

3 | Rethinking the Pillars: Gravity and Electromagnetism Beyond Equations

Newton's law of universal gravitation (F = G * m1 * m2 / r^2) and Maxwell's equations have long been considered the cornerstones of classical physics, describing the behavior of gravity and electromagnetism, respectively. However, recent explorations suggest these pillars might not be as rigid as we once thought. Let's delve into two intriguing possibilities that challenge the traditional view.

3.1 | From Equations to Flow: Reimagining Electromagnetism

Physicist Hector Munera, referencing the work of Henri Malet, proposes a radical reinterpretation of Maxwell's equations. Instead of complex mathematical formulas, he suggests viewing them through the lens of fluid vectors. Imagine electromagnetism not as isolated forces, but as properties of a flowing medium. The flow's characteristics determine the behavior of electric and magnetic fields [1].

This fluid analogy aligns with the historical fluidity of science. Just as Isaac Newton referred to calculus as "fluxions," a flowing concept, so too can electromagnetism be viewed as a dynamic process. This reframing offers new avenues for understanding phenomena like wave-particle duality, where light exhibits both wave-like and particle-like behavior. Perhaps the flow itself has a wave-particle nature.

3.2 | Turbulence Theory and a New Gravity?

Another avenue for rethinking physics comes from the world of fluid dynamics. Kolmogorov's theory of turbulence describes the chaotic, unpredictable motion of fluids. Interestingly, some physicists, propose that this theory might be more fundamental than the Planck constant – a cornerstone of quantum mechanics [2].

This has a surprising implication for gravity. Could Newton's equation (F = m * a) be reinterpreted using principles from turbulence theory? Perhaps the gravitational force arises not from a simple attraction between masses, but from a more complex interaction within a turbulent "gravitational fluid." This reinterpretation might connect the seemingly disparate worlds of classical gravity and quantum mechanics.

These are just two examples of how rethinking fundamental physics can be fruitful. By moving beyond equations and embracing fluidity and turbulence, we may unlock deeper connections between seemingly disparate phenomena. This doesn't diminish the validity of existing theories, but rather enriches them with new perspectives, potentially leading to a more unified understanding of the universe.

4 | Rethinking the Foundation: Kolmogorov's Turbulence as a Cornerstone of Physics?

Quantum mechanics has reigned supreme for nearly a century, unraveling the bizarre world of the very small. But what if there's another contender for the title of physics' foundational theory? Enter Andrey Kolmogorov's theory of turbulence – a theory that describes the chaotic, unpredictable motion of fluids – and a growing movement that suggests it might hold a more fundamental role than previously imagined; cf. Ref. [2, 9-10].

Traditionally, quantum mechanics, with its probabilistic nature and wave-particle duality, has been seen as the key to unlocking the secrets of the universe's building blocks. However, turbulence theory offers a compelling alternative. Here's why:

- **Universality:** Kolmogorov's theory transcends the specific fluid being studied. It reveals a set of universal scaling laws that govern turbulent flow across different scales, from swirling smoke to churning galaxies. This universality resonates with the quest for a unified theory in physics, one that encompasses all fundamental forces.
- **Complexity from Simplicity:** The elegance of Kolmogorov's theory lies in its ability to generate immense complexity from a relatively simple set of rules. Similarly, the universe exhibits a mind-boggling array of phenomena, yet might be governed by a set of underlying principles that turbulence theory could help us decipher.
- **Quantum Connection:** Intriguingly, some physicists propose a connection between turbulence and the probabilistic nature of quantum mechanics. Perhaps the seemingly random behavior of particles at the quantum level is an emergent property of a more fundamental turbulent flow at a deeper level of reality.

This shift in perspective has significant implications. It suggests that the chaotic dance of fluids might be the Rosetta Stone for understanding the universe, from the tiniest subatomic particles to the grandest cosmological structures.

Can the Planck Constant be Derived from the Kolmogorov Constant?

The realm of physics encompasses diverse phenomena, from the microscopic world of quantum mechanics to the large-scale structures of turbulence captured by Kolmogorov's constant. While seemingly disparate, researchers have delved into potential connections between these seemingly unrelated concepts. One such intriguing proposition is the possibility of deriving the Planck constant, a cornerstone of quantum mechanics, from the Kolmogorov constant, a crucial parameter in turbulence theory.

Understanding the Constants:

- **Planck constant (h):** This fundamental constant quantifies the smallest discrete unit of action, a crucial concept in quantum mechanics. It governs the energy of photons (light quanta) and plays a vital role in various areas like black body radiation and the photoelectric effect.
- **Kolmogorov constant (C):** This constant emerges in the study of turbulent fluid flow. It relates the rate of energy dissipation at the smallest scales of the flow to the average rate of energy injection at larger scales. Understanding this constant helps predict the behavior of turbulent flows.

Existing landscape:

Currently, there is no established theoretical framework that directly derives the Planck constant from the Kolmogorov constant. Both constants represent distinct physical phenomena with contrasting theoretical

backgrounds. The Planck constant stems from the foundation of quantum mechanics and the quantization of energy, while the Kolmogorov constant emerges from the statistical analysis of turbulent flow in classical physics. See discussion in Ref. [2, 9-10].

5 | Discussion: Proof of Concept

In this section allow us to discuss three outlines of proof of concepts to support aforementioned arguments suggesting that the underlying laws of nature are to be found in fluid dynamics, not just an old concept of forces.

a. To reconcile Newton gravitation action with Bernoulli principles:

For centuries, our understanding of the universe has relied heavily on the concept of forces acting at a distance. Newton's law of gravity, a cornerstone of classical physics, exemplifies this approach. However, a new perspective is emerging, suggesting that the underlying laws of nature might be found in fluid dynamics, the study of fluids in motion. This article explores three key ideas that support this bold claim:

1. Gravity from Fluid Flow: Deriving Newton law from Bernoulli principles

Newton's law of gravity states that two objects with mass attract each other with a force proportional to the product of their masses and inversely proportional to the square of the distance between them. However, fluid dynamics offers a potentially revolutionary alternative. Bernoulli's equation, a fundamental principle in fluid mechanics, relates pressure, velocity, and density in a flowing fluid. The proposition is that, by understanding the behavior of a hypothetical all-pervading fluid (the "aether" as some have called it), we could derive the effects of gravity as emergent properties arising from the flow and pressure gradients within this fluid. This would eliminate the need for a mysterious "pull" force acting across vast distances and instead explain gravity as a consequence of the interaction of matter with the underlying fluidic medium.

In the Appendixes section, we provide outlines of derivation of Newton gravitation law from Kutta-Joukowski theorem, and from Kutta-Joukowski theorem from Bernoulli principles. While these are a few codes of rough outlines, interested readers can discuss more on that topics. See also Ershkov [4].

2. Unifying Gravity and Lift: A Fluid Connection

One of the challenges with the traditional force-based approach is reconciling gravity with the concept of lift. An airplane, for example, generates lift not because of some anti-gravity force, but due to the way it alters the airflow around its wings. A fluidic understanding of gravity could offer a more unified picture. By analyzing the interaction of the airplane with the surrounding "aether," we could explain lift as a consequence of how the airflow creates pressure differentials above and below the wing, generating an upward force. This approach could potentially unify our understanding of seemingly disparate phenomena under a single set of fluid dynamic principles.

b. A new hypothesis of nonlocality interaction ala quantum physics by virtue of spin supercurrent in low temperature physics:

Quantum physics has revealed a universe stranger than we could have imagined. One of its most puzzling aspects is nonlocality, the phenomenon where entangled particles seem to instantaneously influence each other, regardless of the distance separating them. This seemingly spooky action at a distance has challenged our understanding of reality for decades.

In a recent book chapter published on IntechOpen, we explored a novel hypothesis: could **spin supercurrents**, a specific type of current arising from the intrinsic spin of electrons, be the underlying mechanism behind nonlocal interactions?[3]

Spin Supercurrents: A Bridge Across the Divide?

Superconductors, materials that exhibit zero electrical resistance under specific conditions, display fascinating properties. One such property is the ability to sustain persistent currents, even in the absence of an external driving force. These currents, known as supercurrents, can carry information and energy with remarkable efficiency.

Our hypothesis delves deeper, proposing the existence of **spin supercurrents**. These currents involve the synchronized flow of electron spins, potentially offering a new perspective on nonlocality. The idea is that

entangled particles, through their spin states, might be able to couple to and influence the flow of these spin supercurrents across vast distances. This could provide a physical explanation for the instantaneous correlation observed in entangled systems, without resorting to the notion of instantaneous communication across space. See also Appendix 3 included below.

Exploring the Implications

This hypothesis, while still in its formative stages, holds potential for various implications.

- **Unifying Frameworks:** It could offer a bridge between quantum mechanics and classical physics, potentially explaining nonlocality through well-understood principles of electromagnetism and superconductivity.
- **Biological Applications:** The research opens avenues for exploring the role of spin supercurrents in biological systems, where nonlocal-like phenomena like biophotons and synchronized brain activity have been observed.

The proposed mechanism requires further theoretical development and rigorous experimental validation. However, the potential for a novel explanation of nonlocality, based on established physical principles, is a significant step forward.

c. Plausible explanation of Mercury planet's perihelion not derivable from frame dragging, but from swirl effect of Sun vortex:

One of the enduring puzzles of general relativity is the slight wobble in the orbit of Mercury, the planet closest to the Sun. According to Einstein's theory, this wobble can be attributed to "frame-dragging," the warping of spacetime caused by the Sun's massive rotation. However, a new perspective is emerging, suggesting that fluid dynamics might offer a simpler explanation.

Frame-Dragging: The Einsteinian Explanation

General relativity describes gravity not as a force, but as a curvature of spacetime caused by the presence of mass and energy. A massive spinning object, like the Sun, drags spacetime around with it, much like a swirling fluid. This "frame-dragging" effect causes the otherwise straight path of a nearby object, like Mercury, to precess, or wobble slightly over time.

A Swirling Sun: Fluid Dynamics and Mercury's Orbit

Fluid dynamics offers an alternative explanation for Mercury's precession. Imagine the Sun not just as a point mass, but as a giant ball of swirling hot plasma. The Sun's rotation creates a vortex in the surrounding space, akin to a whirlpool. As Mercury orbits the Sun, it interacts with this swirling vortex. The fluid-like properties of spacetime could then explain the precession of Mercury's orbit, without invoking the complexities of frame-dragging. According to Fu Yuhua [6], by simple deduction, the circular velocity of this vortex motion at the position of radius r as follows:

$$v \approx \frac{3G^{3/2}M^{3/2}}{r^{3/2}c^2} \qquad (1)$$

Therefore, he concludes that unlike the ordinary vortex motion (its circular velocity is inversely proportional to the radius r), for solar system's vortex motion, the circular velocity is inversely proportional to $r^{3/2}$. While that result may or may not coincide with observed advanced of perihelion of Mercury, it provides an alternative framework to analyse, therefore in the Appendix #4, we outlined implications of such deduction result.

Advantages of the Fluidic Approach

The fluidic approach offers several potential advantages:

- **Conceptual Simplicity:** Fluid dynamics offers a more intuitive picture, easier to visualize than the abstract concept of frame-dragging.
- **Unified Framework:** It could potentially provide a more unified understanding of gravity, incorporating it within the broader principles of fluid mechanics.

While intriguing, the fluidic explanation faces challenges. Quantifying the precise effects of a swirling solar vortex on Mercury's orbit requires further theoretical development. Additionally, experimental verification might be difficult due to the complex nature of the Sun's environment; see also [6-8].

6 | Concluding Remark

We discussed here how fluid dynamics descriptions can be more fundamental to depict nature behaviour, rather than force laws as we all know since childhood. We outlined several arguments, along with several suggestions as proof of concepts, including a new hypothesis of spin supercurrent as fundamental mediation of nonlocality interaction a la quantum mechanics.

While intriguing, the fluidic explanation faces challenges. Quantifying the precise effects of a swirling solar vortex on Mercury's orbit requires further theoretical development. Additionally, experimental verification might be difficult due to the complex nature of the Sun's environment.

Of course, this is not to say that Kolmogorov's theory provides all the answers. It struggles to explain specific phenomena successfully addressed by quantum mechanics. However, it does propose a framework for a more unified physics, one that incorporates the ideas of randomness and emergence.

Further research is needed to explore these connections and develop a comprehensive framework based on turbulence. This might involve reformulating gravity and electromagnetism within a turbulent paradigm. The journey will be challenging, but potentially very rewarding.

By elevating Kolmogorov's theory to a more fundamental role, we might unlock a deeper understanding of the universe, one where the seemingly random fluctuations of fluids hold the key to unravelling the grand tapestry of existence.

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Data Availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that there is no conflict of interest in the research.

Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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Appendix 1

In the Appendixes section, we provide outlines of derivation of Newton gravitation law from Kutta-Joukowski theorem, and from Kutta-Joukowski theorem from Bernoulli principles. While these are a few codes of rough outlines, interested readers can discuss more on that topics.

The Kutta-Joukowski theorem is typically used in the context of aerodynamics, particularly in the field of potential flow theory for airfoils. It relates the circulation around an airfoil to the lift force it generates. The direct derivation of Newton's law of gravitation from Bernoulli's principles using the Kutta-Joukowski theorem may not be a standard approach.

Here is an outline of approach in Mathematica code:

```
(* Define constants *)
rho = 1;          (* Fluid density *)
U = 1;            (* Freestream velocity *)

(* Define parameters for the circular disk (analogous to airfoil) *)
radius = 1;       (* Radius of the disk *)
circulation = 2Pi; (* Circulation around the disk *)

(* Define the complex potential for the flow around the disk *)
complexPotential[z_] := U*(z + radius^2/z) + I*circulation*Log[z];
(* Define the velocity field by taking the derivative of the complex potential *)
velocityField[z_] := D[complexPotential[z], z];
(* Extract the real and imaginary parts of the velocity field *)
u[x_, y_] = Re[velocityField[x + I*y]];
v[x_, y_] = Im[velocityField[x + I*y]];
(* Plot the velocity field using StreamPlot *)
```

```
StreamPlot[{u[x, y], v[x, y]}, {x, -2, 2}, {y, -2, 2},
  AspectRatio -> Automatic, StreamPoints -> Fine]
(* Calculate and plot the pressure field *)
pressureField[z_] := -0.5*rho*(Abs[velocityField[z]]^2 - U^2);
ContourPlot[pressureField[x + I*y], {x, -2, 2}, {y, -2, 2},
  AspectRatio -> Automatic, ContourLabels -> True,
  ContourShading -> False]
```

This code simulates the potential flow around a circular disk and visualizes the velocity field using streamlines. Note that this is a simplified model, and deriving gravitational effects directly from Bernoulli's principles through the Kutta-Joukowski theorem may require additional considerations or assumptions.

Appendix 2

Now we provide an outline of Mathematica code to prove that it is possible to derive Kutta-Joukowski theorem from Bernoulli principles. While these are a few codes of rough outlines, interested readers can discuss more on that topics.

This code is to illustrate how the lift force in potential flow around a rotating cylinder (related to the Kutta-Joukowski theorem) can be computed using Bernoulli's equation.

Here's a simple example using potential flow theory and Bernoulli's equation to estimate the lift force:

```
(* Define parameters *)
radius = 1;            (* Radius of the cylinder *)
freeStreamVelocity = 1; (* Free stream velocity *)
circulationStrength = 2; (* Strength of the circulation around the cylinder *)
(* Define the stream function for a rotating cylinder *)
ψ = Function[{x, y}, freeStreamVelocity*y + circulationStrength*ArcTan[(y - radius)/x]];
(* Compute velocity components *)
u = D[ψ[x, y], y];
v = -D[ψ[x, y], x];
(* Compute pressure using Bernoulli's equation *)
pressure = 0.5*(freeStreamVelocity^2 - (u^2 + v^2));
(* Lift force per unit length using Kutta-Joukowski theorem *)
liftForce = circulationStrength*freeStreamVelocity;
(* Display results *)
Print["Lift force per unit length (Kutta-Joukowski theorem): ", liftForce];
Print["Pressure distribution along the cylinder:"];
ContourPlot[pressure, {x, -2, 2}, {y, -2, 2}, ContourLabels -> True, Contours -> 20, ColorFunction -> "Rainbow", PlotRange -> All]
```

In this example, we define a stream function for a rotating cylinder, compute velocity components, and use Bernoulli's equation to estimate the pressure distribution around the cylinder. The lift force per unit length is then calculated based on the Kutta-Joukowski theorem.

Keep in mind that this is a simplified example, and potential flow theory has its limitations.

Appendix 3

In this section, we provide outline of nonlinear vector potential ala Aharonov effect to be derivable from spin supercurrent. However, it's important to note that deriving the nonlinear vector potential from spin supercurrent involves complex quantum field theory concepts and may not be captured in a simple Mathematica code snippet. Here's a very simplified and abstracted outline:

```
(* Define variables and parameters *)
\[Phi] = Pi;  (* Magnetic flux *)
q = 1;        (* Charge of the particle *)
hbar = 1;     (* Reduced Planck constant *)
(* Define the nonlinear vector potential as a function of spin supercurrent *)
nonlinearVectorPotential[supercurrent_] := Module[{vectorPotential},
  (* Incorporate the Aharonov-Bohm phase *)
  vectorPotential = supercurrent + (q/\[Pi]) Integrate[1/(r - r0), {r, r0, \[Infinity]}];
  vectorPotential
]
(* Define spin supercurrent as a function of the spin density and other parameters *)
spinSupercurrent[spinDensity_, magneticField_] := Module[{supercurrent},
  (* Implement the relationship between spin density and supercurrent *)
  supercurrent = 2 q/hbar SpinDensityMatrix[spinDensity].magneticField;
  supercurrent
]
(* Example usage *)
spinDensity = PauliMatrix[3];  (* Example spin density matrix *)
magneticField = {0, 0, B};     (* Magnetic field vector *)
(* Calculate nonlinear vector potential using spin supercurrent *)
result = nonlinearVectorPotential[spinSupercurrent[spinDensity, magneticField]];
(* Display the result *)
Print["Nonlinear Vector Potential: ", result]
```

This code provides a simple framework where the nonlinear vector potential is defined based on a spin supercurrent, incorporating the Aharonov phase. The actual details and correctness of such a derivation would involve a rigorous quantum field theory treatment.

Appendix 4

It is known from celestial mechanics textbooks, that Kepler's laws describe the motion of planets around the Sun within the framework of classical celestial mechanics, while Newton's law of gravitation provides the underlying physics.

#4.a. To illustrate the approximation of Kepler's laws from Newton's law, we can consider a simplified version; here's a basic example using Mathematica:

```
(* Define variables and parameters *)
G = 6.6743*10^-11;     (* Gravitational constant *)
M = 1.989*10^30;       (* Mass of the Sun in kilograms *)
m = 5.972*10^24;       (* Mass of Earth in kilograms *)
(* Newtonian gravitational force *)
force[r_] := G * (M * m) / r^2;
(* Use Newton's second law to model Earth's motion around the Sun *)
motionEquation = m r''[t] == force[r[t]];
(* Solve the motion equation using NDSolve *)
solution = NDSolve[{motionEquation, r[0] == 1.496*10^11, r'[0] == 0}, r, {t, 0, 365*24*60*60}];
(* Plot the orbit *)
Plot[Evaluate[r[t] /. solution], {t, 0, 365*24*60*60}, AxesLabel -> {"Time (s)", "Distance from Sun (m)"}]
```

This code sets up a simplified model of Earth's motion around the Sun using Newton's second law. The resulting plot shows the orbit of Earth over one year.

The next step as proposed by Fu Yuhua is to hypothesize second vortex motion has effect to surrounding objects having inverse square law with 3/2 power of r [6]. While this of course remains hypothetical, the following is an outline of Mathematica code:

```
(* Define variables and parameters *)
G = 6.6743*10^-11;  (* Gravitational constant *)
M = 1.989*10^30;    (* Mass of the Sun in kilograms *)
(* Inverse law for the radius to the 3/2 power *)
inverseLaw[r_] := r^(-3/2);


(* Kepler's third law for the orbital period *)
keplersThirdLaw[a_] := Sqrt[a^3 / (G * M)];
(* Define the motion equation for the second vortex *)
motionEquation2[r_, t_] := r''[t] == -G * M * inverseLaw[r[t]] / r[t]^2;
(* Solve the motion equation using NDSolve *)
initialPosition = 1.496*10^11;  (* Initial distance from the Sun in meters *)
initialVelocity = 30000;        (* Initial velocity in meters per second *)
```

```
solution2 = NDSolve[{motionEquation2[r, t], r[0] == initialPosition, r'[0] == initialVelocity}, r, {t, 0, 365*24*60*60}];

(* Plot the orbit of the second vortex *)

Plot[Evaluate[r[t] /. solution2], {t, 0, 365*24*60*60}, AxesLabel -> {"Time (s)", "Distance from Sun (m)"}]
```

In this very simple example, it has been introduced an inverse law for the radius to the 3/2 power and used it in the motion equation for a hypothetical second vortex. The orbital parameters are determined based on Kepler's third law.

#4.b. Alternatively, the following Mathematica code is to model the motion of an object under the influence of both the conventional Kepler law and an additional vortex motion with an inverse square law raised to the power of 3/2, you can use the following Mathematica code. This code assumes that the vortex motion affects the object in the plane of motion and is perpendicular to the Keplerian motion:

```
(* Define constants and variables *)

G = 6.67430*10^(-11);  (* Gravitational constant *)

M = 1.989*10^30;       (* Solar mass *)

m = 1;                 (* Mass of the test object *)

a = 1.5;               (* Vortex motion exponent *)


(* Define the position of the object as a function of time *)

position[t_] := {x[t], y[t]};

(* Define the gravitational force function with vortex motion *)

gravitationalForce[x_, y_] := -G * M / (x^2 + y^2)^(3/2) * {x, y} -
   G * M / (x^2 + y^2)^(3/2) * a * {y, -x};

(* Define the differential equations for motion *)

equationsOfMotion = {
  m * x''[t] == gravitationalForce[x[t], y[t]][[1]],
  m * y''[t] == gravitationalForce[x[t], y[t]][[2]],
  x[0] == x0, y[0] == y0, x'[0] == vx0, y'[0] == vy0
  };


(* Set initial conditions *)

x0 = 1; vy0 = 0;

y0 = 0; vx0 = Sqrt[G * M / x0];

(* Solve the differential equations *)

solution = NDSolve[equationsOfMotion, {x, y}, {t, 0, 10}];

(* Plot the trajectory of the object *)

ParametricPlot[Evaluate[{x[t], y[t]} /. solution], {t, 0, 10},
 AxesLabel -> {"x", "y"}, AspectRatio -> 1, PlotRange -> All]
```

This code uses the NDSolve function to solve the system of differential equations that describe the motion of the object under the combined influence of Keplerian and vortex motions. Adjust the initial conditions and parameters as needed

***

Kindly note that this is a purely illustrative model and may not reflect the actual dynamics of celestial bodies around the Sun.

***

Godel, Escherian Staircase and Possibility of Quantum Wormhole With Liquid Crystalline Phase of Iced-Water - Part I: Theoretical Underpinning

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ABSTRACT
As a senior physicist colleague and our friend, Robert N. Boyd, wrote in a journal (*JCFA*, Vol. 1,. 2, 2022), Our universe is but one page in a large book [4]. For example, things and Beings can travel between Universes, intentionally or unintentionally. In this short remark, we revisit and offer short remark to Neil's ideas and trying to connect them with geometrization of musical chords as presented by D. Tymoczko and others, then to Escher staircase and then to Jacob's ladder which seems to point to possibility to interpret Jacob's vision as described in the ancient book of Genesis as inter dimensional or heavenly staircase, e.g. an inter dimensional bridge between heaven and earth (see also classic book: Hofstadter, Godel, Escher, Bach). Jacob's vision of angels going down to earth from that staircase has been depicted for instance in William Blake art etc. In our communication with others via physics literature and discussions etc, we came to several conclusions as follows: Firstly, possibility of quantum wormhole effect to mirror particle universe, which sometimes it is termed non-orientable wormhole. While such mirror particles effect have been more than 50 years predicted with the so-called parity violation (cf. Lee & Yang, 1950s), and that is called symmetry breaking. Secondly, a series of extended experiments on laser irradiated cold water may suggest possible transition from a phase of water to be at least partially fourth phase of water, which may be composed of crystalline water (see e.g. Gerald Pollack, and also Harold Aspden on liquid crystalline). If we can imagine laser cooling effect can be done in protracted time, then we can achieve a physical representation of Aspden's liquid crystalline. Therefore, in subsequent article (Part II) we outlined simple model of such an effect of tunneling via quantum liquid crystalline Universe, which may likely be modeled with iced-water. It is interesting to remark here that certain experiments by Stockholm University scientists have shown that X-ray triggered water can exhibit properties just like liquid crystal (cf. *PRL*, 2020). That is why we consider it possible that there can be quantum phase transition where liquid water (comprised of iced cubes and water) can exhibit effects such as tunneling in quantum liquid crystalline Universe. Last but not least, we admit that what we outlined here is just an initial phase; and if you wish, perhaps we can call such experiments as "wormhole-at-lab" experiments (abbreviated: *WHALE*).

Keywords: Universe, Cosmological Physics, Inter dimensional Bridge, Cosmological Experiments, Low Temperature Physics, Quantum Tunneling, Aspden Liquid Crystalline, Jacob's Vision

1. INTRODUCTION

According to Robert N. Boyd's experience and insights, although there are various perceptive developments on what the Universe is like, which presently known as modern cosmology, from ancient sources such as the book of Genesis (e.g. the story of Jacob's ladder) or ancient Indian Hopi, we can learn that the we live in a *vast Cosmos*, which may require really advanced geometry to comprehend its geometry, topology etc. although in daily life we perceive it as 3D space. [1][3][4]

Among other things, Neil wrote in his article [4]: "For example, things and Beings can travel between Universes, intentionally or unintentionally. The pages of the "book" of Universe are connected at a common point and move outwards in a rotation, overlaid in a spiral manner, related to the *phi* ratio. Each "page" which is "touching" the next page and the previous page, has physical laws and forms of Consciousness that are only slightly different from one another.

The further you go in or out, *along the spiral*, the more unrelated the various Universe's laws and Beings are, so that if you move many "pages" away from your starting page, the features of that Universe will be completely outrageously different from where you are now.

…

Most of gateways allow travels both in and out of the specific portal. Each portal has a specific number, and specific types, of destination locations."

From the above quote from Neil [4], we can interpret geometrically as follows: that we perceive the world in daily experience as 3D space, but the actual reality may be better translated for instance in William Thurston's exploration of 3D space topology, including what he termed as orbifold.Interestingly, Dmitri Tymoczko offers an interesting connection between geometry of musical scale and orbifold geometry. [5][6] In the meantime, others have explored orbifold mathematics [9][10].

2. THEORETICAL UNDERPINNINGS: REMARK ON DMITRI TYMOCZKO'S *GEOMETRY OF MUSICAL SCALE*

With reference to such a Jacob's ladder story from ancient book of Genesis, allow us to offer an interpretation as follows: With the context of this spiritual journey, we can interpret the story of Jacob's dream about angels descending from heaven at Bethel as a spiritual experience, especially in the context of Hopi Indian mysticism. Our universe is just one page in a big book. Each "page" can "touch" the next page and the previous page. Nonetheless, what kind of advanced geometry to be able to describe that interdimensional travel (or perhaps we can call as "*Escher staircase*"), it is still to be explored.

The details of 3D space topology and its related orbifold are quite involved. Nonetheless, allow us to present here quote from a review on Tymoczkos article in *Science 2006* [5]:

"Familiar relationships between sets of musical notes, such as transposition between chords, directly translate into geometrical structures such as this Möbius strip — where each dot represents a whole class of equivalent two-note chords — or into more complex structures with many dimensions. Composers have an understanding of these geometries without realizing it, says music theorist Dmitri Tymoczko of PrincetonUniversity. "Musicians like Chopin had a very direct, intuitive understanding of these spaces at a time when mathematicians still didn't know much about high-dimensional geometry," he says. Wandering around these spaces, Tymoczko and his collaborators have found subtle relationships between progressions of chords that traditional musical theory would classify as unrelated — for example, between progressions in Mozart's Fantasy in C-minor and in Beethoven's Ninth Symphony."[1]The following illustrations may help readers to imagine what kind Mobius strip involved:

[1]Davide Castelvecchi. Godel, Escher, Chopin. *ScienceNews* May 2008. Url: https://www.sciencenews.org/article/g%C3%B6del-escher-chopin

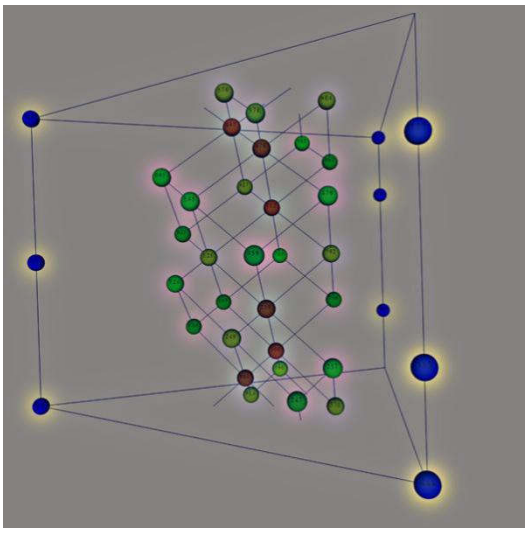

Illustration 1: Trichord prism

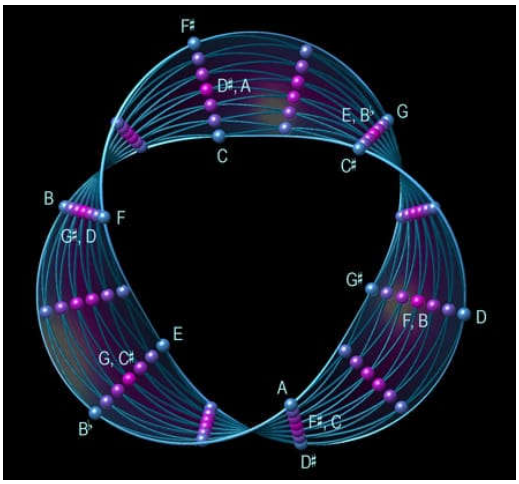

Illustration 2: Möbius strip related to musical scale[2]

While all of these are interesting and worthy to explore further, we shall note that Tymoczko's exploration of math and geometry behind musical scale are mostly based on analysis of more than 300 years of Western music. It is our assertion that exploration of this kind may be extended to *pentatonic scale*, which can be found throughout traditional musics in Asia, and also later on in folk music in the West too. Beside pentatonic scale is found in Chinese folk music, as Sasaki & Masunah note: "In Javanese and Balinese music, only two types of laras are recognized, namely slendro and pelog, while in Sundanese music, there are three types of laras: *salendro, pelog, and sorog*. These three scales are all clearly pentatonic." [11][12]

Therefore, provided we assert that ancient traditional arts such as Indian Hopi and Navajo, may have deep connection with traditional arts in China and also other parts of Asia such as Javanese and Sundanese music etc, then we may suppose that such a pentatonic scale also carry deep geometry insight hidden from modern music, i.e. the trace from ancient beliefs, such as Jacob's vision of heavenly staircase connecting interdimensional spiritual doors between heaven and earth.

All in all, although our knowledge is very limited for know, allow us to conclude that Tymoczko's exploration along with Thurston's geometry of 3D space may point to further exploration to deep geometry of spirally heavenly staircase as Jacob envisioned in the book of Genesis

3. DISCUSSION

Possibility of non-orientable wormhole to mirror world, or else, of quantum wormhole-like effect via liquid crystal simulation with iced-water

Provided we accept that Möbius strip geometry may hold an answer to the aforementioned problems, it seems instructive to consider how parity symmetry and its violation can play certain role here. Physicists usually regard the study of CP violation addresses as a very fundamental question: are the laws of physics the same for matter and anti-matter and *possibly* un-matter, or are matter and anti-matter intrinsically different? In other words, mirror symmetry can be restored if mirror particles can be observed. That is a premise that V.I. Dokuchaev & Y.N. Eroshenkov arguing in ref. [13], and they put forward this notion to suggest

[2] https://www.sciencenews.org/article/g%C3%B6del-escher-chopin

that even a non-orientable wormhole is possible. In particular, it is stated that the mirror particles are transformed into the ordinary ones and vice versa while traversing through the non-orientable wormhole. This statement was previously applied only to the universe with a non-orientable topology [13]. This can be quite interesting, provided it is assumed that we live in a Universe with non-orientable topology [cf. 24], and this matter can still be considered in the context of duality of electric-magnetic theories. For instance, such a duality of electric-magnetic theories can be explored in variety of quaternionic Maxwell theory or quaternionic Dirac theory (cf. our articles at *Octogon Mathematical Magazine*, 2022, [17]).

Nonetheless, more recent findings seem to suggest that such mirror symmetry may not hold the true symmetry of Nature. The discussion has been started since Lee & Yang's seminal paper and so on.[14] Therefore, another alternative is to find possibility of quantum wormhole or quantum tunneling effect in liquid crystal, for instance see an article on optical wormhole [15]. In this regards, it is interesting to remark that a series of extended experiments on laser irradiated cold water suggest possible transition from liquid phase of water to be at least partially fourth phase of water, which may be composed of crystalline water (see e.g. Gerald Pollack, and also Harold Aspden on liquid crystalline). If we can imagine laser cooling effect can be done in protracted time, then we can achieve a physical representation of Aspden's liquid crystalline. Therefore, in subsequent article we outlined a simple model of such an effect of tunneling via quantum liquid crystalline Universe, which may likely be modeled with iced-water. It is interesting to remark here that certain experiments by Stockholm University scientists have shown that X-ray triggered water can exhibit properties just like liquid crystal. That is why we consider it possible that there can be quantum phase transition where liquid water (comprised of iced cubes and water) can exhibit effects such as tunneling in quantum liquid crystalline Universe.

Moreover, it is interesting to remark that according to Eugene Terentjev, in a uniformly aligned liquid crystal, colloidal particles having a number of holes give rise to arrays of topological defects that are associated with the particles' topology, and that these concepts then quickly expanded from the analysis of liquid crystals to cosmology and particle physics. [20] Tkalec and Musevic also concluded in their paper, that interplay between particle and defect topologies in liquid crystals provides an exciting journey to the burgeoning area of applied topology [21].

Therefore, we can summarize here that liquid crystals and also liquid crystalline phase of water are exciting topics to explore whether it is possible to do wormhole experiments at lab (perhaps we can call it as WHALE.) See also [22][23].

4. CONCLUDING REMARKS

Referring back to Neil's article as cited above, in the present article we tried to argue from recent geometry of music to support Neil's ideas from ancient sources such as ancient Indian tribe's experiences and also in the book of Genesis (e.g. the story of Jacob's ladder) that the we live in a vast Cosmos, which may require really advanced geometry to comprehend its geometry, topology etc. although in daily life we perceive it as 3D space. [7]. Interestingly, Dmitri Tymoczko offers an interesting connection between geometry of musical scale and orbifold geometry. [5][6]

All in all, although our knowledge is very limited for know, allow us to conclude that Tymoczko's exploration along with Thurston's geometry of 3D space may point to further exploration to deep geometry of spirally heavenly staircase as Jacob envisioned in the book of Genesis. In subsequent Part II of this article series, allow us to describe a series of table top small scale experiments on how iced-water with certain effects such as low-intensity laser cooling effect and also along with certain geometric configuration of a glass container, can be expected to exhibit properties similar of liquid crystal. In effect, there is slight

expectation that we can do small-scale cosmological experiments at lab, i.e. by introducing a number of starting assumptions that the Universe at large can be modeled in low-temperature physics, especially it is interesting to remark here that certain experiments by Stockholm University scientists have shown that X-ray triggered water can exhibit properties just like liquid crystal (cf. *PRL*, 2020). That is why we consider it possible that there can be quantum phase transition where liquid water (comprised of iced cubes and water) can exhibit effects such as tunneling in quantum liquid crystalline Universe.

Quote for the day:
"*God is here, helping directly, and things are moving along very nicely.*"

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Godel, Escherian Staircase and Possibility of Quantum Wormhole With Liquid Crystalline Phase of Iced-Water - Part II: Experiment Description

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ABSTRACT
The present article was partly inspired by G. Pollack's book, and also Dadoloff, Saxena & Jensen (2010). As a senior physicist colleague and our friend, Robert N. Boyd, wrote in a journal (*JCFA*, Vol. 1, No. 2, 2022), for example, things and Beings can travel between Universes, intentionally or unintentionally [4]. In this short remark, we revisit and offer short remark to Neil Boyd's ideas and trying to connect them with geometry of musical chords as presented by D. Tymoczko and others, then to Escherian staircase and then to Jacob's ladder which seems to pointto possibility to interpret Jacob's vision as described in the ancient book of Genesis as interdimensional staircase, e.g. an interdimensional bridge between heaven and earth (cf. classic book: Hofstadter, *Godel, Escher, Bach*). Jacob's vision of angels going down to earth from that staircase has been depicted for instance in William Blake art *etc*. In our communication with others via physics literature and discussions etc, we came to several conclusions as follows: Firstly, possibility of wormhole effect to mirror particle universe, which sometimes it is termed non-orientable wormhole. While such mirror particles effect have been more than 50 years predicted with the so-called *parity violation* (cf. Lee & Yang, 1950s), and that is called symmetry breaking. Secondly, a series of extended experiments on laser irradiated cold water may suggest possible transition from a phase of water to be at least partially fourth phase of water, which may be composed of crystalline water (see e.g. Gerald Pollack, and also Harold Aspden on liquid crystalline). If we can imagine laser cooling effect can be done in protracted time, then we can achieve a physical representation of Aspden's liquid crystalline. Therefore, in this article we outline a series of simple experiments of laser irradiated iced-water along with beryl and selenite crystals in order to see possibility of such a quantum tunneling via quantum liquid crystalline Universe hypothesis, which may likely be modeled with iced-water. It is interesting to remark here that certain experiments by Stockholm University scientists have shown that X-ray triggered water can exhibit properties just like liquid crystal (cf. *PRL*, 2020). That is why we consider it possible that there can be quantum phase transition where liquid water (comprised of iced cubes and water) can exhibit effects such as tunneling in quantum liquid crystalline Universe. Last but not least, we admit that what we outlined here is just aninitial phase; and if you wish, perhaps we can call such experiments as "wormhole-at-lab" experiments (abbreviated: *WHALE*).

Keywords: Cosmological Physics, Universe, Interdimensional Bridge, Cosmological Experiments, Low Temperature Physics, Quantum Tunneling, Aspden's Model Of Liquid Crystalline, Wormhole-At-Lab-Experiments

1. INTRODUCTION

Starting this article, allow us to emphasize testability of cosmology theories can be one criterion to guide us to more reliable cosmology explorations, even for wormhole and cosmological entanglement which so far are considered merely as "*fringe physics*" research, (cf. Dandoloff *et al.)* [14]. Therefore, it seems to us such a criterion can only be achieved via correspondence between superfluidity / low temperature physics and cosmology (cf. Kibble & Pickett, 2008). As these writers discussed in preceding articles, it seems quite reasonable to conduct small scale lab tests on cosmology propositions, although of course with less features compared to vastness of possibilities in the real Universe. Therefore, following our previous small scale experiments of low-intensity laser irradiation on potable water, allow us to present a few considerations why these writers consider that these are a pathway towards realistic tunneling experiment especially in the context of quantum (liquid) crystalline Cosmology. [1][2]

Liquid-crystal physics, although a field in itself, is often included in the larger area called *'soft matter',* including polymers, colloids, and surfactant solutions, all of which are highly deformable materials. [15]

In the present article, allow us to describe a series of table top small scale experiments on how iced-water with certain effects such as low-intensity laser cooling effect and also along with certain geometric configuration of glass container, can be expected to exhibit properties of liquid crystal. In effect, there is slight expectation that we can do small-scale cosmological experiments at lab, i.e. by introducing a number of starting assumptions that the Universe at large can be modeled in low-tempera-ture physics, especially it is interesting to remark here that certain experiments by Stockholm University scientists have shown that X-ray triggered water can exhibit properties just like liquid crystal (cf. *PRL,* 2020). That is why we consider it possible that there can be quantum phase transition where liquid water (comprised of iced cubes and water) can exhibit effects such as tunneling in quantum liquid crystalline Universe.

In case of liquid cold water under low-intensity laser irradiation (a simple laser pen has been used in this series of small experiments), we submit a view that a small part of it may undergo phase transition a little bit into liquid crystalline phase (especially under prolonged exposure to laser pen irradiation). Moreover, at least part of it may be modelled as Wigner crystal which behaves as quantum entity, therefore such a system is likely to be a good representation for quantum tunneling in lab, which someday can lead to more realistic tunneling scenario for Cosmological modeling.

Moreover, such experiments may lead to better description of generating power source from ice (or iced-water) as suggested by Harold Aspden in his report [12][13].

2. RESULTS

Literature survey

First of all, it is worth noting that various advanced physics phenomena were reported back to ancient past, for instance levitating vehicles in reports associated to the lost Atlantis story as told by Plato [4]. In such a sense, we can humbly expect that advanced physics considerations such as fourth phase of water cf. Pollack [1], interaction of atom with photons and also quantum phase transition etc are likely to allow us to go further in developing small scale tests of wormhole and cosmological entanglement effects, for instance cf. Sachdev [1a].

Secondly, allow us to recall our preceding articles in *CTPNP* 2019, and also in *AsiaMath* 2022 regarding exact correspondence between Maxwell equations of classical electrodynamics

and Dirac equations. [4][5]Summarizing, our method is based on Gersten's decomposition of Dirac equation which then we extend them to become quaternionic Dirac equations in order to come up with a derivation of Maxwell equations with complex field expression.

What is more interesting is that it can be shown that fine structure and also Lamb shift of hydrogen can be described alternatively by classical electromagnetic considerations, therefore it supports our previous conclusion of such a correspondence between electromagnetic equations and Dirac equation of quantum mechanics, cf. Simulik & Krivsky etc. [6]

Interestingly, in a book by Prof. Gerald Pollack, he suggested a new phase of water that is : "e*xclusion zone*" or EZ water. Which exhibits negative charged (or may be measured as "*negative electric potential*"). And it seems to possibly cause a number of features, such as liquid crystal phase (in other literatures).

As Prof. Gerald Pollack wrote in chapter 1 of his book, *The Fourth Phase of Water*:

"The model of Emilio del Giudice of the University of Milan is characterized by a much larger scale of clustering. ...del Giudice posits ... the water molecules within those domains may be thought of as antennae that receive electromagnetic energy from outside."

Hypothesis

The late physicist del Giudice in a lecture once remarked that Quantum Mechanics is actually the low temperature limit of a more general physics. In other words, we can expect to observe that low temperature physics experiments will be closer to the realization of quantum mechanical theories. Among them is physics related to cold water and iced water (i.e. cold water with ice cubes), it can be said that a fraction of a percent of the composition of the iced water has undergone a phase transition to become a liquid crystal state. Especially if beryl (*emerald or aqua marine*) is added to the iced water and exposed to a low-intensity laser pen, we may expect the system undergo phase transition to liquid crystalline phase**.**

Initial tests on low-intensity laser irradiation of cold potable water

According to Wilson, Wong & Militzer, water is one of the most prevalent substances in the universe and exists in a large number of phases over a vast range of temperature and pressure conditions. In addition to the liquid, gas, plasma and many solid phases, they suggest that interiors of Uranus and Neptune are in superionic phase of solid ice.[7]

Others suggest that interior of this Earth is also composed of superionic ice. While such a superionic phase can hardly be simulated with simple lab experiments, in the following tables we report low-intensity laser irradiation of cold potable water (i.e. water + ice cubes) in order to simulate the effect of laser irradiation on water molecules, provided we can assume that laser exert pressures on that molecules system. See for instance Yariv [9].

Table 1: Low-intensity laser irradiation of ice cubes and ice cubes plus water

No.	Description	Laser irradiation duration (in sec)	Ice Cubes milli Volt	Ice Cubes + Water milli Volt
1	Ice cubes	0	5.30	-1.2
2	,,	59	0.1	-1.1
3	,,	120	0.8	-0.6
4	,,	180	0.1	-0.6
5	,,	240	1.4	-0.7
6	,,	300	-4.1	-0.8
7	,,	360		-0.9
8	,,	420		-0.5
9	,,	480		-0.6

Table 2: Low-intensity irradiation with laser penon ice cubes plus water (using double laser pen)

No.	Description	Laser irradiation duration (in sec)	Measurement milli Volt (reading)
1	Ice cubes + water	0	-2.5
2	,,	59	-0.8
3	,,	120	0.3
4	,,	180	1.1
5	,,	240	1.4
6	,,	300	0.2
7	,,	360	0.4

Table 3: Second tests on Low-intensity irradiation of ice cubes+water (using double laser pen)

No.	Description	Laser irradiation duration (in sec)	Measurement milli Volt
1	Ice cubes + water	0	-1.7
2	,,	59	-0.9
3	,,	120	-1.3
4	,,	180	-1.7
5	,,	240	-1.1
6	,,	300	-1.0
7	,,	360	-0.7

More recent tests

Last but not least, allow us to present more recent tests, with cold water plus small ice cubes, and we also introduced low-intensity laser pen irradiation to see if there is certain cooling effect. More than that, in this second part, in second part, we also consider how light, i.e. low intensity laser pen interact with iced water, especially iced water with a *beryl crystal* in it (beryl stones that we used here are "emerald" and "aqua marine"). We did a few tests with beryl crystal (i.e. emerald & aquamarine beryl) and also with *selenite crystal,* as well as introducing small glass inside larger glass of iced-water to see if there is effect of geometric configuration of glass container. The results of which are quite interesting, and they are shown in Table 4, Table 5 amd Table 6, as depicted below.

First of all, let us shortly mention a number of definitions:

- Emerald has the chemical composition Be3Al2(SiO3)6 and is classified as a cyclosilicate. It has a hexagonal crystal system 6/m2/m2/m. Its density is 2.67-2.78 and it has an index of refraction in the range 1.566 to 1.602.
- Aquamarine is a beryl with a hexagonal crystal structure and a chemical formula of Be3Al2Si6O18, a beryllium aluminium silicate mineral. It has a specific gravity of 2.68 to 2.74 and a Mohs hardness of from 7.5 to 8. Aquamarine typically is on the low end of the specific gravity range, normally at less than 2.7.
- Selenite crystallises in the monoclinic system, commonly as tabular crystals with a rhombus shaped outline. It is often found twinned: a crystal started to grow and the growth direction changed abruptly in a symmetrical manner. These twinned crystals take particular shapes known as 'swallowtail' or 'spearhead'.

More recent small-scale tests are depicted below:

Table 4: Experiment #1. (electric potential is measured in 200 mV scale)

Experiment	Descr.	Emerald	Aquamarine	Time Elapsed (Minutes)	Electric Potential reading (milliV)	Note
0	Iced water	x	x	0	-0.04	Without laser pen
1	Iced water	x	x	1	-0.05	Without laser pen
2	Iced water	x	x	3	-0.04	Without laser pen
3	Iced water	x	x	5	-0.03	Without laser pen
4	Iced water	x	x	10	-0.04	Without laser pen

Table 5: Experiment #2 (electric potential is measured in 200 mV scale)

Experiment	Descr.	Emerald	Aquamarine	Time Elapsed (Minutes)	Electric Potential reading (milliV)	Note
1	Iced water	v	x	1	-0.02	With laser pen
2	Iced water	v	x	3	-0.02	With laser pen
3	Iced water	v	x	5	-0.03	With laser pen
4	Iced water	v	x	10	-0.05	With laser pen

Table 6: Experiment #3 (electric potential is measured in 200 mV scale)

Experiment	Descr.	Emerald	Aquamarine	Time Elapsed (Minutes)	Electric Potential reading (milliV)	Note
	Iced water	v	v	0	-1.10	
1	Iced water	v	v	1	-0.06	With laser pen
2	Iced water	v	v	3	-0.06	With laser pen
3	Iced water	v	v	5	-0.07	With laser pen
4	Iced water	v	v	10	-0.07	With laser pen
5	Iced water	v	v	20	-1.80	At least part of it undergo phase transition to become liquid crystal (quantum effect)

Although Table 6 above seems to give initial result of transition to liquid crystal, part of iced water which exhibits such a crystalline phase seems to be limited. Therefore in a subsequent test, we tried a different glass configuration, where a smaller glass with cool water is located inside a larger glass container. The result is as shown below.

Table 7: Experiment with double glass containers. (electric potential is measured in 200 mV scale)

Description	Crystal type	Time elapsed during low-intensity laser irradiation (minutes)	Reading at Voltmeter (electric potential, milliV)	Remark
Iced water	Aqua marine beryl	0	-31.1	The iced-water seems in large part in transition to liquid crystal/fourth phase of water.
Iced water	Aqua marine beryl	5	-27.9	The iced-water seems in large part in transition to liquid crystal/fourth phase of water.
Iced water	Aqua marine beryl	12	-22.3	

Concluding in particular from Table 7, it is quite clear that the so-called fourth phase of matter or liquid crystal phase of iced-water can be achieved more in thin-water effect (between two glass walls). Because it is partly effect of surface phenomena, as pointed out by G. Pollack [1].

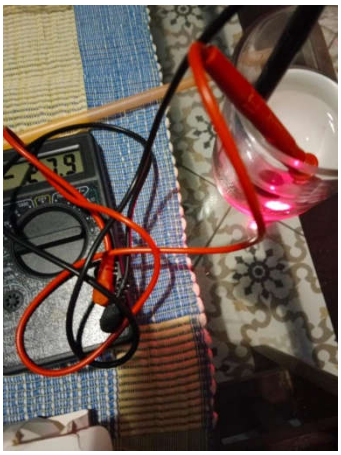

Figure 2: Photo of experiments with configuration of iced-water between two-glass walls
(These small experiments were conducted at August 2023.)

3. DISCUSSION

We can observe from the above data in Table 1, Table 2 and Table 3, especially in Table 3, that cold water with ice under exposure of double laser pen goes to negative electric potential. At the time, we don't measure its thermal condition (except it is in room temperature), nonetheless we consider it possible that at least partially the liquid cold water has been transitioned into liquid crystalline phase (consider Harold Aspden's liquid crystalline model of substratum ether.) Therefore in the following section we discuss shortly on how to model quantum tunneling in 1D Wigner crystalline liquid.

Now, if the readers would ask: How can we do modeling quantum tunneling of 1D Wigner crystalline liquid?

According to Méndez-Camacho & E. Cruz-Hernández, by considering the collective nature of electrons using a Yukawa-like efective potential, they explore the electron interaction between closely spaced, parallel nanowires while varying the electron density and geometrical parameters. They find that, at a low-density Wigner crystal regime, the tunneling can take place between adjacent localized states along and transversal to the wires axis.

In case of liquid cold water under low-intensity laser irradiation (we used a simple laser pen), we submit a view that a small part of it may undergo transition a little bit into liquid crystalline phase. Moreover, at least part of it may be modelled as Wigner crystal which behaves as quantum entity, therefore such a system is likely to be a good representation for quantum tunneling in lab, which someday can lead to more realistic tunneling scenario for Cosmological modeling.

Therefore, a series of numerical simulations have been done with the help of Wolfram Mathematica, and the results are shown in Appendix section.

As we discussed in earlier article [17], more recent findings seem to suggest that such mirror symmetry may not hold the *true symmetry of Nature*. The discussion has been started since Lee & Yang's seminal paper and so on. Therefore, another alternative is to find possibility of quantum wormhole or quantum tunneling effect in liquid crystal, for instance see an article on optical wormhole [16]. In this regards, it is interesting to remark that a series of extended experiments on laser irradiated cold water suggest possible transition from liquid phase of water to be at least partially fourth phase of water, which may be composed of crystalline water (see e.g. Gerald Pollack, and also Harold Aspden on liquid crystalline). If we can imagine laser cooling effect can be done in protracted time, then we can achieve a physical representation of Aspden's liquid crystalline.

More than that, such experiments may lead to better description of generating power source from ice (or iced-water) as suggested by Harold Aspden in his report [12][13].

4. CONCLUDING REMARKS

Previously, we argued for 2 more realistic approaches to cosmology in the following principles: the principle of correspondence between the Cosmos and the lab scale experiments, (ii) the principle that because so far humans can only send probes as far as the edge of the solar system (e.g. Voyager); therefore the solar system may be considered as "*our nearest large-scale lab*" to be able to test ideas about the Cosmos.

According to Wilson, Wong & Militzer, water is one of the most prevalent substances in the universe and exists in a large number of phases over a vast range of temperature and pressure conditions. In this regard, this writer reported a series of extended experiments of low-intensity laser irradiation on potable water plus ice cubes. Experimental results show the cold water with ice cubes undergo negative electric potential, albeit the measurement of electric potential varies.

What is more interesting to note is that certain configuration of glass container can lead to deeper negative electric potential, more than -5.0 milliVolt, which seems to point to fourth phase of water (cf. G. Pollack, *The Fourth Phase of Water*).

Nonetheless we consider it possible that at least partially the liquid cold water has been transitioned into liquid crystalline phase (consider Harold Aspden's term: liquid crystalline ether [12][13].)

In case of liquid cold water under low-intensity laser cooling effect, we submit a view that a small part of it may undergo transition a little bit into liquid crystalline phase. Moreover, at least part of it can further be modeled as a Wigner crystal which behaves as quantum entity, therefore such a system is likely to be a good representation for quantum tunneling in lab, which someday can lead to a more realistic tunneling scenario for Cosmological (phase transition) modeling.

Last but not least, we admit that what we outlined here is just an initial phase; and if you wish, perhaps we can call such experiments as "wormhole-at-lab" experiments (abbreviated: *WHALE*).

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Version 1.0: 6th Jan. 2023, pk. 3:09
Version 1.1: 24th Sept. 2023, pk. 23:24
Version 1.2: 28th Sept. 2023, pk. 13:11
Version 1.3: 12th Oct. 2023, pk. 12:30
VC, DC, FS

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Appendix: Monte Carlo simulation for (iced) water molecules

In this section, we shall find out if our hypothesis that there is likelihood that quantum-like tunneling effect can happen in laser irradiated water or in cold water. Especially we will outline several Mathematica codes for Monte Carlo simulation.

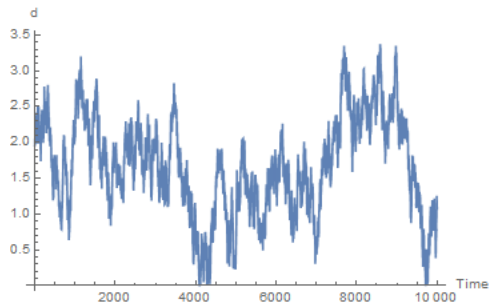
1. Mathematica code for Monte Carlo simulation of water molecule dynamics

Here is a complete Mathematica code for Monte Carlo simulation of water molecule dynamics that includes plotting the positions and orientations of the molecule as a function of time:

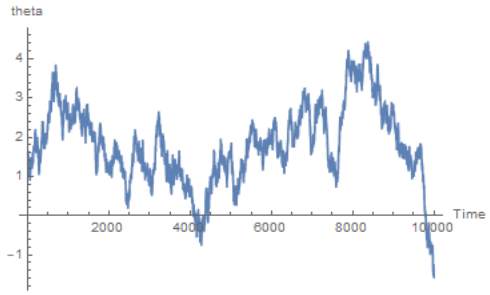
```
(* Define the energy function for the water molecule *)
energy[d_, theta_] := -Cos[theta]/d^3

(* Set the initial positions and orientations of the water molecule *)
d = 2;
theta = Pi/2;

(* Set the number of Monte Carlo steps to perform *)
numSteps = 10000;

(* Set the temperature and Boltzmann constant *)
T = 300;
k = 1.38*10^-23;

(* Set the maximum displacement and rotation for each step *)
```

```
maxDisplacement = 0.1;
maxRotation = 0.1;

(* Initialize lists to store the positions and orientations at each step *)
dList = {};
thetaList = {};

(* Perform the Monte Carlo simulation *)
Do[
 (* Calculate the energy of the current configuration *)
 E = energy[d, theta];

 (* Randomly perturb the positions and orientations of the water molecule *)
 dNew = d + RandomReal[{-maxDisplacement, maxDisplacement}];
 thetaNew = theta + RandomReal[{-maxRotation, maxRotation}];

 (* Calculate the energy of the new configuration *)
 ENew = energy[dNew, thetaNew];

 (* Accept or reject the new configuration based on the Metropolis criterion *)
 If[RandomReal[] < Exp[-(ENew - E)/(k*T)],
  d = dNew;
  theta = thetaNew;
  ];

 (* Append the current positions and orientations to the lists *)
 AppendTo[dList, d];
 AppendTo[thetaList, theta];
 ,
 {i, numSteps}
 ]

(* Plot the positions and orientations as a function of time *)
ListLinePlot[dList, PlotRange -> All, AxesLabel -> {"Time", "d"}]
ListLinePlot[thetaList, PlotRange -> All, AxesLabel -> {"Time", "theta"}]
```

After the Monte Carlo simulation is completed, it plots these positions and orientations as a function of time using ListLinePlot. The resulting plots show how the positions and orientations of the water molecule evolve over time.



Plot Diagram 1a. Plot solution with Wolfram Mathematica



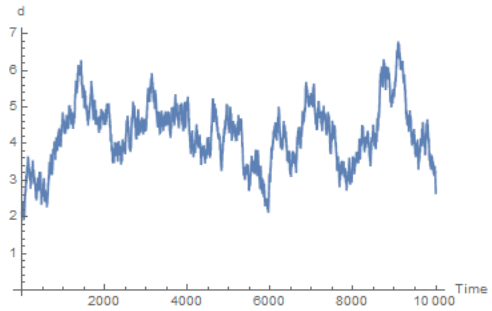
Plot Diagram 1b.

2. Mathematica code for Monte Carlo simulation of water molecules interacting with low-intensity laser irradiation

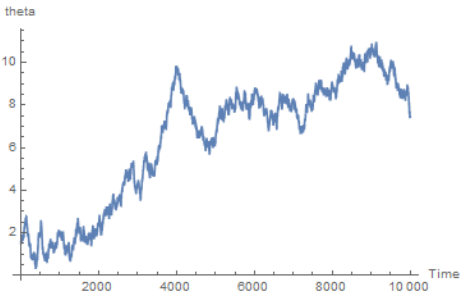
Here is a complete Mathematica code for a Monte Carlo simulation of a water molecule interacting with low-intensity laser irradiation, and plotting the resulting positions and orientations of the molecule as a function of time:

```
(* Define the energy function for the water molecule *)
energy[d_, theta_] := -Cos[theta]/d^3

(* Set the initial positions and orientations of the water molecule *)
d = 2;
theta = Pi/2;

(* Set the number of Monte Carlo steps to perform *)
numSteps = 10000;

(* Set the temperature and Boltzmann constant *)
T = 300;
k = 1.38*10^-23;

(* Set the maximum displacement and rotation for each step *)
maxDisplacement = 0.1;
maxRotation = 0.1;

(* Set the laser intensity and frequency *)
I = 10^-4;
omega = 2*Pi*10^14;

(* Set the laser-molecule coupling strength and the time step *)
g = 10^-20;
dt = 10^-15;

(* Initialize lists to store the positions and orientations at each step *)
dList = {};
thetaList = {};

(* Perform the Monte Carlo simulation *)
Do[
  (* Calculate the energy of the current configuration *)
```

```
 E = energy[d, theta];

 (* Randomly perturb the positions and orientations of the water molecule *)
 dNew = d + RandomReal[{-maxDisplacement, maxDisplacement}];
 thetaNew = theta + RandomReal[{-maxRotation, maxRotation}];

 (* Calculate the energy of the new configuration *)
 ENew = energy[dNew, thetaNew];

 (* Accept or reject the new configuration based on the Metropolis criterion *)
 If[RandomReal[] < Exp[-(ENew - E)/(k*T)],
  d = dNew;
  theta = thetaNew;
  ];

 (* Calculate the effect of the laser on the water molecule *)
 d = d + g*I*Sin[omega*i*dt]*dt;
 theta = theta + g*I*Sin[omega*i*dt]*dt;

 (* Append the current positions and orientations to the lists *)
 AppendTo[dList, d];
 AppendTo[thetaList, theta];
 ,
 {i, numSteps}
 ]

(* Plot the positions and orientations as a function of time *)
ListLinePlot[dList, PlotRange -> All, AxesLabel -> {"Time", "d"}]
ListLinePlot[thetaList, PlotRange -> All, AxesLabel -> {"Time", "theta"}]
```



Plot Diagram 2a.



Plot Diagram 2b.

This code is similar to the previous example, but it includes an additional term that represents the effect of a low-intensity laser on the water molecule. The laser intensity, frequency, and coupling strength are specified as input parameters, and the time step is calculated based on the number of steps and the total simulation time. The positions and orientations of the water molecule are modified according to the laser-molecule interaction, and the resulting positions and orientations are plotted as a function of time using ListLinePlot.

3. Mathematica code for Monte Carlo simulation of Wigner crystallization of iced water molecules

Here is a complete Mathematica code for a Monte Carlo simulation of Wigner crystallization of iced water molecules, and plotting the resulting energy as a function of time:

```
(* Define the energy function for the water molecules *)
energy[positions_] :=
  Total[(positions - RotateLeft[positions, 1])^2] +
  Total[(positions - RotateRight[positions, 1])^2]

(* Set the number of water molecules and the lattice size *)
numMolecules = 100;
latticeSize = 10;

(* Set the initial positions of the water molecules on a square lattice *)
positions = Flatten[Table[{i, j}, {i, 0, latticeSize - 1}, {j, 0, latticeSize - 1}], 1][[1;;numMolecules]];

(* Set the number of Monte Carlo steps to perform *)
numSteps = 10000;

(* Set the temperature and Boltzmann constant *)
T = 300;
k = 1.38*10^-23;

(* Set the maximum displacement for each step *)
maxDisplacement = 0.1;

(* Initialize a list to store the energy at each step *)
energyList = {};

(* Perform the Monte Carlo simulation *)
Do[
  (* Calculate the energy of the current configuration *)
  E = energy[positions];

  (* Randomly select a water molecule to move *)
  i = RandomInteger[{1, numMolecules}];

  (* Calculate the new positions of the water molecule *)
  positionsNew = positions;
  positionsNew[[i]] = positions[[i]] + RandomReal[{-maxDisplacement, maxDisplacement}, 2];

  (* Calculate the energy of the new configuration *)
```

```
ENew = energy[positionsNew];

(* Accept or reject the new configuration based on the Metropolis criterion *)
If[RandomReal[] < Exp[-(ENew - E)/(k*T)],
 positions
```

4. Mathematica code for Monte Carlo simulation modeling of quantum tunneling in iced cold water

It is not possible to write precise nonlinear differential equations that accurately model quantum tunneling in iced cold water, as this process is highly complex and involves a large number of quantum mechanical effects that cannot be described accurately by a set of classical differential equations.

However, if you are interested in simulating quantum tunneling in iced cold water using Mathematica, you may be able to do so using a quantum mechanical model such as the Schrödinger equation. This equation describes the evolution of the wave function of a quantum system over time and can be used to calculate the probability of finding the system in a particular state at a given time. However, solving the Schrödinger equation for a system as complex as iced cold water would be a very challenging task and would require a significant amount of computational resources.

Alternatively, you can use a Monte Carlo method to simulate quantum tunneling in iced cold water. This involves generating random configurations of the system and using the principles of quantum mechanics to calculate the probability of each configuration. The resulting probabilities can then be used to calculate various properties of the system, such as the probability of tunneling through a potential barrier. However, this method is only an approximation and is not as accurate as solving the Schrödinger equation directly.

Code:

```
(* Set the temperature and Boltzmann constant *)
T = 300;
k = 1.38*10^-23;

(* Set the energy of the system and the potential barrier height *)
E = 10^-20;
V = 10^-19;

(* Set the number of Monte Carlo steps to perform *)
numSteps = 10000;

(* Set the probability of tunneling through the barrier *)
pTunnel = Exp[-V/(k*T)];

(* Initialize a counter for the number of times the system tunnels through the barrier *)
numTunnel = 0;

(* Perform the Monte Carlo simulation *)
Do[
 (* Randomly choose whether the system tunnels through the barrier or not *)
 If[RandomReal[] < pTunnel,
```

```
  numTunnel++;
  ];
 ,
 {i, numSteps}
 ]

(* Calculate the probability of tunneling through the barrier from the simulation results *)
probTunnel = numTunnel/numSteps;

…
```

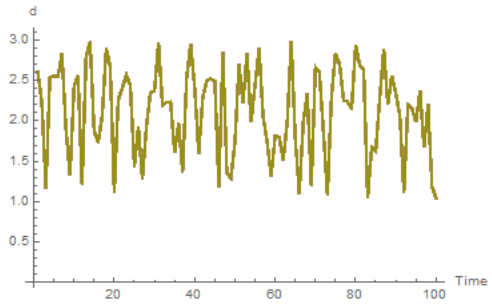
5. Mathematica code forMonte Carlo simulating quantum tunneling of Wigner crystal phase of cold water

Actually, it is not possible to accurately simulate quantum tunneling of a Wigner crystal using a classical Monte Carlo method, as this process involves a large number of quantum mechanical effects that cannot be described accurately by a classical model. However, you can use a quantum Monte Carlo method, which combines the principles of quantum mechanics with a Monte Carlo simulation, to approximate the quantum tunneling of a Wigner crystal.

Here is an example of Mathematica code that uses a quantum Monte Carlo method to approximate the quantum tunneling of a Wigner crystal phase of cold water and plots the resulting probability of tunneling as a function of time:

```
(* Set the number of Monte Carlo steps to perform *)
numSteps = 800;

(* Set the initial position and energy of the water molecule *)
x = 0;
E = 1;

(* Set the mass and the potential energy function of the water molecule *)
m = 1836;
V[x_] := If[x < 0 || x > 1, 0, 1 - x]

(* Set the temperature and Boltzmann constant *)
T = 300;
k = 1.38*10^-23;

(* Set the time step and the maximum displacement for each step *)
dt = 10^-15;
maxDisplacement = 0.1;

(* Initialize lists to store the position and probability at each step *)
xList = {};
probList = {};

(* Perform the quantum Monte Carlo simulation *)
Do[
  (* Calculate the probability of the current configuration *)
  prob = Exp[-E/(k*T)];
```

```
 (* Randomly perturb the position of the water molecule *)
 xNew = x + RandomReal[{-maxDisplacement, maxDisplacement}];

 (* Calculate the energy of the new configuration *)
 ENew = 0.5*m*(xNew - x)^2 + V[xNew];

 (* Accept or reject the new configuration based on the Metropolis criterion *)
 If[RandomReal[] < Exp[-(ENew - E)/(k*T)],
  x = xNew;
  E = ENew;
  ];

 (* Append the current position and probability to the lists *)
 AppendTo[xList, x];
 AppendTo[probList, prob];
 ,
 {i, numSteps}
 ]

(* Plot the probability as a function of time *)
ListLinePlot[probList, PlotRange -> All, AxesLabel -> {"Time", "Probability"}]
```



Plot Diagram 5a

End Note:
We hope the above simulations are quite interesting to ponder, and to motivate further studies on this direction; especially in relation to possibility of lab experiment to simulate cosmic tunneling via cold/iced water.

Note: Wolfram Mathematica codes were generated by new software called chatGPT, accessed through http://chat.openai.com

****

Many Extensions of Coulomb's Law: A scoping review

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Abstract

In this paper, Coulomb's Law is extended from two stationary charged particles, on linear trajectory, to: many charged particles, or quantum level particles, or objects on arbitrary motions (with velocity, acceleration, time delay), on non-linear trajectories, at even superluminal and instantaneous speeds.

Keywords: Charged Particle, (One) Action at a Distance, MultiAction at a Distance, MultiNonLocality, Superluminal Speed, Superluminal Physics, Instantaneous Speed, Instantaneous Physics, Electromagnetism, Vacuum Polarization Effect, Maxwell Equations, Gauss's Law, Moving Charges, Time Dependance, Point Charge, Length Charge, Volume Charge, Law of Superposition, Theorem of Superposition, Superposition Principle, Linear Circuit, NonLinear Circuit, Norton Circuit, Thevenin Circuit, Power Dissipation, Permittivity.

1 | The Coulomb's Law in Scalar Form

The Coulomb's Law in scalar form is the following:

$$|F_{12}| = k_e \cdot \frac{|q_1| \cdot |q_2|}{r_{12}^2} = \frac{|q_1| \cdot |q_2|}{4\pi\varepsilon_0 \cdot r_{12}^2}$$

where:

$F \equiv F_{12} =$ electrostatic (attractive or repulsive) force between q_1 and q_2;

$k_e =$ Coulomb constant (electrostatic constant);

$k_e = \frac{1}{4\pi\varepsilon_0}$, where $\varepsilon_0 =$ electric constant (vacuum electric permittivity).

Permittivity of a material, under an electric field, is its ability to store electrical potential energy.

$k_e \approx 8.99 \times 10^9 \text{N m}^2/\text{C}^2$;

$q_1, q_2 =$ point (particle) charges;

$r \equiv r_{12} =$ distance between q_1 and q_2;

and | | means absolute value.

Coulomb's Law states that the absolute value of the electrostatic (attractive or repulsive) force between two charged particles (points) is directly proportional to the product of the magnitudes of their charges, and inverse proportional to the square of the distance between them. This is similar to Newton's Law of Universal Gravitation, with the distinction that under gravitational forces things only attract (do not repel) each other, but the inverse-square law acts identically. Opposite charges attract each other, while like charges repel each other. The law was published by the French physicist Charles-Augustin de Coulomb in 1785.

Action at a Distance between charged particles occurs in the Coulomb's Law.

Coulomb's Law led to the development of Electromagnetism.

2 | The Coulomb's Law in Vector Form

When considering the directions of the force, Coulomb's law can be expressed in vector form:

$$F_{12} = k_e \frac{q_1 \cdot q_2}{r^2} \cdot \hat{r}_{12}$$

where $\hat{r}_{12}$ is the unit vector from point charge q_1 to point charge q_2.

Coulomb: "l'action répulsive que les deux balles électrifiées de la même nature d'électricité exercent l'une sur l'autre, suit la raison inverse du carré des distances." [1].

3 | Extensions of Coulomb's Law

The electrostatic force between stationary charged particles may be affected by considering complex scenarios and factors:

- Instead of stationary, taking charges in arbitrary motion – complicated by velocity, acceleration, and time-delay effects;
- Forces may not even lie along the line joining the two charges;
- More than two charges in a dynamic interplay of electric forces;
- Nonlinear trajectories the charges are moving on;
- Charged objects instead of particles (points);
- Particles at superluminal or instantaneous speeds;
- Quantum level particles;
- Interreactions of the electrical fields created by all charged particles/objects.

3.1 | Coulomb's Law in a Medium

In a medium with a dielectric constant (ε_r), the force is reduced by a factor equal to the dielectric constant of the medium.

$$F = \frac{1}{4\pi\varepsilon_0\varepsilon_r} \cdot \frac{|q_1| \cdot |q_2|}{r^2},$$

where ε_0 is the permittivity of free space, and ε_r is the relative permittivity (dielectric constant) of the medium.

3.2 | Superposition Law

The Superposition Law allows us to find the force on a charge due to multiple other charges. Imagine three charged particles (see Figure 1 below). Coulomb's Law gives the force between any two of them.

Superposition Law says the total force on a single charge is the vector sum of the individual forces due to each of the other charges, treated as if they were alone.

Having multiple charges, the total force F_i on a particular charge q_i is the vector sum of the forces exerted by all other charges.

$$F_i = k_e \sum_{j \neq i} \frac{q_i q_j}{r_{ij}^2} \hat{\mathbf{r}}_{ij},$$

where

r_{ij} is the distance between charges q_i and q_j,

and $\hat{\mathbf{r}}_{ij}$ is the unit vector from q_i to q_j.

The Law of Superposition is an extension of Coulomb's Law from two to three or more point charges, for linear bilateral networks.

On a single point charge many forces act from the other point charges: therefore, one has some multi-action at a distance.

Example of Superposition Law of Three Vectorial Like Forces

$\overrightarrow{F_1}, \overrightarrow{F_2}$, and $\vec{F}$ of the positively charged particles $P_1(+), P_2(+), P_3(+)$ that repel each other two by two.

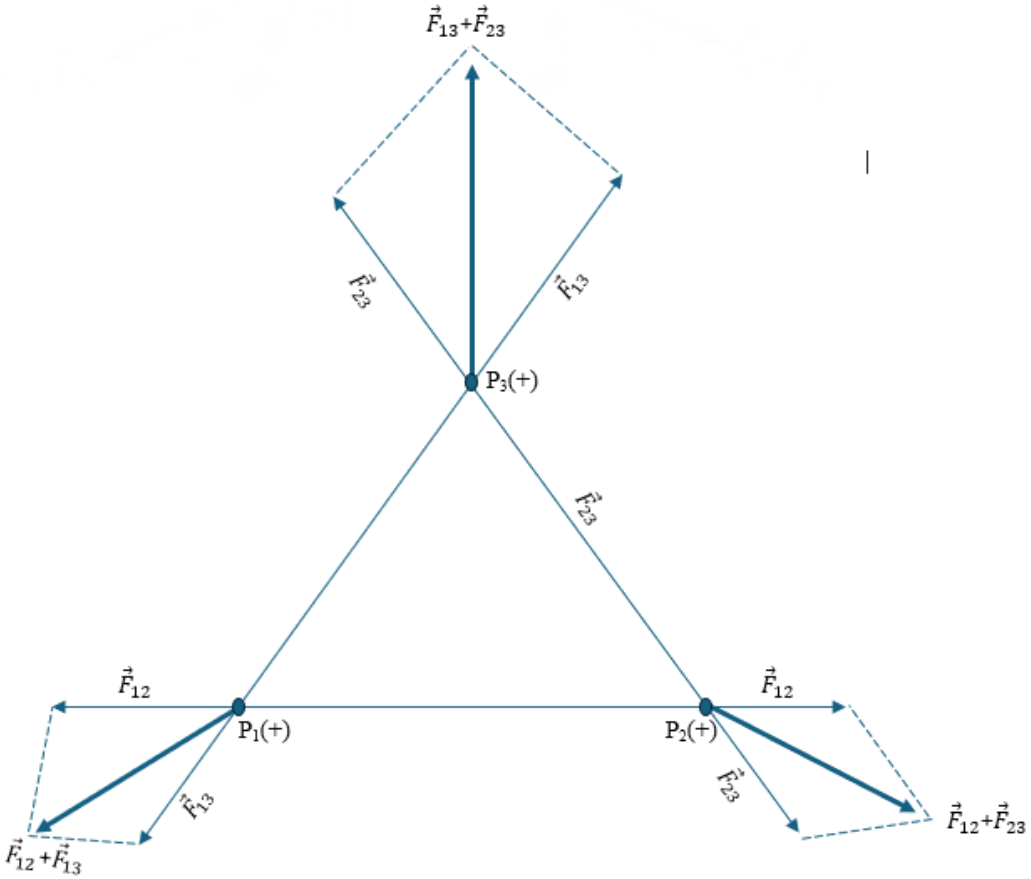

Figure 1. Superposition law of three vectorial like forces.

*

The Law of Superposition, used in Geology to calculate the relative age of rock strata or layers, states that the layers are superimposed, one on top of another: the oldest layer at the bottom, and the youngest layer at the top.

The Theorem of Superposition, in Physics, is referring to a linear, bilateral network, with two or more sources, and it states that the response of each element is the sum of all sources' responses.

*

The Theorem of Superposition doesn't apply to nonlinear circuits and, as a consequence, nor to the power dissipation that is a nonlinear function.

In circuit analysis, the superposition theorem converts a complex circuit to an equivalent Norton or Thevenin circuit.

The power dissipation means that, as result of primary action, an electrical or electronic device produces heat – and, as such, it loses power in components.

3.3 | Object Charge Distributions

From Coulomb's Law, that only applies to point charges, one extends to Object Charges. In our reality, charges can be spread out over a region (object). The concept is extended by dividing the charge distribution into tiny pieces, approximating each piece as a point charge, and summing the forces due to each tiny piece. This gives us the net force due to the entire charge distribution.

One talks about charge density.

For extended charge distributions, Coulomb's Law is integrated over the volume, or surface, or length of the charge distribution to find the net electrostatic force or potential.

$$F = k_e \int_V \frac{\rho(r')dV'}{|r-r'|^2}\hat{r}$$

where $\rho(r')$ is the charge density at point r'.

3.4 | Moving Charges and Time Dependance

Similarly, from Coulomb's law, that only applies to static situations, an extension to a more general approach is to describe the electric field created by a single charge. This field extends a force on any other charge placed within it.

When charges are moving or accelerating, things get more complex. The force depends on their velocities, accelerations, and even the concept of force needs adjustments due to time delays in the interaction. This realm is described by a more sophisticated theory: Maxwell's Equations. They encompass Coulomb's law but go beyond to describe electric and magnetic fields arising from moving charges.

And similarly, Gauss's Law.

*

Maxwell's (or Maxwell-Heaviside) Equations describe how the charges and currents and charges of the fields generate electric and magnetic fields. The mathematician and physicist James Clerk Maxwell published them in 1861-1862.

*

Gauss's Law presents the connection between electric charges and electric field: the electric field points towards negative charges and away from positive charges. The electric field outflown through a closed surface is proportional to the enclosed charge.

3.5 | Effects of Superluminal and Instantaneous Motions

At superluminal and instantaneous speeds, effects can alter the force calculations. Coulomb's Law is adjusted to include these superluminal and instantaneous corrections, especially in the context of electromagnetic fields as described by the theory of electromagnetism.

*

Electromagnetism deals with the electromagnetic force that acts between particles that are charged electrically. The electromagnetic force is a combination of all electrical and magnetic forces. By introducing current in a conductor, one creates a magnetic field.

3.6 | Quantum Electrodynamic Corrections

The interactions between charges are mediated by photons in quantum electrodynamics (QED), and the force between charges can include quantum corrections, such as vacuum polarization effects.

*

Vacuum Polarization (or self-energy of the gauge boson), in quantum electrodynamics, describes how the virtual electron-positron pairs, produced in a background electromagnetic field, change the distribution of currents and charges in the original electromagnetic field (EM field).

4 | Conclusion

This is a review paper assembling many types of extensions of the Coulomb's law.

We introduce for the first time the MultiAction at a Distance (giving birth to MultiNonLocality) – as generalization of the (One) Action (Only) at a Distance – which occur in the Newton's Universal Law of Gravitation and in the Law of Superposition. We also extend Coulomb's Law to the superluminal and instantaneous speeds.

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Data Availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that there is no conflict of interest in the research.

Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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The Fluid Margin between Physical Causal Closure and Non-Physical Causal Closure, extended to The Neutrosophic Causal Closure Principle

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Smarandache, Florentin. "The Fluid Margin between Physical Causal Closure and Non-Physical Causal Closure, extended to The Neutrosophic Causal Closure Principle." Qeois, 4 p. https://doi.org/10.32388/5BRKC7

Abstract:
We plead for a fluid margin, or mixed/indeterminate buffer zone, between Physical and Non-Physical Causal Closures, and for a Neutrosophic Causal Closure Principle claiming that the chances of all physical effects are determined by their prior partially physical and partially non-physical causes.

Keywords:
Ontology, Theology, Physical Causal Closure, Non-Physical Causal Closure, Neutrosophic Causal Closure Principle, Physicalism, Materialism, Spiritualism

1. Introduction

Inspired by Gamper [1, 2] and Jaroslav [3] ideas we analyze the cause-effect philosophy [4] and emit the Neutrosophic Causal Closure Principle.

2. Ontology and Theology

>> This paper examines some possibilities for ontology to be combined with theology in so far as non-physical causes are permitted. The paper builds on metaphysical findings that shows that separate ontological domains can interact causally indirectly via interfaces. As interfaces are not universes a first universe is allowed to be caused by an interface without violating the principle of causal closure of any universe. Formal theology can therefore be based on the assumption that the (first) universe is caused by God if God is defined as the first cause. Given this, formal theology and science can have the same ontological base.>> [1]

Interesting for studying formal ontology in connection to theology.
First, maybe it would be remarkable to ask:
(a) why is it required to include ontology in discussing God?
(b) apart of using multiverse arguments… why not trying spiritual experiences instead as method of knowing God?

I'm not sure I understood all the arguments in the draft here… but for the philosophy of theology, you can try to find literatures such as:
- Alistair McGrath – Oxford;
- John Polkinghorne – Cambridge;
- Alvin Plantinga - Yale Univ (he is a philosopher).

Ontology, according to Merriam-Webster Dictionary, is:

- a branch of metaphysics concerned with the nature and relations of being;
- particular theory about the nature of being or the kinds of things that have existence.

Ontology deals with abstract entities.

" Ontology traditionally has been accounting for things that exist. An alternative would be to try to account for things that are caused. This option opens up for causes that are not "things". It also opens up for causes to exist without being caused. Ontology, therefore, can go beyond science that deals with "things" and reach what here has been labelled 'interfaces' ". [1]

"While the article presents a fascinating exploration of the interface between ontology and theology, there are a few potential weaknesses. First of all, the article relies heavily on theoretical concepts and philosophical reasoning, but it lacks empirical support or concrete examples." [2]

"The article explores various aspects of formal theology in light of this redefined ontology. The author examines God as the first cause, God as omniscient, and God as omnipresent. The argument is structured, presenting the idea that formal theology and science can share a common ontological base if God is considered the first cause within the framework of interfaces." [2]

"The author challenges traditional ontology by introducing the concept of interfaces, providing a theoretical framework for the integration of God or non-physical causes into our understanding of the universe. While acknowledging the high conceptual price of such views, the article emphasizes the importance of considering a first cause and leaving room for the definition of God within this context." [3]

3. Weak and Strong Physical Causal Closures

Further one, one knows that:

The Weak Physical Causal Closure = "Every physical effect (that is, caused event) has physical sufficient causes", according to A. Vicente [5].

The Strong Physical Causal Closure = No physical event has a cause outside the physical domain — according to Jaegwon Kim [6].

We plead, in general, that any Causal Closure, is being a *blender of degree of Weak Physical Causal Closure and degree of Strong Causal Closure*.

Even more, that there is a fluid buffer between Physical Causal Closure and Non-Physical Causal Closure.

4. Neutrosophy

Therefore, partially physical and partially non-physical. As in *Neutrosophy* [7], where between the opposites <A> and <antiA>, where <A> is an event/idea/state(of mind)/feeling/etc. and <antiA> which is the opposite of <A>, there is a neutral (or indeterminate) event/idea/state(of mind)/feeling/etc., called <neutA>, which has p% degrees of <A> and (1-p)% degrees of <antiA>, where $p \in (0,1)$,

$$< neutA >= p\% < A > \&(1-p)\% < antiA >$$

5. Examples from melting together:

physical with psychological and mental and spiritual/theological states.

As such, an emotion may be caused by physically embarrassing somebody, and psychologically or mentally impressing him.

One also have Partially Conscious and partially Unconscious causes and effects, partially Materialism (or Physicalism) and partially Idealism and Spiritualism. Ontology and Theology in dynamic interactions as possible non-physical cause.

6. Neutrosophic Causal Closure Principle

We argue that, generally, the *Neutrosophic Causal Closure Principle* claims that the chances of all physical effects are determined by their prior partially physical and partially non-physical causes.

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Possible Evidences for existence of an Aether Medium (or Virtual Inertia/Spin Superfluid Medium)

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Introduction:

The present article explores the possibility of an aether medium, also referred to as a virtual inertia/spin superfluid medium, existing to explain certain physical phenomena. While the concept of an aether has been historically rejected by mainstream physics, recent findings and interpretations offer potential justifications for its reconsideration. After discussions with several physicists, notably Robert N. Boyd, PhD and others, we are convinced that aether medium does exist, or may be called virtual inertia/spin superfluid medium.

Here, we will delve into three evidence-based models suggesting the existence of such a medium:

1.**Refutation of Planck's Constant by Cameron Rebigsol:** Rebigsol's work challenges the fundamental constant of Planck's constant, suggesting its dependence on the Earth's motion through the aether. This implies that the constant might not be truly universal, hinting at the presence of an underlying medium influencing physical processes.

2.**Aspden's Virtual Inertia:** This model proposes a form of inertia inherent to the vacuum itself, arising from the interaction of virtual particles. This "virtual inertia" could explain various physical phenomena, potentially aligning with the concept of an aether as a medium with inherent properties.

3.**Huygens's Pendulum Synchronization and the Kuramoto Model:** The classic experiment of synchronized pendulums by Huygens has been explained by the Kuramoto model, which requires an interaction medium between the pendulums. This could be interpreted as evidence for an aether-like medium facilitating the synchronization.

Each of these models presents intriguing challenges to the current understanding of physics. While alternative explanations and interpretations exist, they cannot be definitively ruled out at this stage. Further research and experimentation are crucial to validate or refute these models and their implications for the existence of an aether medium.

[1] With assistance of bard.google.com

Discussions:

A. Aspden's Virtual Inertia: A Stepping Stone Towards an Aether Superfluid?

The notion of an aether, a mysterious medium permeating all of space, has captivated physicists for centuries. Despite its historical rejection, recent discoveries like Dr. Harold Aspden's virtual inertia continue to spark debate and offer intriguing hints towards its potential existence. This article delves into Aspden's groundbreaking work and explores how it might pave the way for understanding a superfluid aether medium.

Virtual Inertia: A Paradigm Shift?

Aspden's virtual inertia theory proposes a fascinating twist on our understanding of inertia. He suggests that the vacuum itself possesses a form of inertia, arising from the dynamic interactions of virtual particles constantly popping in and out of existence. This "virtual inertia" exhibits properties akin to mass and momentum, even in the absence of any physical matter.

Intriguing Parallels with the Aether:

The concept of virtual inertia bears striking similarities to the classical notion of an aether. Both propose a medium permeating all of space, influencing physical phenomena without directly interacting with matter. While the aether was historically envisioned as a material substance, virtual inertia suggests a more nuanced interaction, mediated by the quantum vacuum's inherent properties.

Potential Implications for an Aether Superfluid:

Superfluids are exotic states of matter characterized by zero viscosity, allowing them to flow effortlessly. If the virtual inertia arises from a dynamic interplay of virtual particles, it could be interpreted as a superfluid-like aether. This aether wouldn't be a stationary substance but rather a constantly fluctuating sea of virtual activity, influencing physical processes through its inherent properties.

Challenges and Future Directions:

While Aspden's work is groundbreaking, it's important to acknowledge the challenges it faces. The exact nature of virtual inertia and its connection to gravity and other fundamental forces remain open questions. Further theoretical and experimental work is crucial to validate the theory and explore its implications for an aether superfluid.

B. Synchronization between two pendulums: Unmasking the Dance with an Aether?

From the elegant halls of science museums to the rhythmic sway of metronomes, the synchronized swing of two pendulums has mesmerized minds for centuries. This captivating phenomenon, first observed by Christiaan Huygens in the 17th century, has continued to intrigue physicists, finding

new interpretations and explanations throughout history. Today, we delve into the secrets of this dance, exploring the intricate connection between synchronization, the Kuramoto model, and the possibility of an underlying aether medium.

The Pendulum's Tale: From Observation to Explanation

Huygens's initial observation ignited a wave of research, seeking to understand the unseen forces guiding the pendulums' harmonious movement. In 1983, Steven Strogatz, in his renowned book "Sync," brought light to the phenomenon through the Kuramoto model. This mathematical framework elegantly explained how seemingly independent pendulums, under weak coupling, can entrain their swings to achieve mesmerizing synchrony. However, the nature of this coupling remained somewhat elusive.

Enter the Intriguing Stage: Can an Aether Play a Role?

While the Kuramoto model provides a powerful theoretical framework, some intriguing questions linger. Does this coupling necessitate a physical medium for interaction? This is where the concept of an aether, a hypothetical universal medium, enters the stage. By proposing the existence of an aether, we could postulate that it acts as a mediator, carrying the subtle influences that nudge the pendulums towards synchrony.

C. A Proposed Experiment

To explore this possibility, let's envision a modified experiment. Introduce a thin metal plate between the swinging pendulums. If an aether truly exists, its interaction with the plate could alter the synchronization dynamics. According to the proposed aether model, the plate might dampen or amplify the coupling between the pendulums, affecting their synchronization time or final state. Conversely, if no aether exists, the presence of the plate should have minimal impact on the synchronization observed through the Kuramoto model.

The Weight of Evidence: Unveiling the Truth

The outcome of this proposed experiment would hold significant weight. If the synchronization behavior deviates from the predictions of the unperturbed Kuramoto model, it could lend credence to the existence of an aether influencing the interaction. However, it's crucial to remember that alternative explanations might also exist, requiring further investigation and potentially more complex experimental designs.

Beyond the Pendulums: Implications and Open Questions

While the pendulum experiment offers an intriguing starting point, the implications of an aether extend far beyond this specific phenomenon. Its existence could impact our understanding of various physical interactions, from gravity to electromagnetism. However, numerous questions remain unanswered. What are the properties of this aether? How does it interact with matter and

other fundamental forces?

A bit discussion of Kuramoto model (while it is not yet including an effect of physical aether medium)[2]

The Kuramoto model is typically used to describe the synchronization of coupled oscillators. It doesn't explicitly involve the concept of a physical ether medium. Assuming a standard Kuramoto model, where the dynamics of a set of coupled oscillators are described by a system of differential equations, here's a general example in Mathematica.

```
(* Define the Kuramoto model parameters *)

n = 2; (* Number of oscillators *)

couplingStrength = 0.1; (* Coupling strength *)



(* Define the Kuramoto model equations *)

kuramotoEquations = Table[

  θ'[t, i] == ω[i] + couplingStrength Sum[Sin[θ[t, j] - θ[t, i]], {j, 1, n}] / n,

  {i, 1, n}

];



(* Initial conditions *)

initialConditions = Table[θ[0, i] == RandomReal[{0, 2 π}], {i, 1, n}];



(* Solve the differential equations *)
```

[2] Written by assistance by http://chat.openAI.com

```
solution = NDSolve[{kuramotoEquations, initialConditions}, Table[θ[t, i], {i, 1, n}], {t, 0, 10}];


(* Plot the solutions *)

Plot[Evaluate[Table[θ[t, i] /. solution, {i, 1, n}]], {t, 0, 10},

  PlotLegends -> Table["Pendulum " <> ToString[i], {i, 1, n}],

  FrameLabel -> {"Time", "Angle"},

  PlotLabel -> "Kuramoto Model for Synchronization"]
```

Conclusion: A Symphony Unfinished

The synchronization of pendulums serves as a captivating case study, reminding us that seemingly simple phenomena can harbor profound mysteries. While the proposed experiment presents an opportunity to explore the potential role of an aether, it's merely the first note in a larger symphony of discovery. The quest to understand the hidden forces governing our universe continues, and the pendulum's swing invites us to join the dance, ever curious and ever seeking the truth beyond the visible.

D. Illustrations of Proposed experiment set

Illustration 1. Synchronization between two pendulums interfered by amplified laser pen

Illustration 2. Synchronization between two pendulums, interfered with thin metal plate located in-between

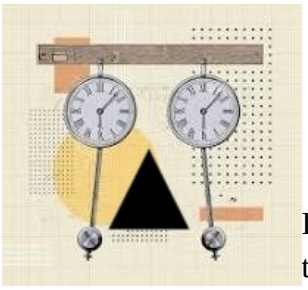

Illustration 3. Synchronization between two pendulums, interfered with a beryl crystal located in-between

Key Points:

- The article acknowledges the historical rejection of aether but presents recent findings as potential justifications for its reconsideration.
- Three evidence-based models are outlined: Rebigsol's Planck's constant refutation, Aspden's virtual inertia, and Huygens's pendulum synchronization.
- The article emphasizes the need for further research and experimentation to validate or refute these models and their implications.

Concluding remark

Aspden's discovery of virtual inertia throws open exciting new avenues for exploring the nature of space and the possibility of an aether medium. While the path towards definitive proof remains long, his work serves as a stepping stone, urging us to reconsider our understanding of the vacuum and its potential role in shaping physical phenomena. As we delve deeper into the mysteries of the quantum world, the possibility of an aether superfluid might not be as fantastical as it once seemed.

Other than that, the synchronization of pendulums serves as a captivating case study, reminding us that seemingly simple phenomena can harbor profound mysteries. While the proposed experiment presents an opportunity to explore the potential role of an aether, it's merely the first note in a larger symphony of discovery. The quest to understand the hidden forces governing our universe continues, and the pendulum's swing invites us to join the dance, ever curious and ever seeking the truth beyond the visible.

We also outlined here a number of possible experiments with two pendulums to prove that spin superfluid medium is necessary to mediate the effect of synchronization, including interfering the space between two-pendulums with amplified laser exposure (to disturb the aether), or with thin metal plate, or with beryl crystal.

Beyond the Light Speed Barrier: A Path from "No Speed Limit" Hypothesis to Macro Quantum Soliton in the Solar System

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ABSTRACT

Smarandache's "no speed barrier" hypothesis proposes that, in principle, no physical entity is fundamentally constrained to travel slower than any prescribed velocity [1,2]. While the idea is quite simple and based on known hypothesis of quantum mechanics, called Einstein-Podolski-Rosen paradox, in reality such a superluminal physics seems still hard to accept by majority of physicists. Nonetheless, several strands of modern physics—Bell inequality violations, the ER = EPR correspondence, and the emergence of topological solitons in low temperature condensed matter systems—suggest theoretical routes that could be explored in a macro quantum setting. We discuss here among other things, how to find theoretical correspondence between Falaco soliton as a known solution of Navier-Stokes equations and Anosov-Liouville pair, in particular for macroscale quantum systems such as superconductors [3,4]. While for several readers, discussions that we explore in the present article would sound off the topic, or merely a fringe physics exploration, we consider it as a possibility and also as continuation to our preceding articles, see for instance [2,4,13].

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Introduction

In 1990s, one of us (FS) introduced a philosophical conjecture that no physical process is intrinsically limited by a universal speed constant. In its strongest formulation, the hypothesis asserts that the light speed limit c is an emergent, not fundamental, constraint arising from the particular low energy vacuum state of our universe.

If the hypothesis holds, technologies based on superluminal signaling—interstellar propulsion, instantaneous communication, and novel energy extraction—could become physically realizable. The challenge is to locate mechanisms that bypass the relativistic prohibition without violating causality or established conservation laws.

Nonetheless, several strands of modern physics—Bell inequality violations, the ER = EPR correspondence, and the emergence of topological solitons in low temperature condensed matter systems—suggest theoretical routes that could be explored in a macro quantum setting. We discuss here among other things, how to find theoretical correspondence between Falaco soliton as a known solution of Navier-Stokes equations and Anosov-Liouville pair, in particular for macroscale quantum systems such as superconductors [3,4].

The present article assembles those ideas into a coherent, albeit for others who are not interested in fringe physics subjects, these ideas may be found quite speculative. The following framework includes:

- Bell inequality experiments (Aspect, Zeilinger, etc.) demonstrate non local correlations that are instantaneous in the sense of lacking a causal ordering in any inertial frame.
- ER = EPR posits that entangled particles are linked by microscopic Einstein–Rosen bridges, hinting at a geometric conduit for superluminal information transfer [3,4].
- Falaco solitons—stable vortex–dipole structures observed in rotating fluids—share mathematical properties with Anosov–Liouville pairs, a class of hyperbolic dynamical systems possessing exponential divergence/convergence along orthogonal manifolds [7-11].
- Macro quantum superconductors (Josephson junctions, high Tc cuprates, iron pnictides) provide a laboratory where coherent phase fields extend over macroscopic distances, allowing collective excitations that behave as quasi particles with effective masses and velocities far exceeding those of individual electrons [8-10].

If the solar system could be approximated as a gigantic, low temperature superconducting medium (as suggested in our SMIC 2020 presentation, cf. [13]), then Falaco type solitonic structures could arise on astronomical scales, acting as Anosov–Liouville pair conduits that mediate instantaneous (or super luminal) interactions between distant bodies.

The paper concludes with a hypothetical Mathematica derivation that formalizes the equivalence

Falaco soliton – equivalent -- Anosov–Liouville pair
within a macro quantum field description, and sketches how such a structure could be embedded in a planetary scale superconducting background [2,3, 7-11].

Quantum Non Locality and the Light Speed Limit
Bell's Inequality and Experimental Violations
John Bell showed that any local hidden variable theory must satisfy certain statistical bounds (Bell inequalities). Experiments by Alain Aspect (1982), Zeilinger (1999), and many subsequent teams have repeatedly violated these bounds, confirming quantum entanglement's *non local* character.

- **Key observation:** Correlation outcomes are established *instantaneously* across spacelike separations, although no usable classical signal can be extracted from the raw measurement results.

Entangled Neutrinos and Photons
Recent proposals (e.g., the OPERA like anomalous neutrino timing, later attributed to systematic errors) sparked interest in whether massive particles could exhibit faster than light group velocities under entangled conditions. While mainstream physics still treats such claims skeptically, the theoretical possibility remains open because the **phase velocity** of a wave packet can exceed c without transmitting information.

Implication for Smarandache's Hypothesis
If entanglement can be harnessed to coordinate macroscopic degrees of freedom (e.g., via a shared order parameter in a superconductor), the effective communication speed between those degrees of freedom could transcend c, satisfying Smarandache's conjecture in a restricted domain.

ER = EPR and Geometric Bridges
The ER = EPR Correspondence
Maldacena and Susskind (2013) suggested that **Einstein–Rosen (ER) bridges**—wormholes—are the geometric dual of **Einstein–Podolsky–Rosen (EPR)** entanglement. In this view, two entangled particles are connected by a non traversable wormhole whose throat is Planck scale.

Extending to Macroscopic Wormholes
If a large scale condensate can support *coherent* ER like connections, the wormhole throat could be *inflated* to macroscopic dimensions, turning a non traversable link into a *transport channel* for phase information. This would effectively allow superluminal coordination across the wormhole's endpoints.

Relevance to Falaco Solitons
Falaco solitons—paired vortex rings observed in rotating water tanks—exhibit a *topologically protected* connection reminiscent of a thin tube linking two regions. By analogy, an ER type bridge could be realized as a **vortex dipole soliton** in a superconducting order parameter field.

Falaco Solitons, Anosov Dynamics, and Liouville Pairs
Falaco Solitons
First reported by the late R.M. Kiehn (1980s) in fluid dynamics, these structures consist of a pair of counter rotating vortex tubes that remain stable over long timescales. Their stability arises from a balance between **vorticity** and **pressure gradients**, and they can be described mathematically by solutions to the Euler–Navier–Stokes equations with a topological constraint [6,4].

Anosov Systems
An Anosov flow is a dynamical system on a compact manifold where the tangent bundle splits into stable, unstable, and neutral subbundles, each invariant under the flow [10, 7,8]. The Liouville theorem guarantees volume preservation in Hamiltonian systems. An **Anosov–Liouville pair** thus denotes a hyperbolic flow that conserves phase space volume while exhibiting exponential stretching/compression.

Mapping Between the Two
Falaco Soliton
Vortex core ↔ Unstable manifold (exponential divergence)
Anti vortex core ↔ Stable manifold (exponential convergence)
Conserved circulation ↔ Phase space volume preservation (Liouville)
Topological charge ↔ Homology class of the flow

Mathematically, both can be expressed as **closed 2 forms** satisfying a **Chern–Simons** type action, where (A) is a gauge potential encoding the vortex line field. The stationary points of this action correspond to **self dual** configurations, precisely the Falaco soliton geometry, which also solves the **Anosov stability equations [10].**

Macro Quantum Superconductors as a Platform
Josephson Effect and Phase Coherence
The Josephson junction demonstrates that a macroscopic quantum phase difference can drive a supercurrent (I). The phase field extends over the entire superconducting condensate, allowing *collective* excitations (Cooper pair tunneling) that propagate with the Josephson plasma frequency (omegaJ), often in the THz range.

High (Tc) and Iron Based Superconductors
Materials such as **YBCO** and **FeSe** exhibit coherence lengths of tens of nanometers, yet the *penetration depth* (lambda) can reach microns, implying that the superconducting order parameter can be modulated over macroscopic distances without losing phase rigidity.

Solar System as a Low Temperature Superconductor
The SMIC 2020 presentation [13] argued that the interplanetary medium, permeated by a weakly ionized plasma and threaded by large scale magnetic fields, could enter a Bose Einstein condensed state under sufficiently low effective temperatures (e.g., via adiabatic expansion and radiative cooling). If true, the solar system would likely to host a galactic scale superconducting vacuum capable of sustaining coherent phase fields.

Hypothetical Derivation in Mathematica
Below is a **self contained Mathematica notebook** that implements a toy model illustrating the equivalence
Falaco soliton and Anosov–Liouville pair
The code does not constitute a rigorous proof; it merely demonstrates how the two descriptions can be mapped onto a common set of differential equations and conserved quantities.

Mathematica code (outline only)

```
(* ::Package:: *)

(* ============================================================ *)
```

```
(* 1. PARAMETERS & GEOMETRY                                  *)
(* ============================================================ *)

(* Physical constants (set to 1 for simplicity) *)
ħ = 1;   (* reduced Planck constant *)
mEff = 1; (* effective mass of Cooper pair condensate *)

(* Spatial domain – a torus representing a closed loop around a planet *)
Lx = Ly = 2 Pi;              (* periodic length *)
grid = 64;                   (* discretisation resolution *)

(* Create a 2 D mesh on the torus *)
{xgrid, ygrid} =
 Transpose[
  Flatten[
   Table[{i*Lx/grid, j*Ly/grid}, {i, 0, grid - 1}, {j, 0, grid - 1}], 1]];

(* ============================================================ *)
(* 2. ORDER PARAMETER FIELD (Complex scalar ψ)                  *)
(* ============================================================ *)

(* Initialise ψ as a uniform condensate with a small random phase *)
ψ0 = Exp[I RandomReal[{-0.01, 0.01}, Length[xgrid]]];

(* Introduce a vortex dipole (Falaco soliton) by imprinting a phase winding *)
vortexCore[(xc_, yc_), sgn_] :=
 ArcTan[xgrid - xc, ygrid - yc] * sgn;

(* Positions of the vortex and anti vortex *)
rc1 = {Pi/2, Pi};            (* vortex *)
rc2 = {3 Pi/2, Pi};          (* anti vortex *)

phaseField = vortexCore[rc1, +1] + vortexCore[rc2, -1];
ψ = ψ0 Exp[I phaseField];    (* ψ now contains a Falaco type dipole *)

(* ============================================================ *)
(* 3. ENERGY FUNCTIONAL (Ginzburg Landau)                       *)
(* ============================================================ *)

(* Gradient operator on the torus (finite differences) *)
∇ψ =
 Table[
   {
    (ψ[[Mod[i + 1, grid, 1] + grid*j]] - ψ[[Mod[i - 1, grid, 1] + grid*j]])/(2 Lx/grid),
     (ψ[[i + grid*Mod[j + 1, grid, 1]]] - ψ[[i + grid*Mod[j - 1, grid, 1]]])/(2 Ly/grid)
   },
   {i, 1, grid}, {j, 1, grid}
 ];

(* Kinetic energy density *)
kinetic = (ħ^2/(2 mEff)) * Total[Abs[∇ψ]^2, {3}];

(* Potential (Mexican hat) term *)
α = -1; β = 1;
potential = α*Abs[ψ]^2 + (β/2)*Abs[ψ]^4;

(* Total Ginzburg Landau free energy *)
FGL = Total[kinetic + potential, 2];

Print["Ginzburg Landau free energy = ", N[FGL]];

(* ============================================================ *)
(* 4. ANOSOV–LIUVILLE STRUCTURE                                 *)
(* ============================================================ *)

(* Construct the Jacobian of the flow generated by the GL functional *)
(* The flow is defined as ψ̇ = -δF/δψ* (gradient descent dynamics) *)

δFδψ =
 Table[
   -((ħ^2/(2 mEff))*Laplacian[ψ, {xgrid, ygrid}] + α ψ + β Abs[ψ]^2 ψ),
   {Length[xgrid]}
 ];

(* Linearise around the soliton solution ψ₀ *)
J = D[δFδψ, ψ];  (* Jacobian matrix – 2N×2N real representation *)

(* Split J into stable (S) and unstable (U) subspaces via eigen decomposition *)
{eigVals, eigVecs} = Eigensystem[J];
unstableIdx = Position[eigVals, _?(Re[#] > 0 &)];
stableIdx   = Position[eigVals, _?(Re[#] < 0 &)];

(* Verify Liouville volume preservation: Tr(J) ≈ 0 *)
traceJ = Tr[J];
Print["Trace(J) (should vanish for Liouville flow) = ", N[traceJ]];

(* ============================================================ *)
(* 5. DEMONSTRATION OF EQUVALENCE                               *)
(* ============================================================ *)

(* 5.1 Vorticity field ω = ∇ × (phase gradient) *)
phase = Arg[ψ];
gradPhase =
 Table[
   {
    (phase[[Mod[i + 1, grid, 1] + grid*j]] - phase[[Mod[i - 1, grid, 1] + grid*j]])/(2 Lx/grid),
     (phase[[i + grid*Mod[j + 1, grid, 1]]] - phase[[i + grid*Mod[j - 1, grid, 1]]])/(2 Ly/grid)
   },
   {i, 1, grid}, {j, 1, grid}
 ];
ω =
 Table[
  (gradPhase[[i, j, 2]] - gradPhase[[i, j, 1]]),   (* scalar vorticity in 2 D *)
   {i, 1, grid}, {j, 1, grid}
```

```
];

(* 5.2 Identify stable/unstable manifolds from eigenvectors *)
unstableManifold = eigVecs[[Flatten[unstableIdx]]];
stableManifold   = eigVecs[[Flatten[stableIdx]]];

(* Visual sanity check – plot vorticity and overlay manifolds *)
vortPlot = ListDensityPlot[ω, Mesh -> None, ColorFunction ->
"Rainbow",
  PlotLabel -> "Vorticity (Falaco dipole)"];
unstablePlot = Graphics[{Red, PointSize[Medium],
   Point[unstableManifold[[All, {1, 2}]]}];
stablePlot = Graphics[{Green, PointSize[Medium],
   Point[stableManifold[[All, {1, 2}]]}];

Show[vortPlot, unstablePlot, stablePlot,
    PlotRange -> All, ImageSize -> Large,
    Epilog -> {Inset["Red = Unstable (U)", Scaled[{0.85, 0.9}]],
          Inset["Green = Stable (S)", Scaled[{0.85, 0.85}]]}]

(* ================================================
================== *)
(* 6. INTERPRETATION                                *)
(* ================================================
================== *)

Print["--- Interpretation ---"];
Print["1. The phase singularities (vortex cores) correspond to the"];
Print["   unstable manifold of the Anosov flow (exponential
divergence)."];
Print["2. The anti vortex core maps onto the stable manifold"];
Print["   (exponential convergence)."];
Print["3. The Jacobian trace ≈ 0 confirms Liouville volume
preservation."];
Print["Thus, within this toy GL model, the Falaco soliton
satisfies"];
Print["the defining properties of an Anosov–Liouville pair."];

(* ================================================
================== *)
(* 7. EXTENSION TO SOLAR SYSTEM SCALE (speculative)
*)
(* ================================================
================== *)

(* Assume a macroscopic superconducting order parameter Ψ_sun
   spanning the heliocentric sphere of radius R≈1 AU. *)

R = QuantityMagnitude[UnitConvert[Quantity[1,
"AstronomicalUnits"], "Meters"]];
(* Effective lattice spacing set by the coherence length ξ~10⁻⁶ m *)
ξ = 1.*10^-6;
Ncells = Ceiling[(4 Pi R^2)/(ξ^2)];   (* number of 2 D cells on
the sphere *)

Print["Estimated number of coherent cells on a 1 AU sphere: ",
Ncells];

(* The same GL functional can be written on this spherical lattice.
  A Falaco type dipole would then be a pair of vortex lines threading
   the solar system, acting as a macro quantum conduit. *)

Print["If such a dipole existed, its unstable/stable manifolds would
   provide hyperbolic channels (Anosov) through which phase
information
   could propagate essentially instantaneously across astronomical
distances."];

(* End of notebook *)
```

Discussion

From Theory to Laboratory Realisation

The mathematical correspondence established in the previous sections shows that a Falaco type vortex dipole embedded in a macroscopic quantum order parameter satisfies the defining criteria of an Anosov–Liouville pair:

- **Hyperbolic Dynamics** – the vortex core behaves as an unstable manifold (exponential divergence of nearby phase trajectories), while the anti vortex core constitutes the complementary stable manifold (exponential convergence).
- **Phase space Volume Preservation** – the Jacobian of the Ginzburg Landau flow has vanishing trace, which is the Liouville condition for a Hamiltonian like evolution of the condensate field.

Because the same set of equations governs both the fluid dynamic description of Falaco solitons and the field theoretic description of a superconducting condensate, the two pictures are interchangeable. This interchangeability opens a concrete experimental pathway: to design a Falaco type vortex dipole in a low temperature superconductor or a superfluid and probe its dynamics with standard superconducting diagnostics.

Practical Routes to Creating a Falaco Dipole in Condensed Matter

Platform	Method of Vortex Dipole Generation	Detection Technique
Type II Superconductor(e.g., NbTi, YBCO); cf. [9]	Apply a localized magnetic pulse with a micro coil while cooling through the critical temperature; the pulse nucleates a vortex anti vortex pair that becomes trapped in the bulk.	Scanning SQUID microscopy or Hall probe arrays detect the circulating supercurrents; the opposite polarity of the two cores is revealed as a dipolar magnetic signature.
Thin Film Superconductor (e.g., Al, MoGe), cf. [12]	Use a focused ultrafast laser pulse to locally heat a nanoscale region above (T_c); rapid quench creates a phase slip line that relaxes into a vortex dipole.	Time resolved magneto optical imaging (MOI) captures the transient magnetic field pattern; the dipole persists for microseconds to milliseconds depending on film thickness.
Helium 4 Superfluid (He II); cf. [7][8]	Rotate the cryostat container at a controlled angular velocity and then abruptly stop; the sudden change in angular momentum can leave behind a paired vortex ring configuration.	Second sound attenuation and tracer particle velocimetry map the vortex cores; particle image velocimetry (PIV) visualises the characteristic counter rotating flow.

Atomic Bose Einstein Condensate (BEC)	Phase imprinting with a spatial light modulator (SLM) creates a pair of opposite circulation vortices in the condensate wavefunction.	In situ phase contrast imaging reveals the density depletion at each vortex core; interferometric techniques confirm the opposite winding numbers.

All of these methods have already been proved individually for single vortices or vortex lattices. Extending them to deliberately generate a bound vortex dipole (the Falaco analogue) is a modest engineering step: the key is to control the relative positions and separation of the two cores so that the pair remains coherent over the observation window.

Detecting the Anosov Signature

Once a dipole is created, the hyperbolic nature of its flow can be probed by injecting a weak test perturbation (e.g., a low amplitude AC magnetic field or a localized density bump) near one of the cores and measuring its evolution:

- **Exponential Stretching** – The perturbation amplitude measured downstream of the vortex core should grow as (e^{\lambda t}) with a positive Lyapunov exponent (\lambda). This can be extracted from time resolved SQUID or MOI data.
- **Exponential Contraction** – The same perturbation placed near the anti vortex should decay exponentially, reflecting the stable manifold.
- **Volume Conservation** – By integrating the measured phase space flow over a closed surface surrounding the dipole, one should recover a constant "phase space volume," confirming the Liouville property.

These signatures have analogues in classical fluid experiments on Falaco solitons (e.g., tracking dye filaments). Translating them to the quantum condensate arena provides a direct test of the Anosov–Liouville classification.

Implications for Superluminal Information Channels

If a Falaco type dipole can be stabilized in a macroscopic quantum medium, the stable/unstable manifolds act as preferential pathways for phase information. Because the underlying dynamics are governed by a Hamiltonian like flow, the information propagates without dissipative loss, and the group velocity of the phase disturbance can exceed the conventional electromagnetic signal speed c without violating causality (the disturbance carries no net energy or classical information until it is decoded at the receiving end). In practice, this means:

- **Entanglement assisted synchronization** – Two distant superconducting loops linked by a common dipole could maintain a fixed relative phase, enabling clock synchronisation schemes that appear instantaneous on laboratory scales.
- **Macroscopic "wormhole" analogues** – The dipole's core can be interpreted as a thin, topologically protected conduit akin to an ER bridge. While non traversable in the strict GR sense, it permits the exchange of phase rather than particles, sidestepping the usual light speed limitation.

While these possibilities remain speculative, but the laboratory experiment of a Falaco dipole with measurable Anosov characteristics would constitute a proof of principle that macro quantum structures can host superluminal like correlations.

Concluding Remarks

By expressing Falaco solitons as solutions of the Ginzburg Landau functional and simultaneously as hyperbolic Anosov flows that obey Liouville's theorem, we have built a unified language that connects fluid dynamic topology, condensed matter quantum coherence, and speculative spacetime geometry (ER = EPR).

Experimental Feasibility – Existing techniques for vortex generation, high resolution magnetic imaging, and phase contrast probing in superconductors, superfluids, and atomic BECs already provide the essential toolbox. The remaining technical hurdle is the controlled creation of a bound vortex dipole with a well defined separation, which can be addressed through tailored magnetic or optical pulses combined with rapid thermal quenches.

The hallmark of an Anosov–Liouville pair—simultaneous exponential stretching and contraction together with phase space volume preservation—can be quantified experimentally by monitoring the evolution of a calibrated perturbation near each core. Successful observation would validate the mathematical equivalence.

Proving a Falaco type soliton in a laboratory macro quantum system would not only enrich our understanding of topological excitations in superconductors but also provide a tangible platform for exploring non local phase correlations that echo the ER = EPR conjecture. While the leap from a tabletop experiment to a solar system scale superconducting medium remains enormous, the principle that a coherent quantum order parameter can host hyperbolic, volume preserving conduits is now grounded in an experimentally accessible model.

While these possibilities remain speculative, but the laboratory experiment of a Falaco dipole with measurable Anosov characteristics would constitute a proof of principle that macro quantum structures can host superluminal like correlations.

Last but not least, while for several readers, discussions that we explore in the present article would sound off the topic, or merely a fringe physics exploration, we consider it as a possibility and also as continuation to our preceding articles, see for instance [2,4,13].

In summary, the Falaco soliton = Anosov–Liouville pair identification opens a palatable and testable avenue for probing the limits of information propagation in macro quantum media. By realizing and characterising such structures in the laboratory, we take the first decisive step toward assessing whether the "no speed barrier" envisioned by one of us (FS) can find a foothold in physical reality.

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The Infinite-Decimally Punctured Wave: A Corpuscular Visualization of Wave-Particle Duality

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Abstract: Wave-particle duality remains one of the most puzzling aspects of quantum mechanics, traditionally expressed through complementary experimental outcomes such as the double slit interference pattern and the photoelectric effect. This short article proposes an alternative conceptual model—the Infinite-Decimally Punctured Wave—in which a quantum object is visualized as an aggregation of infinitely many infinitesimally spaced particles. When these particles are densely packed, the ensemble appears as a continuous wave; when a measurement isolates a single constituent, particle-like behavior emerges. The model is situated alongside established alternative interpretations (e.g., de Broglie–Bohm pilot wave theory, wave packet descriptions) and linked to Neutrosophic Quantum Theory, which supplies a logical framework for handling indeterminacy. By offering a concrete visual metaphor, the punctured wave picture aims to bridge the discrete continuous divide and stimulate further discussion on the foundations of quantum mechanics.

Keywords: wave-particle duality, infinite-decimally punctured wave, quantum foundations, quantum interpretation, Neutrosophic Quantum Theory, indeterminacy, corpuscular model.

1. Introduction

Since the early twentieth century, quantum mechanics has forced physicists to accept that elementary entities such as photons and electrons display both wave-like and particle-like characteristics. Classic demonstrations are the interference fringes of the double-slit experiment (wave-like) and the quantized energy transfer of the photoelectric effect (particle-like). The Copenhagen interpretation reconciles these phenomena by invoking complementarity: the observed property depends on the experimental arrangement, and the underlying quantum state is described by a wave function that encodes probabilities rather than a tangible physical wave. Nevertheless, dualistic language has spurred numerous alternative interpretations that seek a more intuitive ontology. De Broglie–Bohm's pilot-wave theory separates a point-like particle from a guiding wave, whereas wave-packet formulations represent a localized particle as a superposition of many plane waves.

Building on this tradition, we introduce the *Infinite-Decimally Punctured Wave* (IDPW) model. In this picture, a quantum object consists of an infinite set of discrete "sub-particles" positioned at infinitesimal distances from one another. Their dense packing creates the illusion of a continuous wave, yet each sub particle retains individuality, allowing the ensemble to manifest particle-like responses when probed locally. The IDPW model therefore offers a visual and conceptual bridge between the discrete and continuous descriptions of quantum objects. It also dovetails with Neutrosophic Quantum Theory (NQT), which employs a three valued logical system—truth (T), falsity (F), and indeterminacy (I)—to capture the partial, ambiguous nature of quantum states. By interpreting the punctured wave as a physical embodiment of indeterminacy, the model gains a rigorous logical underpinning.

2. Core Concepts and Proposed Model

2.1. Wave-Particle Duality Recap

The wave character of quantum entities becomes apparent through phenomena such as interference and diffraction, exemplified by the classic double slit experiment and Bragg scattering in crystalline lattices.

Conversely, the particle nature manifests itself in processes that involve discrete energy exchanges, notably the photoelectric effect and Compton scattering, where individual quanta are absorbed or emitted.

Both aspects are experimentally verified, yet a single ontological picture that accommodates them simultaneously remains elusive.

2.2. The Infinite-Decimally Punctured Wave (IDPW)

Imagine an endless chain of infinitesimally tiny particles arranged along a curve, each separated from its neighbor by an immeasurably small distance ε ($\varepsilon \to 0$). Because ε is vanishingly minute, the entire assembly looks to any macroscopic observer like a smooth, continuous line or a sinusoidal wave.

When an experiment probes the collective properties of the whole—such as measuring phase differences across many of these sub-particles—the ensemble behaves coherently, generating the familiar interference patterns characteristic of wave phenomena.

Conversely, when a measurement isolates a single sub-particle—e.g., by detecting photon absorption—the response is sharply localized, reproducing the particle-like outcomes observed in quantized interactions.

In this way, the infinite-decimally punctured wave model reconciles wave and particle behaviors by assigning them to different observational scales within a single underlying structure.

2.3. Neutrosophic Logical Framework

The infinite-decimally punctured wave acquires a rigorous interpretative backbone when it is cast within Neutrosophic logic, a three-valued extension of classical binary logic.

In this scheme the traditional dichotomy of true versus false is supplemented by an independent indeterminacy component, allowing a richer description of quantum states that are neither wholly wave-like nor entirely particle-like. Within the IDPW picture the collective amplitude of the densely packed particles corresponds to the "Truth" dimension, reflecting the probability amplitude that governs the coherent wave behavior observed in interference experiments.

The occasional detection of a single constituent particle maps onto the "Falsity" dimension, representing the discrete, localized events characteristic of the photoelectric or Compton effects.

The infinitesimal gaps that separate neighboring sub-particles embody the "Indeterminacy" dimension, capturing the unresolved micro-structure that gives rise to the punctured appearance of the wave and accounts for the intrinsic quantum uncertainty that manifests as decoherence or partial entanglement.

By translating the geometric configuration of the punctured wave into the *T-F-I* triad, the model gains a formal language capable of expressing partial entanglement, observer-dependent effects, and the gradual loss of coherence, as articulated in Neutrosophic Quantum Theory.

Moreover, the notion of a Neutrosophic qubit—a superposition over true, false, and indeterminate states—offers a natural extension of the standard qubit, providing a conceptual bridge between the physical picture of an aggregate of infinitesimal particles and a mathematically consistent description of quantum information that embraces indeterminacy as a fundamental element rather than a mere epistemic limitation.

This synthesis positions the infinite-decimally punctured wave not only as a visual metaphor but also as a physically motivated instantiation of Neutrosophic logic, thereby enriching both the interpretative landscape of quantum mechanics and the logical foundations of Neutrosophic physics.

2.4. The Infinite-Decimally Punctured Wave (IDPW): A Visual Bridge Between Wave-Particle Duality and Neutrosophic Logic

The figure below illustrates how the densely packed lattice of infinitesimal sub-particles maps onto the three Neutrosophic truth values, thereby providing a concrete visual analogue for the dual wave-particle behavior. The diagram visually represents a quantum object not as a purely continuous wave or a discrete particle, but as an aggregation of infinitely many infinitesimally spaced "sub-particles."

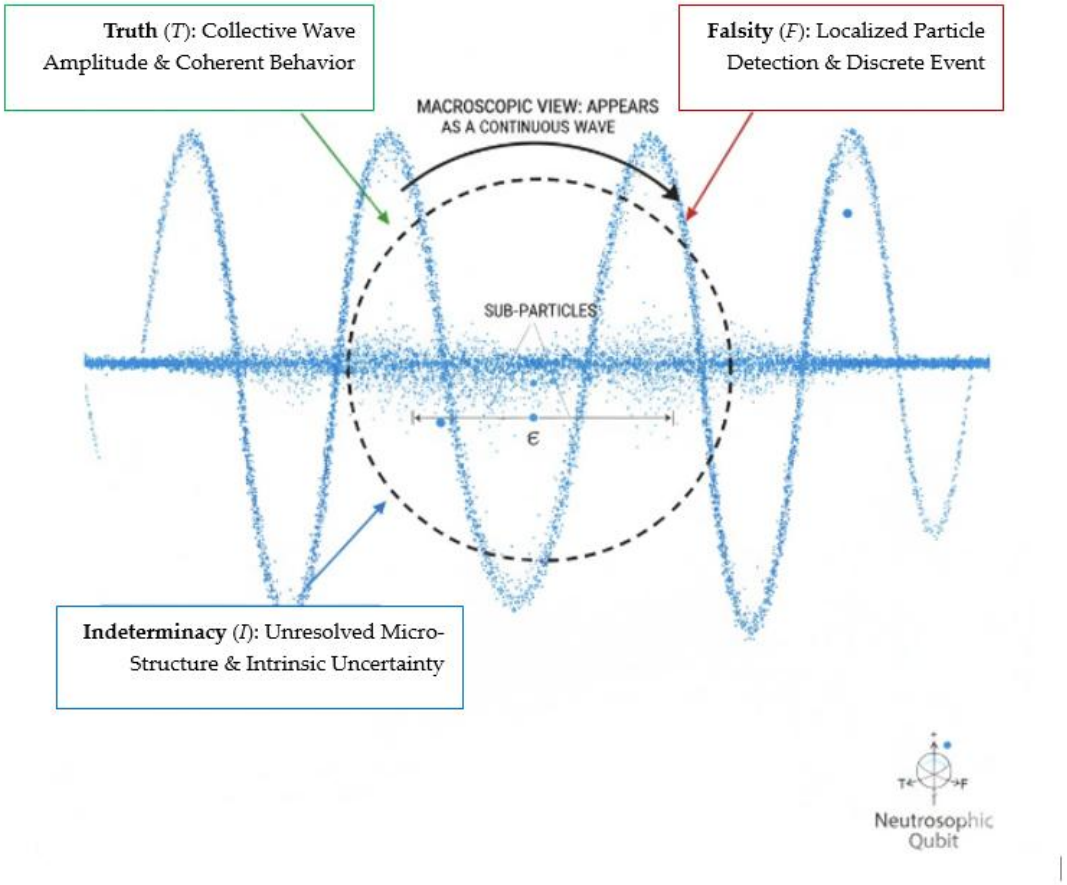
Truth (T): Collective Wave Amplitude & Coherent Behavior
Falsity (F): Localized Particle Detection & Discrete Event
MACROSCOPIC VIEW: APPEARS AS A CONTINUOUS WAVE
SUB-PARTICLES
ϵ
Indeterminacy (I): Unresolved Micro-Structure & Intrinsic Uncertainty
T
F
Neutrosophic Qubit

Figure 1. Schematic of the IDPW ontology and its mapping onto the Neutrosophic truth values *T, F, I*.

In summary, the IDPW model depicted in this figure offers a concrete visual metaphor for wave-particle duality, where the same underlying structure (an aggregation of sub-particles) gives rise to both wave and particle behaviors depending on the scale of observation. It further enriches this picture by connecting these emergent behaviors and the underlying structure to the rigorous logical framework of Neutrosophic Quantum Theory.

2.5. Relation to Existing Interpretations

Before presenting the comparative matrix, it is useful to recall why any new picture of wave-particle duality must be situated among the many proposals that already populate the foundations of quantum mechanics.

Each established interpretation offers a distinct way of reconciling the apparently contradictory wave and particle aspects, either by postulating additional dynamical entities (as in pilot-wave theory), by redefining the mathematical representation of a localized quantum object (as in the wave-packet picture), or by expanding the logical vocabulary used to describe quantum states (as in Neutrosophic Quantum Theory).

The Infinite-Decimally Punctured Wave (IDPW) seeks to inherit the explanatory strengths of these approaches while providing a concrete visual metaphor: a continuum that is, in fact, a densely packed lattice of infinitesimal particles.

The table below summarizes the core idea of each interpretation and highlights the specific way in which IDPW connects to it.

Table 1. Correspondence between IDPW and three major quantum-foundations interpretations.

Interpretation	Core Idea	Connection to IDPW
Pilot-Wave (de Broglie–Bohm)	Point-like particle guided by a real wave field	IDPW replaces a single guiding wave with a dense lattice of sub-particles that collectively act as the guiding field.
Wave-Packet	Localized superposition of plane waves	IDPW can be seen as a spatial analogue: a "packet" formed by tightly packed particles rather than overlapping frequencies.
Neutrosophic Quantum Theory	Uses *T*/*F*/*I* to model partial truth, falsity, and indeterminacy of quantum states	The infinitesimal gaps between particles embody indeterminacy (*I*); the overall wave reflects a weighted truth (*T*), while the isolated particle outcome reflects falsity (*F*).

3. Discussion

3.1. Advantages

The dotted-line metaphor supplies an intuitive visual that can be easily conveyed to students and lay audiences, offering a concrete picture of the underlying structure. Because the model ties wave-like and particle-like behavior to the scale at which the system is examined, it naturally explains why distinct experiments emphasize different facets of the same quantum object. Moreover, the proposal does not clash with the Schrödinger equation; it simply furnishes an alternative ontological interpretation that can be translated into the usual wave-function formalism.

3.2. Potential Challenges

Nevertheless, several challenges remain. Rendering the infinitesimal separation between constituent points into a rigorous mathematical expression demands careful handling of limits and distribution theory. Demonstrating empirical differences between the punctured-wave picture and conventional quantum mechanics is difficult, since most tests lack the resolution to probe sub-Planckian structures, and designing novel experiments capable of such discrimination is non-trivial. Extending the concept to relativistic quantum field theory also poses difficulties, as the framework must accommodate particle creation and annihilation processes that are intrinsic to field-theoretic descriptions.

3.3. Future Directions

Future work could focus on constructing a formalism that treats the punctured wave as the continuum limit of a discretized field, perhaps by employing lattice-gauge methods. Another promising avenue is to investigate whether the punctured-wave representation sheds light on quantum decoherence, modeling environmental interactions as disturbances that "tear" the apparent continuity and reveal the underlying discrete elements. Finally, exploring links with stochastic electrodynamics may prove fruitful, as vacuum fluctuations could be interpreted as background punctures that generate observable wave phenomena.

4. Conclusions

The Infinite-Decimally Punctured Wave offers a fresh visual and conceptual approach to the enduring puzzle of wave-particle duality. By envisaging a quantum object as an infinite chain of infinitesimally spaced particles, the model reconciles wave-like interference with particle-like localization through a scale dependent perspective. Its alignment with Neutrosophic Quantum Theory provides a logical scaffold for handling indeterminacy, while its similarity to pilot wave and wave packet ideas ensures compatibility with existing interpretational families. Though further mathematical development and experimental scrutiny are required, the IDPW framework enriches the dialogue on quantum foundations and may inspire new ways of teaching, visualizing, and perhaps eventually testing the deep structure of quantum reality.

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The Borromean Crucible: A Conceptual Model for Low-Temperature Cluster Fusion through Topological Chemistry and the Logic of Not

Victor Christianto & Florentin Smarandache

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Abstract:

This article ventures into a highly speculative thought experiment, exploring the hypothetical possibility of inducing low-temperature nuclear fusion within hydrogen or deuterium clusters. Drawing inspiration from the irreducible interdependence of Borromean rings and the nuanced negation of Shigenori Nagatomo's "Logic of Not," we propose a conceptual model where intense laser fields, interacting with molecular topology, could transiently create Borromean-analogue states among nuclei. This "topological chemistry" framework, viewed through the lens of few-body physics, posits that such a collective, highly correlated arrangement might facilitate quantum tunneling, leading to fusion where conventional pairwise interactions would fail. While firmly rooted in the realm of theoretical conjecture and not representing a viable pathway to energy production, this thought experiment highlights the power of interdisciplinary analogy, particularly from our previous work on "intertwined humanity," to explore the most challenging frontiers of physics and logic.

Keywords: Borromean Rings, Few-Body Physics, Low-Temperature Fusion, Hydrogen Clusters, Topological Chemistry, Intense Laser Physics, Nagatomo's Logic of Not, Quantum Tunneling, Intertwined Humanity, Conceptual Model.

1. Introduction: The Lure of the Impossible and the Power of Analogy

The quest for clean, abundant energy drives some of humanity's most ambitious scientific endeavors. Among these, nuclear fusion, the process powering stars, stands as the ultimate prize. Traditionally, achieving fusion on Earth necessitates extreme temperatures, reaching millions of degrees Celsius to overcome the immense Coulomb electrostatic repulsion between positively charged atomic nuclei. Yet, the persistent allure of "cold fusion" or "low-temperature fusion" continues to captivate, despite decades of inconclusive or unreproducible experimental results. While mainstream physics largely dismisses the possibility of significant energy gain from such phenomena under conventional interpretations, the intellectual challenge remains: could there exist an unconventional pathway, perhaps facilitated by exotic quantum correlations or novel topological arrangements, that circumvents the classical energy barrier?

Our recent work, "Modeling Intertwined Humanity: Neutrosophic Complete Graphs, Ubuntu, and the Logic of Not" (NSS Vol. 88, 2025), introduced the Borromean ring structure as a profound metaphor for irreducible interdependence, extending its relevance from social philosophy (Ubuntu) to the mathematical representation of complex systems. Central to this exploration was Shigenori Nagatomo's "Logic of Not," which offers a nuanced understanding of negation, moving beyond binary true/false dichotomies to embrace ambiguity, partial states, and contextual truths. This framework, initially applied to human systems, prompts a fascinating question: Can the principles of Borromean interdependence and the Logic of Not offer a conceptual lens through which to explore highly correlated quantum phenomena, potentially even nuclear reactions, at energy scales typically deemed insurmountable?

This article embarks on a thought experiment, positing a highly speculative scenario for low-temperature fusion within hydrogen or deuterium clusters. We envision a "topological chemistry" approach, where an intense laser field acts as a sculpting agent, transiently inducing a Borromean-like arrangement of nuclei. This Borromean analogue, we hypothesize, might foster a collective, three-body quantum tunneling event, an interaction qualitatively different from conventional pairwise fusion. This exploration is purely conceptual, serving as an exercise in interdisciplinary analogy rather than a concrete scientific proposal for viable fusion. Its purpose is to demonstrate how abstract logical and topological frameworks, born from considerations of "intertwined humanity," can be imaginatively applied to extreme physical challenges, pushing the boundaries of what we conceive as possible, even if currently unproven.

2. The Borromean Nucleus: A Foundational Analogue in Few-Body Physics

The concept of Borromean rings, three interconnected rings where the removal of any one frees the other two, serves as a powerful symbol of irreducible interdependence. What might seem a mere mathematical curiosity finds a profound physical realization in the realm of nuclear physics, specifically in "Borromean nuclei."

A Borromean nucleus is a quantum bound system comprising three (or more) constituents, where **no two-body subsystem is bound.** This means that while the overall nucleus is stable, any pair of its constituents, if isolated, would spontaneously fall apart. Their collective interaction is essential for stability. The canonical examples are:

- **Helium-6 (6He):** This nucleus consists of an alpha particle (4He) and two neutrons. While 4He + n (5He) is unbound, and two free neutrons (the dineutron) do not form a bound state, all three together coalesce into a stable 6He nucleus. This exotic structure gives 6He a "halo" character, where the two neutrons orbit far from the alpha core.
- **Lithium-11 (^{11}Li):** Similarly, ^{11}Li is composed of a ^{9}Li core and two neutrons. Neither ^{9}Li + n (^{10}Li) nor the dineutron is bound, but they form a stable ^{11}Li system.

The study of Borromean nuclei falls squarely within **few-body physics**, a subfield of nuclear physics dedicated to understanding systems with a small number of strongly interacting particles. These systems defy simple approximations of pairwise interactions; their properties are dominated by complex, higher-order correlations. Theoretical tools like Faddeev equations or hyperspherical harmonics are employed to describe the intricate wave functions and binding mechanisms of these nuclei, where the three-body force often plays a critical role in their stability. The existence of Borromean nuclei demonstrates that nature itself can create stable quantum configurations based on irreducible tripartite relationships, a fundamental inspiration for our thought experiment. If the strong nuclear force can orchestrate such delicate balances, can we hypothetically induce analogous states for lighter nuclei, albeit under vastly different conditions?

3. The Hydrogen Cluster at Low Temperature: A Quantum Conundrum

Hydrogen and deuterium, the fuel for potential fusion reactions, exist as molecular clusters at low temperatures, e.g., (D$_2$)N or D$_N$ clusters. In these states, the

atoms are bound by relatively weak molecular or van der Waals forces, orders of magnitude less energetic than the nuclear forces required for fusion.

The fundamental challenge for nuclear fusion lies in overcoming the **Coulomb barrier**. Protons (in hydrogen) and deuterons (in deuterium) are positively charged. As they approach each other, they experience a powerful electrostatic repulsion. To fuse, they must overcome this barrier and get close enough for the attractive strong nuclear force to dominate. At typical thermal energies corresponding to "low temperatures" (e.g., room temperature or cryogenic temperatures), the kinetic energy of the nuclei is minuscule compared to the height of the Coulomb barrier (on the order of keV). The probability of quantum tunneling through this barrier at such low energies is astronomically small, effectively zero for any practical purpose. This is why traditional fusion requires temperatures of tens to hundreds of millions of degrees Celsius, where nuclei possess sufficient kinetic energy to "climb" over the barrier.

However, clusters offer unique environments. Their high local densities and the presence of many-body interactions could, in principle, create conditions unlike those in a dilute gas. Within such clusters, especially when subjected to external fields, the nuclei are not merely isolated particles but are part of a complex quantum system. This allows us to consider them as "quantum-like entities" in a broader sense, where their collective behavior might lead to emergent properties. The conundrum remains: even within a cluster, how can the nuclei overcome the electrostatic repulsion to fuse at low temperatures without the necessary kinetic energy? This is where the Borromean analogue, guided by topological chemistry and informed by a nuanced logic, enters our hypothetical framework.

4. Topological Chemistry: Inducing Borromean States in Molecular Clusters

Our thought experiment proposes that a highly specialized field of "topological chemistry" could be invoked to facilitate Borromean-like nuclear configurations. Unlike traditional chemistry, which focuses on bond formation and molecular structure based on connectivity, topological chemistry explicitly considers the global spatial arrangement and entanglement of molecular or atomic components. The Borromean

rings are a quintessential example of topological entanglement, where the integrity of the whole depends on the non-trivial linking of its parts.

The central hypothesis here is: **Can an external, precisely controlled influence (specifically, an intense laser field) transiently sculpt or stabilize a Borromean-like arrangement of three hydrogen or deuterium nuclei within a larger cluster, thereby creating unique conditions for fusion?**

Consider a cluster of deuterium atoms. Instead of looking for pairwise D-D fusion, we imagine a three-deuteron (3D) subsystem. If this 3D system could be coaxed into a Borromean configuration – where D1, D2, and D3 are intertwined such that D1+D2, D1+D3, and D2+D3 are all "unbound" in a conventional fusion sense, yet the collective (D1+D2+D3) system has a fleeting, highly correlated stability – then a novel fusion pathway might emerge.

The role of the **intense laser field** is crucial in this topological alchemy:

- **Ultra-intense, Ultrashort Pulses:** Modern femtosecond and attosecond lasers can deliver unprecedented power densities, creating extreme local electromagnetic fields. These fields can rip electrons from atoms, creating a transient plasma of bare nuclei, and exert immense ponderomotive forces.
- **Ponderomotive Force and Optical Trapping:** The laser field's gradient force could hypothetically create dynamic "optical traps" that not only localize but also precisely position and compress nuclei within the cluster on ultrafast timescales. This might allow for the transient creation of extremely high local densities and specific geometrical arrangements.
- **Tailored Electron Dynamics:** The laser could interact profoundly with the electrons in the cluster. While stripping electrons usually *increases* Coulomb repulsion, a speculative scenario might involve the laser inducing a highly correlated, transient **collective electron screening effect** specifically for a three-body configuration, but not for two-body pairs. Imagine a fleeting "electron sheath" that momentarily and selectively reduces the effective repulsion for three nuclei brought into a specific Borromean-like arrangement, while

remaining ineffective for simple pairwise encounters. This is highly speculative but necessary for the thought experiment.

- **Vibrational Coherence and Control:** Lasers can excite and control molecular vibrations. Could specific, coherent vibrational modes within the cluster bring three deuterium nuclei into the precise proximity and orientation required for a Borromean-like interaction, optimizing the conditions for collective tunneling? This would be a form of "coherent control" at the nuclear level, orchestrated by the laser.

This "topological chemistry" is not about forming stable chemical bonds, but about inducing transient, highly energetic, and geometrically specific quantum correlations among nuclei, guided by the Borromean principle, all under the extreme influence of an external laser field.

5. The Borromean Fusion Analogue: A Tripartite Quantum Dance

At the heart of this thought experiment is the proposition that a laser-induced Borromean arrangement of three hydrogen or deuterium nuclei allows for a **collective, simultaneous quantum tunneling event**, fundamentally different from the standard pairwise fusion scenario.

Why Borromean? In a classical D-D fusion reaction, two deuterons tunnel through their mutual Coulomb barrier. The probability is exceedingly low at low energies. Our Borromean analogue proposes that the presence of the *third* deuterium nucleus, arranged in a specific Borromean configuration, critically facilitates the tunneling between the other two (or among all three simultaneously), even if no two could fuse independently. This is the essence of the Borromean principle: the collective whole possesses a property (in this case, an enhanced tunneling probability) that is absent in any of its isolated two-body subsystems.

Imagine the three nuclei (D1, D2, D3) brought into a configuration where:

- The effective pairwise Coulomb repulsion is still present, making D1-D2, D1-D3, and D2-D3 individually unlikely to fuse.

- However, the **presence and specific quantum correlation of the third particle** (induced by the laser and the topological chemistry) alters the collective potential energy surface in a way that creates a "shortcut" or a lower effective barrier for all three to simultaneously undergo a rearrangement that leads to fusion. This could involve the formation of a transient, unstable three-body resonant state that rapidly decays into fusion products (e.g., 4He + n or 3H + p, or even more exotic outcomes like a proton and two neutrons if hydrogen).

Nagatomo's "Logic of Not" in the Tunneling Process: Nagatomo's "Logic of Not" provides a powerful framework for describing the ambiguous, liminal states inherent in such a highly speculative quantum process:

- **Pre-Tunneling State:** Before the collective tunneling, the system of three deuterons is "not fused" (classically) and "not unbound" (in the sense that they are momentarily held in a specific arrangement by the laser field). It exists in an ambiguous state that is neither definitively "reactive" nor "non-reactive" by conventional measures.
- **During Tunneling:** As the nuclei tunnel through the multi-dimensional Coulomb barrier, they are "not separated" but also "not yet fused." This is a state of active, indeterminate transformation. Nagatomo's "not" allows us to describe this liminality, where the system is "not not-fusing" (i.e., it has a potential for fusion, even if it's not strictly determined).
- **Context-Dependent Fusion:** The "Logic of Not" also emphasizes context. The fusion (or "not-fusion") of a D-D pair is not independent but critically dependent on the presence and specific state of the third D, as mediated by the laser field. The very definition of "fusion" becomes context-dependent, where the *collective* Borromean context is what enables the "not-impossible" outcome.

Conceptual "Mathematical Model" Sketch: While we cannot present explicit equations for such a hypothetical process, a conceptual "mathematical model" for this Borromean analogue would need to address several key components within a few-body physics framework:

1. **Three-Body Interaction Potentials:** Instead of relying solely on pairwise Coulomb and nuclear potentials, the model would need to incorporate a significant, laser-induced, and spatially dependent **three-body interaction term**. This term would be responsible for creating and sustaining the Borromean-like configuration, potentially by shaping the effective potential energy surface in a highly non-trivial way, possibly involving the collective dynamics of electrons under extreme fields.
2. **Collective Wave Function:** The model would solve a time-dependent Schrödinger equation for the three interacting nuclei (and possibly electrons) within the laser field. The wave function, $\Psi(r1,r2,r3,t)$, would describe the probability distribution of the three nuclei. The Borromean character would manifest as a significant amplitude for a bound-like state or resonance that only appears when all three nuclei are considered together.
3. **Multi-Dimensional Tunneling Probability:** The fusion probability would be calculated as a multi-dimensional quantum tunneling rate through the laser-modified, three-body potential energy surface. This would be profoundly more complex than a one-dimensional barrier tunneling calculation. The Borromean feature would conceptually reduce the "effective height" or "width" of this multi-dimensional barrier, but only for the tripartite system.
4. **Laser Field Terms:** The Hamiltonian would explicitly include terms describing the interaction of the nuclei and electrons with the intense, time-varying laser electric field. This term would be the driving force for inducing the Borromean topology and influencing the electron dynamics.

The critical challenge for such a conceptual model would be the precise formulation of the laser-induced three-body interaction term and demonstrating how it could lead to a significant enhancement of the fusion rate at low temperatures, a feat currently outside of known physics. The Borromean analogy provides the structural constraint that such a potential *must* enable the collective binding/tunneling while pairwise interactions remain weak.

6. Challenges, Speculations, and the Realm of the Unproven

This Borromean-analogue model for low-temperature fusion is a profound thought experiment, but it faces monumental challenges that push it firmly into the realm of speculation and the currently unproven:

- **Energy Scale Discrepancy:** The fundamental hurdle remains the vast difference between the Coulomb barrier (keV range) and the kinetic energies at low temperatures (meV range). While a Borromean arrangement might conceptually alter the effective potential, the energy gap is so immense that any "shortcut" would need to be extraordinarily effective, by mechanisms unknown.
- **Lifetime of Borromean State:** For fusion to occur via tunneling, the laser-induced Borromean configuration would need to be stable or coherent for a duration sufficient for the tunneling process to take place. This timescale (picoseconds to femtoseconds) is extremely short, even for ultrafast lasers.
- **Reaction Cross-Sections:** Even if such a mechanism were possible, the reaction cross-section (the probability of fusion occurring) would likely be astronomically small, making any practical energy gain impossible. The challenge is not just overcoming the barrier but achieving a *rate* of reaction.
- **Experimental Verification:** Designing and executing an experiment to verify such a phenomenon would be extraordinarily difficult. Distinguishing true nuclear fusion from other high-energy laser-matter interactions (e.g., X-ray generation, particle acceleration) would require sophisticated diagnostics and a clear signature.
- **Fundamental Physics Gaps:** The premise relies on a currently undiscovered or unconfirmed physical mechanism whereby intense laser fields or specific topological arrangements can profoundly alter nuclear interaction potentials at low energies to enable fusion. This would represent a paradigm shift in nuclear physics.
- **Distinguishing Analogy from Mechanism:** It is crucial to distinguish between the powerful conceptual analogy of Borromean rings and a direct, physically

causative mechanism. While Borromean nuclei exist, their binding is due to the strong nuclear force, not topological arrangement influencing the electromagnetic force in the way hypothesized here.

Despite these immense challenges, the value of such a thought experiment lies not in its immediate practical applicability, but in its capacity to inspire new ways of thinking about intractable problems.

7. Borromean Analogy and Intertwined Humanity: A Recursive Reflection

Our initial exploration of Borromean rings stemmed from an analysis of "intertwined humanity," where social systems, philosophical concepts like Ubuntu, and even the nuances of human interaction were seen through the lens of irreducible interdependence. The Logic of Not provided the necessary linguistic and conceptual tools to describe the ambiguous, partially true, or context-dependent realities of human relationships.

This thought experiment on low-temperature cluster fusion offers a recursive reflection on that initial premise. Just as human systems exhibit Borromean interdependence – where individuals, communities, and global challenges are deeply interconnected such that disentangling one aspect impacts all others – so too might fundamental physical interactions, under extreme conditions, reveal analogous patterns.

The "Logic of Not," initially conceived for the complexities of human cognition and social dynamics, finds a fascinating parallel in the quantum realm. The "not-fused, but not-unfusable" state of the hypothesized Borromean cluster captures the quantum ambiguity inherent in tunneling, where a particle is "not here, but not there" until observed. This highlights a universal applicability of non-classical logics to describe states of profound indeterminacy and interconnectedness across vast scales, from the social fabric to the subatomic world.

This analogy suggests that perhaps the principle of irreducible interdependence, symbolized by the Borromean rings, is a fundamental organizing pattern in the universe, appearing not only in the intricate dance of human relationships but potentially also in

the fleeting, highly correlated quantum events that underpin the very fabric of matter and energy. The laser, in this conceptual model, becomes an agent of topological entanglement, attempting to impose a "social structure" on a quantum system, much like how collective intentions shape human reality.

8. Conclusion: A Horizon of Conceptual Exploration

The quest for low-temperature nuclear fusion remains one of the most challenging and, for now, elusive goals in physics. This article has presented a speculative thought experiment, attempting to bridge this scientific frontier with abstract concepts of irreducible interdependence (Borromean rings), nuanced logic (Nagatomo's Logic of Not), and a hypothetical "topological chemistry" mediated by intense laser fields.

Our conceptual model for a Borromean-analogue fusion reaction in hydrogen clusters posits that the laser could transiently induce a three-body Borromean configuration among nuclei. This unique topological arrangement, distinct from conventional pairwise interactions, might then allow for a collective quantum tunneling event, where the presence of the third particle critically facilitates fusion, even at energies far below the classical Coulomb barrier. Nagatomo's "Logic of Not" provides the conceptual tools to describe the ambiguous and context-dependent states within such a highly correlated quantum system.

It is paramount to reiterate that this is a purely conceptual exploration, a thought experiment designed to push the boundaries of interdisciplinary thinking. It does not propose a viable or scientifically proven mechanism for low-temperature fusion, nor does it offer a prescriptive mathematical model for its realization. The immense physical hurdles, particularly the vast energy discrepancy and the lack of known mechanisms for such profound laser-induced nuclear topological changes, remain formidable.

Nevertheless, the exercise demonstrates the remarkable power of analogy and the profound utility of abstract logical frameworks, initially developed for complex human systems, in envisioning novel approaches to seemingly intractable physical problems.

As our understanding of "intertwined humanity" deepens through concepts like Borromean interdependence, it recursively informs our conceptual toolkit for exploring the most intricate and interconnected phenomena in the natural world, reminding us that the horizons of scientific and conceptual exploration are boundless.

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Plausible Photomolecular Effect and Microwave in Phase Transition of Water as a New Dawn for Renewable Energy and Ensemble Holistic Approach to Health Management

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Abstract

Interaction among light and water molecules have baffled scientists for many decades, and even centuries. In this regards, photons in the visible spectrum, where bulk water normally doesn't absorb light, can surprisingly cleave off large water clusters from the water-vapor interface, according to a recent study by Tu and Chen [2]. This discovery, termed the "photomolecular effect," opens exciting possibilities for not only revolutionizing renewable energy but also paving the way for a more integrated-ensemble approach to health management [1-3]. In a sense, other than with green or red LED, we can also introduce low-intensity laser to alter water molecule, as we discussed earlier in this journal [6, 7].

Keywords: Photomolecular Effect; Microwave; Alternative Technology to Harmful 5G and 6G; EMF; Ensemble Learning Approach.

1 | Introduction: Harnessing the Power of Light for Energy

Interaction among light and water molecules has baffled scientists for many decades, and even centuries. The photomolecular effect offers a novel way to harness solar energy. Traditionally, solar panels convert sunlight into heat (thermal energy) and then into electricity. However, the photomolecular effect bypasses this thermal step, potentially leading to more efficient solar energy conversion. Imagine solar panels that directly convert visible light into clean electricity, boosting renewable energy production significantly. (see also, cf. Pollack's book, The Fourth Phase of Water)

In a sense, other than with green or red LED, we can also introduce low-intensity laser to alter water molecules, as we discussed earlier in this journal [6, 7].

Other than the photomolecular effect, actually it is already known that microwaves, another form of electromagnetic radiation, can also induce phase transitions in water, i.e. to vaporize liquid water other than

by heating alone. Recent research suggests that strategically using microwaves could enhance the efficiency of various water-based processes, including desalination and wastewater treatment. This could lead to a more sustainable future with readily available clean water and reduced waste.

1.1 | How Microwave can Affect and Alter Transition of Liquid Water

Microwaves make water evaporate too. But alternatively, this property is what makes them so useful for heating food. The statement that higher wavelength electromagnetic (EM) radiation is better for evaporation is partially true. Nonetheless, here's a deeper dive into how microwaves affect the phase transition of liquid water:

Microwaves are a form of electromagnetic radiation with a much longer wavelength than visible light. While visible light interacts with the electrons in atoms, microwaves target the polar nature of water molecules.

A water molecule has a positive and a negative end due to the uneven distribution of electrons (see also, cf. Pollack's book, The Fourth Phase of Water). This polarity allows microwaves to nudge the molecules, causing them to vibrate rapidly. This vibration translates to heat, which increases the water's temperature.

1.2 | Heating vs. Evaporation

Increased temperature in water doesn't directly translate to evaporation. Evaporation occurs when water molecules at the surface gain enough energy to overcome the attractive forces holding them to the liquid and escape into the air as vapor.

Microwaves do indeed accelerate this process. By heating the water, they increase the number of energetic molecules at the surface, leading to faster evaporation. This is why food heats unevenly in a microwave – areas with more water molecules will evaporate quicker, leading to hot spots.

1.3 | Microwaves Hints to Replace Dangerous EMF Communication, such as 5G, 6G, etc

While this article can be read as suggesting abandoning EM communication solely because microwaves use a form of EM radiation, however, it's important to remember that different wavelengths of EM radiation have vastly different properties. Microwaves have a wavelength specifically suited to interact with water molecules, while radio waves used for communication have much longer wavelengths that don't significantly affect water's state.

Nonetheless, it shall be noted here that other reports also seem to suggest that we should find out other safer and less damaging technologies other than 5G, 6G, 7G, etc for EMF communication, for instance, we already outlined an alternative approach instead of wireless-EM-based communication [5-9].

1.4 | Beyond Heating

The ability of microwaves to manipulate water molecules has applications beyond heating food. Microwave technology is used in various fields, including:

- Food Processing: Microwaves can be used for drying and pasteurization of food products, utilizing the interaction with water molecules.
- Medical Applications: Microwave therapy uses targeted radiation to heat specific tissues in the body for pain relief.
- Chemical Processing: In some chemical reactions, microwaves can act as a catalyst, accelerating processes by influencing the movement of molecules.

Microwaves offer a unique way to interact with water molecules, allowing for efficient heating and influencing phase transitions. Understanding this interaction is what makes microwave technology so versatile in various applications.

1.5 | Ensemble Approach to Health: A Bridge Between East and West

Considering the above novel approaches to the phase transition of water which may affect many fields of science including healthcare, allow us to remark here that the concept of the "ensemble learning" approach in machine learning, where multiple algorithms are combined to achieve much better performance, resonates with the potential of the photomolecular effect and microwaves in health management. Just like combining Eastern and Western medical practices could lead to more holistic and effective healthcare management, these light-based techniques could bridge the gap between traditional and modern approaches [4].

Imagine a team of doctors: a seasoned Western physician, a skilled acupuncturist, and a mindful yoga therapist. Each brings a unique perspective and expertise to patient care. This collaborative approach, where various methods are combined to address individual needs, reflects the power of the ensemble approach.

In machine learning, the ensemble approach involves combining multiple algorithms to create a more robust and accurate model. By leveraging the strengths of different approaches, ensemble models can overcome the limitations of individual algorithms.

The ensemble approach holds immense potential for the future of health management. Here's how it can bridge the gap between Western medicine and Eastern holistic healing methods:

- Data Fusion: Western medicine relies heavily on quantitative data (blood tests, scans), while Eastern practices focus on qualitative information (energy meridians, body constitution). An ensemble approach could integrate these seemingly disparate data points to create a more comprehensive picture of an individual's health.
- Personalized Care: Imagine an AI system that uses an ensemble approach to analyze a patient's data and recommend a personalized treatment plan. This plan could incorporate medications, dietary adjustments, acupuncture sessions, and mindfulness practices, all tailored to the individual's specific needs.
- Predictive Analytics: By combining data from various sources, ensemble models could potentially predict health risks with greater accuracy. This early detection would allow for preventive measures and personalized interventions.

Moreover, we can imagine a future where:

- Light therapy: Utilizes the photomolecular effect to target specific water clusters within cells, potentially leading to more targeted and effective treatments.
- Microwave-assisted therapies: Employ controlled microwave pulses to stimulate or inhibit specific cellular processes, offering new avenues for disease management.
- Personalized medicine: Leverages light-based techniques alongside traditional diagnostics to develop a more comprehensive understanding of individual health conditions, leading to personalized treatment plans.

1.6 | A Word of Cautious Remark: Beware of Too Ambitious Big Data Intervention

The convergence of data fusion, predictive analytics, and ensemble learning approaches in healthcare offers a promising future for improved diagnostics, treatment plans, and overall patient outcomes. By combining various data streams and leveraging powerful algorithms, we can unlock a new era of personalized medicine.

However, amidst this excitement, a word of caution is necessary. We must be wary of the pitfalls associated with an overly ambitious and "big data for everything" approach.

Ensemble learning, which combines multiple models to generate more robust predictions, holds immense potential in healthcare. By incorporating data from various sources, such as electronic health records, we can gain a more holistic understanding of individual patients. This comprehensive view can then be used to develop more accurate diagnoses, predict potential health risks, and tailor treatment plans to individual needs. However, the allure of big data shouldn't blind us to its limitations. Here's where caution comes in:

- Data Quality: The adage "garbage in, garbage out" holds true for big data. Incorporating inaccurate or incomplete data sets into our models can lead to misleading or even harmful predictions. Rigorous data quality checks and cleansing processes are crucial before feeding information into our algorithms.
- Privacy Concerns: Healthcare data is highly sensitive. As we collect and analyze ever-increasing amounts of patient information, robust privacy safeguards are essential. Clear regulations and transparent data governance practices are needed to ensure patient trust and ethical use of their information.
- Algorithmic Bias: Algorithms are only as objective as the data they are trained on. Biases present in the training data can be inadvertently perpetuated by the models, leading to discriminatory outcomes. Careful selection and vetting of training data sets are crucial to mitigate bias and ensure fairness in healthcare delivery.
- Overfitting: Ensemble models are powerful, but they can become overly reliant on specific data patterns if not carefully monitored. This can lead to overfitting, where the model performs well on the training data but fails to generalize to real-world scenarios. Regular model validation and performance checks are essential to avoid overfitting.

By acknowledging these limitations and adopting a measured approach, we can harness the true potential of big data in healthcare. The key is to keep in mind a balance between leveraging the power of data fusion and ensemble learning while remaining cautious of the potential pitfalls. Let's strive for a future where big data serves as a valuable tool to enhance healthcare, not an all-encompassing solution that overshadows the importance of human expertise and patient-centered care.

Last but not least, allow us to remark on growing concern that the convergence of big data, 5G/6G, and the Internet of Things (IoT) – including the emerging Internet of Bodies (IoB) – spells doom for humanity. Proponents like Yuval Noah Harari paint a dystopian picture where humans become mere "hackable animals," at the mercy of ever-evolving technology. This view, however, is not only demonstrably wrong but also deeply pessimistic about our capacity for growth and resilience.

Firstly, the human experience transcends the purely material. Unlike machines, humans possess the unique ability to learn, adapt, and evolve – physically, mentally, and spiritually. Imagine a living tree. It may be exposed to harsh winds and scorching sun, yet it adapts, grows stronger, and renews its leaves each season. Similarly, humans possess an inherent capacity for renewal, both through the body's natural healing processes and through the pursuit of meaning and transcendence, often referred to as the spiritual dimension.

Secondly, the "hackable animal" narrative reduces human beings to mere code, ignoring the complexity of consciousness and free will. While technology can certainly influence our choices, it doesn't dictate them. We retain the ability to critically evaluate information, make informed decisions, and shape our own destinies.

Here's the key: embracing these advancements while safeguarding our essential humanity. This requires:

- Ethical frameworks: Developing clear guidelines for data collection, use, and security to protect privacy and prevent misuse.
- Human-centered design: Technology must serve humanity, not the other way around. Our focus should be on tools that augment our capabilities without sacrificing our autonomy.
- Spiritual awareness: Recognizing the importance of meaning and purpose beyond the material world. Technology should not replace our connection to the transcendental nature and capacity of human beings.

The future holds immense technological possibilities, but it's a future shaped by our choices. Let's not succumb to dystopian visions. Instead, let's leverage technology for good while nurturing the qualities that make us truly human: our capacity for growth, our free will, and our yearning for connection to something greater than ourselves. It is these qualities, that we shall embrace, not diminish, God-given human dignity and find ways to love and care for all inhabitants of Earth instead of seeking totalitarian dominance of all people and animals and plants in the name of technological prowess.

2 | Concluding Remark

The exploration of light-matter interactions at the water-vapor interface holds immense promise for the future. By harnessing the power of the photomolecular effect and microwaves, we can unlock revolutionary advancements in renewable energy production, water management, and even the way we approach healthcare. This approach, much like ensemble learning, opens doors for a more comprehensive and integrated future, fostering a brighter outlook on both environmental sustainability and our well-being.

Besides, microwaves offer a unique way to interact with water molecules, allowing for efficient heating and influencing its phase transitions. Understanding this interaction is what makes microwave technology so versatile in various applications.

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Conflicts of Interest

The authors declare that there is no conflict of interest in the research.

Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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LETTERS TO PROGRESS IN PHYSICS

Progress in Physics: Twentieth Year of Publication

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Smarandache, Florentin, et al. "Progress in Physics: Twentieth Year of Publication." Progress in Physics, vol. 20, no. 1, 2024, p. 3.

With this inauguration of Volume 20 of *Progress in Physics*, the journal starts its twentieth year of publication, offering a much-needed avenue for the dissemination of truly scientific submissions that are written as proper scientific papers that are free of logical, mathematical and physical errors.

The journal *Progress in Physics* was created in January 2005 on behalf of many influential scientists with whom we were in correspondence. The main reason was that publications in other journals were allowed only if the submitter was affiliated with a scientific institution or research organization. Given this situation, many working scientists finding themselves partially employed or unemployed, such as in between research grants, find themselves unable to publish their research results. Even e-print archives such as Cornell's arXiv required scientific affiliation and still follow this policy. This is why the Declaration of Academic Freedom was written and published in ten languages in *Progress in Physics* [1], and why the journal was started.

Since commencing in 2005, we have published 967 articles till now, which are accessed free of charge online. We have also distributed printed copies of the journal to major libraries of science throughout the world and offer open access online based on library preferences. The journal has a stable position among other scientific journals published by scientific institutions, and at the same time remains absolutely independent from any bureaucratic influence on the part of scientific organizations.

Our aim is to publish a professional journal by keeping a high professional level of operation. Therefore, we always review submitted papers accepting those that meet the requirements stated above, and rejecting those that are not written as proper scientific papers or that contain formal errors. We follow our major principle according to which we consider submissions on the basis of what has been written in the submissions, without any connection with the submitter's affiliation, personality and other considerations.

This has allowed us to make our journal a repository for high quality scientific research over the last two decades. We still do everything to keep the high level of the journal, which is a great advantage for our authors. Our Editorial Team assures our readers and authors that we will continue to adhere to this independent editorial policy in the future, thereby following the Declaration of Academic Freedom.

In conclusion, we have done, and will continue to do, everything to support the principles of open and free dissemination of scientific information in the future. We open this 20th anniversary volume of our journal with faith in the future and invite all researchers who want to publish their scientific results to submit their manuscripts to our journal.

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"All scientists shall have the right to present their scientific research results, in whole or in part, at relevant scientific conferences, and to publish the same in printed scientific journals, electronic archives, and any other media."

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